n-1	n-2	...	i	...	1	0

computer word — $\underbrace{\qquad}_{n \text{ bits}}$

Binary fixed-point numbers

- Sign-magnitude (SM)
  - integer: sign, magnitude $2^{n-2} \dots 2^{i} \dots 2^{1}\ 2^{0}$

n-1	n-2	...	i	...	1	0

  - fraction: sign, $2^{-1} \dots 2^{-(n-i)} \dots 2^{-(n-2)}\ 2^{-(n-1)}$

n-1	n-2	...	i	...	1	0

    implicit binary point
- One's complement (C1)
- Two's complement (C2)

	SM	C1	C2
	sign 2^{6} --- 2^{1} 2^{0}	sign 2^{6} --- 2^{1} 2^{0}	sign 2^{6} --- 2^{1} 2^{0}
$N_1 = +117_{10}$	0|1|1|1|0|1|0|1	0|1|1|1|0|1|0|1	0|1|1|1|0|1|0|1
$N_2 = -117_{10}$	1|1|1|1|0|1|0|1	1|0|0|0|1|0|1|0	1|0|0|0|1|0|1|1
	sign 2^{-1} ... 2^{-6} 2^{-7}	sign 2^{-1} ... 2^{-6} 2^{-7}	sign 2^{-1} ... 2^{-6} 2^{-7}
$N_3 = +0,34575$	0|0|1|1|0|1|1|1|0|0	0|0|1|1|0|1|1|1	0|0|1|1|0|1|1|0|0
$N_4 = -0,34575$	1|0|1|1|0|1|1|1|0|0	1|1|0|1|0|0|1|1	1|1|0|1|0|1|0|0

no weights

no weights !

C1

$$X_{C1} = \bar{X} = \begin{cases} \geq 0 \rightarrow \overset{\text{sign}}{0}\ \overset{2^{n-2}}{x_{n-2}} \ldots \overset{2^{i}}{x_i} \ldots \overset{2^{1}}{x_1}\ \overset{2^{0}}{x_0} \\ \leq 0 \rightarrow 1\ \bar{x}_{n-2} \ldots \bar{x}_i \ldots \bar{x}_1\ \bar{x}_0 \end{cases}$$

$\bar{x}_i = 1 - x_i$ the 1 complement

negative numbers

$$X_{C1} = \overset{\text{sign}}{1}\ \overset{2^{n-2}}{1}\ \overset{2^{n-3}}{1} \ldots \overset{2^{i}}{1} \ldots \overset{2^{1}}{1}\ \overset{2^{0}}{1} - \underbrace{x_{n-2}\, x_{n-3} \ldots x_i \ldots x_1\, x_0}_{\text{magnitud } X_M}$$

$$\rightarrow 2^{n-1} + 2^{n-2} + \ldots + 2^{i} + \ldots + 2^{1} + 2^{0} =$$

$$= (2-1)(2^{n-1} + 2^{n-2} + \ldots + 2^{i} \ldots + 2^{1} + 2^{0}) = 2^{n} - 1$$

$$\rightarrow X_{C1} = 2^{n} - 1 - X_M$$

positive numbers $X = 0X_M$ $X_{SM} = X_{C1} = X_{C2}$

negative numbers $X_{C1} = \overline{X} = 1\ \overline{x}_{n-2}\ \overline{x}_{n-3} \ldots \overline{x}_i \ldots \overline{x}_1\ \overline{x}_0$

