# A 'Reproductive Capital' Model of Marriage Market Matching

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## Draft Under Revision

We often consider the toll the ticking biological clock takes on women, but not the economic loss it causes via its impact on marriage market prospects. I develop a matching model of the marriage market where women's human capital investments impact a second dimension, "reproductive capital," to study the implications for aggregate matching patterns. The model predicts that when the fertility loss from investment is large relative to the income gains, the top-earning men may not match with the top-earning women, and that women's spousal income may in fact be non-monotonic in their own human capital. These predictions help rationalize surprising patterns in historical Census data.

**JEL Codes:** J12, J13, J16, D13, C78

## 1 Introduction

The ticking biological clock is often viewed as a central driver of women's decision-making. But, just as a woman's human capital is valued on the labor market, her fertility may have value on the marriage market.[1] Therefore, women whose human capital investments limit their fertility may experience not just a personal loss, but an economic one. This paper introduces the concept of fertility as "reproductive capital" and examines the economic

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[1] Men marry partly to have children, and marriage has economic value to both men and women, through the production of public goods and returns to scale. It may especially improve women's economic circumstances via hypergamy [Edlund and Pande, 2002].

impacts of its depreciation for women, contributing to literature suggesting differential fecundity may help explain women's career choices [Siow, 1998, Dessy and Djebbari, 2010, Zhang, 2019, Gershoni and Low, 2019].

Whereas men's reproductive systems age at the same rate as other bodily systems, women experience sharply declining fecundity beginning in their mid-thirties, ending in menopause. Mirroring this asymmetric biological pattern is a social one: women who marry later (past their mid-twenties) marry poorer men with each passing year, whereas for men later marriage is associated with richer partners, shown in Figure 1.[2] Although the negative relationship between women's age and spousal income is only correlational, the fact that individuals who marry later tend to be positively selected makes it suggestive of a negative impact of age on marriage market outcomes.[3]

Of course, it is difficult to separate age from other factors, including women's own preferences over partners, and correlated traits like physical appearance. Low performs an incentive compatible experiment and finds that fertility drives the age penalty, with a woman needing to make an additional $7,000 annually for each year older she is, in order to receive the same rating.

This finding indicates that men also hear the ticking of the biological clock. Seeking to marry and have children, they naturally prefer more fertile partners. Women thus face a tradeoff: human capital investments increase earnings, but take up crucial time during the reproductive years, as it is difficult to co-process career investments and family formation.[4] This loss of reproductive capital may cost them real economic returns on the marriage market, despite human capital itself being a positive trait.

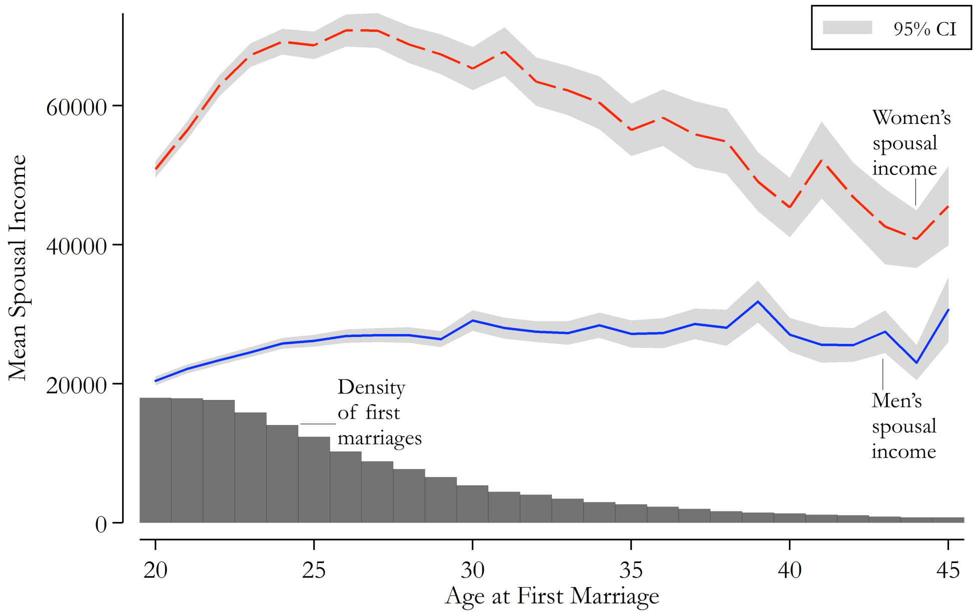
To formally model the consequences of depreciating "reproductive capital," I add fertility as a second dimension to a standard transferable utility matching model. Despite

[2] Appendix Figure A6 shows that in addition to the average pattern, at each level of women's own income, marrying older is linked to marrying a poorer spouse for women (but not for men).

[3] One might worry the pattern stems from unobservable selection, if women who marry later are "leftover". However, the pattern of marriage volume makes this unlikely, since the bulk of marriages—and thus the largest possible sorting—occurs before the decline in husband's income begins. Zhang [2019] additionally notes that the selection of men who marry late tends to be negative, but we do not see the same declining spousal income for men.

[4] E.g., see Michael and Willis [1976], Goldin and Katz [2002], Bailey [2006], Bailey et al. [2012], Adda et al. [2017], Kleven et al. [2019].

Figure 1: Spousal Income by Age at Marriage




*Notes:* Lines represent the average spousal income for first marriages by age at marriage for women versus men. Bars represent the portion of all women's marriages occurring at that age, to check whether selection is driving the effect. Source: 2010 American Community Survey (1 percent sample) marital histories for white men and women, 46-55 years old.

starting from a surplus function that typically predicts assortative matching on income, I find that when fertility is introduced, and human capital investments are time-costly, the richest men do not always marry the richest women. In fact, the model yields an equilibrium condition that for any set of parameters, it is always possible to find a man rich enough such that he would choose a partner with a little extra fertility over one with a little extra income. This provides micro-foundations for negative marriage market effects of age, which have been assumed to better fit marriage data [Díaz-Giménez and Giolito, 2013, Bronson and Mazzocco, 2012, Shephard, 2019].

This equilibrium outcome results in smaller marital surplus shares for women who make large human capital investments. Even if a woman placed no value on children personally, she would still experience a "tax" on her human capital investments via the marriage market equilibrium. Note, that the problem is not in the match she receives *per se*, since she prefers receiving a larger share of the surplus from a lower "quality" mate, but rather that she ultimately receives less marital surplus than she would in the absence of a reproductive cost of time.[5] Despite this penalty, I show that it is possible to sustain an equilibrium where women invest in time-costly human capital, because they value the wage returns over the marriage market penalty.[6]

Reproductive capital helps bridge the gap between the overwhelming evidence that matching tends to be assortative, and literature where men appear to dislike high-earning female partners (e.g., Bertrand et al. [2015], Bursztyn et al. [2017]). The model predicts that matching will be assortative whenever women differ only by income, but need not be when both income and fertility change. This means that the model is able to match a puzzling fact in US Census data that I document: until recently, women with graduate degrees married significantly poorer spouses than women with college degrees, despite every other educational level yielding richer partners. In other words, there was a *non-monotonic* relationship between women's education and spousal income.

This fact cannot be explained by existing models of household formation. If households specialize between market and non-market work, then negative assortative matching is expected, as it would be if female education is somehow a "bad," perhaps because it yields additional bargaining power or creates intra-household conflict. Conversely, if marriage is taste-based, or there are consumption complementarities with one's spouse, positive assortative matching is predicted. Current models fail to match the positive relationship between education and spousal income for those with less than a college degree and the negative relationship for those with college degrees and higher.

A bi-dimensional model with reproductive capital, though, exactly predicts that not all educational attainment is equal on the marriage market. Education that increases income without significantly interfering with fertile years should unambiguously improve matching, while education that might push back marriage to the point of reducing fertility can be

[5] Because in transferable utility models individuals optimize over surplus received, rather than partners, high-skill women in essence *choose* matching with poorer men, with whom they create more relative surplus. But, this optimization is constrained by the fact that they lack an asset highly valued by the richest men, reproductive capital. Thus, the model is entirely compatible with the view that high-skill women might prefer poorer men due to relative bargaining power. Without a reproductive capital penalty, though, they could enjoy a high human capital partner *and* a high surplus share.

[6] This contributes an example where investments can affect multiple dimensions, to the literature on premarital investments [Iyigun and Walsh, 2007, Lafortune, 2013, Dizdar, 2018, Mailath et al., 2013, Cole et al., 2001, Nöldeke and Samuelson, 2015, Mailath et al., 2017].

penalized. Thus, reproductive capital may be an omitted variable in an apparent dislike for "career women" on the marriage market.

This is further supported by examining the recent "reversal in fortune" for educated women on the marriage market [Fry, 2010, Rose, 2005, Isen and Stevenson, 2010, Bertrand et al., 2016]. By decomposing the "college plus" educational category into college versus graduate-educated women, I demonstrate that college-educated women actually never suffered on the marriage market. It was only graduate-educated women who previously matched with lower income men and married less frequently, and now marry better and more. The model predicts such a reversal if either the labor market returns to education rise sufficiently or the fertility costs of investment fall (e.g., due to improved technology or reduced family size desires). This contributes a new explanation to literature examining the causes of increased assortative mating over time [Chiappori et al., 2017b, Hurder, 2013, Greenwood et al., 2016, 2014, Fernandez et al., 2005, Schwartz and Mare, 2005].[7] Moreover, a reduction of the reproductive capital penalty to education on the marriage market can help explain the dramatic rise in women pursuing higher education through a marriage market channel, as discussed in Chiappori et al. [2009] and Ge [2011].

The current matching patterns I document in the Census align with the findings in Low on the tradeoff between income and fertility. Graduate educated women today match with slightly richer spouses than college educated women, while earning $16,000 more than college educated women on average and marrying one year later. This suggests that an age penalty may still exist, but that earnings can make up for depreciation in reproductive capital, as the model predicts.

Together, these results show that while we may presume women value fertility personally, it also has a substantial economic impact. Because "reproductive capital" depreciates at a similar time in the life cycle to when human capital for high-skilled workers appreciates, it is likely to be extremely salient in career investment decisions. Individuals, policymakers, and firms may be able to use a better understanding of this tradeoff to blunt the impact of reproductive capital's decline.

[7] Note that Gihleb and Lang [2016] and Eika et al. [2019] dispute that there has been a large increase in assortativeness over time.

The remainder of the paper proceeds as follows: Section 2 develops a model that incorporates fertility in the marital surplus function, Section 3 establishes the model's relevance to historical Census data, and Section 4 concludes.

# 2 A Theory of the Marriage Market with Reproductive Capital

If men value women's fertility, it will have consequences for matching patterns, as well as women's willingness to invest in human capital. This section studies these effects using a bi-dimensional, transferable utility matching model. In this model, human capital investments yield earnings gains, but can also delay marriage and childbearing, resulting in a lower chance of successful conception. The dimensions of this model cannot be collapsed to an index, because the ability to have children interacts with the household's means of creating surplus through investing income into children. This contributes to a growing literature on truly multidimensional matching problems [Chiappori et al., 2017a, Galichon and Salanié, 2015, Coles and Francesconi, 2011, Lindenlaub and Postel-Vinay, 2017, Dupuy and Galichon, 2014, Galichon et al., 2019, Coles and Francesconi, 2019].

Transferable utility matching models derive matching patterns from the efficient creation and division of surplus [Shapley and Shubik, 1971, Becker, 1973]. The equilibrium payoff of each individual in a marriage is set by the market as "offers" where both spouses are able to attract one another. Thus, the model simply requires assumptions on the form of the marital surplus to establish equilibrium matching patterns and resulting utilities.

## 2.1 General Model

This section demonstrates that a simple bi-dimensional model where couples care about income and fertility can produce non-monotonic matching in incomes. In particular, matching will always be assortative when women's income varies, but fertility stays constant, but need not be when income and fertility vary in tandem.

There is a two sided market with a unidimensional side, "men," and a bi-dimensional side, "women." Men are characterized solely by income, $y \in [0, Y]$, and women are characterized by both income and fertility, $(z, p) \in [0,Z] \times [0,1]$.[8]

[8] Although making men unidimensional is a simplification, it should be noted that other matching

Individuals care about both children and private consumption, which produces a surplus found by maximizing the sum of utilities $s(y, z, p)$, increasing in all arguments.

Let the surplus exhibit:

- Supermodularity in incomes: $\frac{\partial s^2}{\partial y \partial z} > 0$
- Supermodularity between men's income and fertility: $\frac{\partial s^2}{\partial y \partial p} > 0$

Let there be two men, $y$ and $y'$, with $y < y'$.

**Proposition 1.** *For any two women with incomes $z$ and $z'$, $z' > z$, and fertility $p$, the stable matching matches $y'$ with $(z', p)$ and $y$ with $(z, p)$.*

*Proof.* With transferable utility, the stable match will maximize total surplus. Thus, supposing by contradiction that $y$ is paired with $z'$ and $y'$ with $z$, it must be that:

$$s(y, z', p) + s(y', z, p) > s(y', z', p) + s(y, z, p)$$
$$s(y, z', p) - s(y, z, p)) > s(y', z', p) - s(y', z, p)$$

Which would imply that, for a small change in z, $\frac{\partial s(y, z, p)}{\partial z} > \frac{\partial s(y', z, p)}{\partial z}$, which would mean $s$ is submodular, contradicting the premise. □

**Proposition 2.** *There exist two women $(z, p)$ and $(z', p')$ with $z < z'$, $p > p'$ such that the stable matching matches $y$ with $(z, p)$ and $y'$ with $(z', p')$. There exist two women $(\bar{z}, \bar{p})$ and $(\bar{z}', \bar{p}')$ with $\bar{z} < \bar{z}'$, $\bar{p} > \bar{p}'$ such that the stable matching matches $y$ with $(\bar{z}, \bar{p})$ and $y'$ with $(\bar{z}', \bar{p}')$.*

*Proof.* By total surplus maximization, whenever $s(y, z, p) + s(y', z', p') > s(y, z', p') + s(y', z, p)$, $y$ will be matched with $(z, p)$ and $y'$ with $(z', p')$, whereas when the opposite holds, $y$ will be matched with $(z', p')$ and $y'$ with $(z, p)$.

models that feature multiple characteristics are actually unidimensional as long as the characteristics can be collapsed to a single index. I focus on income as that is the key factor usually examined in models looking at societal trends in assortative matching. Many of the predictions here would also hold for other factors that could be part of a quality index, such as height or attractiveness.

