

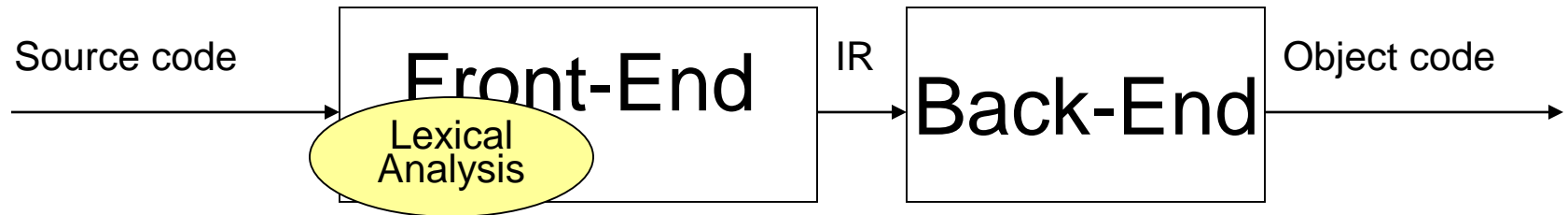
# **Compiler Design**

## **Lecture 3: Lexical Analysis**

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based on the slides of the course book

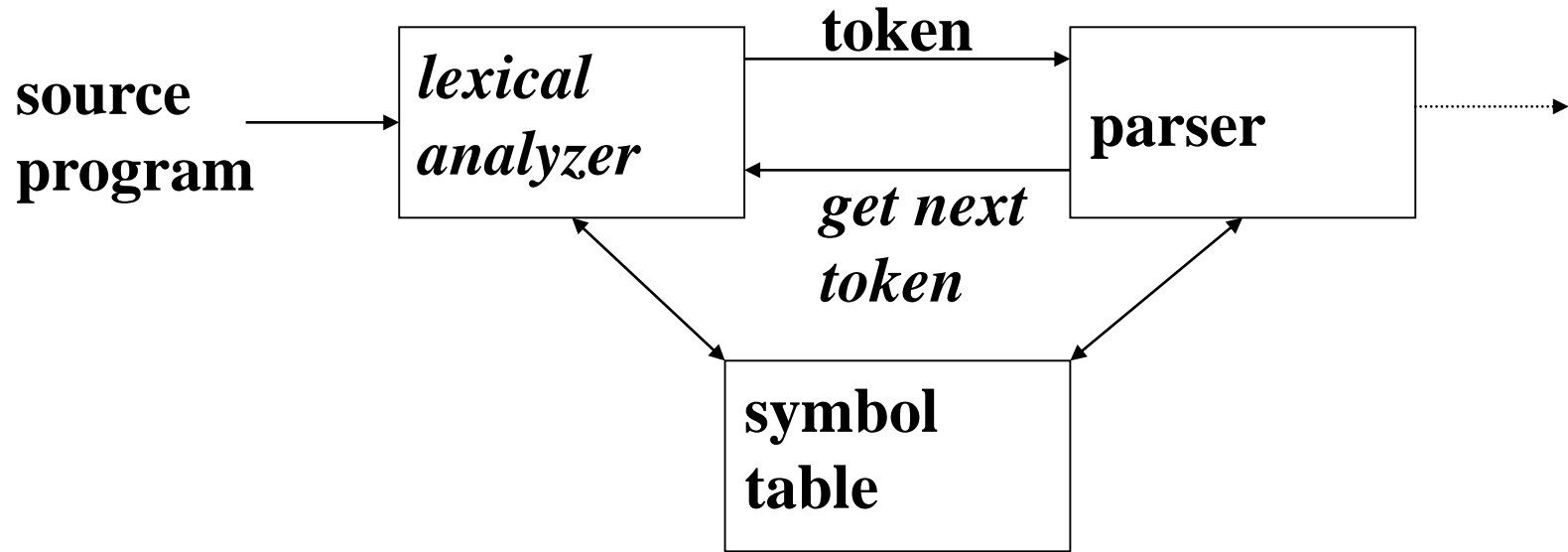
# Lexical Analyzer in Perspective



## ■ Lexical Analysis:

- Reading characters and producing sequences of tokens.

# Lexical Analyzer in Perspective



## ■ Important issue:

- What are responsibilities of each box ?
- Focus on lexical analyzer and parser

# Why to Separate Lexical Analysis and Parsing

- Simplicity of design
- Improving compiler efficiency
- Enhancing compiler portability

# Outline

- **Definition**
- Associating Lexemes with Tokens
- Matching Regular Expressions
- From RE to Automata
- Real-world Application
- Error Recovery
- Toward Automation

# General Definition

- First step in any translation:
  - Determine whether the text to be translated is well constructed in terms of the input language.
  - Syntax is specified with parts of speech - syntax checking matches parts of speech against a grammar.
- In natural languages, mapping words to part of speech is idiosyncratic.
- In formal languages, mapping words to part of speech is syntactic.
  - Reserved keywords are important

# Some Definitions

- A **token** is a pair a token name and an optional token attribute
- A **pattern** is a description of the form that the lexemes of a token may take
- A **lexeme** is a sequence of characters in the source program that matches the pattern for a token

# Why Lexical Analysis?

- We want to specify lexical patterns (to derive tokens):
  - Some parts are easy:
    - *WhiteSpace* → *blank* / *tab* / *WhiteSpace blank* / *WhiteSpace tab*
    - Keywords and operators (if, then, =, +)
    - Comments (*/\** followed by *\*/* in C, *//* in C++, *%* in latex, ...)
  - Some parts are more complex:
    - Identifiers (letter followed by - up to  $n$  - alphanumerics...)
    - Numbers

*We need a notation that could lead to an implementation!*



# Example

Token	Informal description	Sample lexemes
if	Characters i, f	if
else	Characters e, l, s, e	else
relation	< or > or <= or >= or == or !=	<=, !=
id	Letter followed by letter and digits	pi, score, D2
number	Any numeric constant	3.14159, 0, 6.02e23
literal	Anything but “ surrounded by “	“core dumped”

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

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---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

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w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

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w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
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w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

**Lexeme:** the piece of the original program from which we made the token

T\_While

**Token:** an enumerated type representing what logical entity we read out of the source code.



# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

We ignore the lexemes that are not going to be used later.

T\_While

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

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w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
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T\_While

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w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

(

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

(



# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

(

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

(

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

(

# Scanning a Source File

w	h	i	l	e		(	1	3	7		<		i	)	\n	\t	+	+	1	;
---	---	---	---	---	--	---	---	---	---	--	---	--	---	---	----	----	---	---	---	---

T\_While

(

T_IntConst
137

Some tokens can have **attributes** that store extra information about the token.

# Tokenizer

- What tokens are useful here?

```
for (int k = 0; k < myArray[5]; ++k) {  
    cout << k << endl;  
}
```

for	{	[
int	}	]
<<	<	=
(	++	Identifier
)	;	IntegerConstant

# Choosing Good Tokens

- Very much dependent on the language.
- Typically:
  - Give keywords their own tokens.
  - Give different punctuation symbols their own tokens.
  - Group lexemes representing identifiers, numeric constants, strings, etc. into their own groups.
  - Discard irrelevant information (whitespace, comments)

# Scanning Difficulties

- FORTRAN: Whitespace is irrelevant



DO 5 I = 1.25

# Scanning Difficulties

- FORTRAN: Whitespace is irrelevant

DO 5 I = 1.25

DO5I = 1.25

- Can be difficult to tell when to partition input.



# Scanning Difficulties

- C++: Nested template declarations

```
vector<vector<int>> myVector
```

# Scanning Difficulties

- C++: Nested template declarations

`(vector < (vector < (int >> myVector)))`

- Again, can be difficult to determine where to split.

# Scanning Difficulties

- PL/1: Keywords can be used as identifiers.

IF THEN THEN THEN = ELSE; ELSE ELSE = IF



# Scanning Difficulties

- PL/1: Keywords can be used as identifiers.

IF THEN THEN THEN = ELSE; ELSE ELSE = IF

- Can be difficult to determine how to label lexemes.

# Challenges in Scanning

- How do we determine which lexemes are associated with each token?
- When there are multiple ways we could scan the input, how do we know which one to pick?
- How do we address these concerns efficiently?

# Outline

- Definition
- **Associating Lexemes with Tokens**
- Matching Regular Expressions
- From RE to Automata
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# Lexemes and Tokens

- Tokens give a way to categorize lexemes by what information they provide.
- Some tokens might be associated with only a single lexeme:
  - Tokens for keywords like **if** and **while** probably only match those lexemes exactly.
- Some tokens might be associated with lots of different lexemes:
  - All variable names, all possible numbers, all possible strings, etc.

# Sets of Lexemes

- Idea: Associate a set of lexemes with each token.
- We might associate the “number” token with the set  $\{ 0, 1, 2, \dots, 10, 11, 12, \dots \}$
- We might associate the “string” token with the set  $\{ "", "a", "b", "c", \dots \}$
- We might associate the token for the keyword **while** with the set  $\{ \text{while} \}$ .



# Lexeme construction

- How do we describe which (potentially infinite) set of lexemes is associated with each token type?

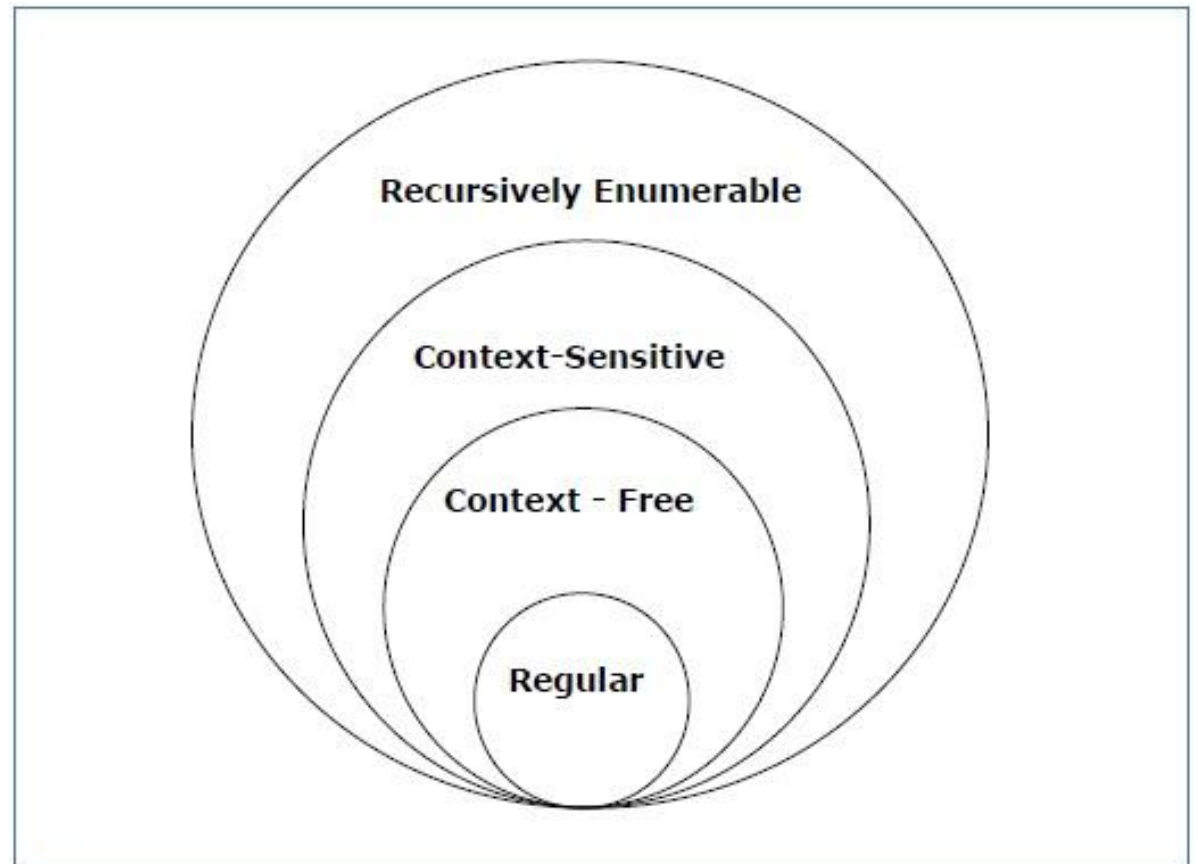
# Formal Languages

- A **formal language** is a set of strings.
- Many infinite languages have finite descriptions:
  - Define the language using an automaton.
  - Define the language using a grammar.
  - Define the language using a regular expression.
- We can use these compact descriptions of the language to define sets of strings.
- Over the course of this class, we will use all of these approaches.

# Some Definitions

- A context-free grammar,  $G$ , is a 4-tuple,  $G=(S,N,T,P)$ , where:
  - $S$ : starting symbol
  - $N$ : set of non-terminal symbols
  - $T$ : set of terminal symbols
  - $P$ : set of production rules
- A language is the set of all terminal productions of  $G$ .

# Chomsky Hierarchy



# Chomsky Hierarchy

1. Unrestricted

$$\alpha \rightarrow \beta$$

2. Context-Sensitive

$$\alpha A \beta \rightarrow \alpha \gamma \beta$$

3. Context-Free

$$A \rightarrow \gamma$$

4. Regular

$$A \rightarrow a \mid aB \text{ or}$$

$$A \rightarrow a \mid Ba$$

# Chomsky Hierarchy

Unrestricted	Turing machine
Context-Sensitive	Linear-bounded non-deterministic Turing machine
Context-Free	Non-deterministic pushdown automaton
Regular	Finite state automaton

# Regular Expressions

- **Regular expressions** are a family of descriptions that can be used to **capture certain languages** (the *regular languages*).
- Often provide a compact and human-readable description of the language.
- Used as the basis for numerous software systems, including the **flex** tool we will use in this course.

# Language & Regular Expressions

- A Regular expression is a set of rules / techniques for constructing sequences of symbols (strings) from an alphabet.
- Let  $\Sigma$  be an alphabet,  $r$  a regular expression. Then  $L(r)$  is the language that is characterized by the rules of  $r$ .



# Atomic Regular Expressions

- The regular expressions we will use in this course begin with two simple building blocks.
- The symbol  $\epsilon$  is a regular expression matches the empty string.
- For any symbol  $a$ , the symbol  $a$  is a regular expression that just matches  $a$ .

# Regular Expressions

## ■ Regular Expression (RE) (over a vocabulary $V$ ):

- $\varepsilon$  is a RE denoting the empty set  $\{\varepsilon\}$ .
- If  $a \in V$  then  $a$  is a RE denoting  $\{a\}$ .
- If  $r_1, r_2$  are REs then:
  - $r_1^*$  denotes zero or more occurrences of  $r_1$ ;
  - $r_1 r_2$  denotes concatenation;
  - $r_1 / r_2$  denotes either  $r_1$  or  $r_2$ ;
- Or more compact as:
  - $[a-d]$  for  $a / b / c / d$ ;
  - $r^+$  for  $rr^*$ ;
  - $r?$  for  $r / \varepsilon$

# Operator Precedence

- Regular expression operator precedence is

$(R)$

$R^*$

$R_1R_2$

$R_1 \mid R_2$

- Example: how to parse  **$ab^*c|d$**  ?

# Operator Precedence

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- Example: how to parse  **$ab^*c|d$**  ?