$\overline{x}_i = 1 - x_i$

$$
\begin{array}{r}
2^{n-1}\,2^{n-2}\,2^{n-3} \quad 2^i \quad 2^1\,2^0 \\
1\ 1\ 1 \ldots 1 \ldots 1\ 1 \\
-\quad 0\ x_{n-2}\ x_{n-3} \ldots x_i \ldots x_1\ x_0 \\
\hline
\end{array}
$$

$$1\times2^{n-1} + 1\times2^{n-2} + 1\times2^{n-3} + \ldots + 1\times2^i + \ldots + 1\times2^1 + 1\times2^0 =$$

$$= (2-1)\left(2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots + 2^i + \ldots + 2 + 1\right) =$$

$$= 2^n - \cancel{2^{n-1}} + \cancel{2^{n-1}} - \cancel{2^{n-2}} + \cancel{2^{n-2}} - \cancel{2^{n-3}} + \ldots + \cancel{2^{i+1}} - \cancel{2^i} + \ldots - \cancel{2^2} + \cancel{2^2} - 2 + 2 - 1 =$$

$$= 2^n - 1$$

$$X_{C1} = \overline{X} = 2^n - 1 - X_M$$

negative numbers $X_{C2} = -X = X_{C1} + 1$ for integers

$$X_{C2} = 2^n - 1 - X_M + 1 = 2^n - X_M$$

$$X_{C2} = -X = X_{C1} + 00\ldots0\ldots01 \quad \text{for fractions}$$

$1\times 2^{-n+1}$

$$X_{C1} = \begin{array}{l} \overset{2^0}{1}\ \overset{2^{-1}}{1}\ \overset{2^{-2}}{1} \ldots \overset{2^{-i}}{1} \ldots \overset{2^{-n+2}}{1}\ \overset{2^{-n+1}}{1} \\ \hline 0\ x_{n-2}\ x_{n-3} \ldots x_i \ldots x_1\ x_0 \end{array}$$

$$\to 1\times 2^0 + 1\times 2^{-1} + 1\times 2^{-2} + \ldots + 1\times 2^{-i} + \ldots + 1\times 2^{-n+2} + 1\times 2^{-n+1} =$$

$$= 2^{-n+1}(2-1)\left(2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots + 2^i + \ldots + 2 + 1\right) =$$

$$= 2^{-n+1}\left(2^n - 1\right) = 2 - 2^{-n+1}$$

$$\to X_{C2} = 2 - \cancel{2^{-n+1}} - X_M + \cancel{2^{-n+1}} =$$

$$= 2 - X_M$$

↑ name of C2

$$\text{SM} \xrightarrow{\text{complement}} \text{C1} \xrightarrow{+1} \text{C2}$$

* C2

$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$\boxed{1|1|0|1|1|0|0|0}$ sign, magnitude $\qquad \boxed{1|0|1|0|0|1|1|1}$ sign $\qquad \boxed{1|0|1|0|1|0|0|0}$ sign

$N = -88_{10}$

$\boxed{1|0|1|0|1|0|0|0}$

James Robertson

$$\begin{cases} X_{C2} = -x_{n-1}\cdot 2^{n-1} + \sum\limits_{i=0}^{n-2} x'_i \cdot 2^i & \text{for } X \text{ integer} \\ X_{C2} = -x_{n-1}\cdot 2^{0} + \sum\limits_{i=1}^{n-1} x'_{n-i-1} \cdot 2^{-i} & \text{for } X \text{ fractional} \end{cases}$$

where the bits x'_i and x'_{n-1-i} coincide with x_i and x_{n-1-i} in case of positive numbers ($x_{n-1} = 0$) and correspond to the bits of C2 code of X in case of negative numbers ($x_{n-1} = 1$)