Define:

$$\begin{aligned}\bar{\Delta} &= s(y,\, z,\, p) + s(y',\, z',\, p') - s(y,\, z',\, p') - s(y',\, z,\, p)\\ &= s(y,\, z,\, p) - s(y,\, z',\, p') - (s(y',\, z,\, p) - s(y',\, z',\, p'))\end{aligned}$$

For $\epsilon = z' - z$ and $\eta = p - p'$ positive and small enough, the sign is the same as:

$$\begin{aligned}\bar{\Delta} &= -\frac{\partial s(y,\, z,\, p)}{\partial z}\epsilon + \frac{\partial s(y,\, z,\, p)}{\partial p}\eta - \left(-\frac{\partial s(y',\, z,\, p)}{\partial z}\epsilon + \frac{\partial s(y',\, z,\, p)}{\partial p}\eta\right)\\ &= \left[-\lambda\frac{\partial s(y,\, z,\, p)}{\partial z} + \frac{\partial s(y,\, z,\, p)}{\partial p} - \left(-\lambda\frac{\partial s(y',\, z,\, p)}{\partial z} + \frac{\partial s(y',\, z,\, p)}{\partial p}\right)\right]\eta\end{aligned}$$

where $\lambda = \frac{\epsilon}{\eta}$. Then:

$$\bar{\Delta} = \int_y^{y'} \left(\lambda\frac{\partial^2 s(\theta,\, z,\, p)}{\partial\theta\partial z} - \frac{\partial^2 s(\theta,\, z,\, p)}{\partial\theta\partial p}\right)\eta\; d\theta$$

Now, define:

$$\begin{aligned}m_z &= \min_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]} \frac{\partial^2 s(\theta,\, z,p)}{\partial\theta\partial z},\; M_z = \max_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]} \frac{\partial^2 s(\theta,\, z,p)}{\partial\theta\partial z},\\ m_p &= \min_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]} \frac{\partial^2 s(\theta,\, z,p)}{\partial\theta\partial p},\; M_p = \max_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]} \frac{\partial^2 s(\theta,\, z,p)}{\partial\theta\partial p}.\end{aligned}$$

Under the assumptions made, $m_z > 0$ and $m_p > 0$. Then:

- for $\lambda \geq M_p/m_z$, $\bar{\Delta} > 0$, and the first pattern obtains.
- for $\lambda \leq m_p/M_z$, $\bar{\Delta} < 0$, and the second pattern obtains.

□

Taken together, these two propositions mean that in a distribution with fertility weakly decreasing in income, with some women with higher income and equal fertility while others have higher income and lower fertility, there can always be non-monotonic matching. Meaning, it is possible to find a distribution such that some $y' > y$ is matched with a richer woman than is $y$, while some $y'' > y'$ is matched with a poorer woman than is $y'$.

Furthermore, with any surplus function where the relative complementarity of income compared to fertility with men's income goes to zero, one can always find a man rich enough such that he matches non-assortatively in the stable match.

**Lemma 1.** *Special case: For* $s(y,\, z,\, p)$ *such that* $\lim_{y\to\infty} \frac{\frac{\partial^2 s(y,z,p)}{\partial y \partial z}}{\frac{\partial^2 s(y,z,p)}{\partial y \partial p}} = 0$ *, for any* $\lambda$ *there exists a* $Y$ *large enough that the stable match does not match him with the highest-income woman.*

*Proof.* Assume by contradiction assortative matching everywhere. Then the richest man $Y$ is matched with the richest woman, with income and fertility $(z',\, p')$. Define man $\bar{y}$ that is $\epsilon$ below $Y$ and matched with woman with income and fertility $(\bar{z},\, \bar{p})$, where $z' > \bar{z}$ and $p' < \bar{p}$. Then it must be that $s(Y,\, z',\, p') + s(\bar{y},\, \bar{z},\, \bar{p}) > s(Y,\, \bar{z},\, \bar{p}) + s(\bar{y},\, z',\, p')$. From above, we know the sign of this is the same as:

$$\int_{y}^{Y} \left( \lambda \frac{\partial^2 s(\theta,\, z,p)}{\partial \theta \partial z} - \frac{\partial^2 s(\theta,\, z,p)}{\partial \theta \partial p} \right) \eta \; d\theta$$

For Y high enough, this will always be negative, since $\lambda$ is fixed and $\frac{\frac{\partial^2 s(y,z,p)}{\partial y \partial z}}{\frac{\partial^2 s(y,z,p)}{\partial y \partial p}}$ is positive and goes to zero as $y$ increases. Then the richest man cannot be matched with the richest woman. $\square$

### 2.2 Parameterized model

In order to further explore and illustrate the model's implications, this section presents an explicit surplus function that can be parameterized. Because we know that matching is generally assortative in income, the model uses as its foundation a surplus function that would yield assortative matching in a unidimensional setting. Introducing the second dimension of fecundity, that children only occur with some probability, leads to a prediction of non-monotonic matching: generally richer men match with richer women, but the *richest* men may match with women who are lower human capital, but more fertile, than the richest women.

### 2.2.1 Assumptions

Men are characterized by income, $y$, and women are characterized by both income, $z$, and fertility, $p$.

Women are divided into three types: low income and high fertility, $L$, medium income and high fertility, $M$, and finally high income and low fertility, $H$. This captures a key feature of biological fecundity, that it declines non-linearly past a certain age. As a result, some amount of human capital can be acquired without incurring reproductive capital losses, but larger human capital investments incur a reproductive capital penalty.

Appendix A.3 shows that a model with continuous female skill produces highly similar predictions for aggregate matching patterns. Thus illustrating with three types does not limit the model's generality, but has the advantage of mapping well onto empirical exercises, where education is typically used as women's "type," since income is chosen endogenously post-marriage. Roughly, you can think of the three types as being high school, college, and graduate-educated women.

The three types of women have the following income–fertility pairs:

| | $z$ | $p$ |
|---|---|---|
| $L$: | $\gamma-\mu_\gamma$ | $\pi+\delta_\pi$ |
| $M$: | $\gamma$ | $\pi+\delta_\pi$ |
| $H$: | $\gamma+\delta_\gamma$ | $\pi$ |

In other words, $\delta_\gamma$ is the income premium to being the high versus medium type, and $\delta_\pi$ is the fertility penalty. $\mu_\gamma$ is the income premium to being the medium versus low type. The mass of the three types of women is first assumed to be exogenously given, as $g^K$, where $K \in L,M,H$. Section 2.4 extends the model to allow for endogenous human capital investment.

There is a total measure 1 of women: $g^L+g^M+g^H=1$. I assume there are more men than women, and thus only measure 1 of men can be matched.[9] Define the poorest man who receives a match as $y_0$ and the richest man as $Y$. Assume the income parameters are

[9] This is for simplicity in pinning down explicit utilities. There could also be excess $L$-type women.

such that the poorest matched man's income plus the poorest woman's income is greater than 1 (this ensures interior solutions for the amount invested in children).

Individuals value private consumption, $q$, and children as a public good, $Q$. I follow Lam [1988] in incorporating complementarity between investment in children and adults' consumption, which produces an underlying force toward assortative matching.[10] With a single public and private good, the necessary and sufficient condition for transferable utility is generalized quasi-linear (GQL) utility [Bergstrom and Cornes, 1983, Chiappori and Gugl, 2014]. The simplest form of GQL is Cobb-Douglas utility, or "$qQ$" utility [Chiappori, 2017, Chiappori et al., 2017b]. Because children realize stochastically in my model, I make the minor adjustment of including $Q+1$ so that the couple cares about private consumption even in the absence of children. The utilities are thus:

$$u_m = q_m(Q+1)$$
$$u_w = q_w(Q+1).$$

The impact of biological fecundity is captured by only allowing households to invest in $Q$ if a child is born, with probability $p$.

### 2.2.2 Household Problem

Because utility is fully transferable with these utilities, the allocation of income between children and private consumption can be found by maximizing the sum of utilities subject to the budget constraint. Assuming they have children, the couple's problem, once married, is thus:

$$\max_{q,Q} q(Q+1)$$
$$\text{s.t. } q+Q = y+z.$$

[10]This can be thought of as the human tendency to want children to have similar levels of consumption as parents, a driving force in quantity-quality tradeoff models.

Accordingly, the utility maximizing level of $Q$ and the sum of private consumptions, $q$, is given by:

$$q^* = \frac{y+z+1}{2}$$
$$Q^* = \frac{y+z-1}{2}.$$

If children were born with certainty, this would result in a very standard surplus function that is supermodular in incomes, and would thus result in assortative mating on the marriage market. However, because children are only realized with probability $p$, the joint expected utility from marriage, $T$, is a weighted average between the optimal joint utility if a child is born and the fallback position of allocating all income to private consumption:

$$T(y,z,p) = p\frac{(y+z+1)^2}{4} + (1-p)(y+y_w).$$

If individuals remain single, they simply consume their incomes, and so the surplus from marriage, is $s(y,z,p) = p\frac{(y+z+1)^2}{4} + (1-p)(y+z) - y - z$, yielding the household surplus function:

$$s(y,z,p) = \frac{1}{4}p(y+z-1)^2. \tag{1}$$

### 2.2.3 Properties of the Surplus Function

The household surplus function given in equation (1) is supermodular in incomes, and is also supermodular in income and fertility. The key trait that will produce non-assortative matching is that the value of fertility *relative* to income increases in income.

If couples with richer men value fertility less relative to income, then there is no tradeoff in matching the richest, and least fertile, women with the richest men. However, if couples with richer men value fertility more relative to income, there is a fertility–income tradeoff that may make matching non-assortative on income.

To see this tradeoff mathematically, we can examine the change in surplus with regard to women's income compared to the change in surplus with regard to fertility. This ratio is essentially a marginal rate of substitution between the two traits in the surplus function.

$$\begin{aligned} MRS &= \frac{\frac{\partial s}{\partial z}}{\frac{\partial s}{\partial p}} \\ &= \frac{\frac{1}{2}p(y+z-1)}{\frac{1}{4}(y+z-1)^2} \\ &= \frac{2p}{y+z-1}. \end{aligned}$$

Next, we need to examine how this rate of substitution changes in men's income:

$$\frac{\partial(MRS)}{\partial y} = -\frac{2p}{(y+z-1)^2} < 0.$$

So, the richer the husband is, the more valuable fertility is relative to wife's income. This will create a counter-pressure on the force of supermodularity, and, if women with higher income have lower fertility, can yield non-assortative stable matches in equilibrium.

Note that in a transferable utility model, individuals maximize the surplus they receive, rather than the "quality" of their partner. So, from the woman's perspective, a non-assortative equilibrium can be viewed as women choosing relationships in which they have more "bargaining power," thus receiving a larger share of a slightly smaller pie, rather than the best match "on paper." They can command this higher surplus share when they create more relative value in relationships with lower earning partners.

### 2.3 Matching Equilibrium

A matching is defined as the probabilities over the distribution of $y$ types for matching with each $(z,p)$ type, and value functions $u(y)$ and $v(z,p)$ such that for each matched pair:

$$u(y)+v(z,p) = s(y,z,p).$$

That is, their individual surplus shares add up to the joint surplus created by a match. A matching is stable if two conditions hold:

$$u(y)+v(z,p) \geq s(y,z,p)$$

for all individuals in the marketplace, and

$$u(y) \geq 0, \quad v(z,p) \geq 0$$

for all individuals matched in equilibrium.

That is, the utility received by any two individuals in their current matches must be jointly higher than the surplus they could create by matching together (the equation holds with equality if the pair is married to each other), and all individuals receive a positive benefit to marriage.

In this way, the surplus shares can be thought of as prices that clear the market for marriage partners. Thus, just like in a market with goods and prices, there exists the equivalent of the first welfare theorem: any stable match must maximize the aggregate marital surplus over all possible assignments [Shapley and Shubik, 1971]. It is intuitive that if the objective is total surplus maximization, supermodular surplus functions naturally lead to assortative mating in models with a single characteristic on each side of the market. However, in this market with two characteristics on the women's side, we will not necessarily observe assortative mating in equilibrium if there is negative co-movement between the woman's two characteristics.

**General traits of equilibrium** The principle of surplus maximization allows us to think about the stable equilibrium in terms of the relative benefit men of different incomes receive from switching between types. The three relative surplus differences, as a function of men's income, are as follows.

Medium versus low:

$$\begin{aligned} \Delta^{M-L}(y_m) &= s(y_m,\gamma,\pi+\delta_\pi) - s(y_m,\gamma-\mu_\gamma,\pi+\delta_\pi) \\ &= \frac{1}{4}(\pi+\delta_\pi)\mu_\gamma(2y_m+2\gamma-\mu_\gamma-2). \end{aligned}$$

High versus medium:

$$\begin{aligned}\Delta^{H-M}(y_m) &= s(y_m,\gamma+\delta_\gamma,\pi) - s(y_m,\gamma,\pi+\delta_\pi) \\ &= \frac{1}{4}\pi\delta_\gamma(2y_m+2\gamma+\delta_\gamma-2) - \frac{1}{4}\delta_\pi(y_m+\gamma-1)^2.\end{aligned}$$

High versus low:

$$\Delta^{H-L}(y_m) = \Delta^{H-M}(y_m) + \Delta^{M-L}(y_m).$$

$\Delta^{M-L}(y_m)$ is linear, and monotonically increasing in men's income. Thus, there is always a higher surplus benefit from pairing a higher income man with a higher-income-type woman, corresponding to the supermodularity in the surplus function. Thus, any stable match must match $M$ women with higher income men than $L$ women.