**$((a(b^*))c)|d$**

# Formal Language Operations

OPERATION	DEFINITION
union of L and M written $L \cup M$	$L \cup M = \{s \mid s \text{ is in } L \text{ or } s \text{ is in } M\}$
concatenation of L and M written $LM$	$LM = \{st \mid s \text{ is in } L \text{ and } t \text{ is in } M\}$
Kleene closure of L written $L^*$	$L^* = \bigcup_{i=0}^{\infty} L^i$ <p><math>L^*</math> denotes “zero or more concatenations of “L”</p>
positive closure of L written $L^+$	$L^+ = \bigcup_{i=1}^{\infty} L^i$ <p><math>L^+</math> denotes “one or more concatenations of “L”</p>

# Formal Language Operations Examples

$$L = \{A, B, C, D\} \quad D = \{1, 2, 3\}$$

$$L \cup D =$$

$$LD =$$

$$L^2 =$$

# Formal Language Operations Examples

$$L = \{A, B, C, D\} \quad D = \{1, 2, 3\}$$

$$L \cup D = \{A, B, C, D, 1, 2, 3\}$$

$$LD = \{A1, A2, A3, B1, B2, B3, C1, C2, C3, D1, D2, D3\}$$

$$L^2 = \{AA, AB, AC, AD, BA, BB, BC, BD, CA, \dots DD\}$$

# Formal Language Operations Examples

$$L = \{A, B, C, D\} \quad D = \{1, 2, 3\}$$

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$$L^2 = \{AA, AB, AC, AD, BA, BB, BC, BD, CA, \dots DD\}$$

$$L^4 = L^2 L^2 = ??$$

$$L^* = \{ \text{All possible strings of } L \text{ plus } \epsilon \}$$

$$L^+ = L^* - \epsilon$$

$$L(L \cup D) = ??$$

$$L(L \cup D)^* = ??$$



# Examples

- $integer \rightarrow (+ \mid - \mid \varepsilon) (0 \mid 1 \mid 2 \mid \dots \mid 9)^+$
- $integer \rightarrow (+ \mid - \mid \varepsilon) (0 \mid (1 \mid 2 \mid \dots \mid 9) (0 \mid 1 \mid 2 \mid \dots \mid 9)^*)$
- $decimal \rightarrow integer.(0 \mid 1 \mid 2 \mid \dots \mid 9)^*$
- $identifier \rightarrow [a-zA-Z] [a-zA-Z0-9]^*$

*Not all languages can be described by regular expressions. But, we don't care for now.*

# Regular Expressions Example

**$(0 \mid 1)^*00(0 \mid 1)^*$**

# Regular Expressions Example

$$(0 \mid 1)^*00(0 \mid 1)^*$$

- A regular expression for strings containing **00** as a substring:

**11011100101**

**0000**

**11111011110011111**

# Regular Expressions Example

**$(0|1)(0|1)(0|1)(0|1)$**

# Regular Expressions Example

**$(0|1)(0|1)(0|1)(0|1)$**

- A regular expression for strings of length exactly four:

**0000**

**1010**

**1111**

**1000**

# Regular Expressions Example

- A regular expression for strings that contain at most one zero:

# Regular Expressions Example

- A regular expression for strings that contain at most one zero:

**11110111**

**111111**

**0111**

**0**

# Regular Expressions Example

$$1^*(0 \mid \varepsilon)1^*$$

- A regular expression for strings that contain at most one zero:

**11110111**

**111111**

**0111**

**0**



# Regular Expressions Example

**$1^*0?1^*$**

- A regular expression for strings that contain at most one zero:

**11110111**

**111111**

**0111**

**0**

# Regular Expressions Example

- A regular expression for email addresses (alphabet is **a**, **@**, and **.**, where **a** represents “some letter.”):

# Regular Expressions Example

- A regular expression for email addresses (alphabet is **a**, **@**, and **.**, where **a** represents “some letter.”):

**cs143@cs.stanford.edu**

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**barack.obama@whitehouse.gov**

# Regular Expressions Example

**$aa^* (.aa^*)^* @ aa^*.aa^* (.aa^*)^*$**

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# Regular Expressions Example

**$a+ (.a+)^* @ a+ (.a+)^+$**

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# Regular Expressions Example

- A regular expression for even numbers:

# Regular Expressions Example

- A regular expression for even numbers:

**42**

**+1370**

**-3248**

**-9999912**

# Regular Expressions Example

**$(+|-)?(0|1|2|3|4|5|6|7|8|9)^*(0|2|4|6|8)$**

- A regular expression for even numbers:

**42**

**+1370**

**-3248**

**-9999912**



# Regular Expressions Example

**$(+|-)?[0123456789]^*[02468]$**

- A regular expression for even numbers:

**42**

**+1370**

**-3248**

**-9999912**

# Regular Expressions Example

**$(+|-)?[0-9]^*[02468]$**

- A regular expression for even numbers:

**42**

**+1370**

**-3248**

**-9999912**

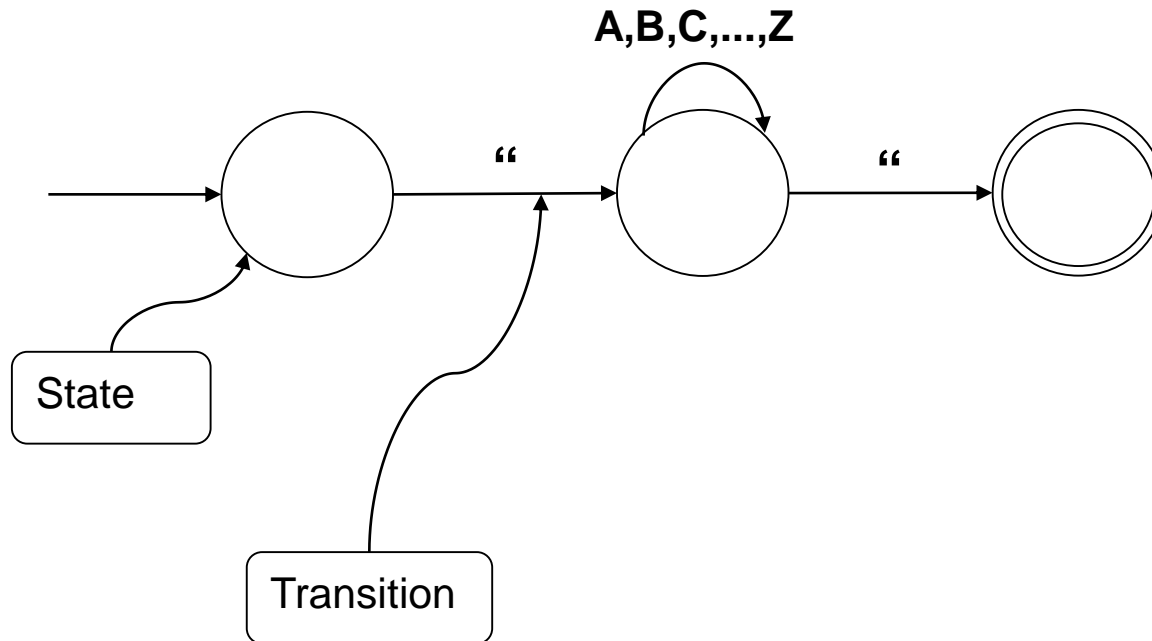
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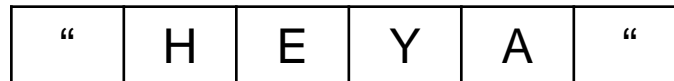
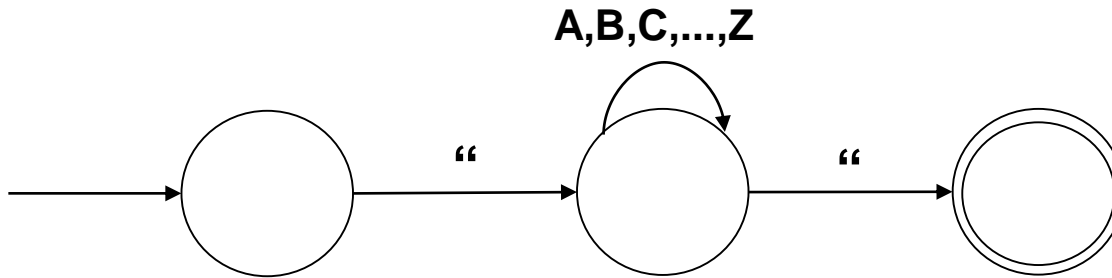
# Implementing Regular Expressions

- Regular expressions can be implemented using **finite automata**.
- There are two main kinds of finite automata:
  - NFAs (**nondeterministic** finite automata)
  - DFAs (**deterministic** finite automata)
- Automata are best explained by example...

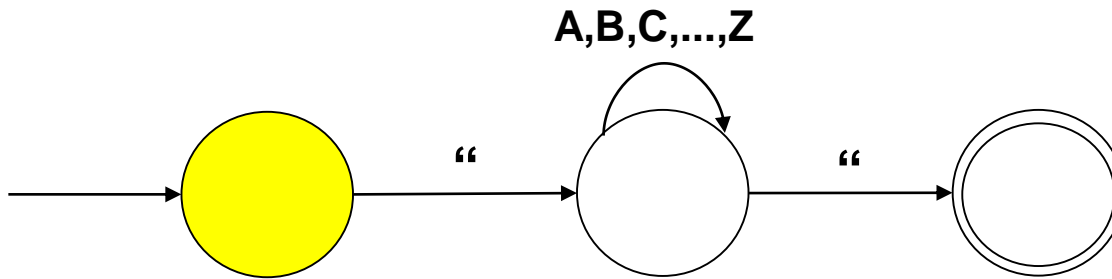
# A Simple Automaton



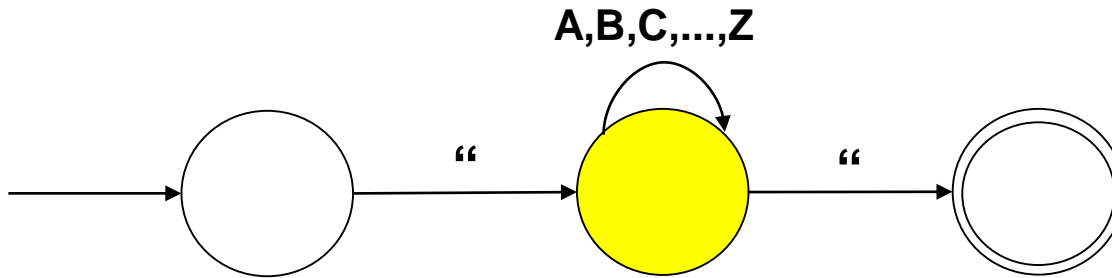
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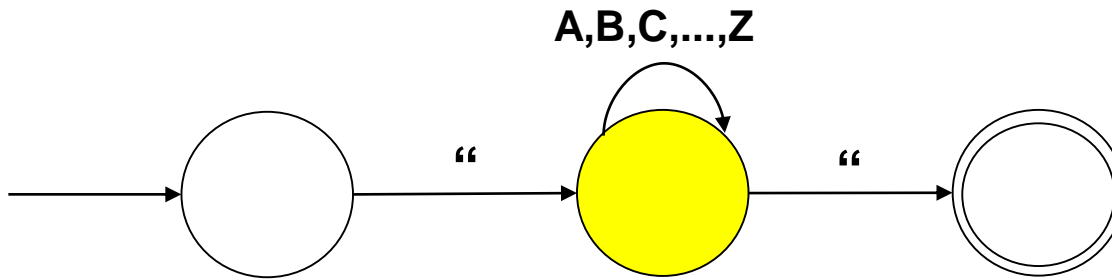


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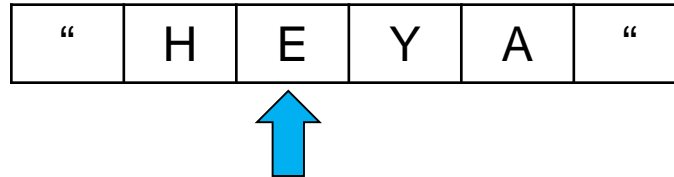
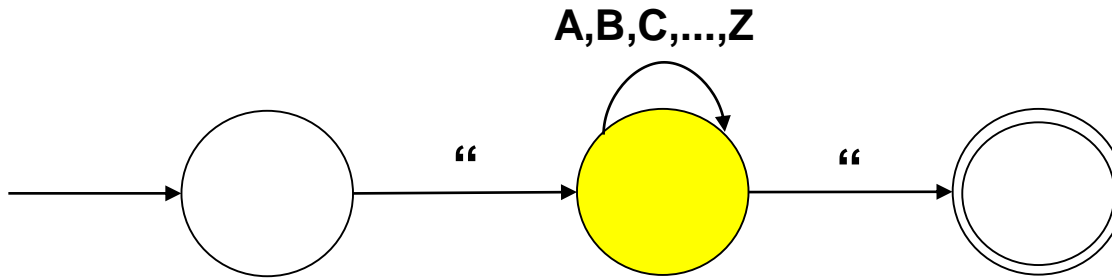




