Controlling Room Temperature in a Building

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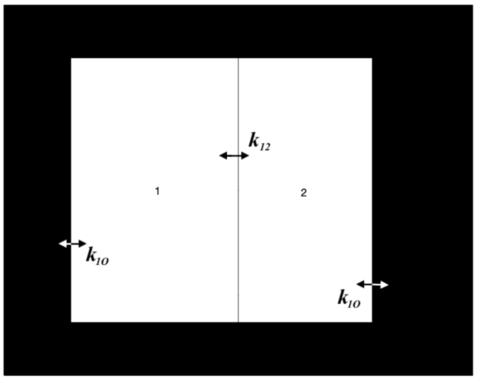


Fig. 1: Image of a 2 room system, annotated with heat transfer coefficients between rooms k_{12}, k_{1O}, and k_{2O}.

I. Introduction

One of the largest forms of daily energy usage for the average person is controlling the temperature in their home. However, the temperature of a building is dynamic, as heat is constantly flowing throughout the building and out of the building, so maintaining the temperature at a desired setting requires taking these dynamics into account. In this report, we model a building's temperature control system and design two control schemes to regulate the temperature in each room to the desired temperature while minimizing the total energy consumption. While we model both heating and cooling in the report, for simplicity, we refer to the temperature control system as a "heating system", and refer to the ventilation units in a given room as "heaters".

II. System Description and Linear Model

We aim to control the temperature in each room of a building by adjusting the heat contributed by each heater. We model the temperature in each room as a function of the heat lost to the outside, the heat flow between adjacent rooms, and the heat added by the heating system.

A. Modeling Thermodynamics

For this system, we assume the heating is provided by a simple ventilation system that can both add and subtract heat from a system. For the sake of simplicity, we assume perfect heat flow within a given room, so the internal temperature within a room is the same at every point. Further, we define temperature with respect to the outside temperature, which we define to be zero. For now we assume that every room has a ventilation system that is independently controlled, but we adjust that assumption later.

Let n be the number of rooms in the system. For each room $i \in \{1 \dots n\}$ in the system, we can define the following variables:

T_i	Temperature in room [°C above outside temperature]
C_i	heat capacity of the room [J/°C]
$\dot{Q}_i$	Rate of heat generated by heater in room [J/s]
$\dot{Q}_{out,i}$	Rate of heat leaving the room [J/s]

From [1], the temperature of the room is related to the change in heat by

$$C_i \dot{T}_i = \dot{Q}_i - \dot{Q}_{out,i} \tag{1}$$

To model this as a linear system, the rate of heat flow between two rooms is approximated as proportional to the temperature difference between the two rooms. The rate of heat flow per difference in temperature per unit area is called the heat transfer coefficient [2], and it takes into account the heat transfer within the walls, known as the the conduction heat transfer coefficient, as well as the heat transfer through the flow of air on either side of the walls, known as the convection heat transfer coefficient. The rate of heat flow through conduction is a linear function of the temperature, so the conduction heat transfer coefficient can be determined directly from the construction of the walls. The heat flow through convection is not an entirely linear process, so estimates of the convection heat transfer coefficient often depend on the temperature difference between inside and outside the building [3]. However, these coefficients change relatively slowly with respect to the temperature difference, so we assume the coefficient is constant.

Therefore, if we define $\dot{Q}_{ij}$ to be the rate of heat flow from room i to room j, we can state

$$\dot{Q}_{ij} = k_{ij}(T_i - T_j)$$

where $k_{ij} = k_{ji} > 0$ is a constant representing the rate of heat transfer between room i and room j, with units of $J/(s \cdot \deg C)$. Similarly, we can express the rate of heat flowing from room i out of the building as

$$\dot{Q}_{iO} = k_{iO} T_i$$

where k_{iO} is a constant representing the rate of heat transfer from room i to the outside.

We can therefore express $Q_{out,i}$ as the sum of the heat losses to the outside and to all the other rooms in the building:

$$\dot{Q}_{out,i} = k_{iO}T_i + \sum_{j=1,\, j\neq i}^{n} k_{ij}(T_i - T_j) \tag{2}$$

B. Basic State Space Model

Combining equations (1) and (2), we can write the following differential equation for the temperature in each room:

$$\dot{T}_i = -\frac{k_{iO}}{C_i}T_i - \sum_{j\neq i}\frac{k_{ij}}{C_i}(T_i - T_j) + \frac{\dot{Q}_i}{C_i} \tag{3}$$

We define our state vector $x = [T_1 \dots T_n]^\top$ to contain the temperature in each room, and the input $u = [\dot{Q}_1 \dots \dot{Q}_n]^\top$ as the power added or removed by the heater in each room. We define the output of the system to be the temperatures in each room.

Based on equation 3, we can describe the system with the following state space equation:

$$\dot{x} = Ax + Bu \tag{4}$$

$$y = Cx \tag{5}$$

where A, B, $C \in \mathbb{R}^{n\times n}$ are defined as

$$A_{ij} = \begin{cases} -\frac{1}{C_i}\left(k_{iO} + \sum_{j\neq i} k_{ij}\right) & i = j \\ \dfrac{k_{ij}}{C_i} & i \neq j \end{cases}$$

$$B_{ij} = \begin{cases} \frac{1}{C_i} & i = j \\ 0 & i \neq j \end{cases}$$

$$C = I_{n\times n}$$

Here, the notation M_{ij} defines the entry in the i-th row and j-th column of a matrix M.