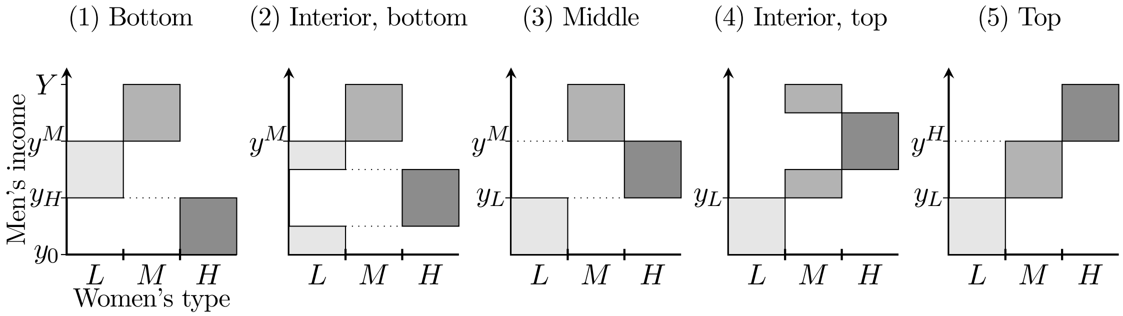
$\Delta^{H-M}$ is quadratic, giving it a unique maximum. The implication of this is that there is a single interval of men that it is maximally beneficial to pair with high-income women. However, this interval may not be the richest men. Note that even when $\Delta^{H-M}(y_m)$ is positive over the full range of $y$, indicating that all men prefer $H$ women to $M$ women, the richest men may not be matched with the richest women, because they do not value the match sufficiently to pay the "price" these women command. This quadratic form is related to the tension between the supermodularity of the surplus function and the decreasing marginal rate of substitution between income and fertility.

$\Delta^{H-L}(y_m)$ is also quadratic, implying that there is a unique interval of men that should be matched with $H$ versus $L$ women, when relevant. From this we can derive the following lemma.

**Lemma 2.** *Any stable matching will exhibit the following three characteristics:*

1. *All matched men will be higher income than all unmatched men.*
2. *All men matched with $M$ women must be higher income than all men matched with $L$ women.*
3. *The set of men matched with $H$ women must be connected.*

Figure 2: Possible matches: $H$ women match with...




*Proof.* Item (1) follows from the fact that the surplus function is monotonically increasing in men's income (as long as the total household income exceeds 1, which was assumed). Item (2) follows from the fact that the benefit to matching with an $M$ type versus an $L$ type is monotonically increasing in income. Item (3) follows from the fact that the benefit of matching with an $H$ type versus $M$ or $L$ type is single-peaked: If there is a gap in the men who are matched with $H$ women, then the men in the gap must be matched with $L$ or $M$ women. But, as the benefit to matching with $H$ women over $L$ or $M$ women is single-peaked, it cannot simultaneously be better to be matched with $H$ women on both sides of the gap than in the gap. □

The options for the match that meet these criteria are illustrated in Figure 2. In all options except for the final one, the match features non-monotonicity in income-matching. *Some* richer men are matched with richer women than some poorer men, while other still richer men are matched with poorer women. The possible equilibria show the strong tendency of the model to produce non-monotonic matches when the highest income women also have low fertility.

**Full equilibrium characterization** Which form the stable match will take depends on the parameter values. The rules for the match outlined in Lemma 1 allow the surplus maximization problem to be written as a single-variable optimization problem: "sliding" up the segment of men matched with $H$-type women from the bottom to the top until surplus is maximized.

To write down this maximization problem, we need a bit of additional notation. Recall the mass of each female type is $g^K$, where $g^L+g^M+g^H=1$, and men from $y_0$ to $Y$ are matched. Define $F(y)$ as a CDF of matched men, where $F(y_0)=0$ and $F(Y)=1$.

As labeled in Figure 2, when $M$-type women match at the top of the distribution, call the lower male income threshold for matching with an M woman $y^M$. When instead $H$-type women match at the top, call the male income threshold $y^H$. When the $H$-type women match at the bottom of the male income distribution, call the upper male income threshold for matching with an $H$woman $y_H$. When instead $L$-type women match at the bottom, call the male income threshold $y_L$.[11]

The optimization will be over the bottom man to receive an $H$-type match: call this $\underline{y}$. Define $h$ as the length of the segment of men who match with $H$-type women, so that the top man who receives an $H$-type match will be $\underline{y}+h$.[12] Finally, let $s^K(y)$ represent the surplus obtained from a match with a man of income $y$ and a woman of type $K \in L,M,H$.

We can now write down the single variable optimization problem to maximize the total surplus by choosing $\underline{y}$:

$$\max_{\underline{y}\in y_0,y^H} \begin{cases} \displaystyle\max_{\underline{y}\in y_0,y_L} \int_{y_0}^{\underline{y}} s^L(y)f(y)\mathrm{d}y + \int_{\underline{y}}^{\underline{y}+h} s^H(y)f(y)\mathrm{d}y + \int_{\underline{y}+h}^{y^M} s^L(y)f(y)\mathrm{d}y + \int_{y^M}^{Y} s^M(y)f(y)\mathrm{d}y, \\ \displaystyle\max_{\underline{y}\in y_L,y^H} \int_{y_0}^{y_L} s^L(y)f(y)\mathrm{d}y + \int_{y_L}^{\underline{y}} s^M(y)f(y)\mathrm{d}y + \int_{\underline{y}}^{\underline{y}+h} s^H(y)f(y)\mathrm{d}y + \int_{\underline{y}+h}^{Y} s^M(y)f(y)\mathrm{d}y. \end{cases}$$

Intuitively, this maximization divides the problem into two cases: one where the segment of men matching with $H$-type women bisects the segment matching with $L$-type women (or there is a corner solution), and one where the segment of men matching with $H$-type women bisects the segment matching with $M$-type women (or there is a corner solution). Because the surplus gain from switching to an $H$ type is quadratic and concave, we need to find a segment of length $h$ on either side of the maximum benefit from an $H$ match. Thus, the first order conditions for this problem reduce to finding a $\underline{y}$ for which the surplus gain is equal to that of $\underline{y}+h$. When one cannot be found, there is a corner solution, which are

[11] Note that these thresholds have specific definitions in terms of the distributions, but I "name" them for notational simplicity. $y^M=F^{-1}(1-g^M)$, $y^H=F^{-1}(1-g^H)$, $y_H=F^{-1}(g^H)$, and $y_L=F^{-1}(g^L)$.

[12] $h=F^{-1}(F(\underline{y})+g^H)-\underline{y}$

the match types 1, 3, and 5. The boundaries for the equilibria are in terms of the surplus gain from switching from either $L$ or $M$ to an $H$-type woman at the ends of each segment. This provides a full characterization of exactly which form the stable match will take.

**Proposition 3.** *The unique stable match is fully characterized by Lemma 1 and the following conditions:*

- *If* $\Delta^{H-L}(y_H) \leq \Delta^{H-L}(y_0)$,

  *$H$ women match with poorest men, from $y_0$ to $y_H$.*

- *If* $\Delta^{H-L}(y^M) < \Delta^{H-L}(y_L)$ *and* $\Delta^{H-L}(y_H) > \Delta^{H-L}(y_0)$,

  *$H$ women match with men interior to the set matching with $L$ women, where* $\Delta^{H-L}(\underline{y}^*) = \Delta^{H-L}(\underline{y}^* + h)$.

- *If* $\Delta^{H-L}(y^M) \geq \Delta^{H-L}(y_L)$ *and* $\Delta^{H-M}(y^M) \leq \Delta^{H-M}(y_L)$,

  *$H$ women match with middle men, from $y_L$ to $y^M$.*

- *If* $\Delta^{H-M}(Y) < \Delta^{H-M}(y^H)$ *and* $\Delta^{H-M}(y^M) > \Delta^{H-M}(y_L)$,

  *$H$ women match with men interior to the set matching with $M$ women, where* $\Delta^{H-M}(\underline{y}^*) = \Delta^{H-M}(\underline{y}^* + h)$.

- *If* $\Delta^{H-M}(Y) \geq \Delta^{H-M}(y^H)$,

  *$H$ women match with richest men, from $y^H$ to $Y$.*

*Proof.* The conditions in Lemma 1 create a single-variable maximization problem that has a unique solution for any given parameters. The solution is found through the first-order conditions of the problem, and the cutoffs for each equilibrium type is found through the boundaries for corner solutions. □

These conditions are illustrated in Appendix Figure A1. Note that these cutoffs are simply conditions on the underlying parameters, which can be solved for by plugging in the $\Delta$ functions. For example, the condition for assortative matching to occur, as in equilibrium 5, is as follows:

$$\frac{\pi}{\delta_\pi}\delta_\gamma \geq \frac{1}{2}(Y+y^H)+\gamma-1.$$

Thus, the condition of assortative matching relies on the impact of investment on income and fertility relative to men's highest possible income, the number of women who invest, and women's baseline income.[13]

Figure 3 illustrates the boundary conditions with a uniform distribution and some parameters fixed, with a given distribution of types, showing the range of $\delta_\gamma$, the financial return to investment, and $\delta_\pi$, the fertility penalty, that support different equilibria types. As $\delta_\gamma/\delta_\pi$ increases, the equilibrium progresses to assortative matching.

**Tendency toward non-monotonic matching** As previously noted, the non-monotonicity present in all but the final equilibrium is an inherent product of the surplus function, specifically the combination of supermodularity and a decreasing marginal rate of substitution between income and fertility. In fact, for any set of parameters a non-monotonic equilibrium will arise as long as the richest man is "rich enough."

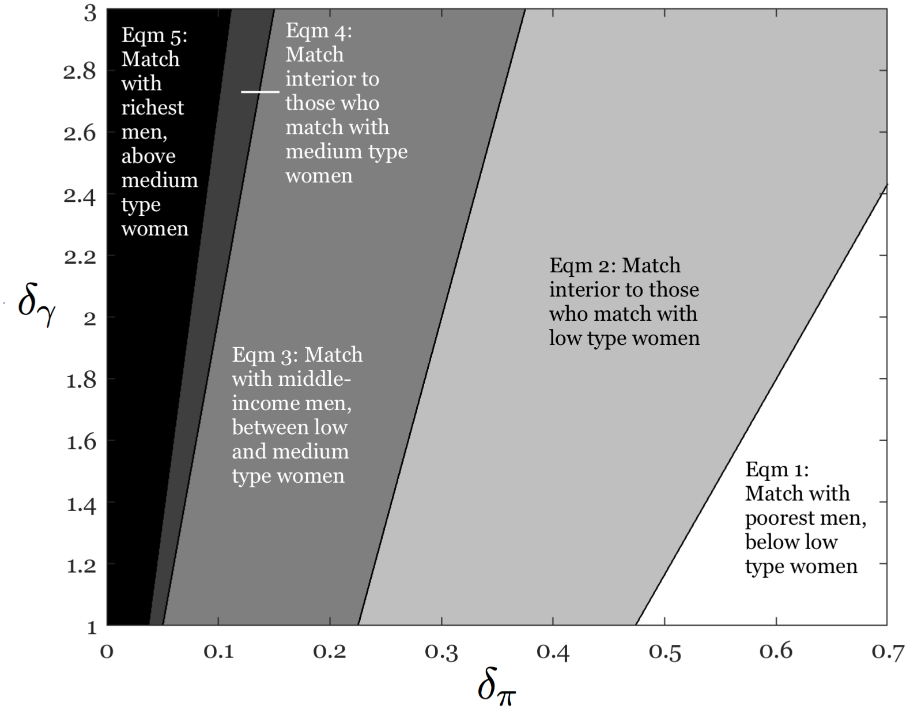
**Proposition 4.** *Let $Y$ represent the income of the richest man. For any set of parameters, it is possible to find a $Y$ large enough such that the equilibrium match is non-monotonic in income.*

See Appendix A.1 for proof. Intuitively, we can also see that because the condition for assortative mating, $\frac{\pi}{\delta_\pi}\delta_\gamma \geq \frac{1}{2}(Y+y^H)+\gamma-1$, relies linearly on $Y$, it is possible to increase $Y$ sufficiently such that the condition is never met, resulting in non-monotonic matching.

**Welfare implications** Transferable utility matching models allow the direct calculation of each individual's equilibrium utility (value function). This is done through using the equilibrium stability condition that $u(y)+v(z,p) \geq s(y,z,p)$, and that marriages improve welfare over singlehood. This procedure is shown in Appendix A.2. Let women's value

[13] All conditions with $\Delta^{H-M}$ will be of the same form. Conditions with $\Delta^{H-L}$, for example for $H$ women to marry the poorest men, will be of the form $\frac{\pi}{\delta_\pi}(\delta_\gamma+\delta_\mu) \leq \frac{1}{2}(y_0+y_H)+\gamma-\mu_\gamma-1$.

Figure 3: Matching equilibrium by return to investment, $\delta_\gamma$ and fertility penalty, $\delta_\pi$




*Notes:* Men's income ranges uniformly from 0 to 6 (total mass of 1), and for women there is a mass of 0.35 $L$ types, and 0.35 $M$ types, and 0.3 $H$ types. $M$-type income is 4, $L$-type is 2. Baseline fertility is a 0.3 chance of conceiving.

functions for each type be denoted $U^K$, and the marital surplus each type receives as $v^K$ , constants that depend on the underlying parameters, including $\delta_\pi$. Then women's equilibrium utility for each type will be:

$$U^H = \gamma + \delta_\gamma + v^H,$$

$$U^M = \gamma + v^M,$$

$$U^L = \gamma - \delta_\mu + v^L.$$

The $H$-type equilibrium utility function is affected by the fertility loss associated with education, both through her own lower utility from children, and through the equilibrium channel of a smaller marital surplus. Appendix Figure A4 uses a back-of-the-envelope calculation (using the continuous version of the model) to show that the equilibrium channel results in about one-third of the total welfare loss from reduced fertility, relative to no fertility consequences from investment.

### 2.4 Endogenous Human Capital Investment

Both the personal and marriage market impacts of human capital investment will influence women's willingness to invest in human capital in the first place. The reproductive capital loss creates an extra "tax" on women's human capital investments, reducing the returns to intensive human capital investments. However, it is still possible to sustain an equilibrium where women invest in costly human capital, even if in doing so they forego the most favorable marriage market matches.