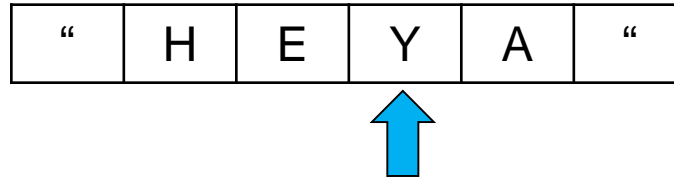
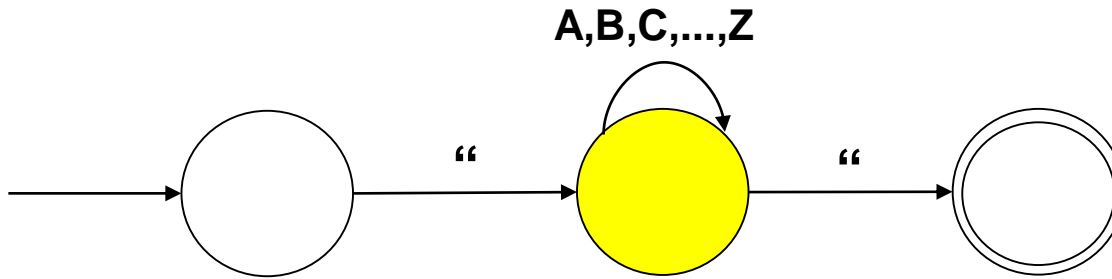
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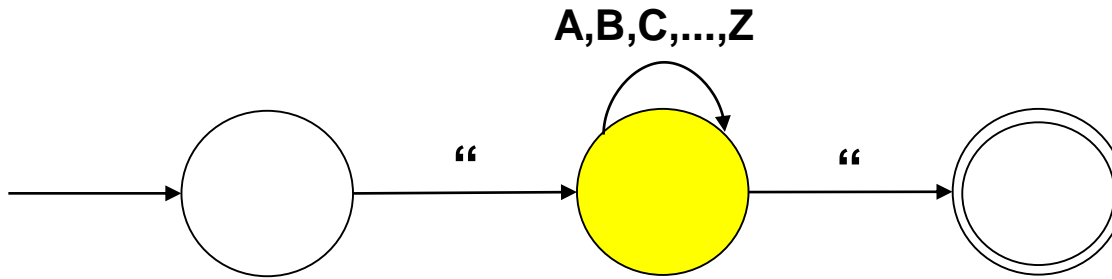
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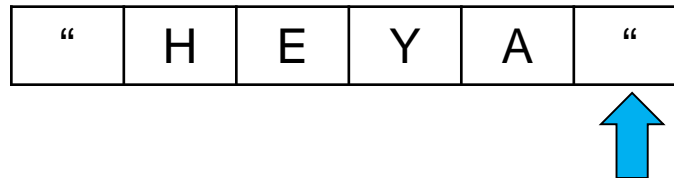
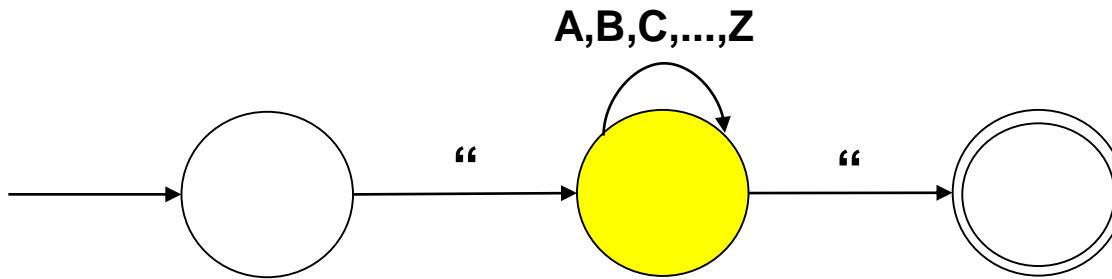
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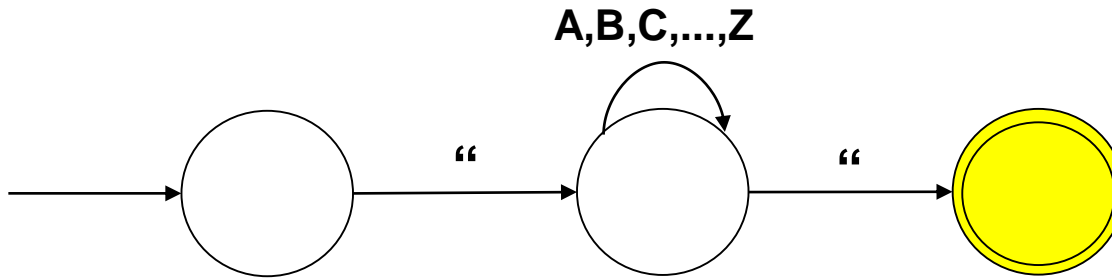
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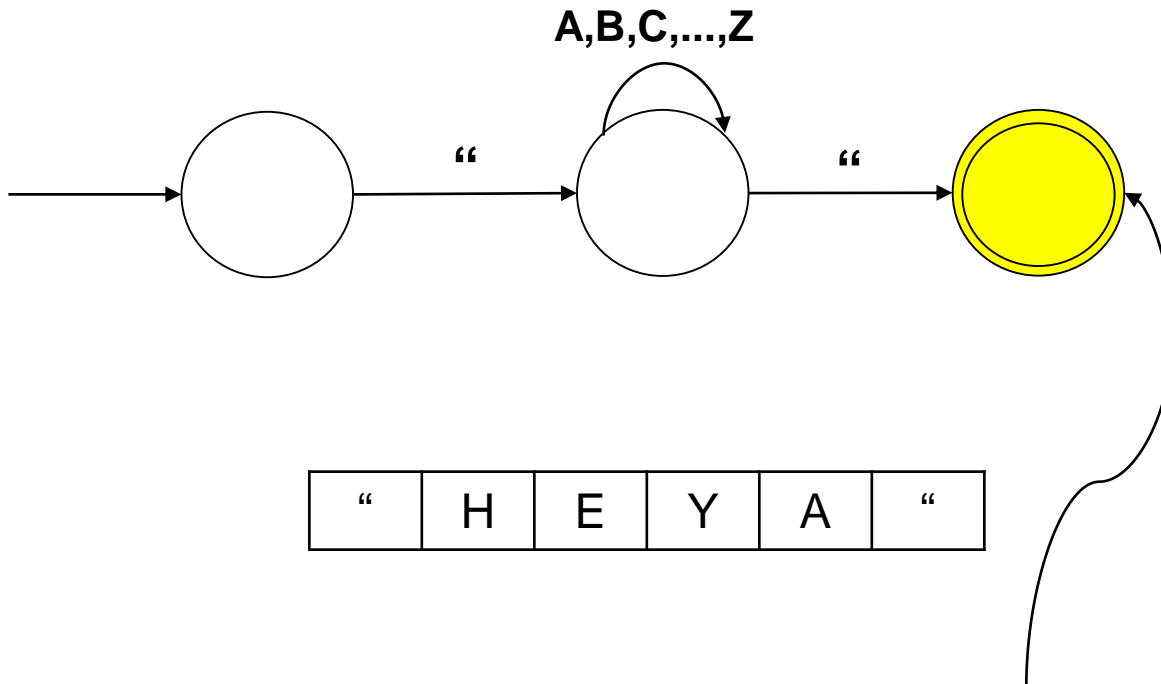
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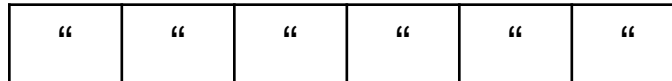
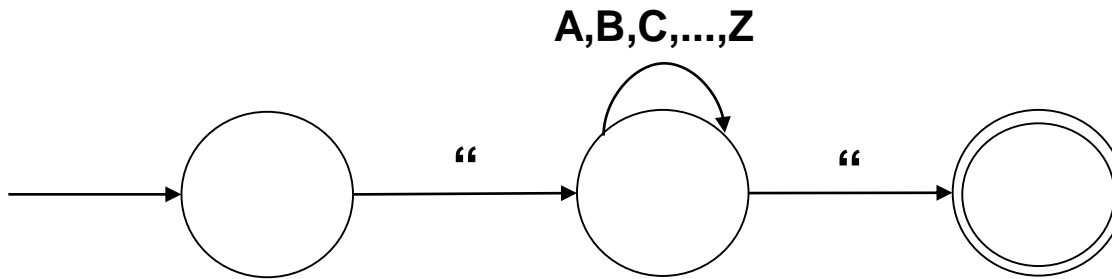


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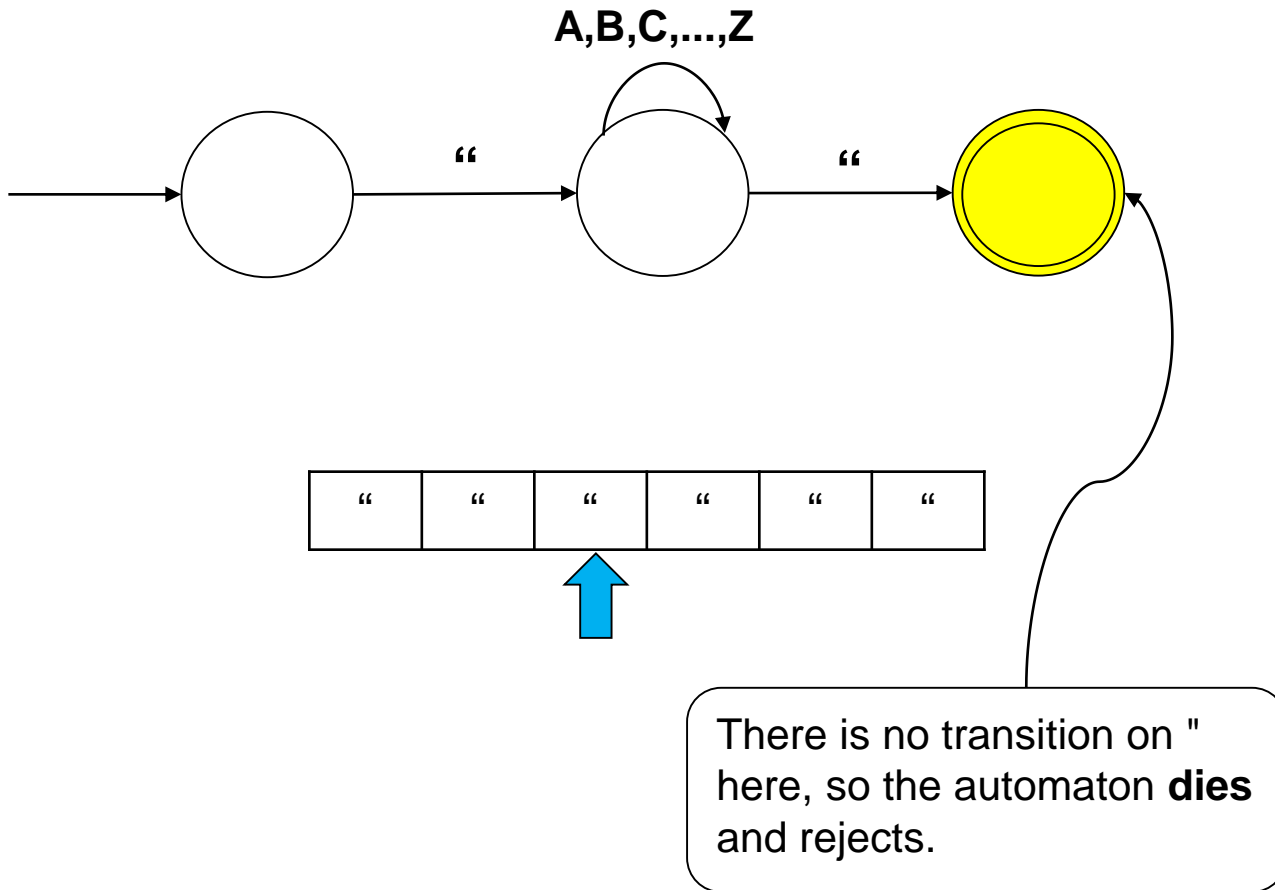
The double circle indicates that this state is an **accepting state**. The automaton accepts the string if it ends in an accepting state.