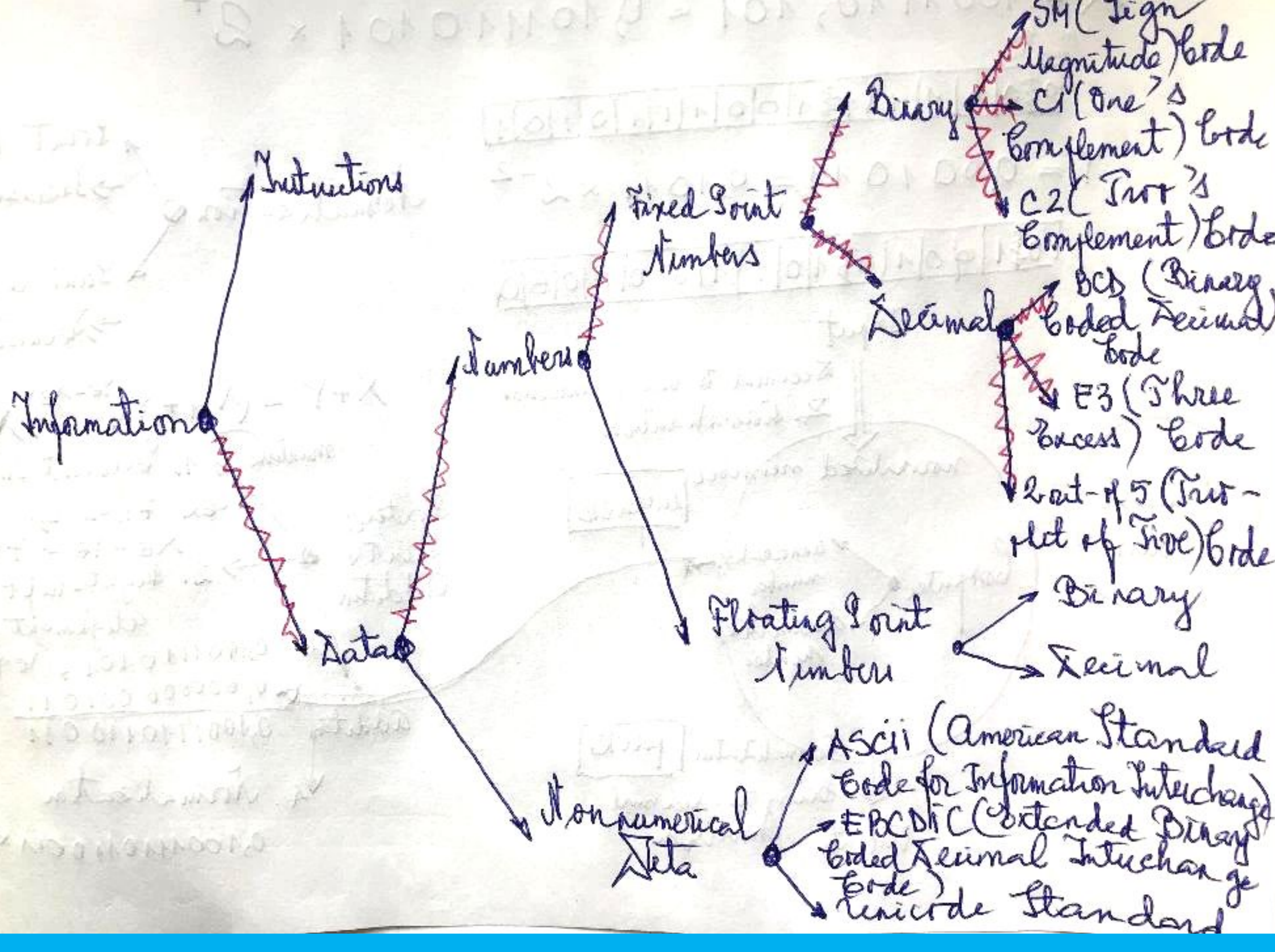
$$\begin{array}{cccccccc} 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$N = -88_{10}$ (C2) $\boxed{1|0|1|0|1|0|0|0} \rightarrow X_{C2} = -1 \times 2^7 + \sum\limits_{i=0}^{6} x'_i \cdot 2^i = -1 \times 2^7 + (1 \times 2^5 + 1 \times 2^3) =$

