PSYCHOMETRIKA—VOL. 7, NO. 4
DECEMBER, 1942

ON DETERMINING THE RELIABILITY AND SIGNIFICANCE OF A TETRACHORIC COEFFICIENT OF CORRELATION*

J. P. GUILFORD AND THOBURN C. LYONS
UNIVERSITY OF SOUTHERN CALIFORNIA

In this note are presented facilitating tables for the estimation of the standard error of a tetrachoric r and also tables providing significant and very significant tetrachoric coefficients for various sizes of samples and various combinations of proportions in the dichotomized distributions.

The tetrachoric coefficient of correlation has been coming more and more into use in recent years, particularly since the Thurstone computing diagrams (1) are generally available. There is reason to believe that the popularity of this statistic will continue. It is important, therefore, that consideration be given to the question of the reliability and the statistical significance of the tetrachoric r.

The complete formula for estimating the standard error of this kind of coefficient is so forbidding in terms of labor that rarely does a textbook on statistics present it. And yet, because the standard error is so much larger than that for an ordinary Pearson r under similar circumstances, it is important that the research worker be aware of its magnitude when he computes a tetrachoric r.

The present trend in sampling theory is to use the standard error rather than the probable error of an estimated parameter, so that practice will be observed here. According to Kelley (2), the standard error of a tetrachoric r is given by the formula

$$\sigma_r = \frac{\sqrt{pp'qq'}}{yy'\sqrt{N}} \sqrt{\left[1 - \left(\frac{\sin^{-1} r}{90^\circ}\right)^2\right](1 - r^2)}\ , \tag{1}$$

in which p is the proportion of the cases in one of the two main categories for one of the correlated variables,

p' is the similar proportion for the other variable,

$q = 1 - p$, and $q' = 1 - p'$,

y and y' are ordinates in the normal distributions of unit

* The task of computing the values in the accompanying tables should be credited to Mr. Lyons.

area at the deviates which correspond to p and p' respectively,

r is the tetrachoric coefficient of correlation,

and $\sin^{-1}r$ is the angle whose sin is equal to r .*

For convenience in what follows we have envisaged this formula as being factored as follows:

$$\sigma_r = \frac{1}{\sqrt{N}} \cdot \frac{\sqrt{pq}}{y} \cdot \frac{\sqrt{p'q'}}{y'} \cdot \sqrt{\left[1 - \left(\frac{\sin^{-1}r}{90^\circ}\right)^2\right] \cdot (1 - r^2)} \, . \tag{2}$$

Let us call the five factors A , B , C , D , and E , respectively. The equation then reads

$$\sigma_r = A \cdot B \cdot C \cdot \sqrt{D \cdot E} \, . \tag{3}$$

It is our purpose to present, first, tabled values for factors B , C , and $\sqrt{DE}$ which will facilitate decidedly the computation of σ_r . Similar tables for this purpose have appeared before, but not collectively or in complete form, and only then for the purpose of estimating PE_r rather than σ_r (2,3). Table 1 gives values for both B and C . Entering this Table with either p or q (whichever is .50 or larger) and then with p' or q', we can read values of B and C . Table 2 provides values for the factor $\sqrt{DE}$ for values of r ranging from 0.00 to 0.99 in steps of 0.01. Factor A is readily determined from general tables of square roots and reciprocals or by computation. The product of the four factors yields σ_r . The use and interpretation of this estimated parameter is of course subject to the same restrictions and qualifications as in the case of any σ_r .

Probably of greater utility in interpreting a coefficient of correlation is the practice of determining whether an r is far enough removed from zero to be indicative of a genuine correlation in the population from which the sample was drawn. The practice of computing σ_r , which was discussed above, is based upon the assumption that the true r , or population r , is identical with the sample r . Furthermore, as Fisher has often pointed out, the distribution of the sample r's is skewed when r is large so that the usual interpretations of the fluctuations of sample r's are sometimes most unsatisfactory. To the knowledge of the writers, there is no provision for translating a σ of a tetrachoric r into terms of a z parameter which is symmetrically distributed, as is true for the ordinary Pearson r . The best solution, therefore, seems to be the assumption of a null hypothesis. This means to

* A misprint appearing in Kelley's presentation of the formula has been corrected here.

suppose that the population correlation is actually zero and to compute σ_r to fit this assumption. A distribution of such sample r's would be symmetrical, and from the size of this σ_r and of the obtained coefficient we can infer the probability that the null hypothesis is tenable or untenable.