# A Simple Automaton

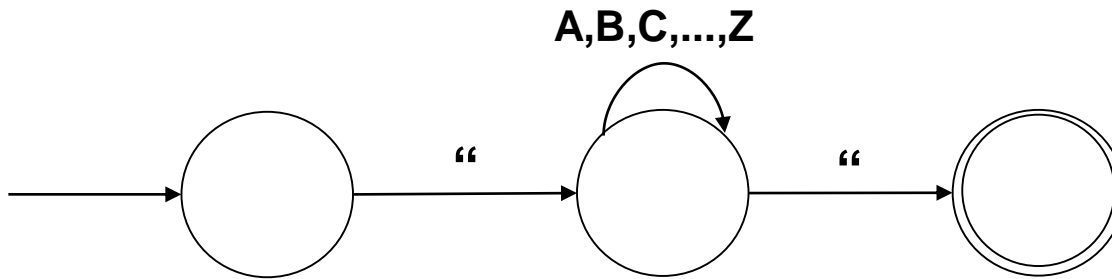




# A Simple Automaton

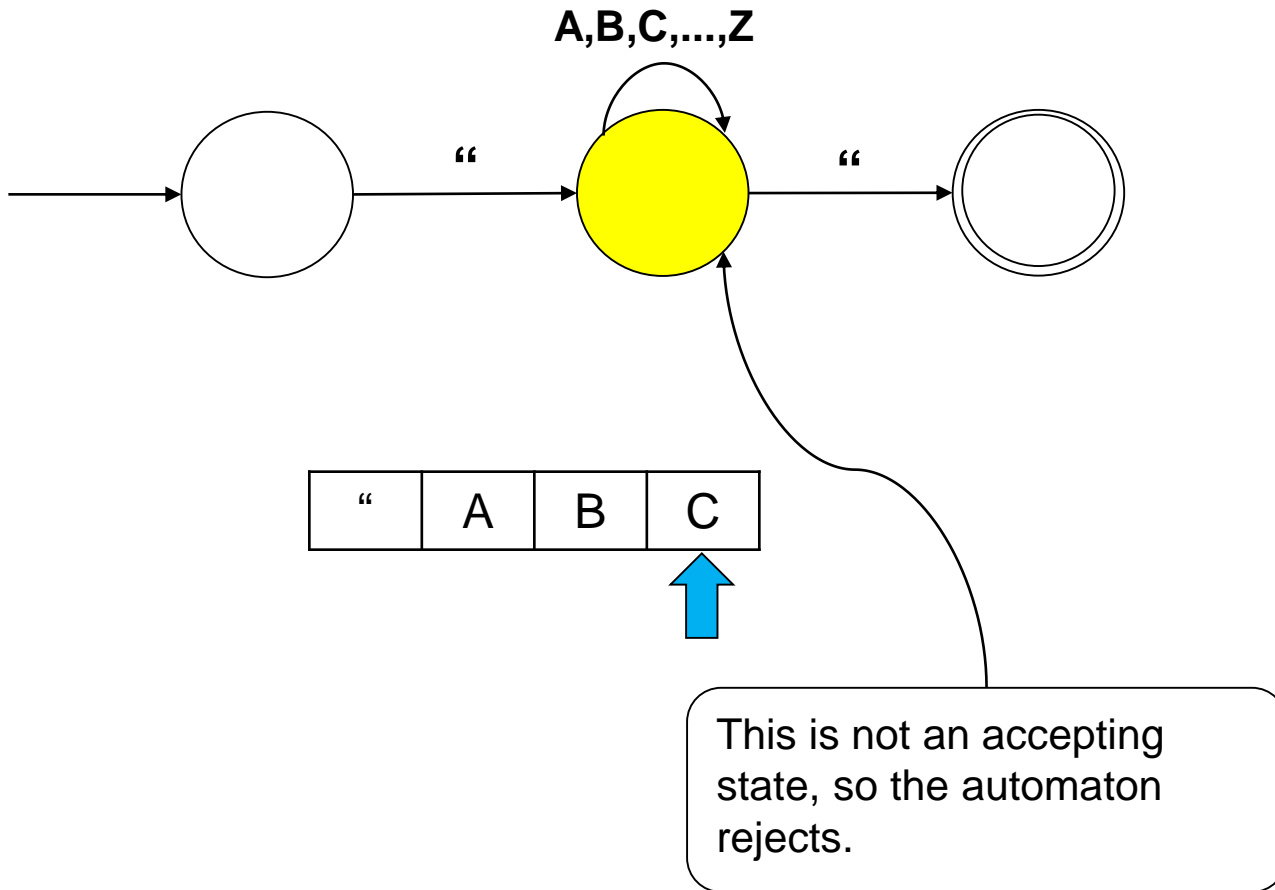


# A Simple Automaton

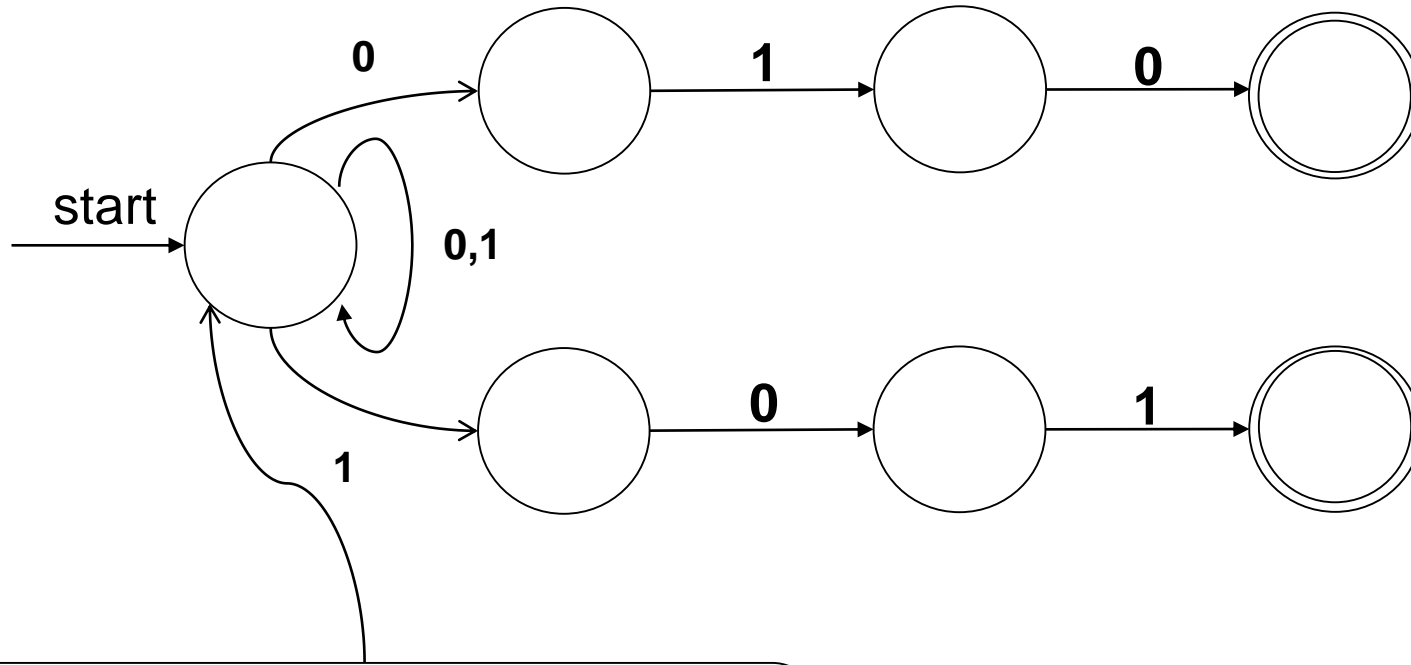


"	A	B	C
---	---	---	---

# A Simple Automaton

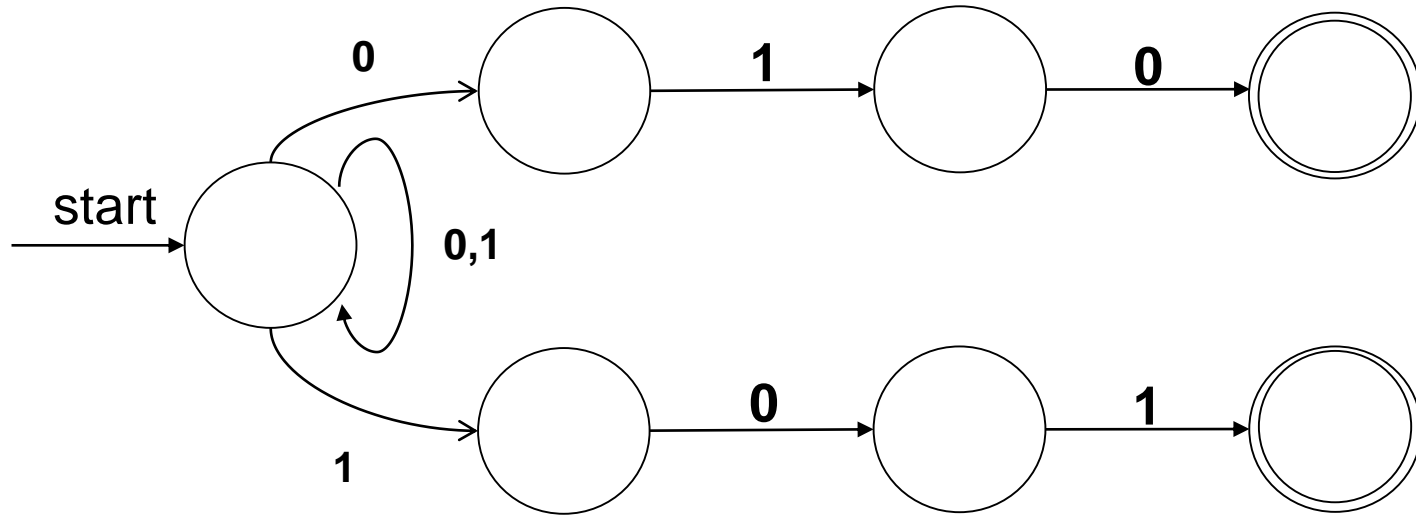


# A More Complex Automaton



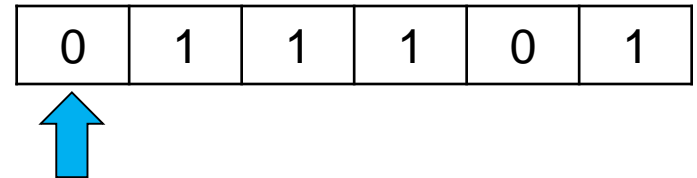
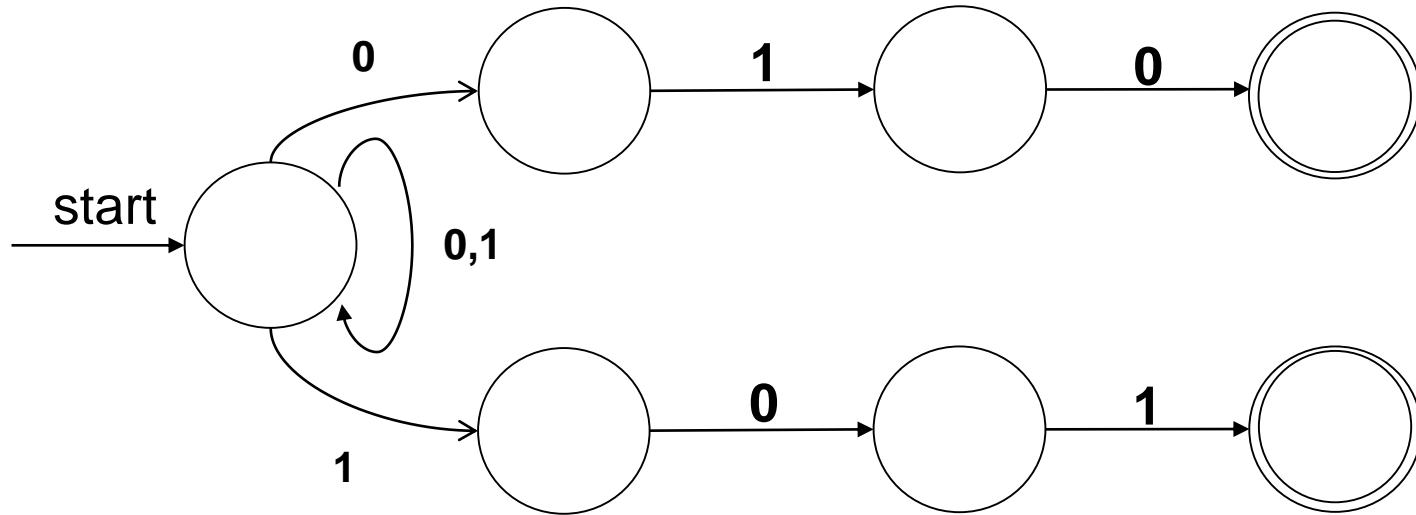
There are multiple transitions defined here on 0 and 1. If we read a 0 or 1 here, we follow both transitions and enter multiple states.

# A More Complex Automaton

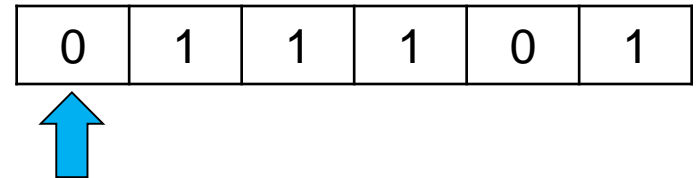
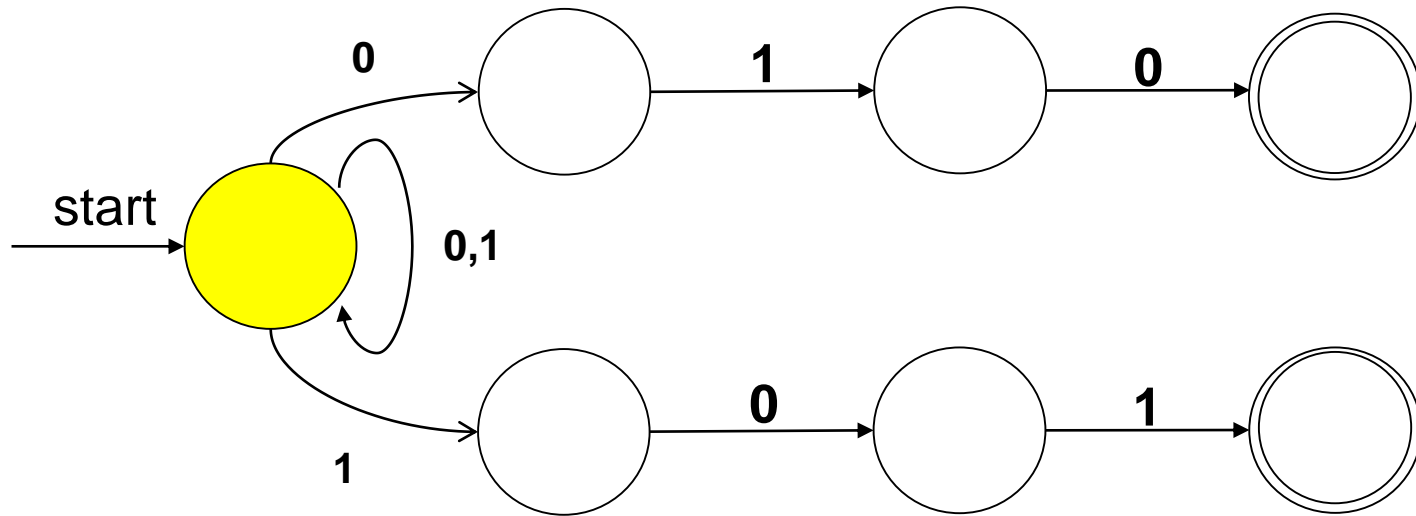


0	1	1	1	0	1
---	---	---	---	---	---

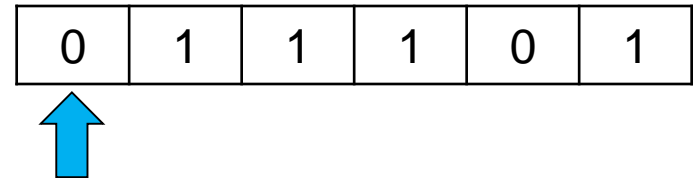
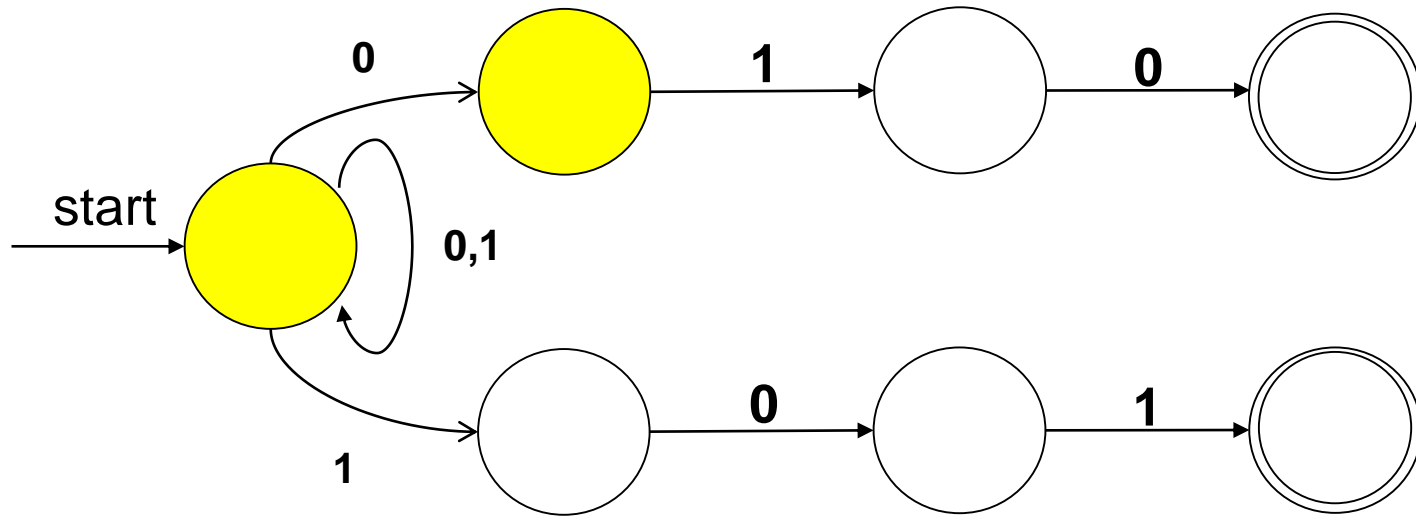
# A More Complex Automaton



# A More Complex Automaton

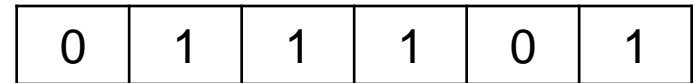
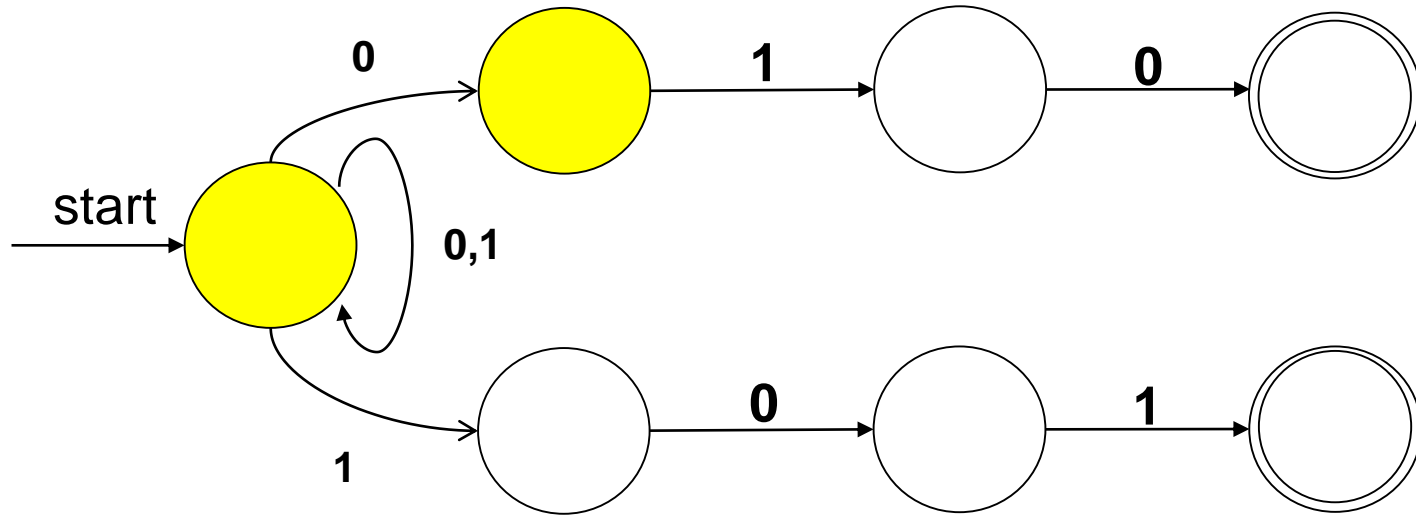


