No Deep Thought Required. Let a, b, c be positive real numbers such that $\frac{bc}{a} = \frac{1}{2}$, $\frac{ca}{b} = 8$, and $\frac{ab}{c} = 128$. What is the value of $a + b + c$?

(Austin Math Circle Practice MATHCOUNTS 2018, Countdown #48)

Is That Your Final Answer? Steven has a solid cube of side length 1. He glues six right square pyramids, each with base side length 1 and altitude $\frac{1}{2}$, to the six faces of his cube. How many faces does the resulting solid have?

(Austin Math Circle Practice MATHCOUNTS 2020, Team #1)

Expect the Unexpected. Eric randomly picks a positive integer n from 1 to 1000. What is the expected number of digits in the decimal representation of n? Express your answer as a decimal to the nearest thousandth.

(Austin Math Circle Practice MATHCOUNTS 2018, Team #4)

The Strangest Thing. Lucas rolls a fair eight-sided die and Dustin rolls a fair twelve-sided die. What is the expected value of the sum of the two die rolls?

(Austin Math Circle Practice MATHCOUNTS 2018, Countdown #38)

Hello There. In equilateral pentagon $HOWDY$, $\angle H = 90^\circ$ and $\angle W = 120^\circ$. Compute $\angle Y$ (in degrees).

(Texas A&M High School Math Contest 2019, Best Student Exam #5)

Oh, Henry. Della has some coins worth a total of \$1.87, exactly n of which are pennies. For how many values of n is this possible?

(Austin Math Circle Practice MATHCOUNTS 2018, Countdown #33)

Po-Ta-Toes. Sam likes potatoes. Every day at dinnertime, he cooks potatoes either by boiling them, mashing them, or sticking them in a stew. (Each of these three ways is chosen randomly, equally likely, and independent of what he chooses on other days.) What is the probability that Sam serves mashed potatoes at least twice this week?

(Austin Math Circle Practice MATHCOUNTS 2022, Countdown Championship #9)

JJJ. Joshua randomly picks a positive perfect square less than 2017, Jay randomly picks a positive perfect cube less than 2017, and Jonathan randomly picks a positive perfect sixth power less than 2017. What is the probability that all three picked the same number?

(Texas A&M High School Math Contest 2017, Best Student Exam #3)

I Regret Nothing. In the spring interhigh volleyball tournament, Tsubakiduba High School and Shiratirazira High School play each other in a best-of-three match. (Thus, they play three sets, and whichever team wins two or more sets wins the match.) If Shiratirazira wins each set with probability 60% (independently of all other sets), what is the percentage probability that they win the match?

(Austin Math Circle Practice MATHCOUNTS 2022, Countdown Championship #11)

Knightlife. In chess, a rook attacks by moving any number of squares in one of the four cardinal directions, while a knight attacks by moving two squares in one direction, then one square in a perpendicular direction. A rook and a knight are randomly and independently placed in distinct squares on an 8×8 chessboard. What is the probability that one of them can attack the other?

(Texas HMMT November TST 2019, General Round #4)

Lost in Space. Major Tom is orbiting the Earth at an altitude of 450 miles. Assuming the Earth is a perfect sphere with radius 4000 miles, compute the largest possible distance (in miles) between Major Tom and any point on Earth's surface that he can see from his current location.

(Austin Math Circle Practice MATHCOUNTS 2020, Countdown #39)

Reach for the Summit. Madeline is climbing a 2000-meter tall mountain. She starts climbing from the base of the mountain at noon, and ascends at a constant speed of 10 meters per minute. After she has been climbing for some time, the wind begins blowing and slows her ascent to 6 meters per minute. If it is 4:00 PM on the same day when Madeline reaches the summit, how many minutes past noon was it when the wind started blowing?

(Austin Math Circle Practice MATHCOUNTS 2021, Sprint #15)

A Prime Problem. Chris thinks of three different prime numbers, and notices that their product is equal to 19 times their sum. Compute the sum of the three prime numbers Chris must be thinking of.