Assume that the distribution of $L$ types is fixed, but that $M$ types can invest to become $H$ types. Further assume that women considering investing face a utility cost, $c_i$, of investment. Using the equilibrium value functions, women will invest in becoming the high type when:

$$
\begin{aligned}
c_i &\leq U^H - U^M \\
c_i &\leq v^H - v^M + \delta_\gamma
\end{aligned}
$$

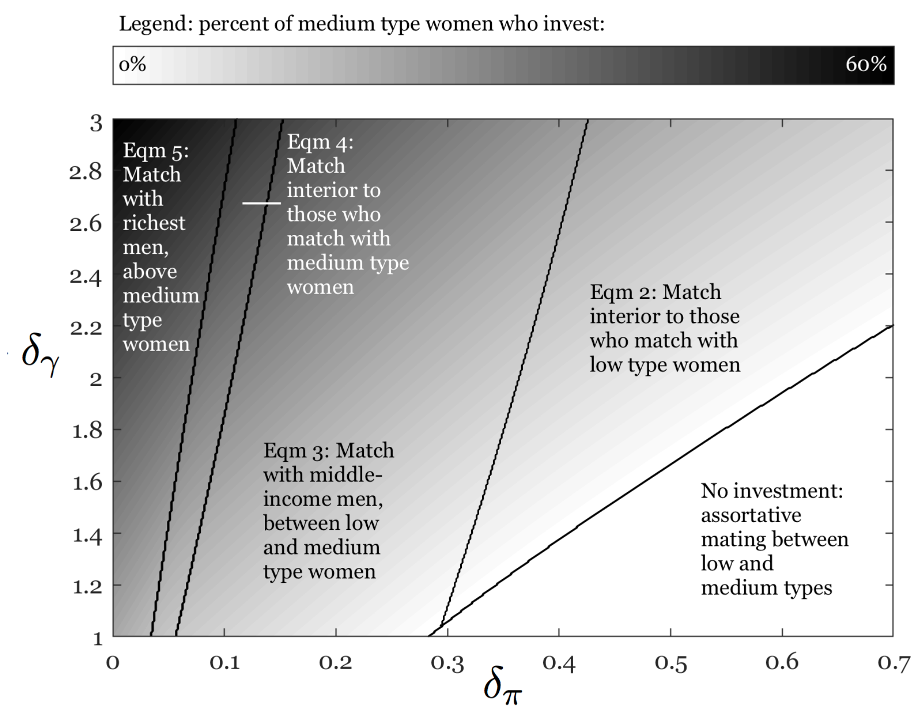
The mass of $H$ types will now be endogenously determined as a function of the underlying density of $c_i$. This mass affects the marital surpluses for each female type through its impact on the $y$ at the boundary between different wife types. Thus, the cutoffs for women investing can be solved for as a fixed point of $c_i = v^H(c_i) - v^M(c_i) + \delta_\gamma$. Call the solution to this equation $\hat{c}$. There will be a unique equilibrium where all women with costs below $\hat{c}$ invest in becoming the $H$ type, and then match according to Proposition 1. If no women invest, the matching will be assortative between $L$ and $M$ types. The threshold cost for investment $\hat{c}$ is decreasing in $\delta_\pi$ (fewer women invest as the fertility cost rises) and increasing in $\delta_\gamma$ (more women invest as the income premium rises).

Figure 4 illustrates the portion of women that invest and resulting matching equilibria for a simple example, where the costs of investment range uniformly from 0 to $2Y$. Note that the thresholds for the matching equilibria are somewhat different than in Figure 3, as the equilibrium responds endogenously to the number of educated women on the market.

Importantly, some women invest in all possible marriage market equilibria, except when $H$-type women are matched with the absolute lowest income men.

The figure illustrates the interesting difference in the forces driving women's investment decision versus the marriage market equilibrium. Women's investment changes more in $\delta_\gamma$, the financial return to investment, while the marriage market equilibrium is more influenced by $\delta_\pi$ the fertility penalty. This is because women get the direct financial benefit of their investment in addition to the marriage market payoff, and thus receive an extra financial incentive to invest that does not appear in the marital surplus, which is what influences the matching equilibrium.

Figure 4: PORTION OF WOMEN WHO INVEST AND THE MATCHING EQUILIBRIUM OVER $\delta_\gamma$ AND $\delta_\pi$




*Notes:* Men's income ranges uniformly from 0 to 6 (total mass of 1), and for women there is a mass of 0.35 $L$ types, and 0.65 $M$ types who have the option to invest. Women's $M$-type income is 4, $L$-type is 2. Baseline fertility is a 0.3 chance of conceiving. The cost of investment ranges from 0 to 12–thus, when half of women invest, women with a cost up to 6 invest.

### 2.5 Empirical predictions

**Non-monotonic matching** The first empirical prediction of the model is that we may not expect the matching relationship between men's and women's income to be monotonic.

If income and fertility are negatively related in the distribution of women, the richest men need not be matched with the richest women. In terms of predictions for different education levels, as long as the educational investment affects primarily human capital, and not reproductive capital, we expect that matching will be assortative. However, for education levels associated with a loss of reproductive capital, husband's income may decrease.

**Trend toward assortativeness and higher investment** The model predicts that if either $\delta_\pi$, the fertility penalty to investment, falls, or $\delta_\gamma$, the income premium, rises, matching will become more assortative at the top of the female income distribution, and women will invest more in human capital. An increase in $\delta_\gamma$ is natural to think about, as women may experience less discrimination in high-earning professions, and the returns to skill on the labor market are rising. $\delta_\pi$ is also likely to be rising over time, making the time impact of women's investments less costly from a reproductive perspective. The first reason is improved technology, such as the introduction of *in vitro* fertilization and egg freezing. The second is falling desired family sizes, which would increase *effective* fertility. This change could be driven by an increasing preference for child quality over child quantity, which tends to accompany economic development [Becker et al., 1990, Doepke, 2015].[14]

**Other implications** The model could also be extended to make predictions on marriage and divorce rates. In the current model all women marry. But, if we imagine there is a stochastic taste for marriage that is a simple utility bonus or cost to being married versus single, some individuals may choose to stay single. And, if we further imagine that this shock can be redrawn after marriage, some individuals may divorce. Importantly, recall that an $H$-type woman's total utility if married is $U^H = \gamma + \delta_\gamma + v^H$. The "marital premium," $v^H$, is a function of both $\delta_\gamma$ and $\delta_\pi$. If she is unmarried, she is unaffected by $\delta_\pi$, but still gets the benefit of her investment through her own income. So, as fertility technology

[14] If the desired quantity of children falls, and quality increases, later-life fertility becomes less of an issue, since fewer children can be had earlier in life. At the same time, the benefits of women's education for children's quality increases. We can think of this as a fall in $\delta_\pi$ (as well as other potential shifts in the surplus function increasing the value of women's human capital. Thus, if desired child quantity falls, we expect an increase in assortative mating, and an increase in marriage and fall in divorce rates for the highest educated women, who previously experienced a reproductive decline.

improves or desired family size falls, the marital premium for $H$-type women increases, leading to increased marriage and decreased divorce rates in a world with stochastic shocks.

# 3 Model Relevance to Historical Data

## 3.1 New Stylized Facts

The model suggests that time-consuming human capital investments represent a double-edged sword for women: on the one hand, human capital is a positive marriage market trait, and likely to help attract a high-income spouse. On the other hand, income-increasing investments take time, decreasing what could be another valuable asset on the marriage market, reproductive capital.

This means that not all human capital investments are created equal; those that take place later in life and take longer are more likely to carry potentially negative marriage market effects, whereas short investments before childbearing years, such as earning a college degree, would be unambiguously positive. To see if there is support for the marriage market reproductive capital–human capital tradeoff in the data, I examine how the marriage outcomes of women with graduate degrees compare to those of women with college degrees.[15]

Essentially, I consider women with college degrees "type $M$" women, since they increase their earning potential without substantially affecting their reproductive capital, and women with graduate degrees (MAs, MDs, JDs, PhDs, MBAs, etc.) "type $H$" women, since these investments likely affect both traits. Although not all graduate educated women will delay marriage, and not all will be higher earning, the presence of a graduate degree indicates on average later marriage and higher earning. Indeed, data from the US Census shows that graduate educated women in the 2010 Census married about one year later and earned \$16,000 more than college educated women, shown in Table 1. The one-year average delay in marriage blends women who likely did not delay marriage at all with those who may have made longer term investments, such as joining the partner or tenure track, with longer delays.

[15]Much empirical work categorizes all women with college degrees as "college plus." However, the "reproductive capital" hypothesis suggests women with college degrees and graduate degrees may have very different marriage market outcomes, since women with college degrees only could still marry quite young and have large families.

Table 1: Income, age at marriage, and children by education

| | 1980 | | | 2010 | | |
|---|---|---|---|---|---|---|
| | College Ed. | Highly Ed. | Difference | College Ed. | Highly Ed. | Difference |
| Income | $18,462 | $28,653 | $10,190*** | $32,326 | $48,030 | $15,703*** |
| AFM | 23.01 | 23.75 | 0.73*** | 26.28 | 27.23 | 0.95*** |
| Children | 1.98 | 1.46 | -0.52*** | 1.64 | 1.52 | -0.12*** |

*Notes:* "Highly Educated" constitutes all formal education beyond a college degree. Income in 1999 USD. Children measured as children in household (this may be downward biased by older children leaving the household, but this bias will be stronger for college educated women, who have children younger. Children ever born is available for 1980 only, and shows the same pattern, with a difference of 0.50 between college and highly educated). Source: 1 percent samples of 1980 US Census and 2010 American Community Survey. Sample consists of white women, age 36-45, with children in household measured for age 36-40 (to avoid children leaving) and age at first marriage measured for ages 41-45, to ensure most marriages are complete.

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

**Spousal income** In Figure 5, we see that in the 1970, 1980, and 1990 Censuses, college educated women were married to richer spouses than non-college-educated women, but graduate educated women were married to *poorer spouses* than college-educated women.[16] This pattern mirrors the non-monotonic matching predicted by the model. This "penalty" to graduate education is significant in all three decades, although somewhat smaller by 1990, and is also present in the 1960 data, although very few women received graduate degrees at the time.[17]

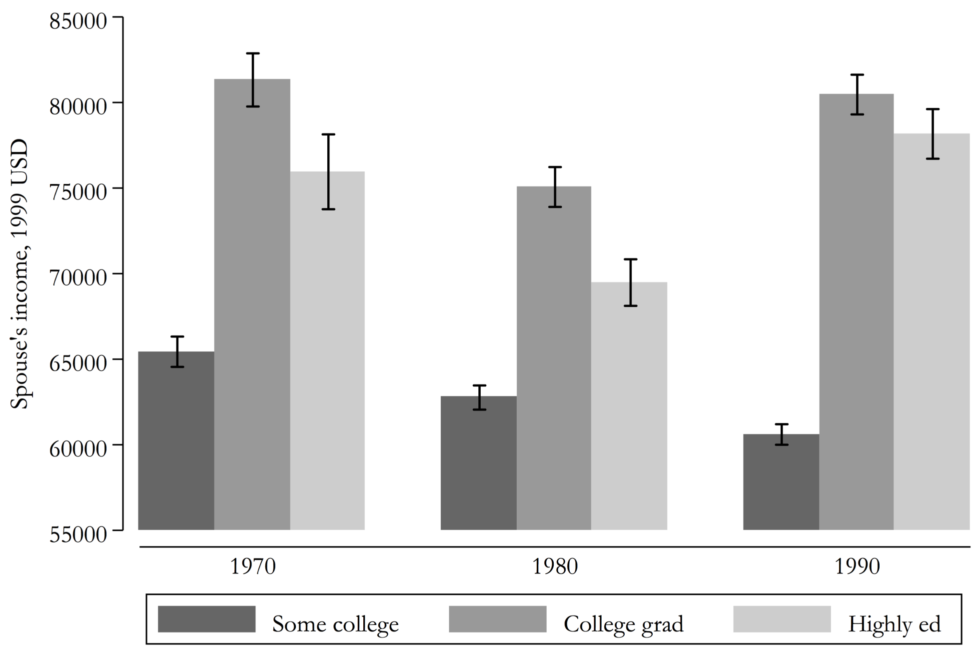
Standard models fail to match this non-monotonicity. Division of labor, and thus substitutability between men's and women's incomes, could explain the negative relationship between education and spouse's income for college and graduate educated women, but not the positive relationship at other education levels. Complementarity in spouse's incomes, social class, or education (whether because of consumption complementarities, or convex returns to certain investments) could explain the positive relationship in most of the data, but not the apparent "penalty" to graduate education.

A model with reproductive capital can account for this non-monotonic matching, while

[16] For empirical results in this section: US Census and ACS sample is restricted to white individuals in their 40s, so that the vast majority of first marriage matching activity and educational investments have already taken place by the time they are observed. I analyze each ten-year cohort in a single Census year, rather than analyzing multiple groups retrospectively, which allows greater homogeneity of current life situation, since most variables, such as income, are reported for the present time only. I restrict to first marriages when showing results for only 1980 and 2010, but use all marriages when showing results across Census years, to allow for comparability with 1990 and 2000 data, which do not contain a variable for marriage number.

[17] Appendix figure A7 shows the pattern remains when restricting to first marriages in 1970 and 1980–number of marriages is not available for 1990.

Figure 5: Non-monotonicity in spousal income by wife's education level




*Notes:* Income of spouse based on wife's education level, with "highly educated" constituting all formal education beyond a college degree. Difference between college and highly educated women's spousal earnings is significant in all three samples. 95% confidence interval shown by black lines. Source: 1 percent Census data from 1970, 1980, and 1990. Sample consists of white women, ages 41-50 years old.

also predicting that if either the returns to time-costly investments have risen or the reproductive costs have fallen, this matching will realign toward assortativeness. Market opportunities for women have naturally risen dramatically in the past 50 years [Hsieh et al., 2019]. However, a more dramatic shift might come from changes in the fertility penalty associated with investment. One reason would be changing technology, in terms of increased technological assistance for reproduction, including fertility drugs, in vitro fertilization, surrogacy, egg donation, and egg freezing (see, for example, Gershoni and Low [Forthcoming, 2019]). Second, there is the decrease in family sizes, likely driven by a substitution from child quantity to child quality [Doepke and Tertilt, 2009, Gould et al., 2008, Isen and Stevenson, 2010, Preston and Hartnett, 2010]. Not only did actual family sizes fall, but *desired* family sizes have fallen substantially. During the 1970s, there was a rapid transition from "four or more" as the modal answer for ideal family size to "two," shown in Appendix Figure A9 [Livingston et al., 2010].[18] If couples wish to have four children, graduate education may significantly interfere with the probability of reaching this desired family size. With a desire for fewer children, longer delays are possible with less of an impact on reproductive success. One can think of this as a fall in $\delta_\pi$, the fertility penalty to investment, which will drive the model toward a more assortative equilibrium.[19]

Figure 6 graphs the movement of spousal income by wife's educational level over time, to see whether there has been an increase in assortative matching for highly educated women. Indeed, the graph shows that the non-monotonic matching in earlier decades disappeared by the time of the 2000 Census.[20] Separating the "college plus" category into college educated versus graduate educated demonstrates that the reversal in marriage outcomes for educated women that has been noted elsewhere was really driven by highly educated women. The alignment of spousal income for every other educational level remained constant over this period. The trends in spousal income of lower education levels are largely parallel. There is some growth in the incomes of college-educated women's spouses relative to other educational levels, consistent with increasing inequality and returns to skill during this period, but this cannot explain the crossing in college versus graduate women's marriage outcomes.