$= -128 + 32 + 8 = -88$

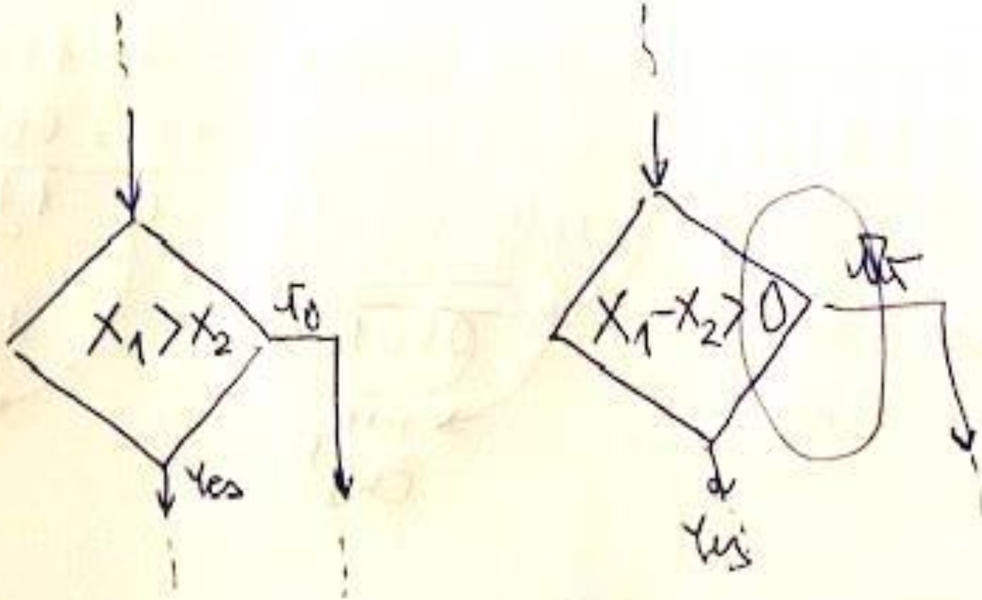
* C2's anomaly

Decimal number	Fixed-point binary codes		
	SM	C1	C2
+7	0111	0111	0111
+6	0110	0110	0110
⋮	⋮	⋮	⋮
+2	0010	0010	0010
+1	0001	0001	0001
(+)0	0000	0000	0000
(-)0	1000	1111	
-1	1001	1110	1111
-2	1010	1101	1110
⋮	⋮	⋮	⋮
-6	1110	1001	1010
-7	1111	1000	1001
-8	-	-	1000


Information
Instructions
Data
Numbers
Fixed Point Numbers
Binary
SM (Sign Magnitude) Code
C1 (One's Complement) Code
C2 (Two's Complement) Code
Decimal
BCD (Binary Coded Decimal) Code
E3 (Three Excess) Code
2 out-of 5 (Two-out of Five) Code
Floating Point Numbers
Binary
Decimal
Nonnumerical Data
ASCII (American Standard Code for Information Interchange)
EBCDIC (Extended Binary Coded Decimal Interchange Code)
Unicode Standard

0's Representation

- SM
  - (+) 0 : $\overset{sign}{0}$0 ... 0 ... 00
  - (−) 0 : $\overset{sign}{1}$ 0 ... 0 ... 00
- C1
  - (+) 0 : $\overset{sign}{0}$ 0 ... 0 ... 00
  - (−) 0 : $\overset{sign}{1}$ 1 ... 1 ... 11
- C2 → (+) 0 : 00 ... 0.00



Amdahl's Law: „Make the common case fast"

Decimal

$$\begin{array}{r} 7+ \\ 6 \\ \hline 13 \end{array}$$

carry 1, 3 sum

10^1 Digits

2^1

$$\begin{array}{r} 0+ \\ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 0+ \\ 1 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1+ \\ 0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1+ \\ 1 \\ \hline 10 \end{array}$$

carry bit, sum bit

Binary

sign magnitude

SM

$$\begin{array}{l} X = X_S\, X_M \; + \\ Y = Y_S\, Y_M \\ \hline Z = Z_S\, Z_M \end{array}$$

$$\begin{array}{l} X = 0\, X_M \; + \\ Y = 0\, Y_M \\ \hline Z = 0\, Z_M \end{array}$$

