$$M = M_{f_2} M_{Prop} M_{f_1} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_1} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 - \frac{L}{f_2} & L \\ \frac{-1}{f_2} & 1 \end{pmatrix} =$$
$$\begin{pmatrix} 1 - \frac{L}{f_2} & L \\ \frac{-1}{f_2} + \frac{L}{f_2 f_2} - \frac{1}{f_2} & 1 - \frac{L}{f_2} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$q = 0 + iz_r$$

$$q' = z' + iz_r{}'$$

$$q' = \frac{Aq+B}{Cq+D} = \frac{iz_r(1-\frac{L}{f_1})+L}{iz_r(\frac{L}{f_1 f_2} - \frac{1}{f_1} - \frac{1}{f_2})+1-\frac{L}{f_2}} = \frac{i\frac{\pi\omega_0^2}{\lambda}(1-\frac{L}{f_1})+L}{i\frac{\pi\omega_0^2}{\lambda}(\frac{L}{f_1 f_2} - \frac{1}{f_1} - \frac{1}{f_2})+1-\frac{L}{f_2}} \stackrel{!}{=} i\frac{\pi\omega_0^2}{\lambda} + z'$$

For a general quotient of complex numbers:

$$\frac{ai+b}{a'i+b'} = \frac{(ai+b)(-a'i+b)}{(a'i+b')(-a'i+b')} = \frac{aa'+bb'+i(ab'-a'b)}{a'^2+b'^2}$$

$$\Rightarrow q' = \frac{bb'+z_r^2(1-\frac{L}{f_1})(\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})}{z_r^2(\frac{L}{f_1 f_2})-\frac{1}{f_2})^2+(1-\frac{L}{f_2})^2} + i\frac{z_r(1-\frac{L}{f_1})(1-\frac{L}{f_2})-z_r(\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})L}{z_r^2(\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})^2+(1-\frac{L}{f_2})^2}$$

$$\Rightarrow z' \stackrel{!}{=} \frac{bb'+z_r^2(1-\frac{L}{f_1})(\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})}{z_r^2(\frac{L}{f_1 f_2})-\frac{1}{f_2})^2+(1-\frac{L}{f_2})^2} \text{ and } \Rightarrow z_r{}' \frac{z_r(1-\frac{L}{f_1})(1-\frac{L}{f_2})-z_r(\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})L}{z_r^2(\frac{L}{f_1 f_2} - \frac{1}{f_2} - \frac{1}{f_1})^2+(1-\frac{L}{f_2})^2}$$

Thus to determine $z_r{}'$

$$z_r{}' = z_r \frac{(1-\cancel{\frac{L}{f_2}}-\cancel{\frac{L}{f_1}}+\cancel{\frac{L^2}{f_2 f_1}}-\cancel{\frac{L^2}{f_2 f_1}}+\cancel{\frac{L}{f_2}}+\cancel{\frac{L}{f_1}})}{z_r^2(\frac{L}{f_1 f_2} - \frac{1}{f_1} - \frac{1}{f_2})^2+(1+\frac{1}{f_2})^2} = \frac{z_r}{z_r^2(z_r^2(\frac{L}{f_1 f_2} - \frac{1}{f_1} - \frac{1}{f_2})^2+(1+\frac{1}{f_2})^2)}$$

using the attached python code, this yields

$$L \approx 0,35m$$