C. Additional Model Constraints

In the system described above, we have heaters and thermometers in every room. However, this may not reflect the reality of a building. In this section, we consider how adjusting these assumptions changes our system. In our later analysis, we consider whether adding these constraints changes our ability to achieve the desired system behavior.

1) Limited Heaters: Assume instead of having heaters in every room, assume we only have heaters in the rooms contained in set $\mathcal{H} \subset \{1 \dots n\}$, where $\mathcal{H}$ has m entries. We can then define our input $u \in \mathbb{R}^m$ to be a vector containing $\dot{Q}_i \;\forall i \in \mathcal{H}$.

We can therefore redefine $B \in \mathbb{R}^{n\times m}$ to add the heat generated by each heater to the equation for the appropriate room:

$$B_{ij} = \begin{cases} \frac{1}{C_i} & u_j = \dot{Q}_i \\ 0 & \text{otherwise} \end{cases}$$

2) Limited Measurement: Now, we no longer assume we can measure the temperature in every room in the building. Instead, we can only measure the temperature in rooms contained in the set $\mathcal{T} \subset \{1 \dots n\}$, where $\mathcal{T}$ has k entries. As a result, the output of our system $y \in \mathbb{R}^k$ only contains the temperatures of the rooms in $\mathcal{T}$. We therefore redefine $C \in \mathbb{R}^{k\times n}$ as

$$C_{ij} = \begin{cases} 1 & j \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases}$$

III. System Analysis

A. Two Room System

For the sake of simplifying the analysis, we use the two room system shown in Fig. 1 to do our symbolic analysis. However, the same principles used to analyze the two room system apply to a system of any size, and we use MATLAB to analyze and simulate a larger, more complex system as an illustrative example.

With a two room system, the full state space model becomes

$$\dot{x} = \begin{bmatrix} -\frac{1}{C_1}(k_{1O} + k_{12}) & \dfrac{k_{12}}{C_1} \\ \dfrac{k_{12}}{C_2} & -\frac{1}{C_2}(k_{2O} + k_{12}) \end{bmatrix} x + \begin{bmatrix} \dfrac{1}{C_1} & 0 \\ 0 & \dfrac{1}{C_2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x$$

B. Stability Analysis

First, we examine the internal stability of the system. By physical intuition this system should be Lyapunov stable, because by the laws of thermodynamics, the temperature of the building should never grow unbounded without external input. Moreover, if the insulation of the building is not perfectly ideal, some heat will leak between the building and the outside so that eventually the temperature of all of the rooms in the building will converge to the temperature outside.

We can prove this by analyzing the eigenvalues of A. Setting the characteristic equation equal to zero, we get

$$C_1C_2\lambda^2 + (C_1(k_{2O} + k_{12}) + C_2(k_{1O} + k_{12}))\lambda + k_{1O}k_{12} + k_{2O}k_{12} + k_{1O}k_{2O} = 0$$

Let $a = C_1C_2$, $b = C_1(k_{2O} + k_{12}) + C_2(k_{1O} + k_{12})$, and $c = k_{1O}k_{12} + k_{2O}k_{12} + k_{1O}k_{2O}$. Note that because for real systems the heat capacities and heat transfer constants must all be positive, $a, b, c > 0$. Applying the quadratic formula, we can express the eigenvalues as

$$\lambda = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

a and c are both positive, so $b^2 - 4ac < b^2$. Therefore, if $\sqrt{b^2 - 4ac}$ is real, it must be strictly less than b^2. In this case, the real part of the eigenvalues can be expressed as

$$\Re(\lambda) = \frac{1}{2a}(-b \pm \sqrt{b^2 - 4ac})$$

which must both be negative. If $\sqrt{b^2 - 4ac}$ is imaginary, the real parts of λ are simply $-\frac{b}{2a}$. Therefore, the eigenvalues have strictly negative real part, so the system must be asymptotically stable.

C. Controllability

To check the controllability of the system, we compute the controllability matrix $\mathcal{C} = [B \ AB]$. For the case where there is a heater in every room, $\mathcal{C}$ trivially has rank 2 because B is full rank, so the system is controllable.

However, the system is also controllable even if only one room contains a heater. Since the system is symmetric, we can arbitrarily choose to analyze the case where Room 2 does not have the heater without loss of generality. In this case,

$$B = \begin{bmatrix} \dfrac{1}{C_1} & 0 \\ 0 & 0 \end{bmatrix}$$

The controllability matrix is then

$$\mathcal{C} = \begin{bmatrix} \dfrac{1}{C_1} & 0 & -\frac{1}{C_1^2}\left(k_{1O} + k_{12}\right) & 0 \\ 0 & 0 & \dfrac{k_{12}}{C_1 C_2} & 0 \end{bmatrix}$$

Since C_1 and C_2 are finite and $k_{12} > 0$, the matrix has rank 2 and the system remains controllable. If the building had no heaters though, B and therefore $\mathcal{C}$ would become zero matrices, and the system would be trivially uncontrollable.