(Texas A&M High School Math Contest 2019, Best Student Exam #3)

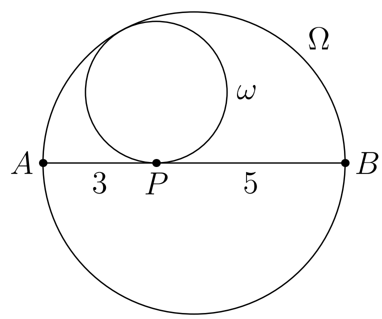
Meanwhile in Iowa. The River City High School marching band has 76 students who play the trombone and 110 students who play the cornet. If every student plays at least one of these two instruments, and exactly half of the students in the band play both, compute the total number of students in the River City High School marching band.

(Austin Math Circle Practice MATHCOUNTS 2020, Countdown #59)

The Evil Vector Problem. Ol' Rock the Good Ag is standing somewhere in the first quadrant, on the line $y = 2x + 1$. If he is also exactly two units away from the line $y = \frac{3}{4}x$, find the coordinates of the point where Ol' Rock is standing.

(A very misplaced MATH 151 quiz question, Fall 2021)

The Death Star. Circle Ω has diameter AB. Circle ω is tangent to segment AB at P, and internally tangent to circle Ω. If $AP = 3$ and $BP = 5$, compute the radius of circle ω. Express your answer as a common fraction.



(Austin Math Circle Practice MATHCOUNTS 2020, Countdown #12)

The Green Light. There are N people playing a very large-scale game. One person is eliminated, and then the remaining people can be divided evenly into groups of 13. Then, four more people are eliminated, and the remaining people can be divided evenly into groups of 11. Finally, one more person is eliminated, and the remaining people can now be divided evenly into groups of 10. What is the least possible value of N such that this could be true?

(Austin Math Circle Practice MATHCOUNTS 2022, Target #7)

The Root of All Evil, Part One. Evaluate

$$\sqrt[3]{1\sqrt[3]{2\sqrt[3]{4\sqrt[3]{8\sqrt[3]{16\sqrt[3]{\cdots}}}}}}$$

(Texas A&M High School Math Contest 2016, EF Exam #10)

The Root of All Evil, Part Two. Compute the least positive integer n such that

$$\sqrt{1+\sqrt{2+\sqrt{3+\sqrt{4+n}}}}$$

is an integer.

(Austin Math Circle Practice MATHCOUNTS 2020, Sprint #20)

The Root of All Evil, Part Three. Evaluate

$$\frac{4}{\sqrt{17+\frac{4}{\sqrt{17+\frac{4}{\sqrt{17+\frac{4}{\sqrt{\cdots}}}}}}}}$$

(Austin Math Circle Practice MATHCOUNTS 2022, Tiebreaker #1)

Sam's Gambit. Wolstenholme and Zsigmondy decide to compete head-to-head to determine which of the two is a better mathematician. They are given a single number theory problem to solve, and both solve it in a two-digit positive integer number of seconds. Afterwards, they observe that

- The product of their times is a perfect square
- The sum of their times is a prime number
- When Wolstenholme's time w and Zsigmondy's time z are concatenated, the result $100w+z$ is a perfect square

How many seconds passed before the first person solved the problem?

(Austin Math Circle Practice MATHCOUNTS 2018, Team #8)

Captain's Log. Find the positive real number x such that

$$\log_4(x)-\log_{16}(x)+\log_{64}(x)-\log_{256}(x)+\log_{1024}(x)-\cdots=1$$

(The bases of the logarithms are consecutive powers of 4.)

Making the Cut. Isabella has a sheet of paper in the shape of a right triangle with sides of length 3, 4, and 5. She cuts the paper into two pieces along the altitude to the hypotenuse, and randomly picks one of the two pieces to discard. She then repeats the process with the other piece (since it is also in the shape of a right triangle), cutting it along the altitude to its hypotenuse and randomly discarding one of the two pieces once again, and continues doing this forever. As the number of iterations of this process approaches infinity, the total length of the cuts made in the paper approaches a real number ℓ. Compute $[\mathbb{E}(\ell)]^2$, that is, the square of the expected value of ℓ.