$Z_M = (X_M + Y_M)$

$X_M > Y_M$

$$\begin{array}{l} X = 0\, X_M \; + \\ Y = 1\, Y_M \\ \hline Z = 1\, Z_M \end{array}$$

$Z_M = (X_M + Y_M)$ ← false → $Z_M = (X_M + Y_M)$

$Z_M = (X_M - Y_M)$ ← correct → $Z_M = (Y_M - X_M)$

$X_M < Y_M$

$$\begin{array}{l} X = 1\, X_M \\ Y = 0\, Y_M \\ \hline Z = 1\, Z_M \end{array}$$

$$\begin{array}{l} X = 1\, X_M \\ Y = 1\, Y_M \\ \hline Z = 0\, Z_M \end{array}$$

$Z_M = (X_M + Y_M)$ false

$Z_M = 1\,(X_M + Y_M)$ correct

3 cases false result!

time penalty!

Z_{n-2}

1 | 0 | 1 | ... |

sign

$$\begin{array}{rcl}
X=+3_{10} &=& \overbrace{0}^{\text{sign}}.\,\overset{2^2}{0}\;\overset{2^1}{1}\;\overset{2^0}{1}_{SM} \quad + \\
Y=+3_{10} &=& 0.\,0\;1\;1_{SM} \\ \hline
Z &=& 0.\,1\;1\;0_{SM}=+6_{10}
\end{array}$$

a

$$\begin{array}{rcl}
X=-3_{10} &=& \overbrace{1}^{\text{sign}}.\,\overset{2^2}{0}\;\overset{2^1}{1}\;\overset{2^0}{1}_{SM} \quad + \\
Y=-3_{10} &=& 1.\,0\;1\;1_{SM} \\ \hline
Z &=& \cancel{1}0.\,1\;1\;0_{SM}=+6_{10}\,(!)
\end{array}$$

b

$$\begin{array}{rcl}
X=+3_{10} &=& \overbrace{0}^{\text{sign}}.\,\overset{2^2}{0}\;\overset{2^1}{1}\;\overset{2^0}{1}_{SM} \quad + \\
Y=-3_{10} &=& 1.\,0\;1\;1_{SM} \\ \hline
Z &=& 1.\,1\;1\;0_{SM}=-6_{10}\,(!)
\end{array}$$

c

$$\begin{array}{rcl}
X=+1_{10} &=& \overbrace{0}^{\text{sign}}.\,\overset{2^2}{0}\;\overset{2^1}{0}\;\overset{2^0}{1}_{SM} \\
Y=-6_{10} &=& 1.\,1\;1\;0_{SM} \\ \hline
Z &=& 1.\,1\;1\;1_{SM}=-7_{10}\,(!)
\end{array}$$

d

Addition SM vs C1 (case $\pm$, $X_M \geq Y_M$)

$$\begin{array}{l} X = +3 + \\ Y = -2 \\ \hline Z = +1 \end{array} \qquad \begin{array}{r} X_{SM} = 0011 + \\ Y_{SM} = 1010 \\ \hline 1101 \end{array}$$

$\underbrace{1101}_{-5}$ false

$X_M \geq Y_M$

(3) (2)

magnitude comparison!

$$\begin{array}{l} X = 0X_M \\ Y = 1Y_M \\ \hline Z = 0(X_M - Y_M) \end{array}$$

$+ (3-2) = +1$

$$\begin{array}{r} X_{C1} = X_{SM} = 0011 + \\ Y_{C1} = 1101 \\ \hline 1\underbrace{0000}_{0} \end{array}$$

must be corrected!

$$Z = X_{C1} + Y_{C1} = X_{M} + 2^n - 1 - Y_{M} = 2^n + (X_M - Y_M) - 1$$

must be corrected

carry out

$$\begin{array}{r} 0011 + \\ 1101 \\ \hline 10000 + \\ 1 \\ \hline 0001 \\ +1 \end{array}$$

end around correction