D. Observability

Like controllability, the observability of the system can be analyzed with the observability matrix $\mathcal{O} = [C^\top (CA)^\top]^\top$. By inspection, if C has full column rank (as is the case when the temperature in every room can be measured), then $\mathcal{O}$ has rank n, and the system is observable.

If the temperature can only be measured in one room (without loss of generality, let this be room 1), the system is still observable. If only the temperature in the first room can be measured,

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Therefore,

$$\mathcal{O} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{C_1}\left(k_{1O} + k_{12}\right) & \dfrac{k_{12}}{C_1} \end{bmatrix}$$

Since k_{1O} and k_{12} are positive and C_1 and C_2 are finite, $\mathrm{rank}(\mathcal{O}) = 2$, so the system is still observable.

If the temperature cannot be measured in any room, there is no output, and the system is trivially unobservable.

IV. Control and Observer Design

A. Controller Design

In this section, we design an open-loop and a closed-loop control scheme for this system. We assume that when the control begins, the system has been off for a long time, so each room's is equal to the external temperature (i.e. $x = 0$). The controller aims to bring the state to x^*, the vector containing the desired temperature for each room, and maintain the temperature at that state. Further, we aim to minimize the energy consumption of the system by minimizing the norm of our input.

1) Open Loop Control: As shown above, the two room system is fully controllable for a system with at least one heater. Therefore, by the properties of a continuous-time LTI system, all states in $\mathbb{R}^2$ must be reachable, so the reachability Gramian may be defined as:

$$W_R(t_0, t_1) = \int_{t_0}^{t_1} e^{A(t_1 - \tau)} B B^\top e^{A^\top (t_1 - \tau)} \, d\tau$$

and must be invertible for all $t_1 > t_0$.

Let $\eta = W_R(0, t_i)^{-1} x^*$ for some $t_i > 0$. The minimum-energy open loop control from $x(0) = 0$ to $x(t_i) = x^*$ is given by $u_{init}(t) = B(t)^\top e^{A^\top (t_i - t)} \eta$. As a result, we can define the optimal open loop control starting from zero as

$$u_{init}(t) = B(t)^\top e^{A^\top (t_i - t)} W_R(0, t_i)^{-1} x^*, \ t \in [0, t_i]$$

However, because x^* generally may not be an equilibrium state of the system, further control is necessary to maintain the state close to x^*. To accomplish this with open loop control, we can periodically control the state back to x^*. This control strategy can be derived from the solution to an LTI state space system. Let $u_m(t)$, $t \in [0, t_m]$ be a control strategy that drives the system from $x(0) = x^*$ to $x(t_m) = x^*$. This problem can be rewritten in terms of a reachability problem:

$$\begin{aligned} x(t_m) &= e^{At_m} x(t_1) + \int_0^{t_m} e^{A(t_m - \tau)} B u(\tau) \, d\tau \\ x(t_m) - e^{At_m} x(0) &= \int_0^{t_m} e^{A(t_m - \tau)} B u(\tau) \, d\tau \\ x^* - e^{At_m} x^* &= \int_0^{t_m} e^{A(t_m - \tau)} B u(\tau) \, d\tau \end{aligned}$$

As seen in the above equation, the $u_m(t)$ that drives the system from $x(0) = x^*$ to $x(t_m) = x^*$ is equivalent to the $u_m(t)$ that drives the state from $x(0) = 0$ to $x(t_m) = \left(I - e^{At_m}\right) x^*$. Using the same process as for $u_{init}(t)$, $u_m(t)$ can be expressed for $t \in [0, t_m]$ as

$$u_m(t) = B^\top e^{A^\top t_m} W_R(0, t_m)^{-1} \left(I - e^{At_m}\right) x^*$$

Now, we can design a full input signal to bring the state to x^* and maintain it at that level. To do so, we define the parameters T_i and T_m as the time it takes to initially bring the system from zero to x^* and the subsequent interval at which we bring the system back to x^*. We then define $u(t)$ over $t \in [0, \infty)$ as u_i until time T_i, followed by repetitions of u_m to return the state at x^* every T_m seconds:

$$u(t) = \begin{cases} u_{init}(t) & 0 \leq t \leq T_i \\ u_m\left((t - T_i) \bmod T_m\right) & T_i < t \end{cases}$$

By tuning the parameters T_i and T_m, we can find an optimal balance between reducing the energy consumption of the system, minimizing the maximum temperature overshoot and undershoot, and getting to the desired state quickly.

2) Closed Loop Control: First, we assume $x^* = x^{eq}$ where x^{eq} is part of some equilibrium point (x^{eq}, u^{eq}), and design a closed loop control scheme to bring the system to that equilibrium. Then we examine the case where x^* is not an equilibrium point and design a closed loop control scheme that stabilizes x at an equilibrium point as close as possible to x^*.