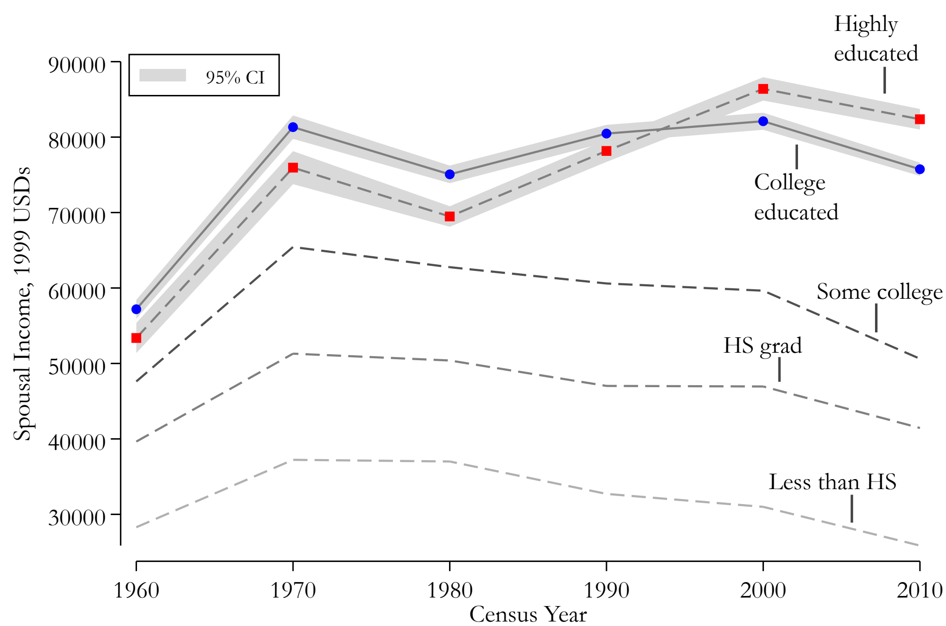
Despite this elimination of the penalty to graduate education, an age penalty can still exist, but with graduate-educated women simply having sufficiently high incomes to compensate partners for higher age. The income-age tradeoff between college and graduate educated women in the Census aligns well with the penalty to aging found in Low, where, women being a year older is equivalent to earning \$7,000 less. In the 2010 Census, women with graduate degrees earned \$16,000 more while marrying on average 1 year later, as shown in Table 1. Thus, it makes sense that they would actually have richer spouses than women with college degrees, as their extra earnings makes up for their later marriage.

[18]This change is clearly endogenous, and driven by multiple sources, including women's rising opportunity costs. However, if a portion was driven by the quality-quantity tradeoff, it would drive an increasing return to human over reproductive capital on the marriage market.

[19]A substitution toward child quality, in addition to lowering the $\delta_\pi$, might also change the surplus function by making women's human capital a more important input into children, as suggested by [Chiappori et al., 2017b]. This is outside the scope of the current paper, but would be a fruitful direction for future research.

[20]This is also evident in a regression with dummies for each cohort, as shown in Appendix Table A2. Appendix figure A8 also shows this pattern for first marriages only, excluding 1980 and 1990 when times married is not available.

Figure 6: Spousal income by wife's education level



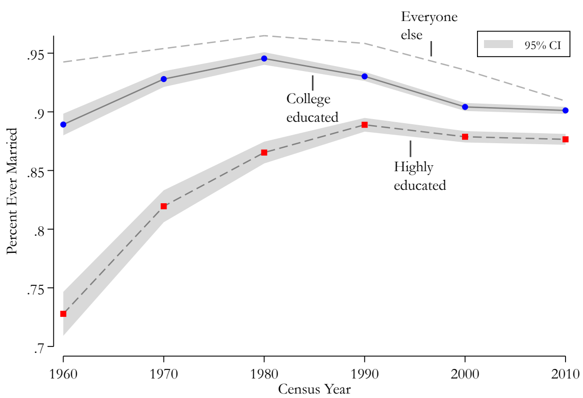

*Notes:* Income of spouse based on wife's education level, with "highly educated" constituting all formal education beyond a college degree. Source: 1 percent Census data from 1960, 1970, 1980, and 1990, and American Community Survey from 2000 and 2010. Sample consists of white women, ages 41-50 years old.

**Marriage and divorce rates** Highly educated women's marriage *rates* have also risen precipitously, shown in Figure 7, another measure of an increasing marriage market premium for highly educated women. Conventional wisdom holds that too-educated, too-high-earning women are punished on the marriage market. However, Figure 7 panel (a) demonstrates that *college* educated women actually always married at rates close to all other educational categories. It is only *highly* educated women who previously had comparatively low rates of marriage, and have now experienced substantial gains.[21] Similarly, as shown in panel (b), highly educated women previously divorced at the highest rates, while college educated women's divorce rates were on par with other educational categories. Since 1990, highly educated women's divorce rates have fallen while college educated divorce rates have leveled off, and all other categories' have risen.

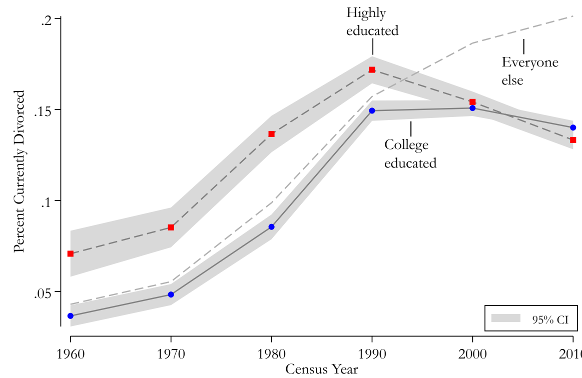
[21] The figures shows the percent of women ages 41-50 who ever married by a given Census year.

Figure 7: Marriage and divorce rates by education level

(a) Marriage




(b) Divorce




*Notes:* Ever married and currently divorced rates by ages 41-50, for women, based on education level, with "highly educated" constituting all formal education beyond a college degree. Ever divorced rates show a similar pattern, but are not available in all years. Source: 1 percent Census data from 1960, 1970, 1980, and 1990, and American Community Survey from 2000 and 2010. Sample consists of white women, ages 41-50 years old.

### 3.2 Alternative explanations and discussion

These findings are consistent with the improvement in educated women's marriage outcomes that has been noted by several authors [Rose, 2005, Isen and Stevenson, 2010, Bertrand et al., 2016, Fry, 2010], as well as increases in assortative mating over time [Chiappori et al., 2017b, Hurder, 2013, Greenwood et al., 2016, 2014, Fernandez et al., 2005, Schwartz and Mare, 2005, Reynoso]. However, existing explanations in the literature for the overall increasing marital assortativeness do not explain the different trajectories seen for graduate versus college educated women.

Stevenson and Wolfers [2007] note that consumption complementarities are likely to overtake the traditional Beckerian household model [Becker, 1973, 1974, 1981] as women enter the labor force and household work becomes more easily substituted by technology. However, Figure 5 shows that matches during the period of Becker's work were not negative assortative, but rather *non-monotonic*. If better technology [Greenwood et al., 2005, Albanesi and Olivetti, 2016, Greenwood et al., 2016] or an increase in the skill premium [Fernandez et al., 2005] alone were responsible for increasing assortativeness, one would expect more movement by college educated relative to less educated women, rather than the strong differential pattern present for college and graduate educated women.

The shift in spousal matching might also be expected if graduate education previously offered little earnings benefit. However, Table 1 shows that even in the 1980 Census, highly educated women earned substantially more than college educated women, with almost 50% higher income.[22]

And, although the non-monotonic matching can reflect women's preference for a higher share of the surplus from a lower quality partner, such patterns would be unlikely to arise solely from high-skill women preferring "low-powered" men. If high-skill women would actually rank lower-earning men above higher-earning men (e.g., due to them being able to spend more time at home), the negative-assortative matching at the top would strengthen, rather than weaken, as female earning power grew.[23] Instead, high-skill women may choose a better relative position with a lower quality partner as a compromise because she cannot command a high "bargaining position" (surplus share) when marrying a high-income man. As the reproductive penalty to career investments dissipate, women can realize more equal partnerships with more assortatively matched mates.[24]

Another possible explanation for these patterns is that the selection of women into post-bachelor's education has changed in a way that could align with the observed matching patterns. If women previously pursued graduate education because they had difficulty marrying, rather than because of higher capability, and this force has lessened over time, one would expect graduate educated women to have been historically less positively selected on skill. I directly test for this using the National Longitudinal Surveys in Appendix Table A1, and show that the "aptitude gap" between graduate educated women and college educated women has remained stable over time.[25] Additionally, as shown in Figure 8, the

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[22]Women's own income in all years is shown in Appendix Figure A10. Results are also unlikely to be driven by measurement error in classifying women as highly educated, as noted by Kominski and Siegel [1993], since the income gap between "highly educated" and college educated has remained relatively constant over time.

[23]Moreover, in appendix **??**, I use data from the dating experiment to test for whether male income is less important for high-income women in evaluating potential partners, and find that high-income women actually care *more* about income.

[24]Bertrand et al. [2016] offers gender norms against career women as a possible explanation, suggesting these norms may have dissipated in recent years. My model demonstrates, though, that such a norm shift could at least partly be driven by economic fundamentals.

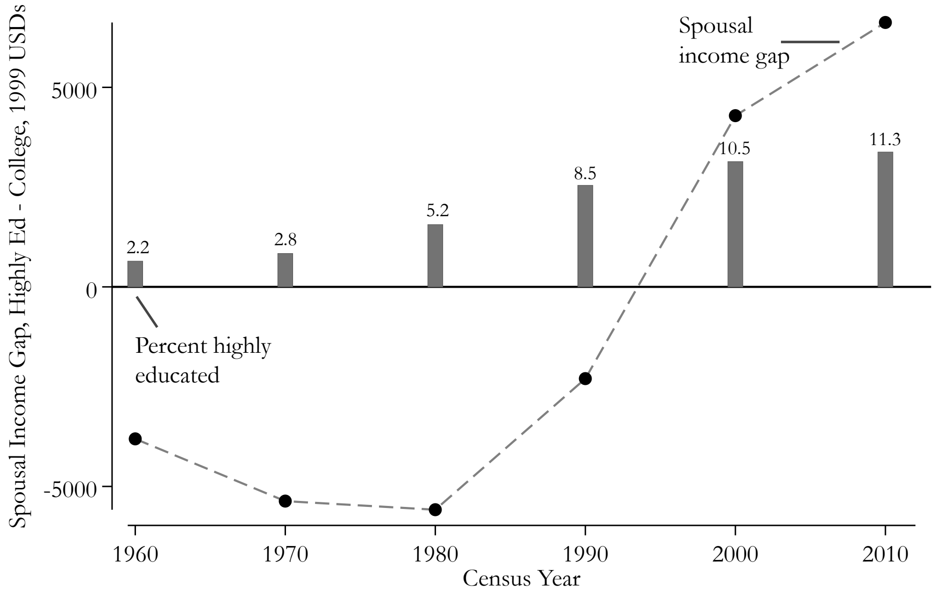
[25]Appendix Table A1 uses data from aptitude scores and educational attainment of three NLS

spousal income gap between college and graduate-educated women does not respond to the percentage of women earning graduate degrees, as would be expected if it were primarily a selection effect. Between the 1970 and 1980 Census, the number of women who achieved post-bachelor's education approximately doubled, while the "penalty" in spousal income compared to college education remained unchanged. From 1990 to 2000 there is a much more modest change in the "pool" of women with graduate degrees, whereas the spousal income gap showed a rapid reversal. Figure 8 is consistent with the predictions of the model, simulated in Figure A5, where as the reproductive costs of investment fall and career returns rise, women first become more willing to pursue graduate education despite the marriage market costs, and then the marriage gap closes, reinforcing this trend.

Table 1 suggests that, in line with the model, falling ideal family sizes may help explain the transition to more assortative matching. In 1980, women with college degrees had households with about 0.5 fewer children than women with graduate degrees. This difference dissipated substantially in 2010, but driven entirely by falling family sizes among the college educated, while graduate educated women's family sizes stayed largely constant. This is consistent with the idea that while graduate educated women continued to delay marriage, falling family size desires meant this delay was less costly in terms of realized fertility.

cohorts—the 1968 Young Women panel, the 1979 Youth panel, and the 1997 Youth panel. (Unfortunately, the earliest cohort with the necessary test score information only captures the tail end of the non-assortative matching group, but nonetheless, the trend should be informative about whether there are large shifts occurring in underlying selection.) The data shows that highly educated women indeed had substantially higher aptitude scores on average than college educated women even in the earliest cohort, and the level of positive skill selection is not systematically increasing.

Figure 8: RATES OF WOMEN'S GRADUATE EDUCATION VERSUS THE SPOUSAL INCOME GAP




*Notes:* "Highly educated" constitutes all formal education beyond a college degree. "Spousal income gap" is defined as the average spousal income for highly educated women minus the average spousal income for college educated women. Source: 1 percent Census data from 1970, 1980, and 1990. Sample consists of white women, ages 41-50 years old.