# A More Complex Automaton

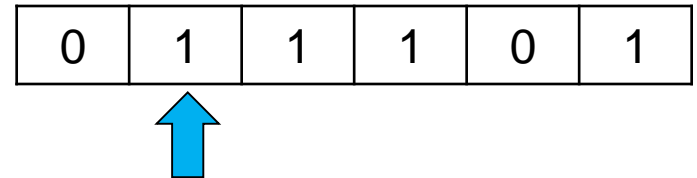
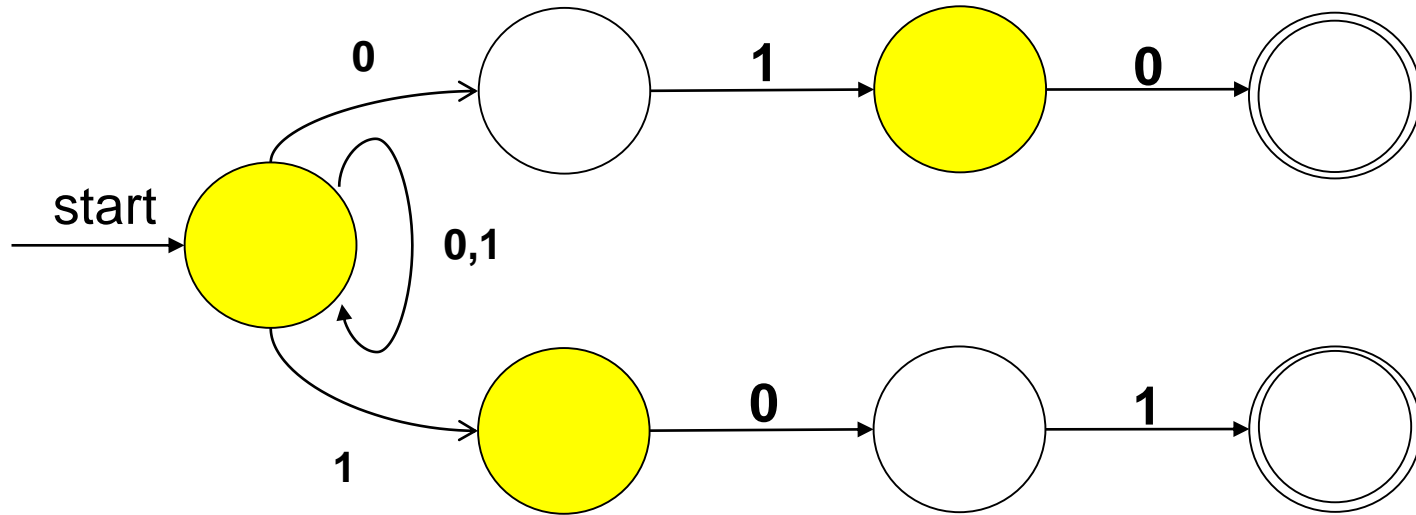




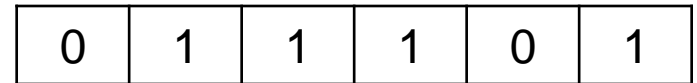
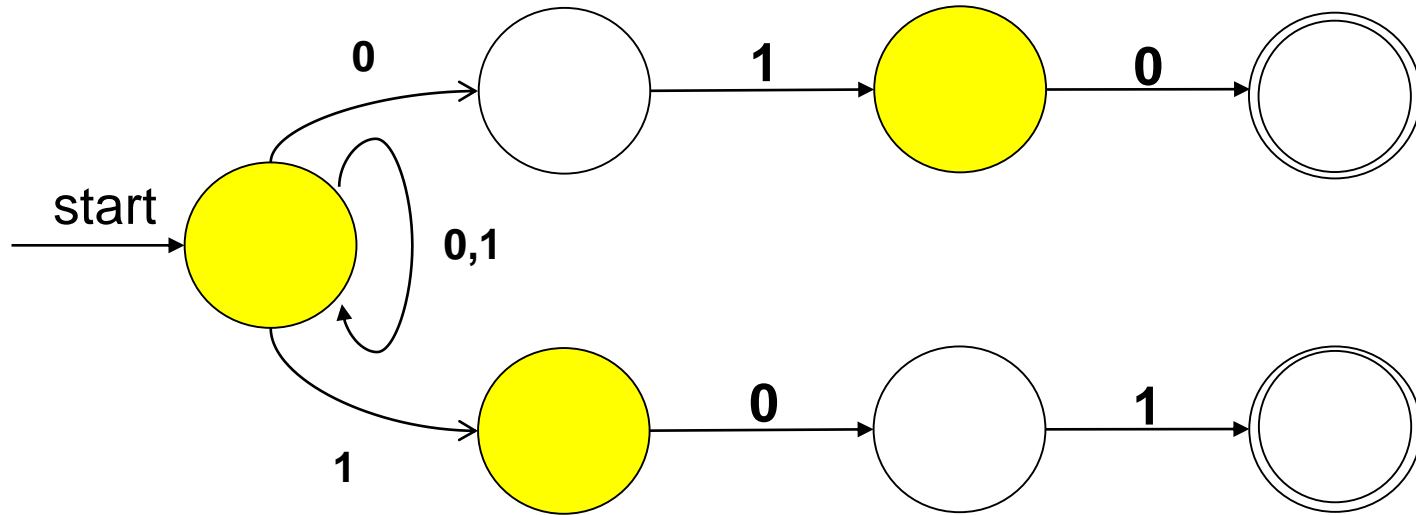
# A More Complex Automaton



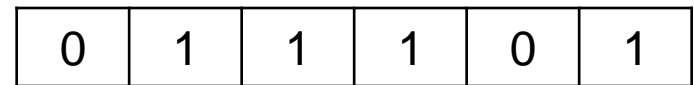
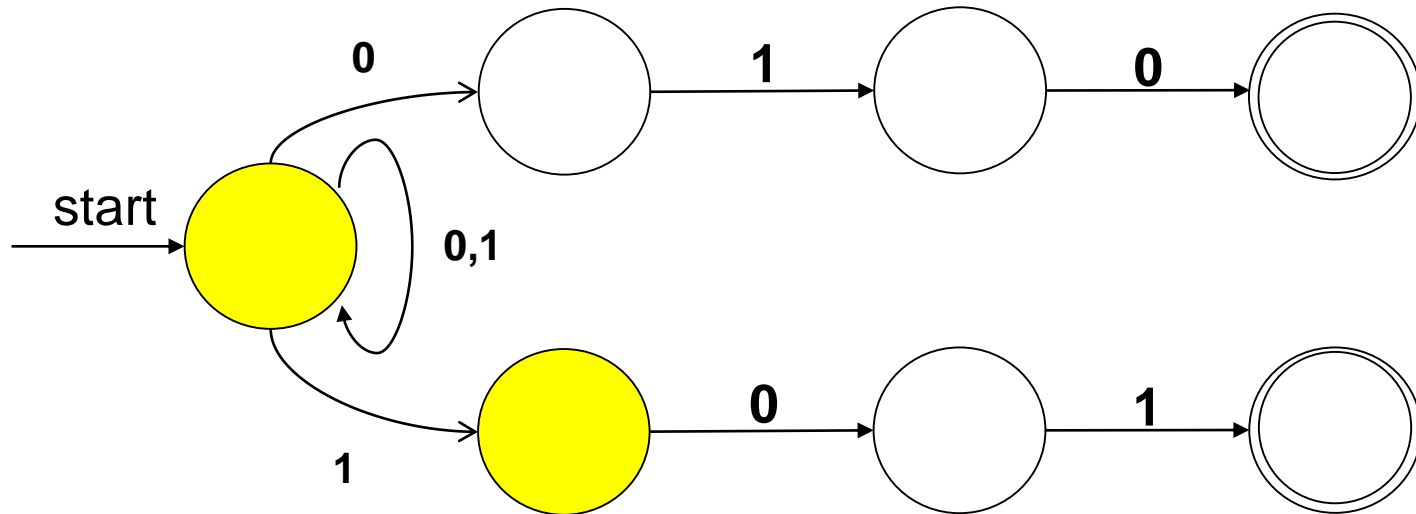
# A More Complex Automaton



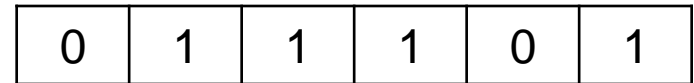
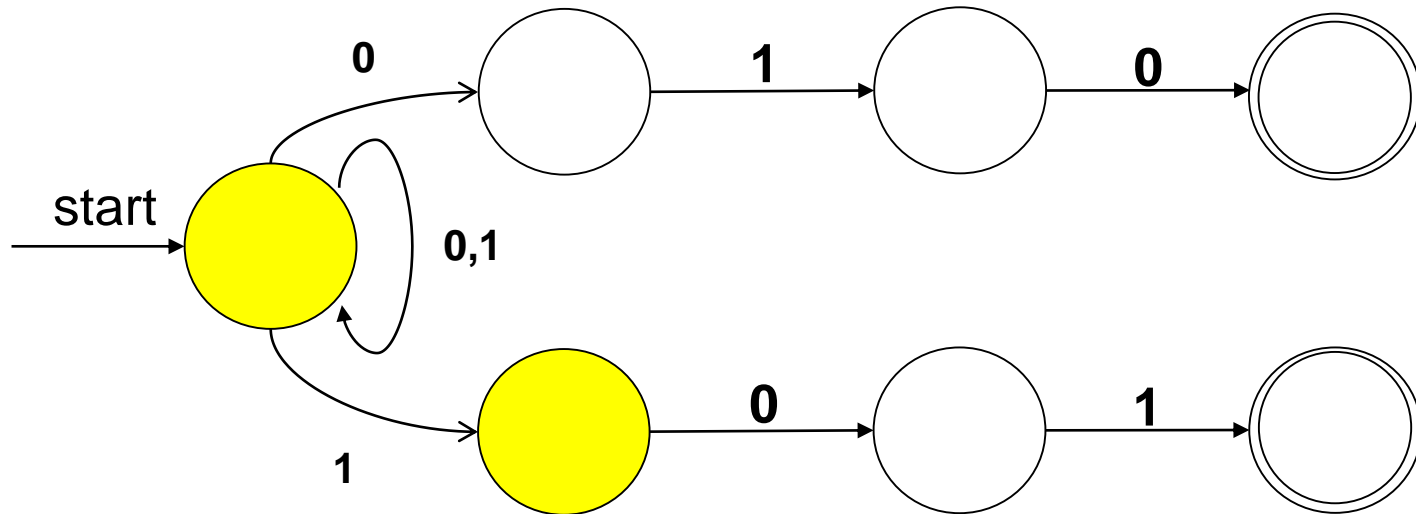
# A More Complex Automaton



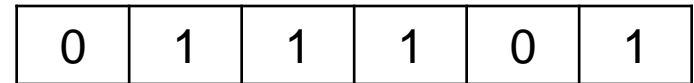
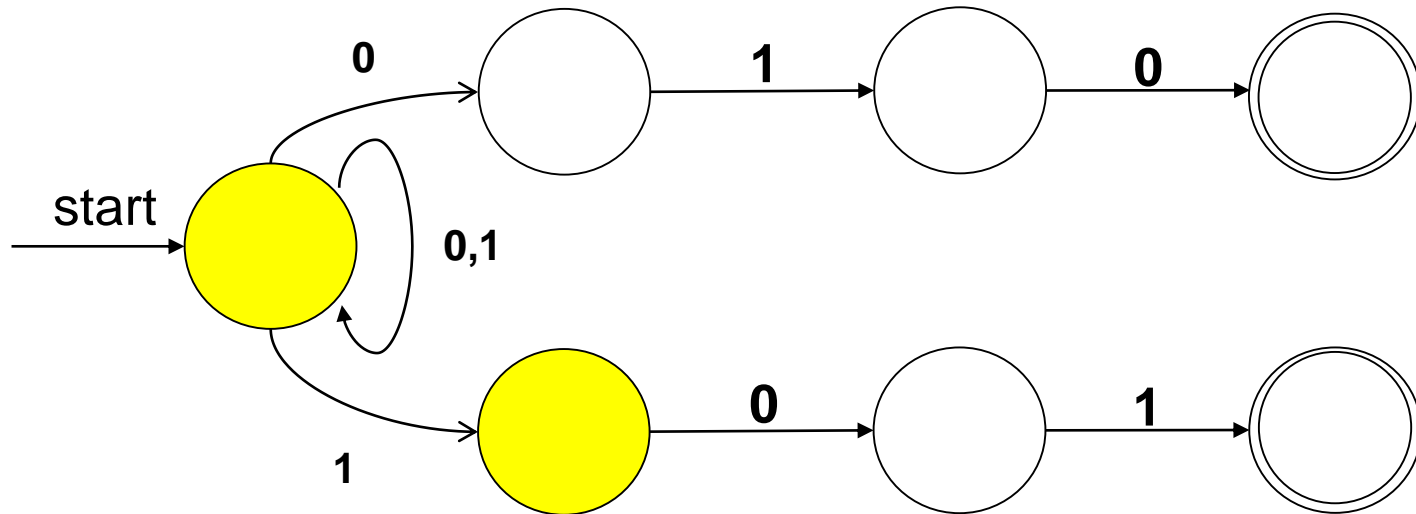
# A More Complex Automaton



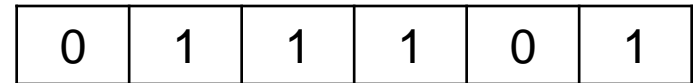
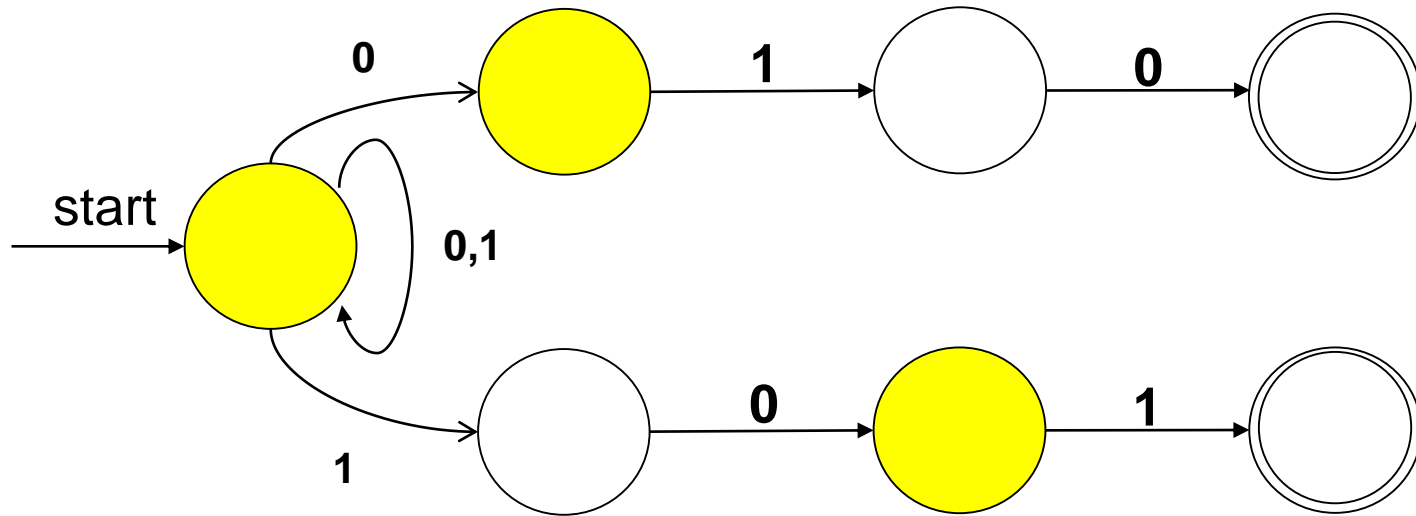
# A More Complex Automaton



