In the task on page 16 we were asked to calculate the theoretical values for the experiment on optical resonators. We were given $R = 50mm$ for both plano-convex lenses, $L = 45mm$ distance between the lenses. The lenses have a width $b = 6.35mm$ and a refraction index $n_L = 1.515$ The wavelength of the light will be $\lambda = 632nm$ for the following calculations.

Using the given equation for the waist within the optical resonator

$$\omega_0^2 = \frac{\lambda}{\pi}\sqrt{\frac{L}{2}(R - \frac{L}{2})} \tag{1}$$

we obtain

$\omega_0^2 = \frac{632nm}{\pi}\sqrt{\frac{4.5mm}{2}(50mm - \frac{4.5mm}{2})} = 4.9 * 10^(- 8)m$

The resulting rayleigh length is then

$z_R = \frac{\pi 4.9*10^(-8)m}{632} = 1.2 * 10^{-8}m$

$D = \frac{n_L - 1}{R}$

Upon hitting the first mirror, for the light we have

$M_{boundary1} = \begin{pmatrix} 1 & 0 \\ D_{12} & 1 \end{pmatrix}$ with $D_{12} = -\frac{n_{mirror1} - n_{air}}{R}$

Then for propagation through the first mirror

$M_{propagation1} = \begin{pmatrix} 1 & \frac{b_1}{n_{mirror1}} \\ 0 & 1 \end{pmatrix}$

When leaving the mirror

$M_{boundary2} = \begin{pmatrix} 1 & 0 \\ D_{23} & 1 \end{pmatrix}$ with $D_{23} = -\frac{n_{air} - n_{mirror1}}{R'}$

Thus in total we have $M_{mirror1} = M_{boundary1} \cdot M_{propagation1} \cdot M_{boundary2}$

$= \begin{pmatrix} 1 & 0 \\ D_{12} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \frac{b_1}{n_{mirror1}} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ D_{23} & 1 \end{pmatrix} = \begin{pmatrix} \frac{n_2 - b \cdot D_{23}}{n_2} & \frac{b}{n_2} \\ -\frac{-D_{12} \cdot n_2 - D_{23}(-D_{12}b + n_2)}{n_2} & \frac{-D_{12}b + n_2}{n_2} \end{pmatrix}$

Since, $n_{mirror1} = n_{mirror2} \equiv n_2$ and $D_{12} = \frac{n_{mirror1} - n_{air}}{R}$ and $R \to \infty$ for the first square boundary, $D_{12} \to 0$

$\Rightarrow M_{mirror1} = \begin{pmatrix} \frac{n_2 - b \cdot D_{23}}{n_2} & \frac{b}{n_2} \\ \frac{D_{23} n_2}{n_2} & 1 \end{pmatrix}$

Applying the same method to the second mirror

$M_{mirror2} = M_{boundary3} \cdot M_{propagation2} \cdot M_{boundary4}$

$$M_{mirror2} = \begin{pmatrix} 1 & 0 \\ D'_{12} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & \frac{b_1}{n_{mirror1}} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{b}{n_2} \\ D'_{12} & \frac{D_{23}b+n_2}{n_2} \end{pmatrix}$$