## 4 Conclusion

From Hollywood films to online dating sites, it seems evident that men prefer younger women, yet the economic cost of this preference to women has not been modeled or measured. If, indeed, younger partners are preferred by men, then time-consuming career investments have an added price for women. By bringing together theory and experimental evidence, this paper demonstrates the dollars-and-cents cost of depreciating reproductive capital to women's economic well-being, and shows how understanding this force sheds light on historical patterns.

This paper treats women's decisions as a tradeoff between two assets: human capital, which grows based on investment, and reproductive capital, which depreciates with time. I develop a bi-dimensional marriage matching model where women's career investments affect both human and reproductive capital. Matching is predicted to be non-monotonic when the fertility cost of career investments are large relative to the income gains. This adds a second cost to women considering time-consuming career investments—not only do they themselves potentially lose out on fertility, but they experience a "tax" on the marriage market as well. I document in US Census data that until recently marriage matching followed the non-monotonic pattern predicted by the model. As family size desires fell, there has been a transition to more assortative mating, and higher marriage and lower divorce rates for graduate educated women.

Women who make the most time consuming career investments may still experience worse matching outcomes, as indicated by the falling spousal income with age in Figure 1. Thus, reproductive capital might help explain the lack of women in top executive positions, or certain fields with rapid human capital depreciation (e.g., tech) or heavy on-the-job training requirements (e.g., surgery). Future research should consider investments of different lengths, rather than the binary investment modeled here.

The reproductive capital framework may also provide useful insights to firms interested in attracting and retaining top female talent. Optimal contracts for women may be very different from those that have evolved in a historically male-dominated workforce. Policy-makers could also utilize a better understanding of reproductive capital to inform efforts to promote women's human capital accumulation, such as parental leave policies and workforce re-entry programs, and calculate the welfare effects of such policies. Government policies that ease access to infertility treatments can create spillover impacts on human capital decisions (see Gershoni and Low [Forthcoming, 2019], Buckles [2007] for example). When viewed through this framework, insurance coverage of infertility treatments becomes a question of not just health policy, but also labor and economic policy.

The fundamental reproductive capital–human capital tradeoff shows the unique costs to women of large human capital investments within the framework of a rational economic model. Even if women themselves do not desire children, they will experience a material loss from lower fertility via the marriage market. This paper shows that reproductive capital is an important consideration in understanding both marriage patterns and women's human capital decisions. More broadly, the concept of reproductive capital suggests substantial welfare costs of aging for women, or of the premature loss of fertility, as may be experienced by women in developing countries facing high rates of illness or childbirth-related infertility. These costs have largely not been considered by policymakers.

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# Online Appendix

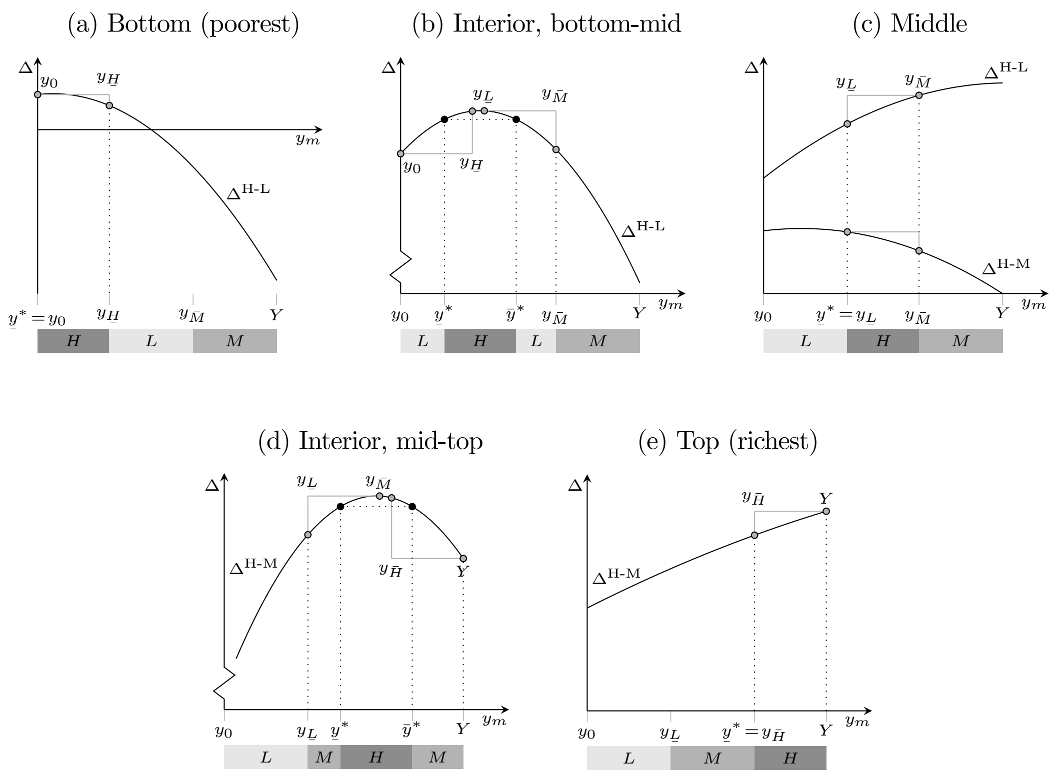
## A Appendix: Model

### A.1 Stable match

**Proposition 5.** *Let $Y$ represent the income of the richest man. For any set of parameters, it is possible to find a $Y$ large enough such that the equilibrium match is non-monotonic in income.*

*Proof.* Suppose not. Then, all $H$ women must be matched with richer men than all $M$ women. For this match to be stable, it must be that $\Delta^{H-M}(Y) \geq \Delta^{H-M}(y^H)$, since otherwise men in the neighborhood of $y^H$ would be able to form a blocking coalition with $H$ women, since they receive more benefit from matching with $H$ types than men

Figure A1: Illustration of surplus difference conditions for different matching equilibria




*Notes:* $\delta_\pi$ and $\delta_\gamma$ vary by panel as follows. 1: $\delta_\pi = 0.65$, $\delta_\gamma = 1.0$; 2: $\delta_\pi = 0.4$, $\delta_\gamma = 2.0$ , 3: $\delta_\pi = 0.19$, $\delta_\gamma = 2.5$, 4: $\delta_\pi = 0.11$, $\delta_\gamma = 2.5$, 5: $\delta_\pi = 0.05$, $\delta_\gamma = 3.0$. $M$-type income is 4, $L$-type is 2. Baseline fertility is a 0.3 chance of conceiving. Men's income ranges uniformly from 0 to 6 (total mass of 1), and for women there is a mass of 0.35 $L$ types, and 0.35 $M$ types, and 0.3 $H$ types. This means $y_{\underline{H}}$ is 1.8, $y_{\underline{L}}$ is 2.1, $y^M$ is 3.9, and $y^H$ is 4.2.

in the neighborhood of $Y$, and thus could offer a greater share of the surplus. Given that $\Delta^{H-M}(y)$ is quadratic, though, it is possible to make $Y$ sufficiently larger such that $\Delta^{H-M}(Y) < \Delta^{H-M}(y^H)$, in which case the total surplus could be increased by matching men near $y^H$ with $H$ women and men near $Y$ with $M$ women, in which case the match would not be assortative everywhere. In fact, the match can be improved by lowering the segment of men matched with $H$ women until the benefit to the first man matched with an $H$ woman is equal to the benefit to the last man matched with an $H$ woman, creating a non-monotonic match where richer men match with richer women until the segment of the richest men who match with $M$ women while poorer men match with $H$ women. □

### A.2 Equilibrium utilities

I first describe the process for calculating the equilibrium utilities, which are needed to back out the payoff to women of investing in human capital. Because the stable match results from a competitive market, we can recover these utilities as the "prices" associated with each individual. That is, we can calculate the surplus share each individual receives, or the utility over and above their counterfactual single utility.

Because at the stable match the sum of any two individuals' utilities must be greater than or equal to the surplus they could create from marrying one another, we can imagine the matching process as each spouse choosing the partner that maximizes his or her own share of the surplus conditional on keeping his or her spouse happy. That is, for women:

$$v(z,p) = \max_y \{s(y,z,p) - u(y)\}.$$

The first order condition of this problem dictates that the slope of the husband's value function must equal the slope of his contribution to the surplus:

$$\begin{aligned} u'(y) &= \frac{\partial s(y,z,p)}{\partial y} \\ &= \frac{1}{2}p(y+z-1). \end{aligned}$$

Because men's partner type does not change locally with their income except at the “boundaries” of a given female type, we can ignore the woman’s type and integrate this function to pin the utility down to an additive constant. Then, we know what men’s surplus share will be when matched with each of the three types of women:

$$u^K(y)=\frac{1}{4}p^K y(y+2z^K-2)+\mu^K$$

where $K\in L,M,H$, and $p^K$ and $z^K$ refer to the fertility and income of a $K$ type woman.

Women’s surplus shares will be a constant for each type, $v^K$. We can solve for each of the constants and the woman’s surplus shares using two sets of restrictions. First, that for each couple the two surplus shares must add up to the surplus produced by the match, and second, that for each male type at a “boundary” between two female types, the utility achieved through each match must be the same. This pins down all values except for the division of surplus between the poorest man and his wife.

Assuming initially that there are more men than women in the market provides this restriction, and allows us to assume the poorest man receives no surplus (since otherwise the unmatched men would compete to take his place), and thus $u(y_0)=0$ (with his total utility simply equaling $y_0$).

I will now go through an example of this process for equilibrium 3, where high-income women are matched with the middle income men, from $y_L$ to $y^M$.

The two “boundary” men, $y_L$ and $y^M$, must be indifferent between their possible partners, as otherwise the match will not be stable. Thus we know $u^L(y_L)=u^H(y_L)$ and $u^M(y^M)=u^H(y^M)$. This allows us to pin down the constants $\mu^M$ and $\mu^H$ relative to $\mu^L$ (as a function of $y_L$ and $y^M$, but recall these are simple functions of the densities of female types, $g^K$). To pin down $\mu^L$, we use the assumption that there are more men than women, and thus the lowest-income man earns 0 surplus, and thus $u^L(y_0)=0$.

From here, we can solve for the female surplus shares in each pairing, which will each be a constant simply using the total surplus restriction:

$$v^K=s^K(y)-u^K(y).$$

We then have a full characterization of women’s and men’s surplus shares from marriage, and can further characterize their full utility based on their single utility plus the surplus share, i.e., for men $y+u^K(y)$ and for women $z^K+v^K$.

Note that a woman's value function responds to fecundity loss through two channels. First, even if the woman's consumption level stayed constant, her utility would be reduced through the lower probability of conceiving, since children directly impact her utility. However, her consumption will also be reduced via the marriage market equilibrium, given that lower fecundity also lowers her husband's utility, and thus he requires a greater share of the available consumption in order to agree to the match.

### A.3 Extension to continuous skill

Rather than having three discrete human capital groups, one could imagine women are endowed with continuous skill, and choose whether to invest in increasing their income relative to their skill. This section briefly outlines the adaptations to the model to accommodate this framework, and demonstrates that results are qualitatively similar to the discrete model.

**Setup** Men and women are each endowed with skill. In the man's case, human capital investment is assumed to be costless, and he thus arrives on the marriage market with a single characteristic, income, $y^h$, distributed uniformly on $[1,Y]$.[26]

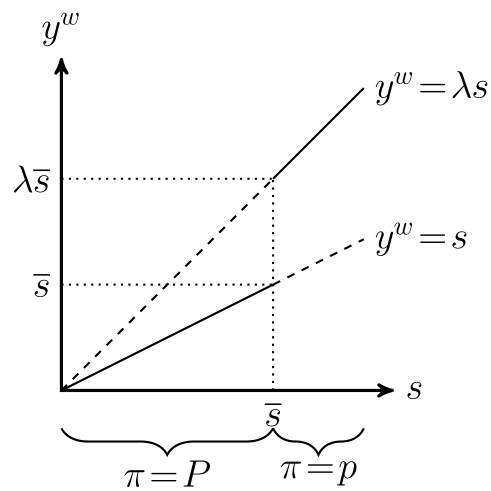
Women, starting with skill $s$ distributed uniformly on $[0,S]$, can choose to improve their level of income, but doing so takes time, and this time is costly in terms of reproductive capital depreciation. As a result, if they choose to make investments, they will have a lower probability of becoming pregnant when they get married. Women are therefore characterized by a pair of characteristics, $(y^w,\pi)$. This pair is equal to $(s,P)$ if the woman marries without investing and $(\lambda s,p)$ if the woman marries after investing, where $\lambda > 1$ and $P > p$. Note that the "fertility penalty" of investment is the same for all women, whereas the wage difference from investment increases with skill. Thus, higher skilled women may have more to gain from investing.

First, I assume an exogenous skill threshold, $\overline{s}$, above which women invest. After determining the equilibrium in the marriage market conditional on $\overline{s}$, I use this equilibrium

[26]Starting at 1 simplifies the model by ensuring all individuals want to marry, because marriage is only "profitable" if total income is greater than 1.

to solve backwards for which women would optimally invest in the first stage. Thus, assume women with $s > \bar{s}$ invest, earn income of $\lambda s$, and have fertility $p$, whereas women with $s < \bar{s}$ earn income $s$ and have fertility $P$, as shown in Figure A2.

Figure A2: WOMEN'S INCOME VERSUS POTENTIAL INCOME: EXOGENOUS $\bar{s}$




*Notes:* Women are endowed with skill, $s$, shown on the x-axis. Their level of income, $y^w$, shown on the y axis, is determined by their investment decision. If women invest, they earn income $\lambda s$, with $\lambda > 1$, but at the cost of reducing their fertility, $\pi$ from $P$ to $p < P$. In this section, we assume women with $s > \bar{s}$ invest.

After couples match, each has a child with probability $\pi$, and allocates their income. This process determines the surplus created by a given marriage, and thus individuals' preferences over different matches. Thus, solving the model requires working backwards from consumption decisions if a child occurs, to the surplus function from marriage, to then determining the optimal match.