First, assuming $x^* = x^{eq}$ is an equilibrium point, there exists u^{eq} such that

$$Ax^{eq} + Bu^{eq} = 0$$

Let $\tilde{x} = x - x^{eq}$, and $\tilde{u} = u - u^{eq}$. The dynamics of $\tilde{x}$ can be expressed as follows:

$$\begin{aligned}
\dot{\tilde{x}} &= \dot{x} = Ax + Bu \\
&= Ax + Bu - Ax^{eq} - Bu^{eq} \\
&= A(x - x^{eq}) + B(u - u^{eq}) \\
\dot{\tilde{x}} &= A\tilde{x} + B\tilde{u}
\end{aligned}$$

By driving $\tilde{x}$ to zero, we drive the system to x^{eq}. To optimally accomplish this with minimal cost, the problem can be formatted as a Linear Quadratic Regulator problem for some $Q \succeq 0$ such that (A, Q) is detectable, and some $R \succ 0$. Solving the algebraic Riccati equation $A^\top P + PA + Q - PBR^{-1}B^\top P = 0$ for P, we can define the optimal $\tilde{u} = -R^{-1}B^\top P\tilde{x}$. Therefore, the closed-loop controller can be defined as

$$u(t) = u^{eq} - R^{-1}B^\top P(x - x^{eq})$$

This closed loop design can be used for any x^* that exists as an equilibrium state of the system, i.e. for any x^* for which $\exists u^{eq}$ s.t. $Ax^* + Bu^{eq} = 0$. This occurs if and only if Ax^* is in the image of B. If B is full-rank, this is always true, so in the case that there is a heater in every room, closed loop control can be used to reach any x^*.

In the case that B is not full-rank (i.e. at least one room does not contain a heater), then x^* may not be an equilibrium point for any u. In this case, the closed loop control system cannot drive x directly to x^*. Instead, the system can be used to drive x to x^{eq}, where (x^{eq}, u^{eq}) is the equilibrium point with state closest to x^*. The x^{eq} corresponding to u^{eq} can be written as $x^{eq} = -A^{-1}Bu^{eq}$, so the u^{eq} minimizing the difference between x^{eq} and x^* can be defined mathematically as the solution to

$$\underset{u}{\operatorname{argmin}} \|x^* + A^{-1}Bu\|,$$

Note that because A is always asymptotically stable, it is always invertible. The least squares solution gives a closed form expression for u^{eq}, where

$$u^{eq} = -((A^{-1}B)^\top A^{-1}B)^{-1}(A^{-1}B)^\top x^*,$$

and by extension,

$$x^{eq} = A^{-1}B((A^{-1}B)^\top A^{-1}B)^{-1}(A^{-1}B)^\top x^* \tag{6}$$

B. Observer Design

For the open loop control scheme, we only need to know the model parameters and the initial state of the system (which can be assumed to be zero if the system has been off for long enough) in order to determine our control inputs. However, for the closed loop control system, we must have access to an estimate of our state variables at all times.

If our system has a thermometer in every room, the output of the system is equal to its state, so this isn't an issue. However, if one or more of the rooms does not have a thermometer, we must design an observer to approximate the temperature of the rooms we cannot measure directly. Using a Kalman filter, we can model the dynamics of $\hat{x}$, the estimate of the state, with the following equations:

$$\begin{aligned}
\dot{\hat{x}} &= A\hat{x} + Bu + L(y - \hat{y}) \\
\hat{y} &= C\hat{x} + Du
\end{aligned}$$

Using certainty equivalence, we can define our closed loop control scheme with respect to $\hat{x}$ as $u = u^* - K(\hat{x} - x^*)$. Because closed loop observer and controller design are separable, the controller remains stable. The Simulink block diagram for this observer, including the closed loop control scheme, is shown in Fig. 2. Letting W^{-1} and V^{-1} be tunable parameters respectively representing the quadratic penalties on the disturbance and noise of our observed system, the Kalman filter defines the optimal L as $PC^\top V^{-1}$, where P is the solution to the dual algebraic Riccati equation

$$W + PA^\top + AP + PC^\top V^{-1}CP = 0$$

Because the system is always observable, (A, C) is always detectable. Therefore, a unique P exists for any W such that (A, W) is stabilizable.

V. Simulation

To verify our analysis, we first simulate the performance of our open loop and closed loop control schemes under different conditions in a smaller, three-room system, as shown in Fig. 3. Then, we apply the control to a larger eight-room system, shown in Fig. 4, to demonstrate how the control generalizes to larger systems.

For all examples in this section unless otherwise stated, the house's temperature is initially set to 0°C (i.e. equal to the external temperature) in all rooms, with a desired temperature set to 20°C above the outside temperature for all rooms (i.e. $x_i^* = 20\ \forall i$). For the three room system, open loop control is simulated for $T = 400$ overall, with $T_i = 300$ and $T_m = 100$. Closed loop control is simulated for $T = 300$ for the three room system. A video for each of the cases of the 3 room simulations evolving over time can be found at here: `https://www.youtube.com/watch?v=jx6y9Nen6_A`.