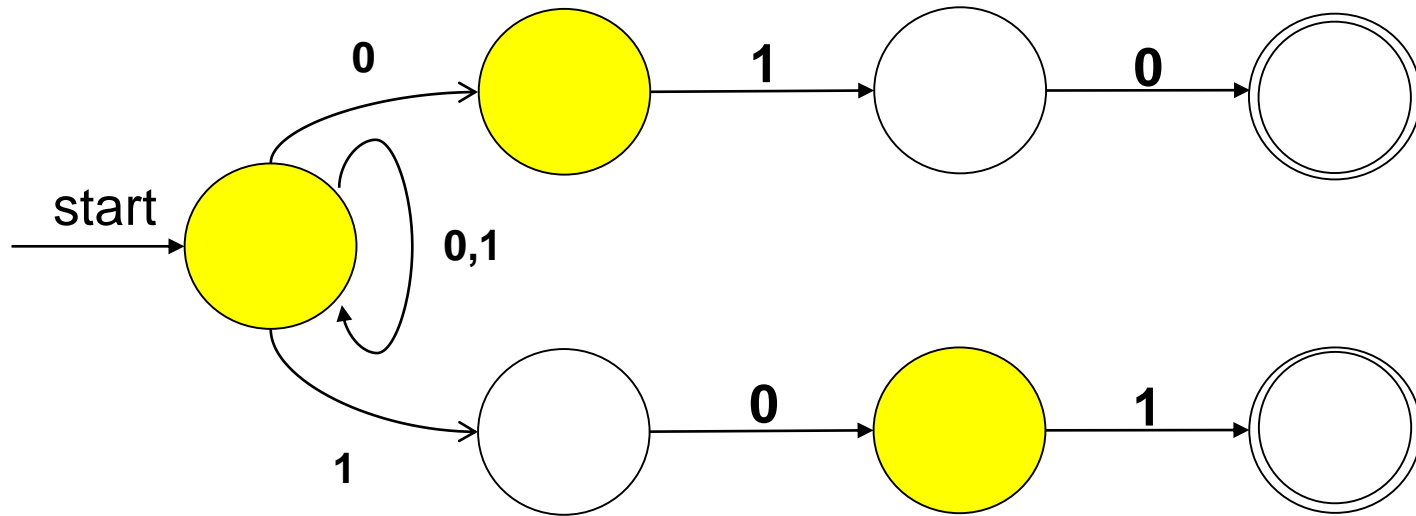
# A More Complex Automaton



# A More Complex Automaton



# A More Complex Automaton

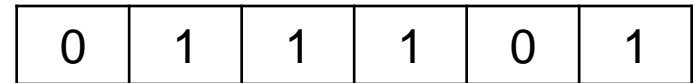
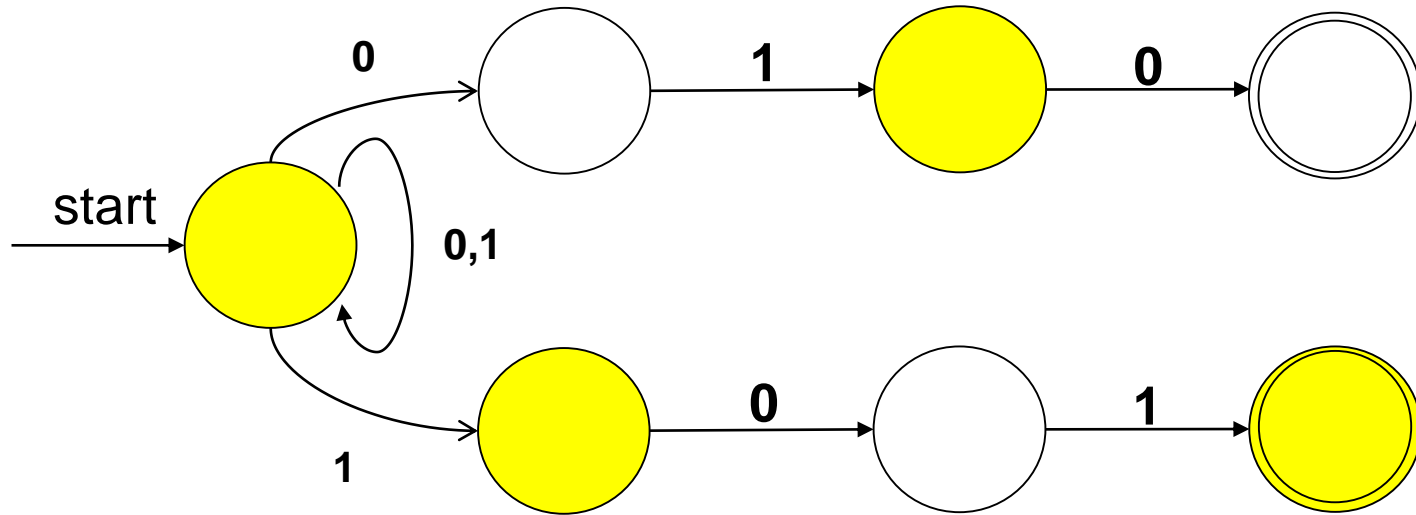


0	1	1	1	0	1
---	---	---	---	---	---

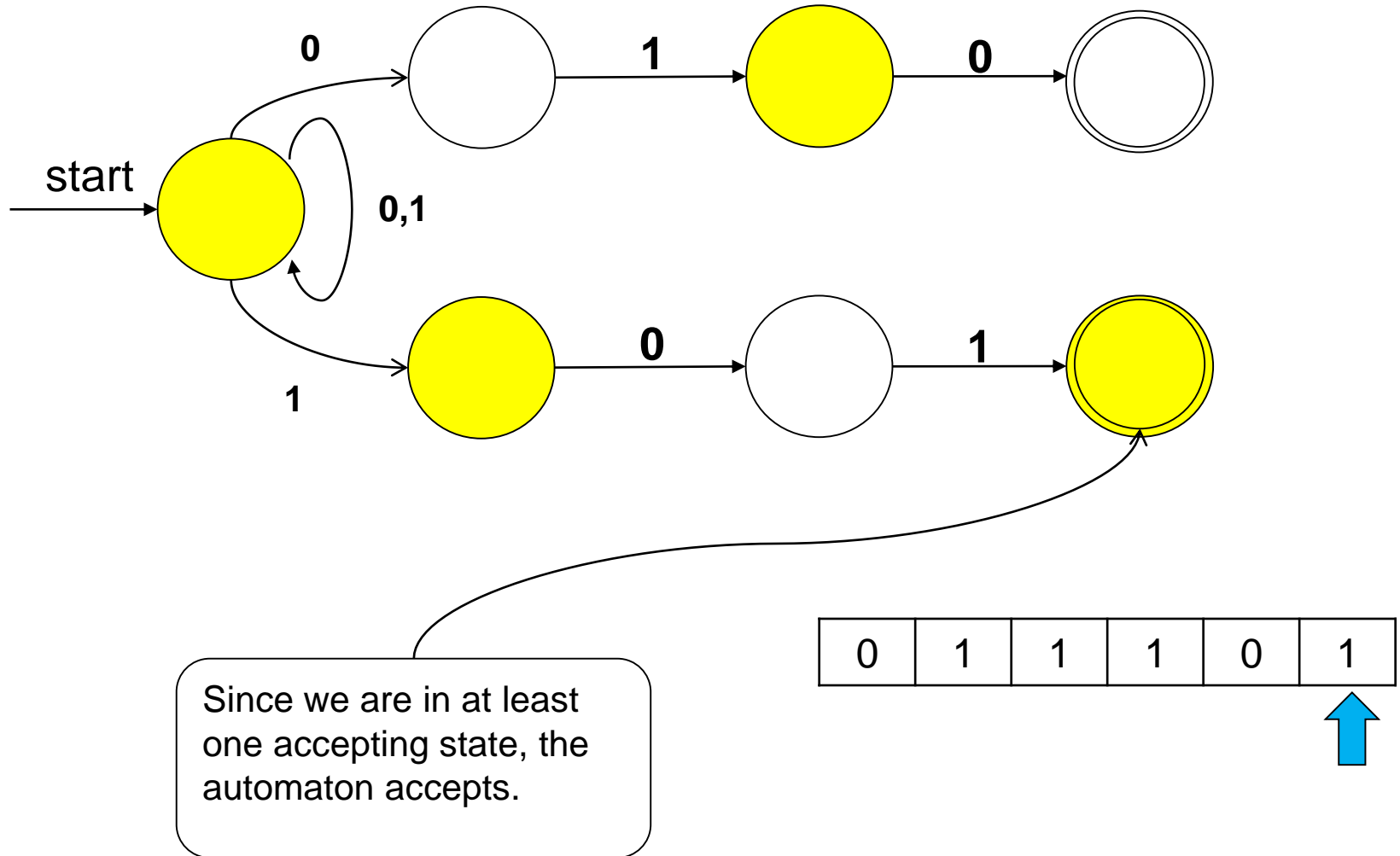




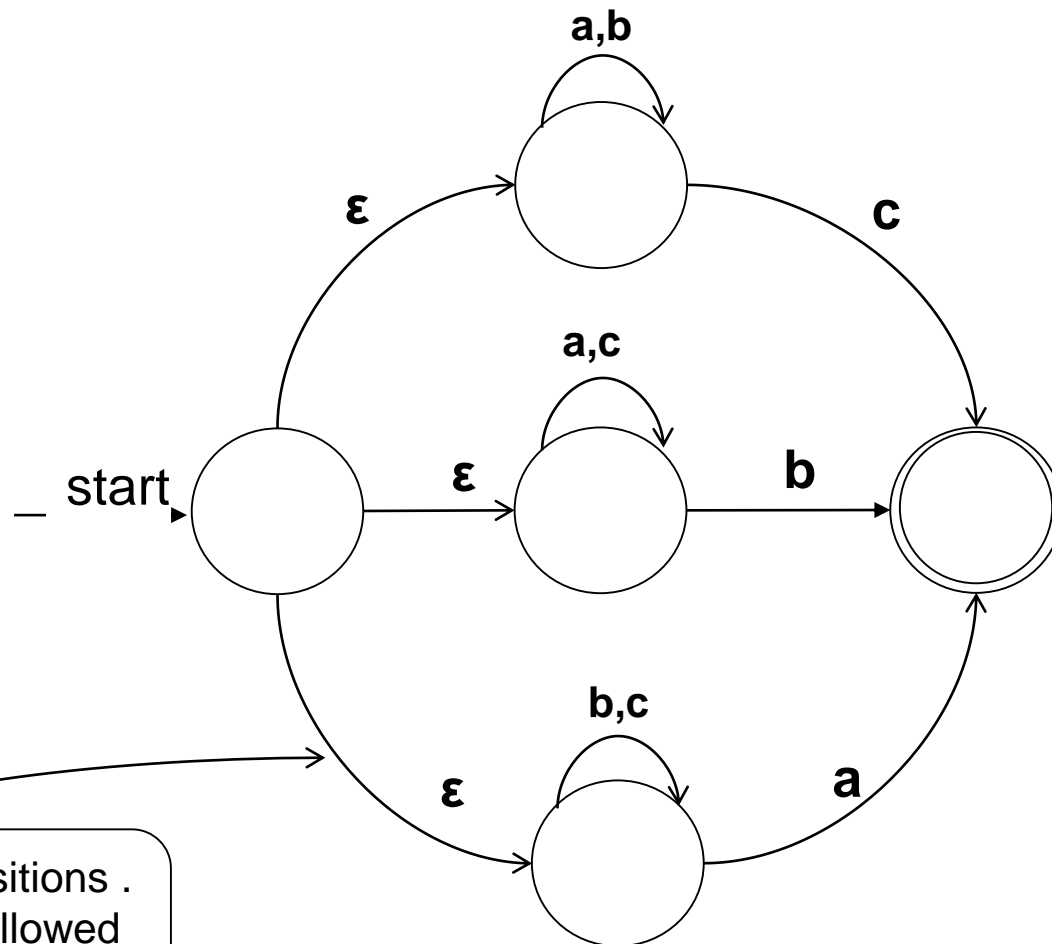
# A More Complex Automaton



# A More Complex Automaton



# An Even More Complex Automaton



These are called  $\epsilon$ -transitions .  
These transitions are followed  
automatically and without  
consuming any input.

# Simulating an NFA

- Keep track of a set of states, initially the start state and everything reachable by  $\epsilon$ -moves.
  
- For each character in the input:
  - Maintain a set of next states, initially empty.
  - For each current state:
    - Follow all transitions labeled with the current letter.
    - Add these states to the set of new states.
  - Add every state reachable by an  $\epsilon$ -move to the set of next states.

# Outline

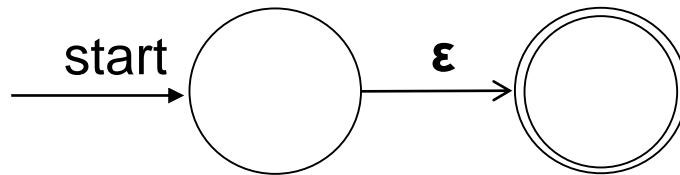
- Definition
- Associating Lexemes with Tokens
- Matching Regular Expressions
- **From RE to Automata**
- Real-world Application
- Error Recovery
- Toward Automation

# From Regular Expressions to NFAs

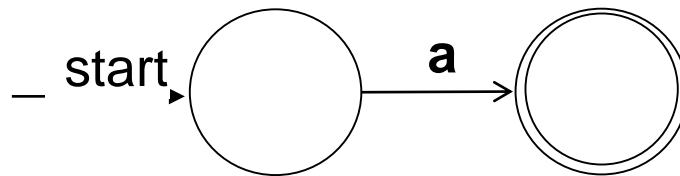
- There is an straightforward procedure from converting a regular expression to an NFA.
- Associate each regular expression with an NFA with the following properties:
  - There is exactly one accepting state.
  - There are no transitions out of the accepting state.
  - There are no transitions into the starting state.
- These restrictions are stronger than necessary, but make the construction easier.