(NIMO Revenge Round #31, Problem #4)

The Breakfast of Champions. Jay can buy two bagels and one muffin with exactly twenty silver coins. Kevin can buy one bagel and two muffins with exactly three gold coins. Together, they can buy five bagels and seven muffins with exactly eleven gold coins and four silver coins. If a gold coin is worth n times as much as a silver coin, compute n.

(Austin Math Circle Practice MATHCOUNTS 2019, Team #7)

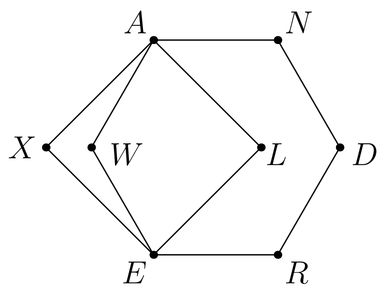
Vote for Pedro. In the election for class president of Preston High School, every student votes for exactly one candidate. Pedro and Summer receive a combined total of exactly 85% of the votes cast, and Pedro receives 43 more votes than Summer. If Pedro wins a strict majority (that is, strictly more than 50%) of the total number of votes, what is the maximum possible number of students who voted for Pedro?

(Austin Math Circle Practice MATHCOUNTS 2020, Challenge Round A3)

You Know What I Heard About Amy? Amy likes spiders. During lockdown, she is bored, so she places one spider on each of the three vertices of an equilateral triangle. Each second, every one of the three spiders simultaneously crawls from its current vertex to either of the other two vertices, choosing independently at random. Compute the expected number of seconds before all three spiders meet at the same vertex for the first time.

(Texas HMMT November 2020 TST, Theme #5)

Arbitrarily Awesome. Regular hexagon $ANDREW$ and square $ALEX$ lie in the same plane, as shown in the figure. What fraction of the area of hexagon $ANDREW$ overlaps with square $ALEX$? Express your answer as a common fraction in simplest radical form.



(Austin Math Circle Practice MATHCOUNTS 2018, Sprint #28)

Crazy Cubics, Part One. Suppose f is a cubic polynomial with roots x, y, and z such that

$$x = \frac{1}{3 - yz}, \qquad y = \frac{1}{5 - zx}, \qquad z = \frac{1}{7 - xy}.$$

If $f(0) = 1$, compute $f(xyz + 1)$.

(Texas A&M High School Math Contest 2018, Best Student Exam #8)

Crazy Cubics, Part Two. Let x, y, z be three pairwise distinct real numbers satisfying

$$x + y = yz$$

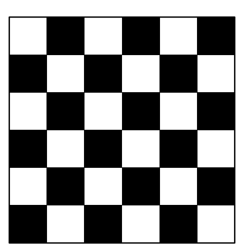
$$y + z = zx$$

$$z + x = xy$$

Compute $(x + 10)(y + 10)(z + 10)$.

(Texas HMMT November 2020 TST, General #6)

When Bishops Come Together. In chess, a bishop can attack by moving any number of spaces in any of the four diagonal directions. Nico wants to place nine identical bishops on the 6×6 chessboard shown below such that no two of them can attack each other. Let N be the number of ways for Nico to do this. Compute the remainder when N is divided by 1000.



(2020 Fake AIME, #9)

Never Forget. Mews the cat and Yertle the turtle are running a race. They run at constant speeds, with Mews running six times as fast as Yertle. To compensate for this, Yertle is allowed to start fifty feet ahead of the starting line, while Mews starts at the starting line. When the start signal is given, they both begin running. Fifteen seconds after the beginning of the race, Mews passes Yertle. However, once Mews reaches the finish line, she turns around and begins running back towards the start at the same speed, and she passes Yertle again fifty-four seconds after the beginning of the race. How many feet long is the race?

(Austin Math Circle Practice MATHCOUNTS 2020, Target #6)

Square Deal. Point J lies in the interior of square $ANDY$ such that $AJ = 8$ and $JD = 4$. Compute the least possible integer value for the area of square $ANDY$.