A. Estimating Heat Transfer Coefficients

To simulate a realistic system, we picked heat transfer coefficient values based in the literature. Because interior and exterior walls generally have different properties, we

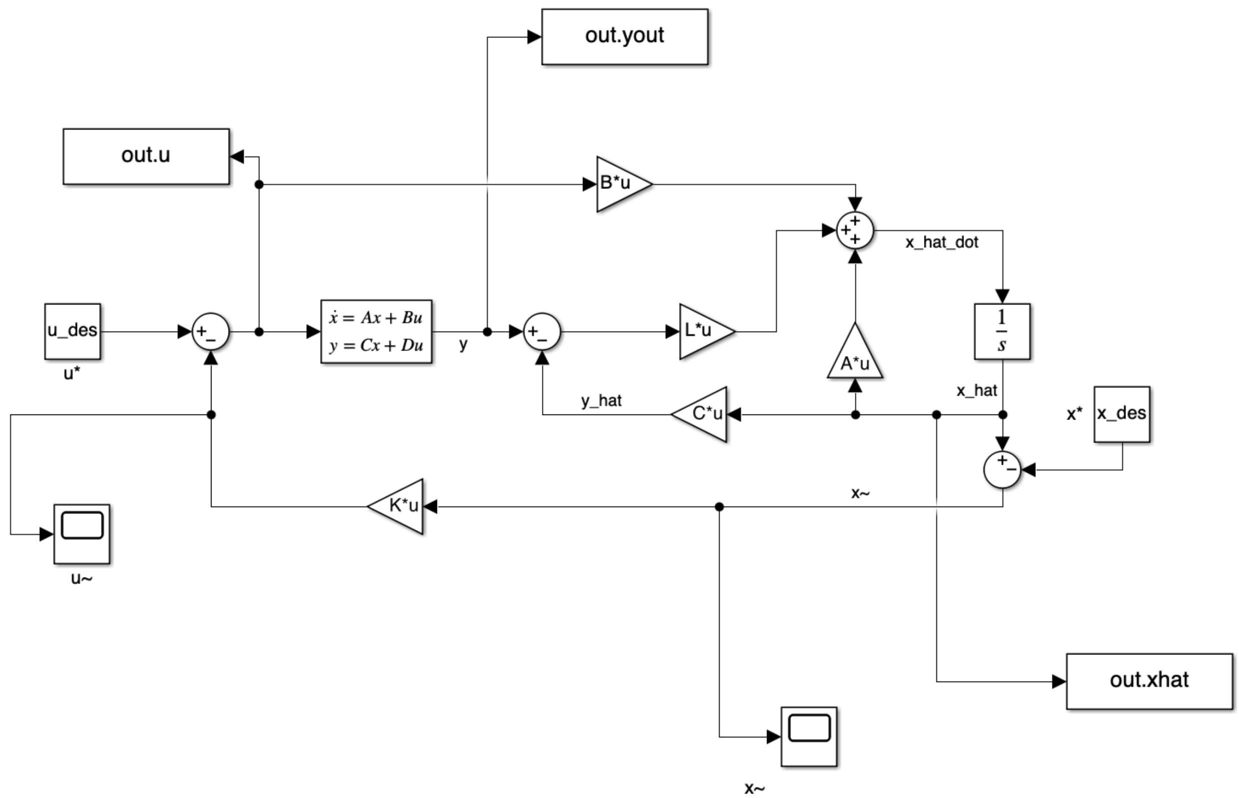


Fig. 2: The full Simulink implementation of the LQR closed loop controller around equilibrium point (x^*, u^*) using a Kalman filter observer.

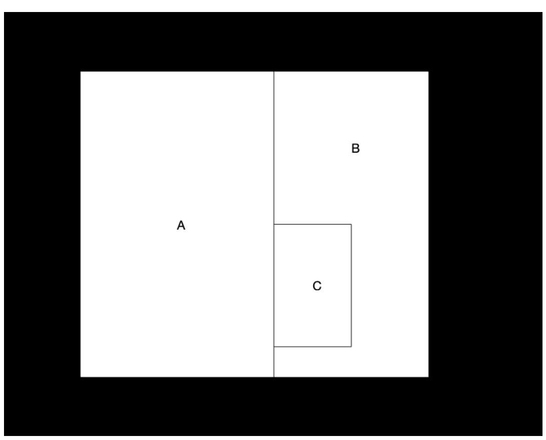


Fig. 3: Layout of the three room system used for this demonstration.

estimate their overall heat transfer coefficients separately. Using [3], we picked reasonable heat transfer convection coefficients for the both sides of interior walls and for the inner side of exterior walls. Using [4], we estimated a reasonable heat convection coefficient for the outer heat convection coefficient for exterior walls. Finally, picking values for the wall materials, we used [2] to estimate the overall heat transfer coefficients for the interior and exterior walls. From these, the values of k were computed by scaling by the surface area of each wall.

B. *Open Loop Control*

1) Full Heating: In this example, there is a heater present in each room. As seen in Fig. 5, the controller is able to bring the system to the desired state, though the temperature does noticeably overshoot for Room 3 when attempting to reach the equilibrium point.

2) Limited Heating: In this example, there is no heater in Room 3. Because the system is still controllable, the system is still able to reach the desired states at 300 and 400 seconds, as seen in the video.