In line with this discussion, we have adopted Fisher's fiducial limits of 5 per cent and 1 per cent and Student's distribution as bases of deciding whether a certain tetrachoric r is significant or very significant. An r is regarded as significant if for a sample of size N there is only 1 chance in 20 of obtaining an r as large or larger in random sampling from the same population. An r is regarded as very significant if there is only 1 chance in 100 of obtaining similarly an r that deviates that much or more from zero. We present in Table 3 the significant tetrachoric r's for various combinations of N and of p and p'. We present in Table 4, similarly, the very significant tetrachoric r's. To be specific, when N is 100, when p (or p') is .6, and p' (or p) is .5, it takes an r of at least 0.315 to be regarded as significant (see Table 3). An r as large as 0.315 or larger, either positive or negative, could occur simply by random sampling in an uncorrelated population 5 times in 100, when the size of sample is 100. For the same population and size of sample, Table 4 tells us that it would take an r of 0.417 to be regarded as very significant. Once in a hundred times a tetrachoric r as large as this or larger could occur when the true correlation is zero.

In using Tables 3 and 4, there is a general rule that p and p' are interchangeable. To take an example, from the column headed $p = .9$ and $p' = .6$, one can also find the significant (or very significant) r for the case in which $p = .6$ and $p' = .9$. Assume that $N = 250$, $p = .9$ and $p' = .6$, and a significant r is 0.270 and a very significant one 0.356. The same values of r would apply when $p = .6$ and $p' = .9$.

Another rule is that p and q are interchangeable. When p is less than .5, one must enter the table with q, which equals $1 - p$. For example, if the obtained p is .2 and p' is .4, in the table we look for $p = .8$ and $p' = .6$. To take another example, if p and p' both equal .1, we look in the column headed $p = .9$ and $p' = .9$. This type of replacement also holds when only one p is less than .5. For the combination $p = .3$ and $p' = .8$, we would look for the heading $p = .7$ and $p' = .8$. But since p is always greater than or equal to p' in these particular tables, we look for $p = .8$ and $p' = .7$.

If the obtained proportions and values of N do not coincide with those offered in the tables, one may perform the necessary interpolations. It is doubtful whether the labor of interpolating is worth while except when the obtained r is quite near the boundary line of significance, however. In other instances, one might be conservative by taking the next smaller N than his sample contained, and by choosing p values nearer to 1.00 than the obtained ones.

Inspection of Tables 3 and 4 shows that within their limits the significant r's range from 0.097 to 0.580, and very significant r's range from 0.128 to 0.767. These facts should impress one with the importance of working only with very large samples when a tetrachoric r is to be the index of correlation. Only then will σ_r be reasonably small and will one be justified in rejecting the null hypothesis when r turns out to be small or even moderate in size.

REFERENCES

1. Chesire, L., Saffir, M., and Thurstone, L. L. *Computing Diagrams for the Tetrachoric Correlation Coefficient.* Chicago: Univ. Chicago Press, 1933.
2. Kelley, T. L. *Statistical Method.* New York: Macmillan Company, 1924.
3. Davenport, C. B. and Ekas, M. P. *Statistical Methods in Biology, Medicine, and Psychology.* New York: John Wiley & Sons, 1936.