(Austin Math Circle Practice MATHCOUNTS 2019, Sprint #29)

You Can't Factor This. Given that there are exactly four primes that divide the number

$$257^4 + 32^3 - 8193^2 - 640^2$$

compute the largest of the four primes.

(Texas HMMT November 2020 TST, General #7)

A Roast. At the Slightly Pointless Ultimate Regional Smackdown (SPURS), there are 100 contestants, numbered 1 through 100. The contestants all take a 30-problem test, with problems numbered 1 through 30. For each m and n, the probability that contestant m solves problem n is $\frac{m}{mn+60}$. (All these probabilities are assumed to be independent.) What is the probability that every contestant solves at least one problem on the test? Express your answer as a common fraction.

(Austin Math Circle Practice MATHCOUNTS 2019, Target #8)

Concentrated Evil. Given that the number

$$N = \frac{20^{22} - 22}{20 + 22}$$

is an integer, how many digits long is it?

(Austin Math Circle Practice MATHCOUNTS 2022, Sprint #27)

Signature Problem. In cyclic pentagon $AKHIL$, U is the intersection of diagonals AH and KI, and M is the intersection of UL and AI. Furthermore, $[AUK]+[HUI] = 5$, $HU = 4 \cdot KU$, and $UM = 2 \cdot LM$. Compute $\sqrt{[HUK] \cdot [ILAU]}$. (The brackets in this problem denote area.)

(Texas A&M High School Math Contest 2016, EF Exam #5)

What's That Sine Doing There. Compute

$$\int_0^{\sqrt{\pi}} (4x^4 + 3)\sin(x^2)\, dx$$

(Texas A&M High School Math Contest 2019, Best Student Exam #17)

Hexed. Tomas writes two three-digit positive integers (no leading zeros) in base 10 on a blackboard such that one is three times the other. Later, Ben sees the blackboard, but assumes the two integers on it are written in base 16. Under this assumption, he converts the two integers to base 10, divides the larger one by the smaller one, and writes the result on the board. What is the difference between the largest and smallest possible values of the rational number Ben writes?

(Austin Math Circle Practice MATHCOUNTS 2018, Target #8)

A Complex Configuration. Triangle ABC is inscribed in circle ω. The line containing the median from A meets ω again at M, the line containing the internal angle bisector of $\angle B$ meets ω again at R, and the line containing the altitude from C meets ω again at L. Given that quadrilateral $ARML$ is a rectangle, compute the degree measure of $\angle BAC$.

(ARML 2018, Individual #6)

Good Intuition. Leon is thinking of a quadratic polynomial $f(x)$ with integer coefficients with the property that $0 < f(1) < f(2) < f(0)$. He defines the function $g(x) = xf(x)f(f(x))$ and calculates $g(1) = 511$. Given this information, compute $g(3)$.

(Austin Math Circle Practice MATHCOUNTS 2021, Team #9)

Unorthodox. In acute triangle ABC, the feet of the altitudes from A, B, C to the opposite sides are R, S, T respectively; and the three altitudes meet at a point H. Point M is such that S is the midpoint of MR, and $MATH$ is a square. What is $\frac{AC}{BC}$? Express your answer as a common fraction in simplest radical form.

(Austin Math Circle Practice MATHCOUNTS 2022, Sprint #30)

A Sum. Evaluate

$$\sum_{n=0}^{100}(n^3 - 150n^2 - 42n - 19)\cos\left(\frac{n+1}{17}\pi\right)$$

(Home-Brewed Math 2021)

A Sum of a Sum. Evaluate

$$\sum_{j=0}^{2020}\sum_{k=0}^{2020}(-1)^{j+k}\binom{4040}{j+k}$$

(Texas HMMT November 2020 TST, General #8)

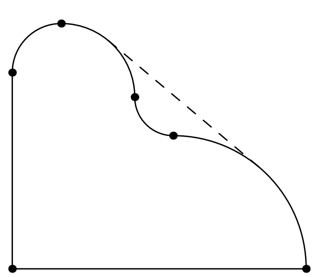
An Integral of an Integral of a Sum. Evaluate

$$\int_0^1\int_0^1\sum_{k=0}^{\infty}x^{(y+k)^2}\,dx\,dy$$

(Texas A&M High School Math Contest 2017, Best Student Exam #15)

Vietastic. It is given that the four complex roots of the polynomial $x^4+2x^3+6x^2+5x+19$ are the vertices of a rectangle in the complex plane whose sides are parallel to the coordinate axes. Compute the area of the rectangle.