3) Lowpass-Filtered Heating Input: While the open loop control is guaranteed to drive the system to the desired state at the specified time, it can require sharp changes in the input, which may not be physically possible for a real heater. Because of this, we also investigate the impact of lowpass-filtering the input to better approximate the input signal achievable by a physical system. In this example, all rooms have heaters, but the input to the heaters is lowpass-filtered to show the effects this would have on the temperature within the system. As shown in the video, the effects of this are mostly negligible.

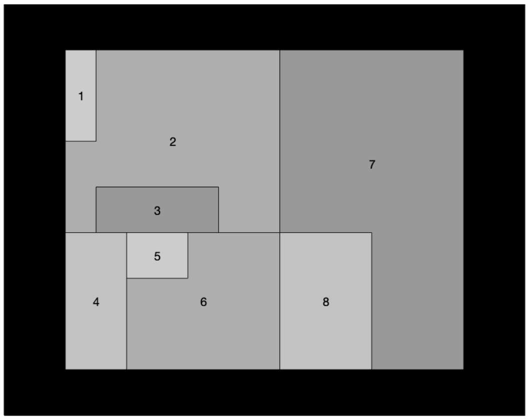


Fig. 4: Layout of the eight room system used for this demonstration.

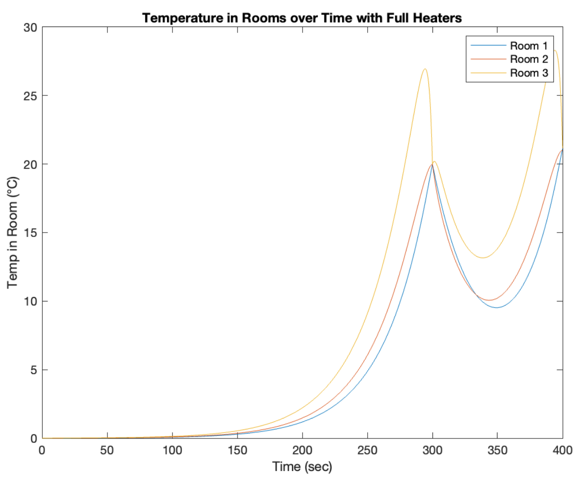


Fig. 5: Temperature over time for open loop control

4) Temperature-Dependent A Matrix: To validate our approximation of the heat transfer coefficient as linear, we use our open loop control based on the linear system on one of the non-linear models for heat transfer presented in [3]. Specifically, we define the heat transfer coefficient h_{12} between rooms $1, 2$ with temperatures T_1, T_2 and wall heat conductance h_{wall} as:

$$h_{12} = 2.2|T_2 - T_1|^{0.22} h_{wall}$$

As shown in the video, the open loop controller designed to bring x to $[20, 20, 20]^\top$ brings the system to $x = [19.917, 19.853, 19.766]^\top$. This is not a significant error, and appears to validate our approximation.

C. Closed Loop Control

For our LQR control, we set $Q = 100I$ and $R = I$ for all of the cases.

1) Full Heating and Sensing: With full sensing, an observer is unnecessary as the output of the system is the state. Because we have full heating and B is therefore full rank, we can always find an equilibrium point (x^*, u^{eq}) for any x^*, so we are successfully able to drive our system to asymptotically towards our desired state $x^* = [20, 20, 20]^\top$, as shown in Fig. 7.

2) Limited Heating: In this case, we examine the system where the temperature can still be directly observed in every room, but not every room contains a heater. In this case, B is not full rank, so the system drives x towards the nearest equilibrium point x^{eq} rather than x^*, as specified by (6). As B's range depends on which rooms contain heaters, heater distribution plays a significant role in determining which x^* the closed loop control system is able to closely approximate. In particular, because we are generally interested in reaching a state where all rooms have the same temperature, spreading the heaters evenly through the system and toward the edges of the building results in a more even temperature throughout the building.

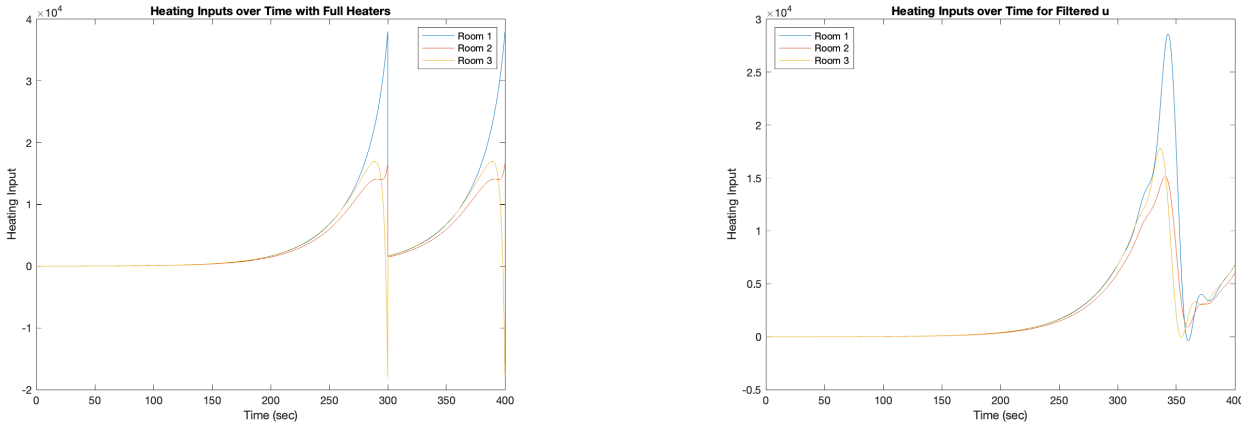
For the three room system, we examine the case where only two rooms contain heaters. First, consider the case where Room 3 does not have a heater, as in the video. In this case, since the room lacking a heater is in the middle, x^* is in fact an equilibrium point. The closed loop system is able to function just as with heaters in all rooms. In the case where Room 2 is missing a heater, x^* is no longer an equilibrium point. Intuitively, because Room 2 is on the edge of the building it cannot be at x^* without other rooms being warmer. As a result, the closest possible equilibrium point is $x^{eq} = [21.44, 15.20, 21.92]^\top$, demonstrating that extra heating from Rooms 1 and 3 is unable to compensate for the lack of heat in Room 2.

