$$\begin{split} M &= M_{f_2} M_{Prop} M_{f_1} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_1} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 - \frac{L}{f_2} & L \\ \frac{-1}{f_2} & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{L}{f_2} & L \\ \frac{-1}{f_2} + \frac{L}{f_2 f_2} - \frac{1}{f_2} & 1 - \frac{L}{f_2} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \end{split}$$

$$q = 0 + iz_r$$

$$q' = z' + iz_r'$$

$$q' = \frac{Aq + B}{Cq + D} = \frac{iz_r(1 - \frac{L}{f_1}) + L}{iz_r(\frac{L}{f_1 f_2} - \frac{1}{f_1} - \frac{1}{f_2}) + 1 - \frac{L}{f_2}} = \frac{i\frac{\pi\omega_0^2}{\lambda}(1 - \frac{L}{f_1}) + L}{i\frac{\pi\omega_0^2}{\lambda}(\frac{L}{f_1 f_2} - \frac{1}{f_1} - \frac{1}{f_2}) + 1 - \frac{L}{f_2}} \stackrel{!}{=} i\frac{\pi\omega_0^2}{\lambda} + z'$$

For a general quotient of complex numbers:

$$\tfrac{ai+b}{a'i+b'} = \tfrac{(ai+b)(-a'i+b)}{(a'i+b')(-a'i+b')} = \tfrac{aa'+bb'+i(ab'-a'b)}{a'^2+b'^2}$$

$$\Rightarrow q' = \frac{bb' + z_r^2 (1 - \frac{L}{f_1}) (\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})}{z_r^2 (\frac{L}{f_1 f_2}) - \frac{1}{f_2})^2 + (1 - \frac{L}{f_2})^2} + i \frac{z_r (1 - \frac{L}{f_1}) (1 - \frac{L}{f_2}) - z_r (\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1}) L}{z_r^2 (\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})^2 + (1 - \frac{L}{f_2})^2}$$

$$\Rightarrow z' \stackrel{!}{=} \frac{bb' + z_r^2 (1 - \frac{L}{f_1}) (\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})}{z_r^2 (\frac{L}{f_1 f_2}) - \frac{1}{f_2})^2 + (1 - \frac{L}{f_2})^2} \text{ and } \Rightarrow z_r' \frac{z_r (1 - \frac{L}{f_1}) (1 - \frac{L}{f_2}) - z_r (\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1}) L}{z_r^2 (\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})^2 + (1 - \frac{L}{f_2})^2}$$

Thus to determine  $z_r'$ 

$${z_r}' = z_r \frac{(1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} \frac{1}{f_1} - \frac{1}{\sqrt{2}} \frac{1}{f_2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{z_r^2 (\frac{1}{f_1} \frac{1}{f_2} - \frac{1}{f_1} - \frac{1}{f_2})^2 + (1 + \frac{1}{f_2})^2} = \frac{z_r}{z_r^2 (z_r^2 (\frac{1}{f_1 f_2} - \frac{1}{f_1} - \frac{1}{f_2})^2 + (1 + \frac{1}{f_2})^2)}$$

using the attached python code, this yields

$$L \approx 0,35m$$