Married couples can spend income on private consumption given by $q^h$ and $q^w$ and a public good, investment in children, denoted by $Q$:

$$U^h(q^h, Q) = q^h(Q+1)$$
$$U^w(q^w, Q) = q^w(Q+1),$$

with budget constraint $q^h + q^w + Q = y^h + y^w$

These utilities satisfy the Bergstrom-Cornes property for transferable utility [Chiappori and Gugl, 2014, Bergstrom and Cornes, 1983], and thus consumption decisions can be found by maximizing the sum of utilities, subject to the budget constraint. Accordingly, the utility maximizing level of $Q$ and the sum of private consumptions, $q$ is given by:

$$q^* = \frac{y^h + y^w + 1}{2}$$
$$Q^* = \frac{y^h + y^w - 1}{2}.$$

(Corner solutions are avoided by restricting $y^h + y^w > 1$ based on the distributions of $y$ and $s$).

The joint expected utility from marriage, $T$, is a weighted average between the optimal joint utility if a child is born and the fallback position of allocating all income to private consumption:

$$T(y^h, y^w, \pi) = \pi \frac{(y^h + y^w + 1)^2}{4} + (1 - \pi)(y^h + y^w).$$

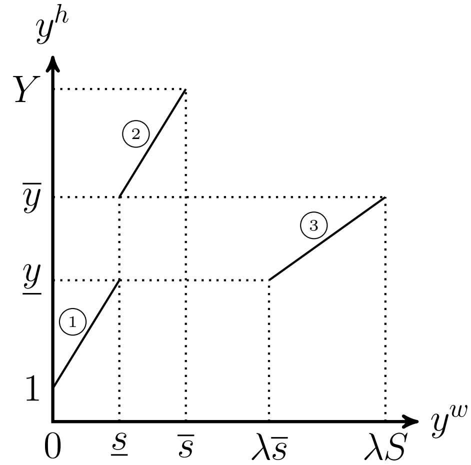
For simplicity, I normalize the utility of singles to be zero, so that $T$ represents the surplus from marriage. The predictions of the model are unchanged if we assume each individual converts his or her income into private consumption when unmarried.

**Equilibrium** An equilibrium displaying assortative matching for women with the same fertility, but non-assortative matching for women with different fertility levels, is shown in Figure A3. Recall $\bar{s}$ is the skill threshold for women becoming educated. Poor men, from 1 to $\underline{y}$, marry low-skill, fertile women (matching assortatively). On the other side of the threshold, the richest group of women matches assortatively with the middle group of men, from $\underline{y}$ to $\overline{y}$. But the richest men, from $\overline{y}$ to $Y$, forego matching with the richest women and instead marry the "best of the rest"—the more high-skilled women among those who have not invested and are thus still fertile.[27]

The equilibrium value functions can be used to show that this is indeed a stable match when $\lambda$, the income gain from investing, is high enough to overcome the fertility cost, $\frac{P}{p} - 1$, for some men, but not high enough that all men prefer women who have invested. In particular, when $\frac{S - \bar{s}}{S + \bar{s}}(\frac{P}{p} - 1)\frac{Y - 1}{S} < \lambda < (\frac{P}{p} - 1)\frac{Y - 1}{S}$, the three-segment match is the unique

[27]The matching functions in this uniform case are linear—in an arbitrary distribution, their form would be determined by the density of individuals, so that the number of women above any point exactly matches the number of men above that point.

Figure A3: Non-monotonic equilibrium match




*Notes:* Women's income, $y^w$ is on the x-axis, and men's income, $y^h$ on the y-axis. The diagonal lines represent matching between men and women. In this non-monotonic matching equilibrium, women with income between 0 and $\underline{s}$ match with men with income between 1 and $\underline{y}$. Women with incomes between $\underline{s}$ and $\overline{s}$ match with men with incomes between $\overline{y}$ and $Y$. Women who have invested, and thus have incomes between $\lambda\overline{s}$ and $\lambda S$, match with men with incomes between $\underline{y}$ and $\overline{y}$.

stable match. For *any* value of $S$, $\overline{s}$, $P$, and $p$, such a $\lambda$ exists, as $\frac{S-\overline{s}}{S+\overline{s}} < 1$. Thus, this model predicts non-monotonic matching.

The matching equilibrium implies that as $\lambda$ grows relative to $\frac{P}{p}$, the world transitions from one where educated women are penalized for their investment, because the additional income they earn is insufficient to compensate wealthy male partners for their loss in fertility, to one where they are able to compensate, and thus match with, partners similarly high in the income distribution.

The lower bound on women's skill for them to be willing to invest, $\overline{s}$, can be found by using the payoff functions resulting from the matching equilibrium, and finding the point at which the investment payoff dominates the non-investment payoff. Adding a small fixed cost of education, $c$, provides a more realistic set of educational investment outcomes, and creates a broader range of parameter values that yield an interior solution (note, as the cost to invest is in this setup a monetary, rather than utility cost, it is also possible that very high-skilled women choose not to invest—i.e., non-monotonicity in investment decisions). To simplify this section, let $Y=2$, $S=1$, and $P=1$.

Using the equilibrium payoff functions, we seek the skill level at which $v_3(\overline{s}) = v_2(\overline{s})$, or $\overline{s}^*(\lambda,c,p)$. Although its functional form is complex, $\overline{s}^*$ varies with the parameters in expected ways: it is increasing in $c$, decreasing in $\lambda$, and decreasing in $p$. In other words, the higher the fixed cost of investment, the higher the skill threshold for pursuing it; the higher the return to investment, the lower the skill threshold; and, the higher the chance of conceiving following investment, the lower the threshold. A higher threshold means fewer women making career investments. A lower threshold means more women making career investments, and this can be spurred by a lower fixed cost of investment, greater returns, or a higher chance of conceiving (e.g., through IVF technology).

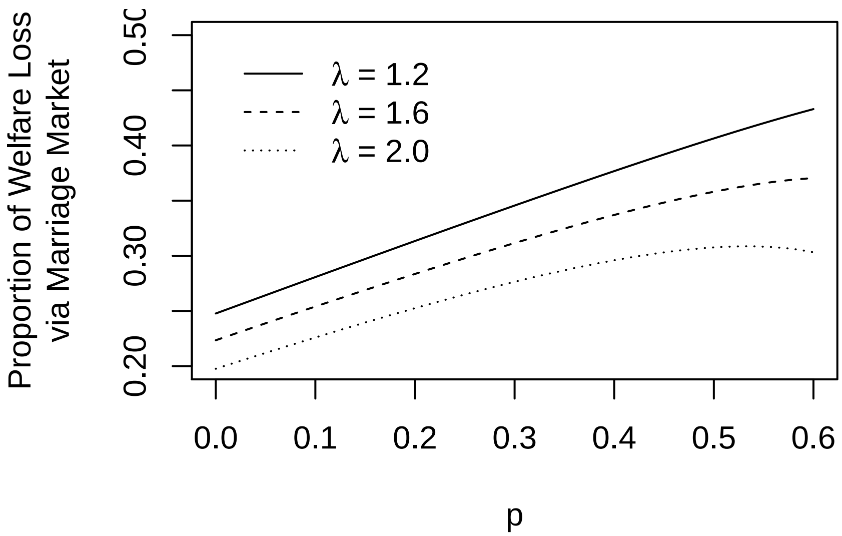
Note that the equilibrium payoff function internalizes not just the individual change in utility from a different fertility level, but also any change in the share of surplus received. This reflects the impact of traits on the overall surplus: someone with traits that yield a large surplus will in exchange receive a favorable match with a high surplus share. Someone with less desirable traits will face a less desirable match and a lower surplus share. Thus, when equalizing the payoff between investing and not investing to find the optimal threshold, both the personal cost of lower fertility and the cost to the marital surplus are considered

**Welfare** Crucially, the model provides a mechanism through which the biological clock impacts women's welfare through a channel other than her own desire for children. That is, even if a woman did not care at all about having a family, she would still be negatively impacted by her fertility loss through her loss of status on the marriage market.

In fact, a back of the envelope calculation using the model suggests that approximately one-third of the utility cost from the post-investment fertility loss comes through the marriage market, rather than directly through women's utility over children. Figure A4 compares the utility loss of lower fertility from the marriage market alone to the loss including women's personal valuation of fertility. The portion of the welfare loss stemming from being matched with a lower quality spouse and needing to cede more of the marital surplus to that spouse ranges from 20-40% of the total utility cost.[28] This simple calculation highlights that the loss of reproductive capital is an economic loss, just as worker disability

[28]The calculation is $1-\frac{v(S|p=P)-v(S|MM)}{v(S|p=P)-v(S)}$ where $v(S|p=P)$ is the woman with skill $S$'s indirect utility if she invests but fertility is unaffected, $v(S)$ is her actual indirect utility, and $v(S|MM)$ is her indirect utility if there is indeed a fertility penalty, but she were to still match with man $Y$ and receive the same *share* of the surplus as if there were no fertility loss.

Figure A4: Proportion of Welfare Loss from Time-Limited Fertility due to Marriage Market



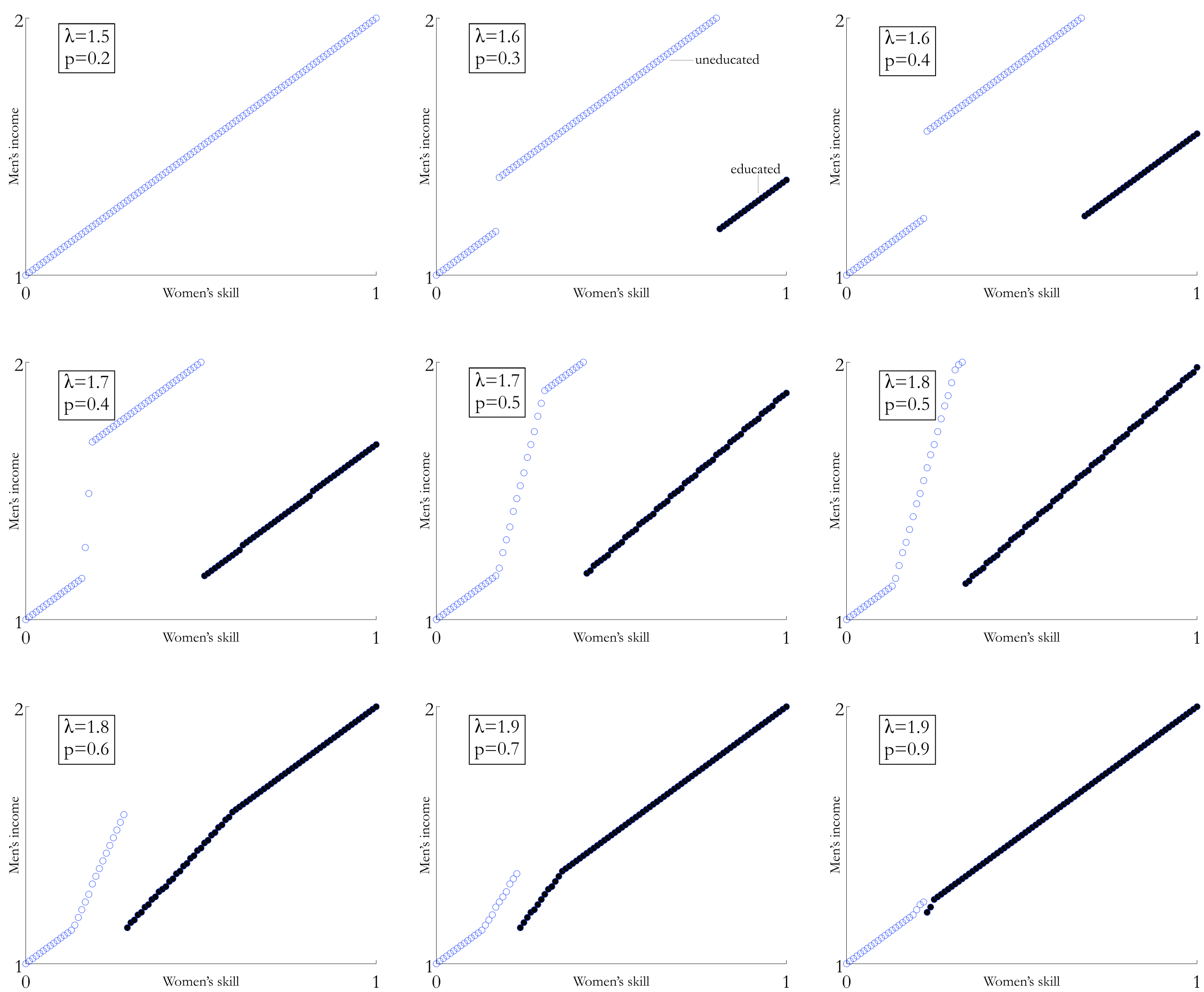

*Notes:* Figure depicts the portion of the welfare loss (y-axis) from lower fertility that comes through the marriage market compared to the total welfare loss, for varying values of $\lambda$, across a range of values for $p$ (x-axis). This is shown for the most-skilled woman, with skill-level $S$, across the range of $p$s where non-monotonic matching results, for parameter values $Y = 2$, $S = 1$, and $P = 1$, with an exogenous $t$, investment threshold, of 0.7. The graph is produced by calculating the change in woman's indirect utility between a scenario with zero fertility cost of investment, to one where post-investment fertility equals $p$, and comparing that to the same effect if her partner and share of the marital surplus were held constant.

is. Because the marriage market creates value for women, the loss of a valuable asset on that market creates real economic impacts.

**Simulation** Figure A5 simulates the model in the presence of growing returns to women's education and falling fertility costs. The first row of images in Figure A5 show that at first, no woman is willing to risk the marriage market costs of investing, so human capital accumulation by women is limited, and matching is assortative. As $\lambda$, the gain from investing, slowly increases while the fertility cost falls (via increasing the success of post-investment conception, $p$), the education and marriage market transforms. The first women to invest, shown by dark blue dots, are penalized through worse marriage matches, creating the non-monotonic equilibrium exhibited in the early Census data. Over time, as labor market returns to investment rise and the fertility cost falls, the marriage matches of these women gradually improve, as seen in the second row of images. This, in turn, creates a feedback loop, with more women being willing to invest (which also matches the dramatic rise in

US women pursuing higher education). Finally, the market becomes essentially assortative, with some "randomization" by the highest earning men: some marry the very richest women, while others still choose the best among the women who have not invested.