3) Limited Sensing: Now we consider the case where there is a heater in every room, but some rooms do not have a thermometer. In this case, the closed loop control must rely on an observer to estimate the state, as shown in Fig. 2. To demonstrate some initial estimation error, for this example $x(0) = [5, 5, 5]^\top$. Fig. 8 displays the results of this system, showing that the observer's estimate of the state is able to successfully converge to the correct state, and the closed loop control functions successfully.

D. Eight Room System

The eight room system, shown in Fig. 4, has sensors in rooms 2, 4, 5, and 8, and is missing heaters in rooms 1, 3, and 5. A video of the eight room system's simulations can be found here: `https://www.youtube.com/watch?v=QuRFb0l5RXg`.

With the open loop control, even though three rooms lack heaters, the system is still able to exactly reach x^* at the required intervals. However, the rooms often spike to incredibly cold or warm temperatures to reach the exact state specified. Fig. 9 shows that increasing the time scale T_m for maintaining the temperature does not seem to greatly reduce these oscillations, as the oscillation peaks in Fig. 9b are indistinguishable from the peaks in Fig. 9a, for which



(a) (b)

Fig. 6: Heating input for the case with heaters in all rooms. Fig. (a) shows unfiltered heating input; Fig. (b) shows filtered heating input.

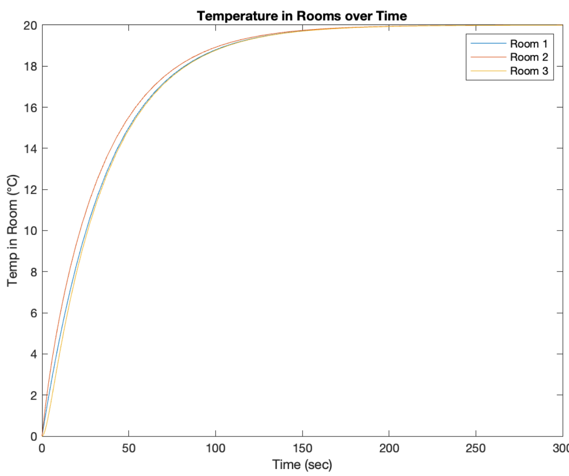


Fig. 7: Temperature in three-room system with full sensing, full heating, and closed loop control.

T_m is twice as long. With closed loop control (Fig. 10), x^* is not an equilibrium point, so the system is projected onto an equilibrium point with (6). This equilibrium is relatively close to x^* for most rooms except for Room 1, where $x^{eq} = 16.4$, and Rooms 2 and 3, where x^{eq} is 21.9 and 21.1, respectively.

As seen in Fig. 10, Room 1 is noticeably cooler than the desired temperature because it does not have a heater and is on the outside of the house. However, by rooms 7 and 8, adjacent to other rooms with heaters, the desired temperatures can be acheived.

E. Comparing Energy Consumption

The energy consumption is defined as

$$E(t) \int_0^t ||u(\tau)||_2^2 d\tau,$$

where the system starts at time $t_0 = 0$ This quantity corresponds to the total energy used by the heating systems. In the case where both the open and closed loop systems have heaters in all rooms, open loop's $E(300) \approx 38 * 10^9$ while we have closed loop's $E(300) \approx 196 * 10^9$. This follows from the literature, as open loop control gives the minimum energy path between two points, and thus we would expect it to be less energy-intensive.

Analyzing the 8-room system, open loop control has energies of $8 * 10^9$ for $T_m = 10000$ and $54 * 10^9$ for $T_m = 5000$ while for closed loop control the energy is a much smaller $2*10^9$. This can be explained by the open-loop control system needing to reach an exact state which requires large fluctuations, while closed-loop is able to maintain a more energy efficient equilibrium.