The Piano Problem. Andy is measuring the grand piano shown below, which consists of four quarter-circle arcs, each of which has rational radius, and two straight line segments. First, he finds that its perimeter is $20 + 6\pi$. Then, he holds his measuring stick in the unique way such that it touches the curved part of the piano twice, and finds that the distance between the two points where it touches is 8. If the area of the piano is $p+q\pi$ where p is an integer and q is rational, compute p.



(Austin Math Circle Practice MATHCOUNTS 2021, Team #10)

Funky Quadrilateral. In convex quadrilateral $ARML$, $AR = 19$, $ML = 15$, $AL = RM$, $\angle A = 60°$, and $\angle R = 75°$. Compute the area of quadrilateral $ARML$.

(2020 Fake AIME, #14)

Happy Mother's Day. Edward and Alphonse are social distancing in the Euclidean plane. Each of them draws a circle in the plane, centered around himself, in such a way that the circles do not intersect and have non-overlapping interiors. There are four distinct lines in the plane, each of which is tangent to both circles and no two of which are parallel. These four lines divide the plane into eleven regions, three of which are bounded. One of the bounded regions is shaped like a quadrilateral and has area 20. The other two are shaped like right triangles, each having a right angle at their common vertex, and each of them has area 15. Compute the sum of the radii of Edward and Alphonse's circles.

(Texas HMMT November 2020 TST, Theme #10)

Why Did I Write This, Part One. In the addition problem

$$\begin{array}{r} GALILEO \\ GALILEO \\ GALILEO \\ GALILEO \\ GALILEO \\ +\quad FIGARO \\ \hline 20222202 \end{array}$$

each letter represents a different nonzero digit. Compute the seven-digit number $GALILEO$.

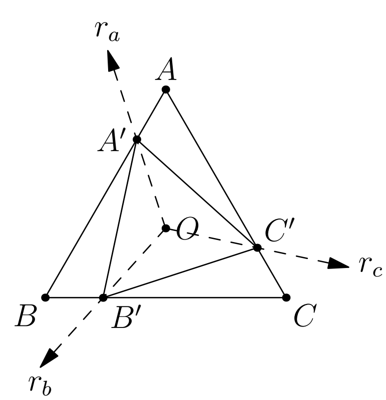
(Austin Math Circle Practice MATHCOUNTS 2021, Challenge Round A5)

Why Did I Write This, Part Two. Richard rolls a fair six-sided die repeatedly. He continues rolling until he sees six consecutive rolls, no two of which have the same value, at which point he stops. What is the expected number of times Richard rolls the die?

Why Did I Write This, Part Three. A certain tetrahedron has the property that all four of its faces are congruent. The tetrahedron has volume 2022, surface area 1200, and the sum of the lengths of its six edges is 160. If the centers of the inscribed circles of its four faces are joined to form a smaller tetrahedron, what is its volume?

(Austin Math Circle Practice MATHCOUNTS 2022, Challenge Round #10)

A Tale of Two Triangles. Equilateral triangle ABC has side length 1 and center O. Let r_a, r_b, r_c denote rays emanating from O and passing through A, B, and C respectively. We rotate these three rays counterclockwise about O through an angle of θ, and denote their respective intersections with the perimeter of ABC as A', B', and C'. If θ is chosen randomly from the interval $[0, 2\pi)$, what is the expected value of the area of $A'B'C'$?



Chaotic Calculus. Evaluate

$$\left[\int_1^\infty \prod_{k=0}^{\infty} \frac{1}{x^{\frac{2^k(5-2k)}{|5-2k|}}+1}\,dx\right]^{-1}$$