Figure A5: Full Two-Stage Optimization Simulation




*Notes:* Figure depicts the results of a simulation of the investment and matching equilibrium as the value of the return on investment, $\lambda$, and post-investment chance of fertility, $p$, increases. Women's skill is shown on the x-axis and men's income on the y-axis, with dots depicting marriage matches. At first, the returns are low enough—and the potential marriage market cost high enough—that no women invest (and thus matching is assortative). As $\lambda$ and $p$ rise, some women invest (shown by dark blue dots, but these top-skilled women are penalized on the marriage market, and matching is non-monotonic. As $\lambda$ and $p$ continue to grow, matching becomes more assortative. Simulation shown for Y=2, S=1, P=1, and c=0.2.

# B Appendix: Census Data

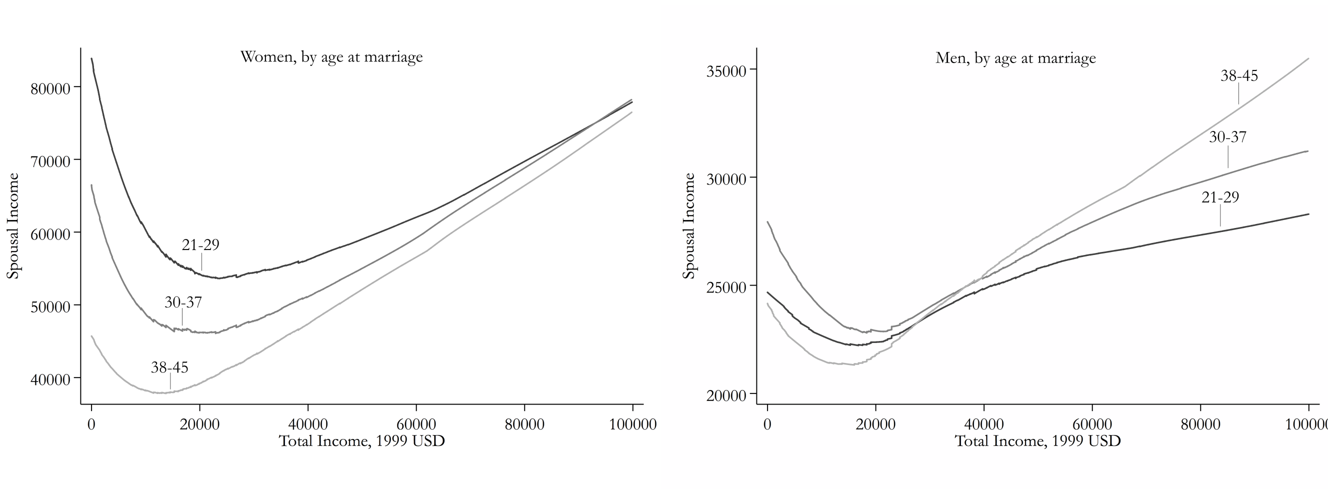
Figure A6 shows that conditional on income, marrying older is always worse for women, but not for men. For women, no matter their own income, women who marry at an older age have a lower-earning spouse. For men, on the other hand, marrying at an older age is linked to a higher-earning spouse when they themselves are high-earning.

Figures A7 and A8 repeat analysis in Figures 5 and A8, but restricting to first marriages. In order to do this, I must exclude data from 1990 and 2000, when number of marriages was not available.

Table A1 examines whether there has been an increasing skill premium among women who attain post-bachelor's education, using data from aptitude scores and educational attainment of three National Longitudinal Surveys NLS cohorts—the 1968 Young Women panel, the 1979 Youth panel, and the 1997 Youth panel. (Unfortunately, the earliest cohort with the necessary test score information only captures the tail end of the non-assortative matching group, but nonetheless, the trend should be informative about whether there are large shifts occurring in underlying selection factors.) If women were previously selecting into post-bachelor's education due to negative selection in other areas, they may be expected to be less positively selected on intelligence and academic potential. The data shows, to the contrary, that highly educated women indeed had substantially higher aptitude scores on average than college educated women even in the earliest cohort, and that the two numbers are not systematically diverging (which would indicate greater skill-driven selection).

Figure A10 shows that women who were highly educated always made higher wages than women who were only college educated, and thus that own income does not show a similar "crossing" as does husband's income. Table A2 demonstrates the change in spousal income based on educational status over time in a regression format.

Figure A6: Lowess-smoothed spousal income by age at marriage



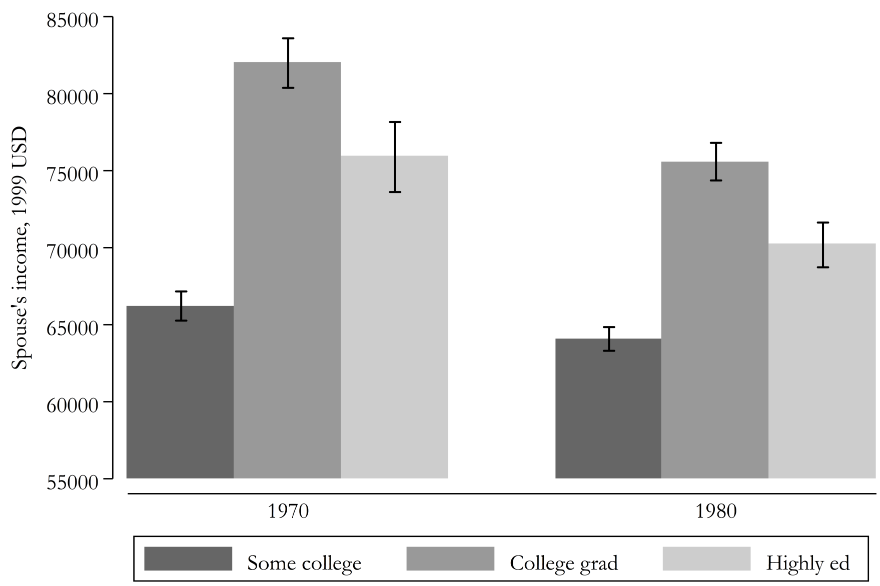

*Notes:* Figure graphs own income versus spousal income for three different age groups. Women who marry older marry poorer men no matter their own income, whereas for wealthy men, those marrying older are matched with higher-earning spouses.

Table A1: Relative college and post-bachelor's average test score percentiles of three NLS cohorts

| | NLS Young Women<br>1944-1954 birth cohort | NLS Youth '79<br>1957-1964 birth cohort | NLS Youth '97<br>1980-1984 birth cohort |
|---|---|---|---|
| College graduate | 66.5 | 70.3 | 63.6 |
| Highly educated | 72.0 | 74.9 | 69.3 |

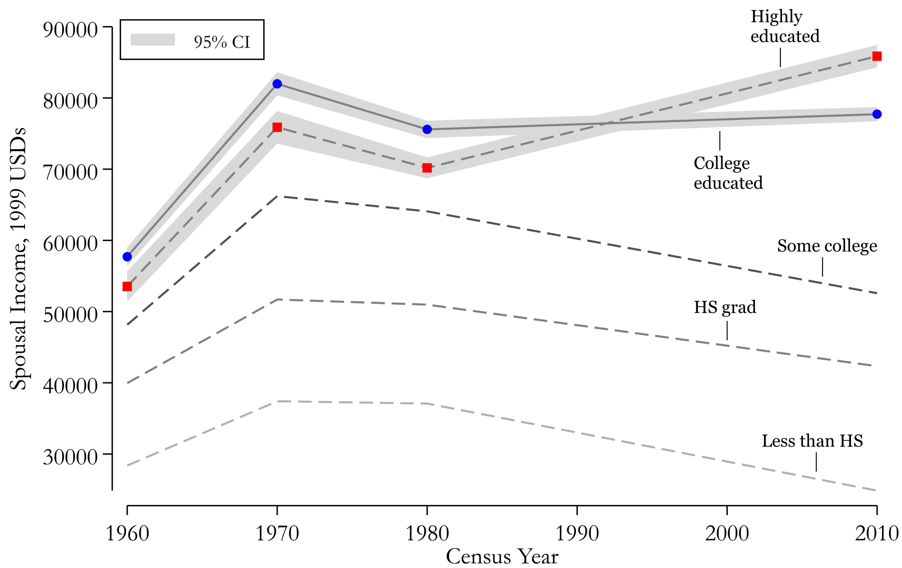
*Notes:* Numbers represent percentiles for test scores by education group, compared to other women of all education levels with test score information available, in three different National Longitudinal Study cohorts. Young Women test score data is from the SAT converted into an IQ measurement. 1979 and 1997 data is from the Armed Forces Qualification Test. Gap in scores between college and graduate-educated women is large and relatively stable. The difference in percentile at both educational levels between the different years may be attributable to score data being available for a different selection of individuals in different survey rounds (e.g., for the Young Women sample, it was only available for individuals who reached the later years of high school).

Figure A7: Non-monotonicity in spousal income by wife's education level, first marriages only



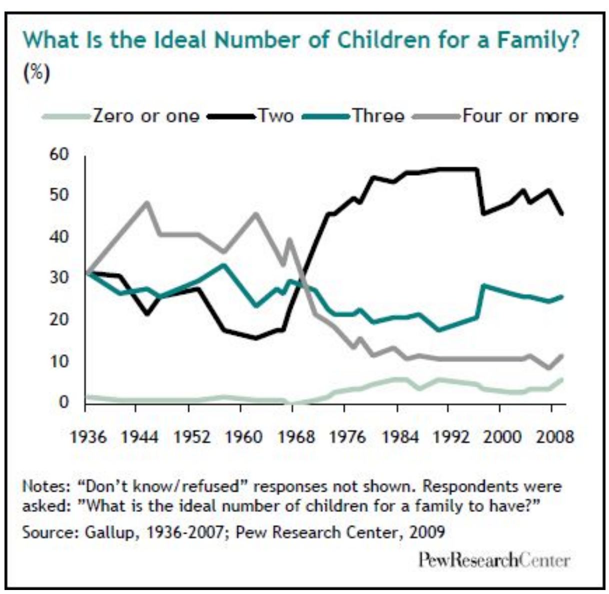

*Notes:* Income of spouse based on wife's education level, with "highly educated" constituting all formal education beyond a college degree. Restricted to first marriages. Difference between college and highly educated women's spousal earnings is significant in bothsamples. 95% confidence interval shown by black lines. Source: 1 percent Census data from 1970 and 1980, white women, ages 41-50 years old.

Figure A8: Spousal income by wife's education level, first marriages only



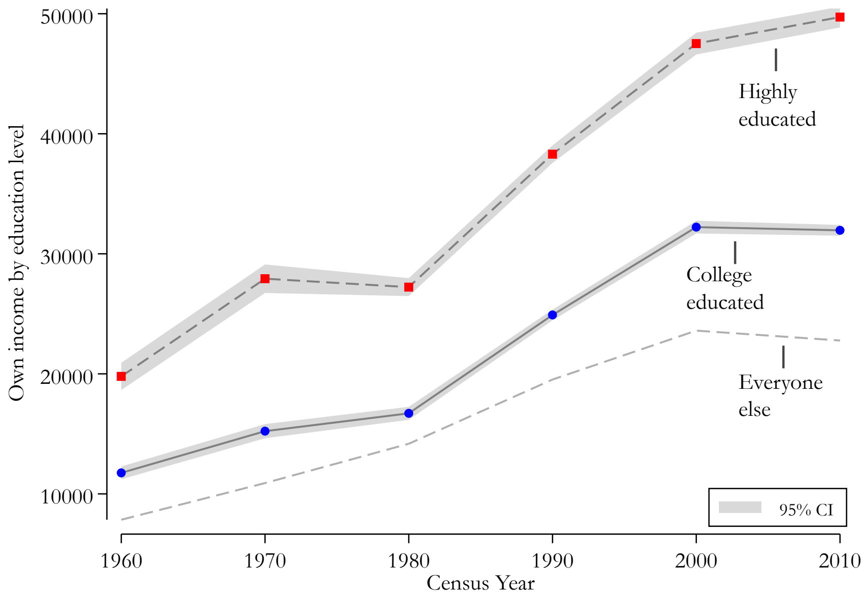

*Notes:* Income of spouse based on wife's education level, with "highly educated" constituting all formal education beyond a college degree. Restricted to first marriages. Source: 1 percent Census data from 1960 and 1970, and American Community Survey from 2000 and 2010. Sample consists of white women, ages 41-50 years old.

Figure A9: Desired family size transition




*Notes:* Figure depicts the rapid transition from four children as the modal desired family size to two children, as evidenced by Gallup polls of men and women. As published in: Pew Center, The New Demography of American Motherhood, August 2010

Figure A10: Own income by education level




*Notes:* Figure shows own income for women by education level, with "highly educated" constituting all formal education beyond a college degree. 1 percent samples of US Census data from 1960, 1970, 1980, 1990 Census, and 2000 and 2010 ACS, white women, ages 41-50. Income in 1999 USDs.

Table A2: Spousal income by wife's education level

| | Dependent variable: Spousal income, 1999 USD | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| 1960 × highly ed | -3,809 | -3,809 | -3,833 |
| | (2,355) | (2,355) | (2,355) |
| 1970 × highly ed | -5,368*** | -5,368*** | -5,386*** |
| | (1,926) | (1,926) | (1,926) |
| 1980 × highly ed | -5,584*** | -5,584*** | -5,580*** |
| | (1,554) | (1,554) | (1,554) |
| 1990 × highly ed | -2,300** | -2,300** | -2,300** |
| | (1,055) | (1,055) | (1,055) |
| 2000 × highly ed | 4,290*** | 4,290*** | 4,268*** |
| | (828.7) | (828.7) | (829.1) |
| 2010 × highly ed | 6,625*** | 6,625*** | 6,623*** |
| | (758.9) | (758.9) | (758.9) |
| Year FEs | Y | Y | Y |
| YOB FEs | | Y | Y |
| Spouse age | | | Y |
| Observations | 115,223 | 115,223 | 115,223 |

*Notes:* Regressions of spousal income on wife's education level interacted with year for women with at least a college degree, with "highly educated" constituting all formal education beyond a college degree. No constant or "highly" dummy is included, so coefficients can be interpreted as the additional spousal income for those in the highly educated category in each sample. Source: 1 percent samples of US Census data from 1960, 1970, 1980, 1990 Census, and 2000 and 2010 ACS, white women, ages 41-50. Standard errors in parentheses.

*** $p<0.01$, ** $p<0.05$, * $p<0.1$