# Basic Cases

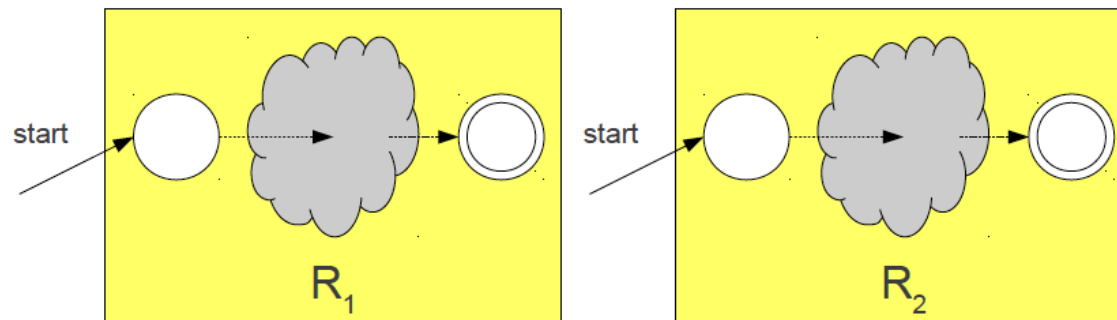
Automaton for  $\epsilon$



Automaton for single character  $a$

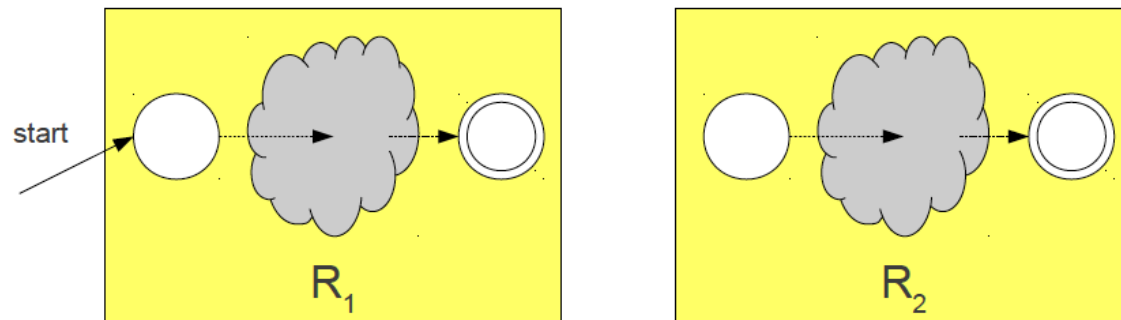


# Construction for $R_1R_2$

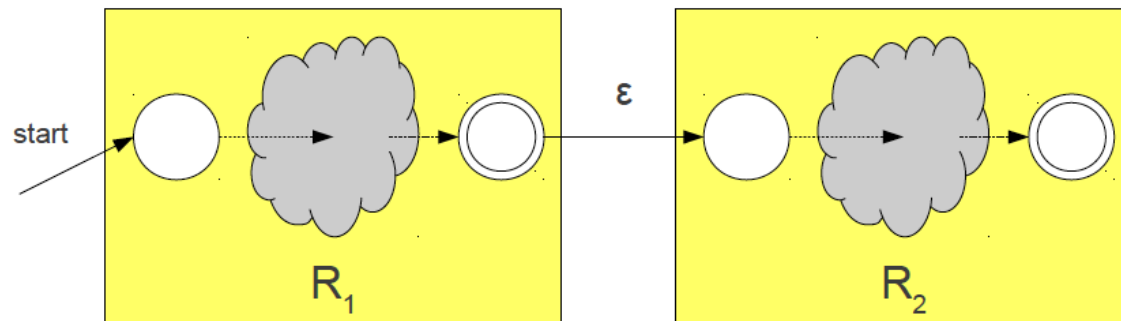




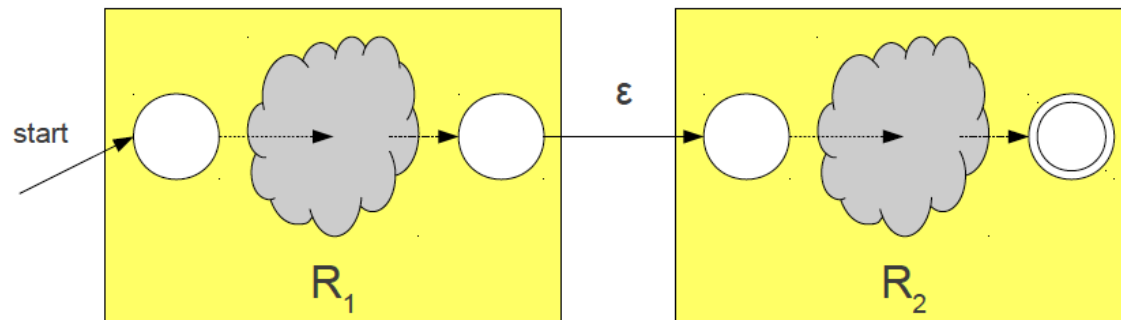
# Construction for $R_1R_2$



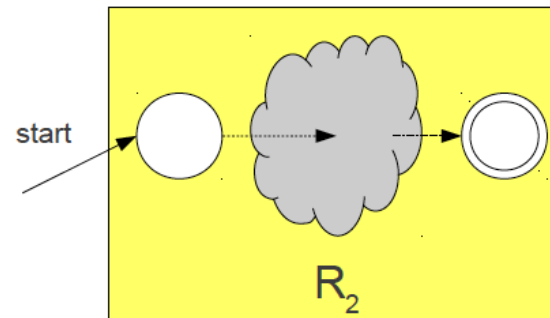
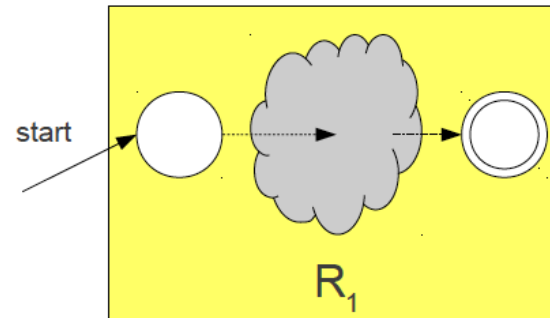
# Construction for $R_1R_2$