TABLE 1
Providing the Values for Factors B and C in Formula (2) Corresponding to Various Values of p or q

p or q	$\frac{\sqrt{pq}}{y}$	p or q	$\frac{\sqrt{pq}}{y}$	p or q	$\frac{\sqrt{pq}}{y}$	p or q	$\frac{\sqrt{pq}}{y}$	p or q	$\frac{\sqrt{pq}}{y}$
.50	1.2533	.60	1.2680	.70	1.3180	.80	1.4287	.90	1.7094
.51	1.2535	.61	1.2712	.71	1.3256	.81	1.4457	.91	1.7623
.52	1.2539	.62	1.2748	.72	1.3338	.82	1.4641	.92	1.8248
.53	1.2546	.63	1.2787	.73	1.3427	.83	1.4844	.93	1.9003
.54	1.2556	.64	1.2830	.74	1.3523	.84	1.5067	.94	1.9936
.55	1.2569	.65	1.2877	.75	1.3626	.85	1.5315	.95	2.1131
.56	1.2585	.66	1.2928	.76	1.3738	.86	1.5590		
.57	1.2604	.67	1.2984	.77	1.3859	.87	1.5897		
.58	1.2626	.68	1.3044	.78	1.3990	.88	1.6245		
.59	1.2652	.69	1.3109	.79	1.4133	.89	1.6640		

TABLE 2
Providing the Values for the Factor $\sqrt{DE}$ in Equation (2) Corresponding to Different Values of the Tetrachoric r

r	$\sqrt{DE}$	r	$\sqrt{DE}$	r	$\sqrt{DE}$	r	$\sqrt{DE}$	r	$\sqrt{DE}$
.00	1.0000	.20	.9717	.40	.8845	.60	.7297	.80	.4844
.01	.9998	.21	.9687	.41	.8784	.61	.7199	.81	.4686
.02	.9997	.22	.9657	.42	.8723	.62	.7099	.82	.4526
.03	.9994	.23	.9625	.43	.8658	.63	.6996	.83	.4362
.04	.9988	.24	.9591	.44	.8594	.64	.6892	.84	.4191
.05	.9982	.25	.9555	.45	.8526	.65	.6784	.85	.4018
.06	.9975	.26	.9520	.46	.8458	.66	.6675	.86	.3838
.07	.9966	.27	.9483	.47	.8388	.67	.6563	.87	.3652
.08	.9955	.28	.9442	.48	.8314	.68	.6448	.88	.3460
.09	.9942	.29	.9401	.49	.8240	.69	.6331	.89	.3262
.10	.9930	.30	.9358	.50	.8165	.70	.6210	.90	.3057
.11	.9915	.31	.9314	.51	.8087	.71	.6087	.91	.2844
.12	.9899	.32	.9268	.52	.8007	.72	.5961	.92	.2620
.13	.9881	.33	.9220	.53	.7926	.73	.5834	.93	.2387
.14	.9862	.34	.9171	.54	.7841	.74	.5702	.94	.2142
.15	.9841	.35	.9122	.55	.7755	.75	.5569	.95	.1881
.16	.9819	.36	.9070	.56	.7669	.76	.5429	.96	.1606
.17	.9795	.37	.9016	.57	.7579	.77	.5288	.97	.1305
.18	.9770	.38	.8961	.58	.7488	.78	.5145	.98	.0973
.19	.9745	.39	.8904	.59	.7394	.79	.4995	.99	.0586

TABLE 3

Tetrachoric Coefficients of Correlation, Significant at the .05 Level, for Various Sizes of Sample and Combinations of p and p'

N	$p =$ .9 $p' =$ .9	.9 .8	.9 .7	.9 .6	.9 .5	.8 .8	.8 .7
100	.580	.485	.447	.430	.425	.405	.374
150	.471	.394	.363	.350	.346	.329	.304
200	.407	.340	.314	.302	.299	.285	.263
250	.364	.304	.281	.270	.267	.254	.235
300	.332	.277	.256	.246	.243	.232	.214
350	.307	.257	.237	.228	.225	.215	.198
400	.287	.240	.222	.213	.211	.201	.185
500	.257	.215	.198	.191	.188	.179	.166
600	.234	.196	.181	.174	.172	.164	.151
800	.203	.170	.156	.151	.149	.142	.131
1000	.181	.151	.140	.134	.133	.127	.117
1500	.148	.124	.114	.110	.108	.103	.095
2000	.128	.107	.099	.095	.094	.089	.083
2500	.115	.096	.088	.085	.084	.080	.074
3000	.105	.087	.081	.078	.077	.073	.067
5000	.081	.068	.062	.060	.059	.057	.052
10000	.057	.048	.044	.043	.042	.040	.037

