2019 Austin Math Circle Practice Mathcounts
Challenge Problems

Problem A1. The planet Buzz is orbited by three moons: Squawk, Squib, and Sump. The three moons complete their orbits in six days, eight days, and ten days, respectively. The three moons and Buzz are collinear today. In how many days will the moons next be collinear along the same line?

Problem A2. Johnny writes three consecutive integers on a board, and for each digit he writes, his sister Julie adds that digit to a running total. At the end, Julie's total is 51. Find the smallest possible value of the first of Johnny's three consecutive numbers.

Problem A3. Faith's fast clock runs x times faster than Norman's normal clock, where x is some integer. At midnight, both clocks tell the same time, and the next time they will show the same time is in $x+5$ minutes. Assuming both clocks are twelve-hour clocks, compute x.

Problem C1. A random nonnegative integer less than 10^5 is chosen such that it has exactly two 1s in its base-10 representation and all of its other digits are zero. What is the expected value of this integer? Express your answer as a common fraction.

Problem C2. Alice and Bob are playing a guessing game. Bob chooses a password from among 24 options. Then, once per minute, Alice can guess Bob's password. Alice wins when she correctly guesses Bob's password. Let M be the expected value of the number of guesses Alice will make if she chooses any of the 24 options, uniformly at random, for each guess. Let m be the expected value of the number of guesses Alice will make if she chooses uniformly at random, but only from the options she hasn't chosen yet. Compute $M+10m$.

Problem C3. Advait and Jordan each arrange the letters in their names in a random order to form a string of six letters (where no distinction is made between uppercase and lowercase letters). What is the probability that for some integer n with $1 \le n \le 6$, the n-th letter of Advait's string is the same as the n-th letter of Jordan's? Express your answer as a common fraction.

Problem N1. Alice writes 9! in base-10 on a board. Bobby sees Alice's number on the board, converts it to base-6, and then he erases all of the trailing 0s. Then, Charlie sees Bobby's number on the board, and he converts it back into base-10. Compute Charlie's number.

Problem N2. Call two positive integers "friends" if they share a common factor more than 1, and call two positive integers n and m "acquaintances" if $|n-m| < 10$. Find the smallest positive integer n such that the number of positive integers that are both friends and acquaintances of n is at a maximum.

Problem N3. Lucy, who counts her age in base 10, has been transported to an alien world where everyone (except her) counts in a different integer base, b. When asked for her age, she responds cryptically by saying that her age is the only two-digit number such that the sum of the digits in her age is one-third of her real age. The aliens think that her age is $2b$ more than what it really is. Compute the value of b in base 10.

Problem G1. What is the distance, in units, from the origin to the line $y = 7x+35$? Express your answer as a common fraction in simplest radical form.

Problem G2. A large, perfectly smooth pool table is shaped like a rectangle with sides of length 2 and 4. It has six point-sized pockets, four of which are at the vertices, and the other two of which are at the midpoints of the two longer sides of the table. Rithvik places a point-sized cue ball at the midpoint of one of the table's shorter sides, and strikes it at an angle so that it begins rolling on the table. It continues rolling at a constant speed of one unit per second, bouncing off the sides of the table repeatedly, until it falls into one of the pockets. Given that the ball rolled for exactly 29 seconds before falling into a pocket, how many seconds passed after Rithvik hit the ball before it bounced off the edge of the table for the first time? Express your answer as a common fraction.

Problem G3. Triangle ABC has circumcenter O, and the midpoints of sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$ are M, N, and Y respectively. If triangle AMY is equilateral and has area 12, compute the area of quadrilateral $AYON$.