VI. Summary and Conclusion

This report models the heat flow within a building as a linear system, and designs two control schemes to control the temperature in every room under varying constraints the number of heaters and sensors. While the open loop control scheme is able to guarantee that the temperature in every room converges exactly to the desired state in any specified time interval and consumes less energy, it may force extreme behavior onto the system in order to get there, resulting in temperatures that may not even be within the range of human survivability. On the other hand, while closed loop control may not be able to exactly achieve every desired state, it guarantees stability of the system, even with limited sensing. Moreover, by placing heaters in outer rooms and distributing them evenly throughout the building, the closed loop control can better achieve even temperatures in all rooms. Therefore, we recommend the closed loop control scheme.

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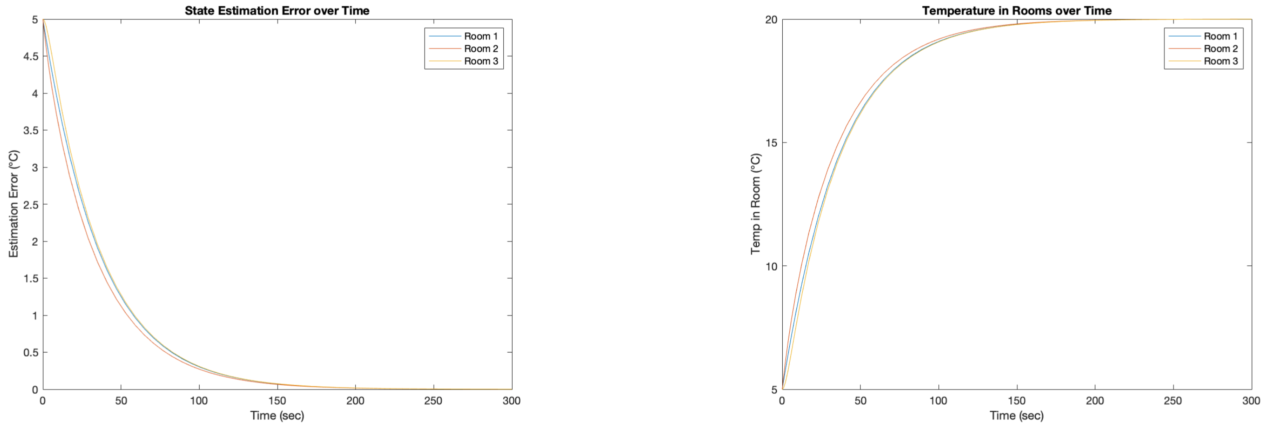
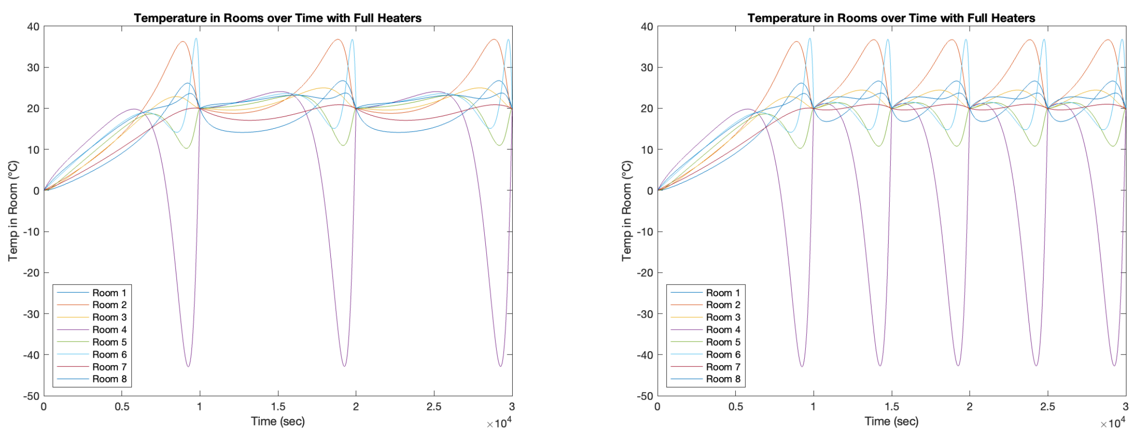


(a) State estimation error, $x - \hat{x}$

(b) State value, x

Fig. 8: Performance of closed loop control with limited sensing. In Figure (a), the state estimation error converges to zero over time, showing that the observer is successfully able to estimate the state. In Figure (b), the state is able to converge to x^*, demonstrating that the controller still works with certainty equivalence.



(a) Open Loop Control for $T_m = 10000$

(b) Open Loop Control for $T_m = 5000$

Fig. 9: Open loop control on the eight room system for two different maintenance intervals. Even when varying the maintenance interval, the system is unable to get to and return to x^* without extreme behavior in some rooms.

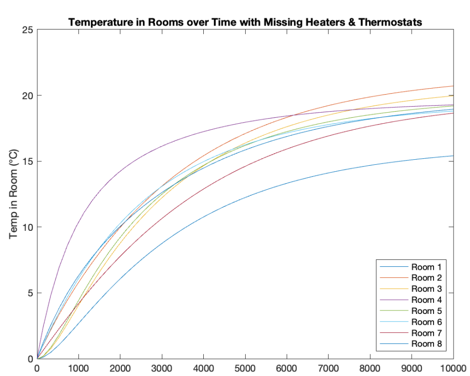


Fig. 10: Closed loop control for eight room system.

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