TABLE 3 (continued)

N	$p =$ .8 $p' =$ .6	.8 .5	.7 .7	.7 .6	.7 .5	.6 .6	.6 .5	.5 .5
100	.360	.355	.345	.332	.328	.319	.315	.312
150	.292	.289	.280	.270	.267	.259	.256	.253
200	.253	.250	.242	.233	.230	.224	.222	.219
250	.226	.223	.216	.208	.206	.200	.198	.196
300	.206	.203	.197	.190	.188	.183	.181	.179
350	.190	.188	.183	.176	.174	.169	.167	.165
400	.178	.176	.171	.164	.162	.158	.156	.154
500	.159	.157	.153	.147	.145	.141	.140	.138
600	.145	.144	.139	.134	.133	.129	.128	.126
800	.126	.124	.121	.116	.115	.112	.110	.109
1000	.112	.111	.108	.104	.102	.100	.099	.097
1500	.092	.091	.088	.085	.084	.081	.080	.080
2000	.079	.079	.076	.073	.072	.071	.070	.069
2500	.071	.070	.068	.066	.065	.063	.062	.062
3000	.065	.064	.062	.060	.059	.058	.057	.056
5000	.050	.050	.048	.046	.046	.045	.044	.044
10000	.036	.035	.034	.033	.032	.032	.031	.031

TABLE 4
Tetrachoric Coefficients of Correlation, Significant at the .01 Level, for Various Sizes of Sample and Combinations of p and p'

N \ $p=$ / $p'=$	.9 / .9	.9 / .8	.9 / .7	.9 / .6	.9 / .5	.8 / .8	.8 / .7
100	.767	.641	.592	.569	.563	.536	.494
150	.622	.520	.480	.462	.456	.435	.401
200	.537	.449	.414	.399	.394	.375	.346
250	.480	.401	.370	.356	.352	.335	.309
300	.437	.365	.337	.324	.321	.305	.282
350	.404	.338	.312	.300	.297	.283	.261
400	.378	.316	.292	.281	.277	.264	.244
500	.338	.282	.260	.251	.248	.236	.218
600	.308	.258	.238	.229	.226	.215	.199
800	.267	.223	.206	.198	.195	.186	.172
1000	.238	.199	.184	.177	.175	.166	.153
1500	.194	.163	.150	.144	.143	.136	.125
2000	.168	.141	.130	.125	.123	.118	.109
2500	.151	.126	.116	.112	.110	.105	.097
3000	.137	.115	.106	.102	.101	.096	.089
5000	.106	.089	.082	.079	.078	0.74	.069
10000	.075	.063	.058	.056	.055	.053	.049

TABLE 4 (continued)

N \ $p=$ / $p'=$	.8 / .6	.8 / .5	.7 / .7	.7 / .6	.7 / .5	.6 / .6	.6 / .5	.5 / .5
100	.476	.470	.456	.439	.434	.422	.417	.413
150	.386	.381	.370	.356	.352	.343	.339	.335
200	.333	.329	.320	.307	.304	.296	.292	.289
250	.298	.294	.285	.275	.271	.264	.261	.258
300	.271	.268	.260	.250	.247	.241	.238	.235
350	.251	.248	.240	.231	.229	.223	.220	.217
400	.234	.232	.225	.216	.214	.208	.206	.203
500	.209	.207	.201	.193	.191	.186	.184	.182
600	.191	.189	.183	.176	.174	.170	.168	.166
800	.165	.163	.158	.152	.151	.147	.145	.143
1000	.148	.146	.142	.136	.135	.131	.130	.128
1500	.121	.119	.116	.111	.110	.107	.106	.105
2000	.104	.103	.100	.096	.095	.093	.091	.091
2500	.093	.092	.099	.086	.085	.083	.082	.081
3000	.085	.084	.082	.079	.078	.076	.075	.074
5000	.066	.065	.063	.061	.060	.059	.058	.057
10000	.047	.046	.045	.043	.043	.041	.041	.040