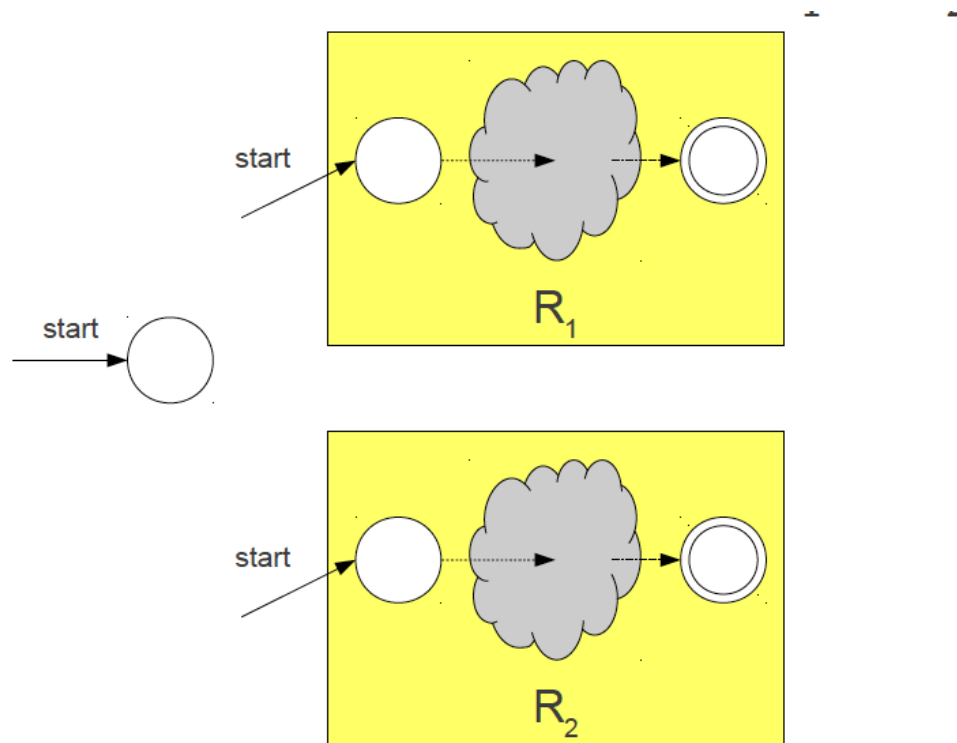
# Construction for $R_1R_2$



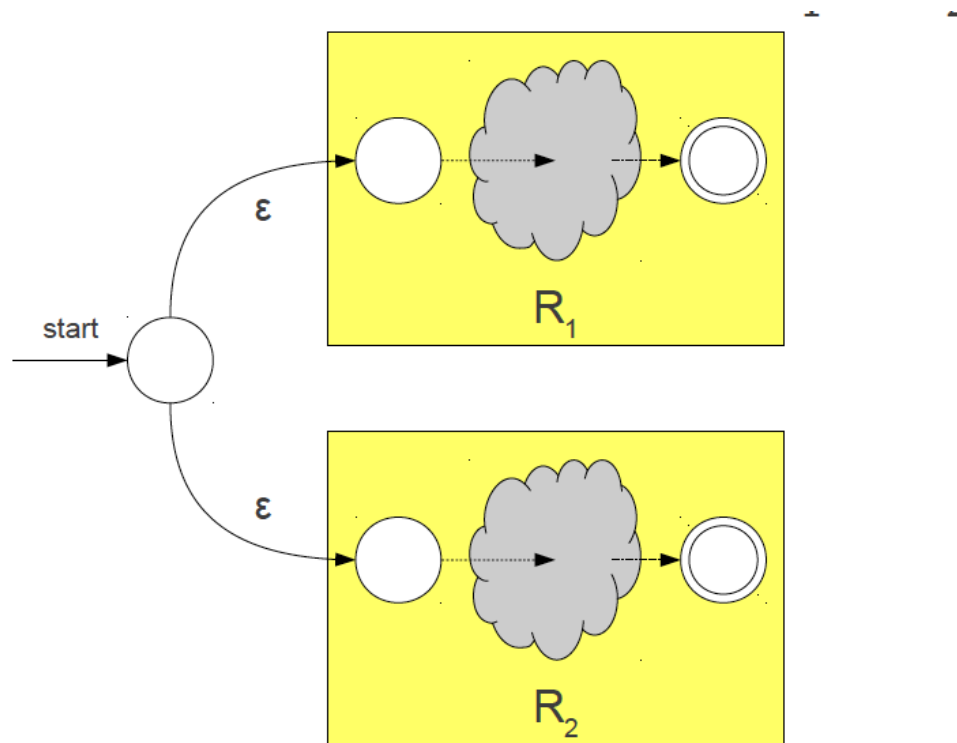
# Construction for $R_1 \mid R_2$



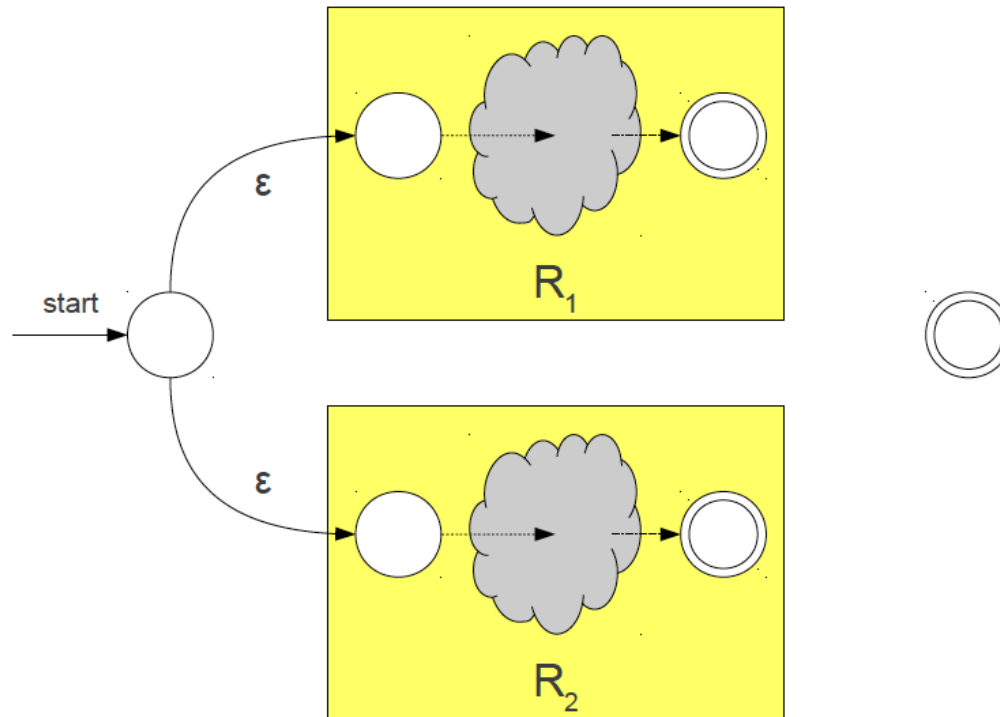
# Construction for $R_1 \mid R_2$



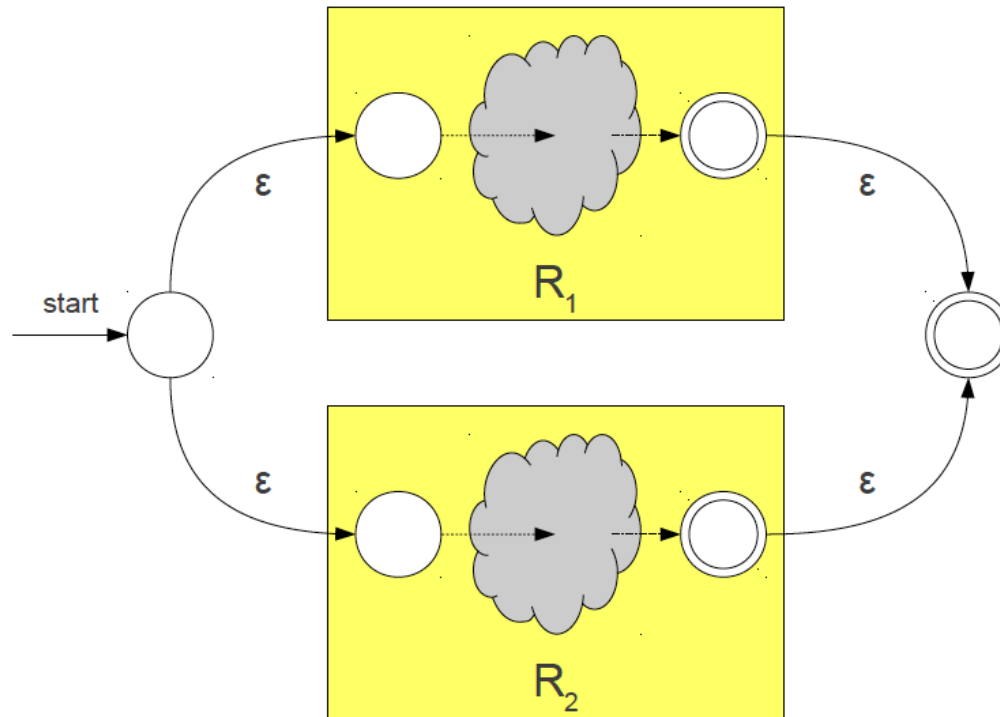
# Construction for $R_1 \mid R_2$



# Construction for $R_1 \mid R_2$

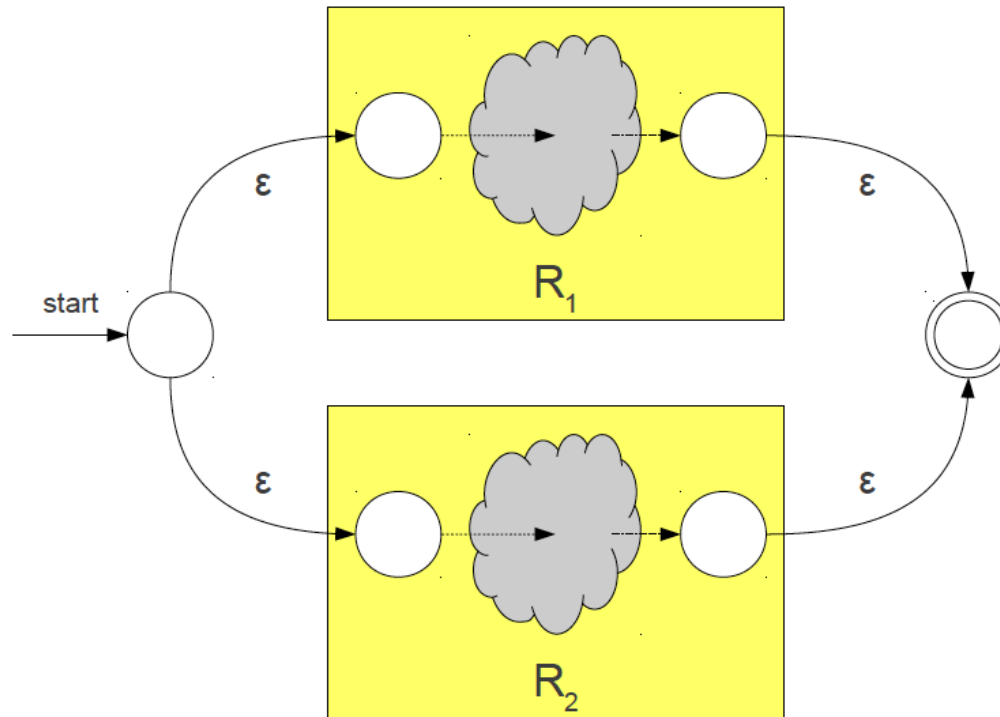


# Construction for $R_1 \mid R_2$

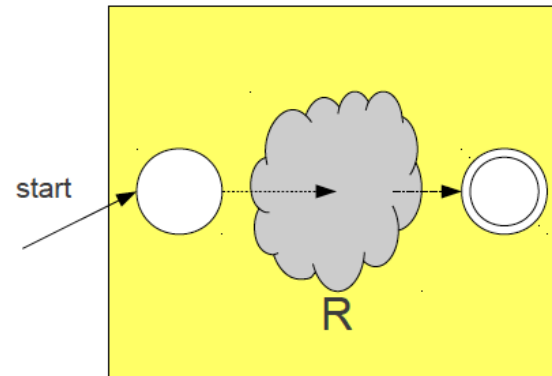




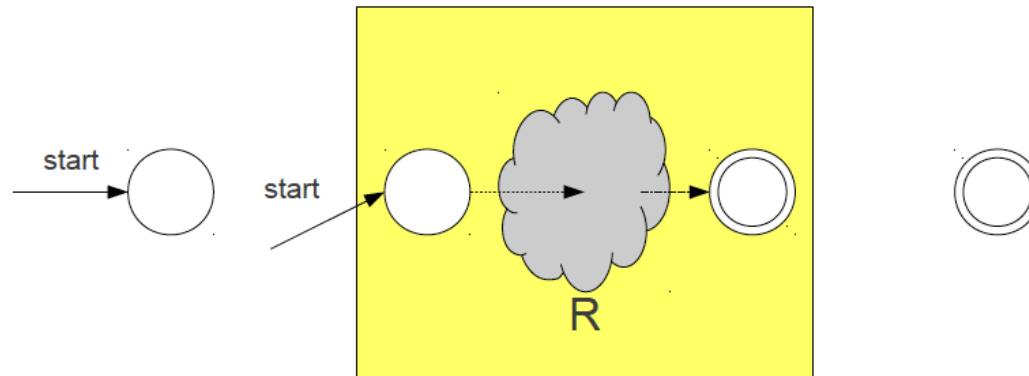
# Construction for $R_1 \mid R_2$



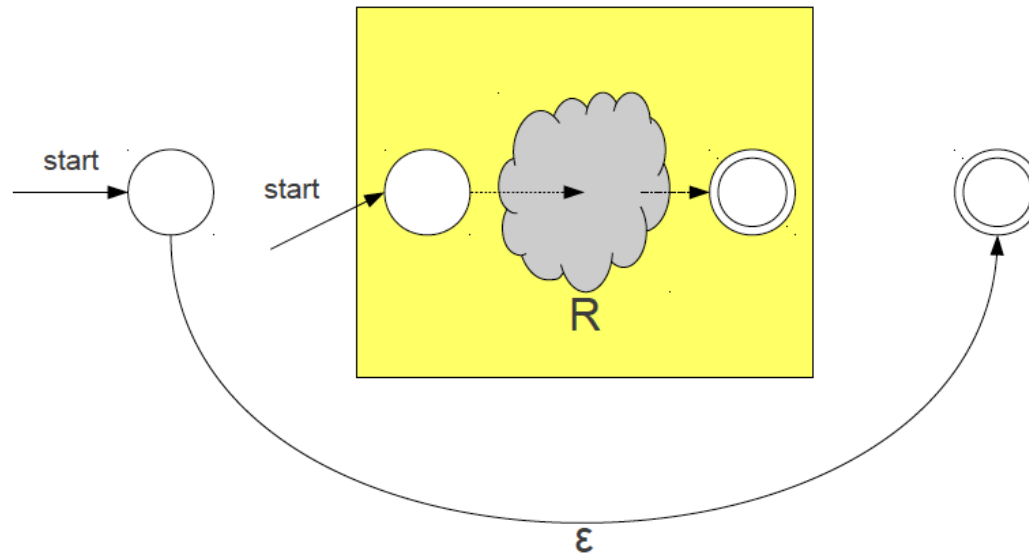
# Construction for $R^*$



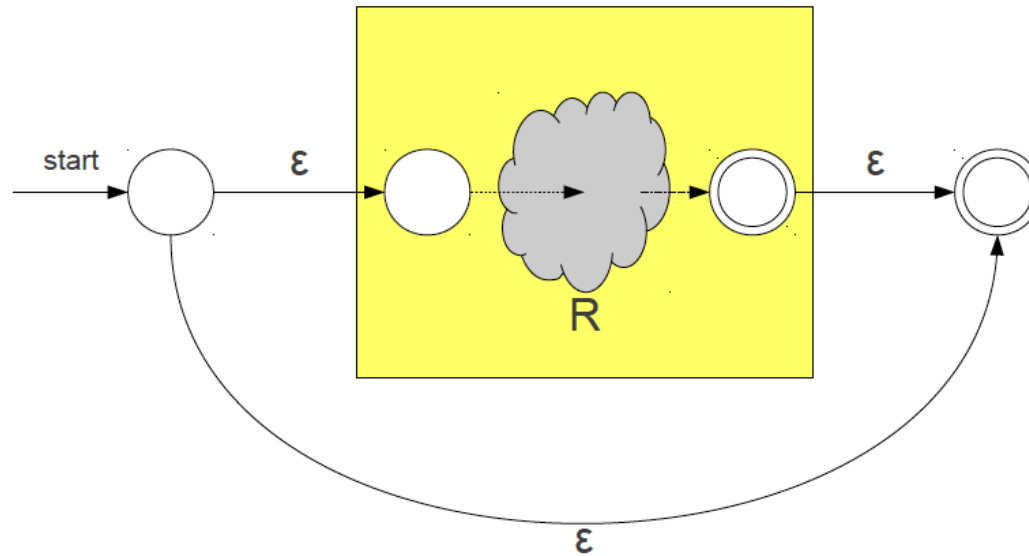
# Construction for $R^*$



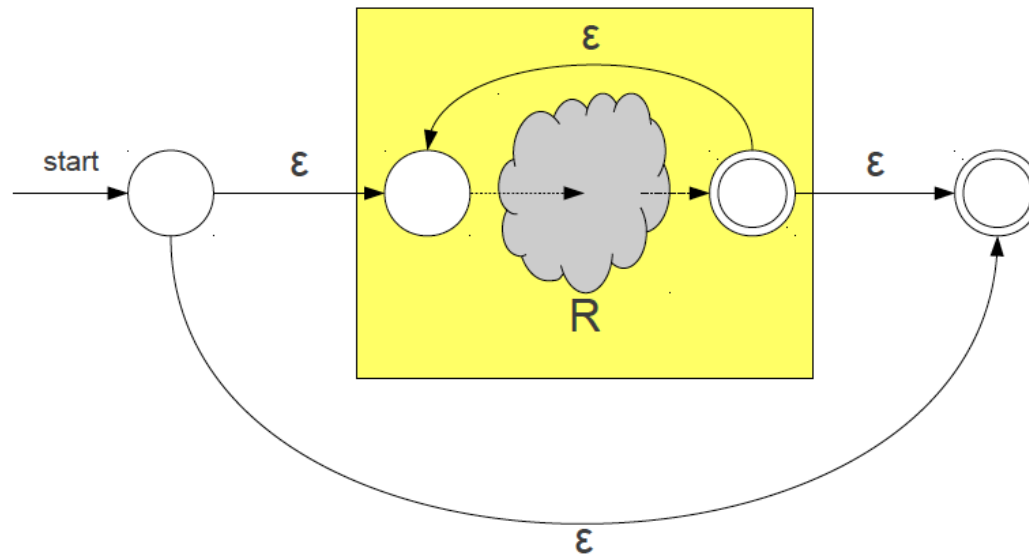
# Construction for $R^*$



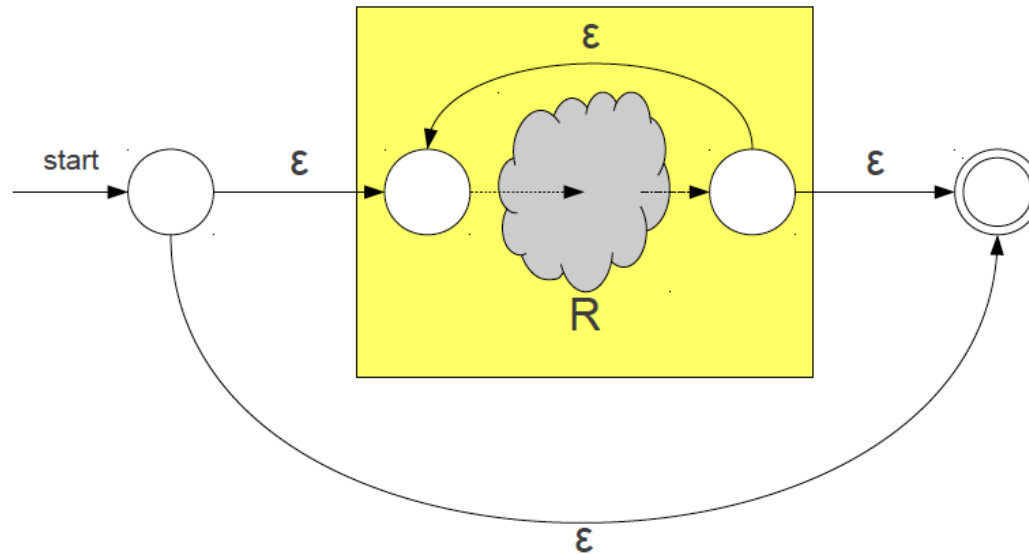
# Construction for $R^*$



# Construction for $R^*$

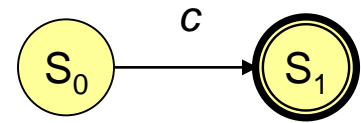
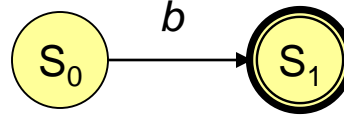
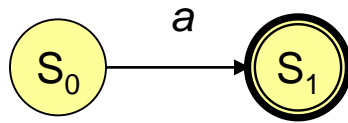


# Construction for $R^*$



# Example: Construct the NFA of $a(b/c)^*$

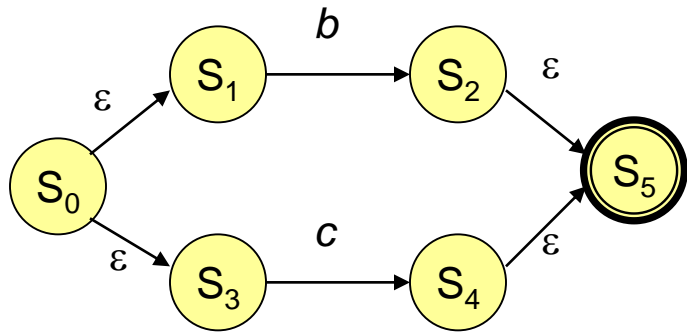
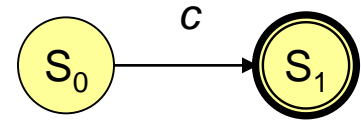
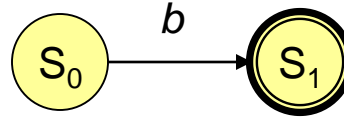
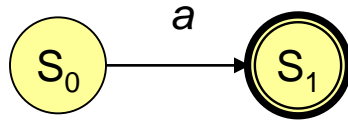
1)NFAs  
for  $a, b, c$





# Example: Construct the NFA of $a(b/c)^*$

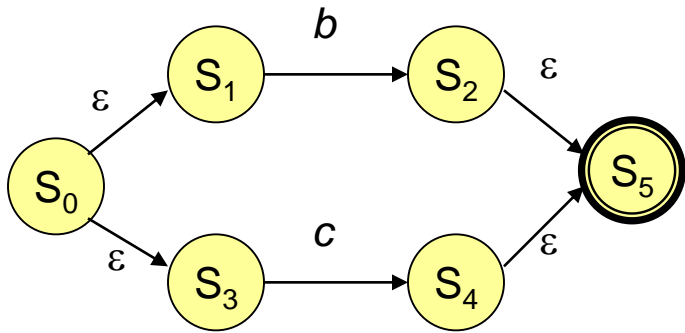
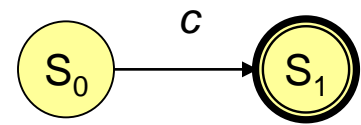
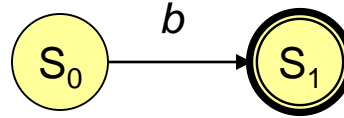
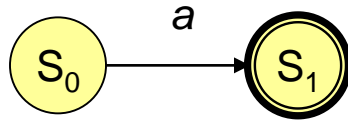
1) NFAs  
for  $a, b, c$



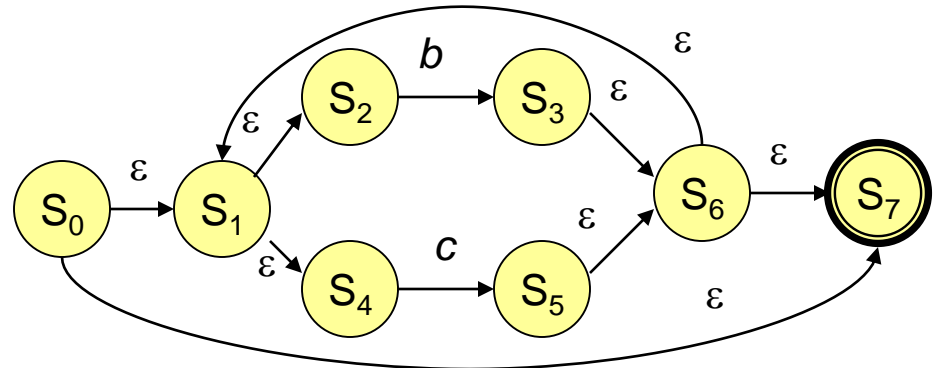
2) NFA for  $b/c$

# Example: Construct the NFA of $a(b/c)^*$

1) NFAs  
for  $a, b, c$



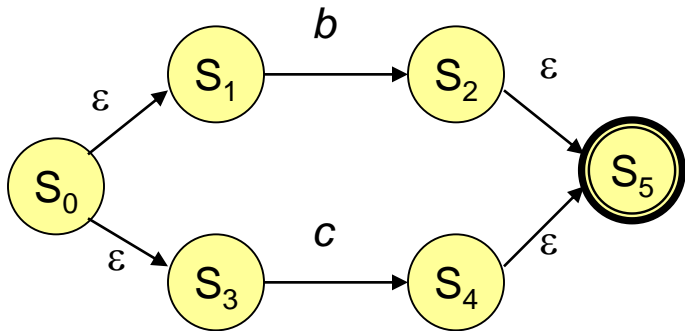
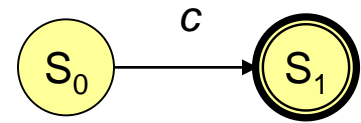
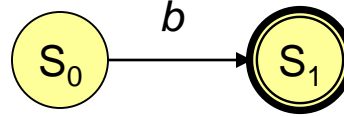
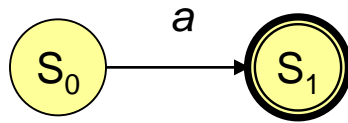
2) NFA for  $b/c$



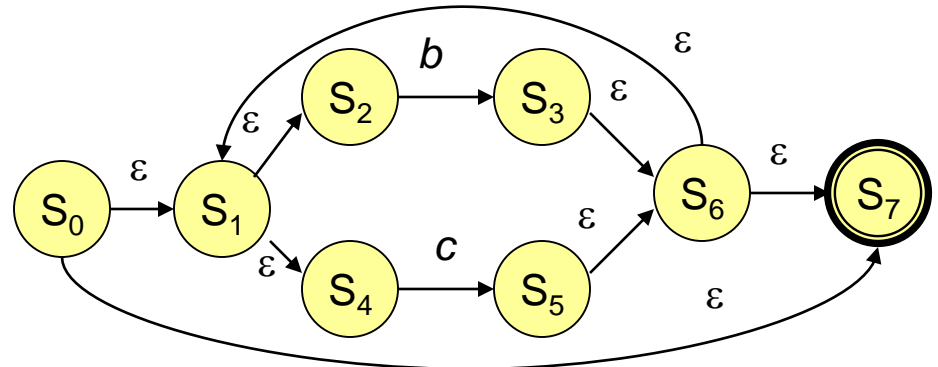
3) NFA for  $(b/c)^*$

# Example: Construct the NFA of $a(b/c)^*$

1) NFAs  
for  $a, b, c$

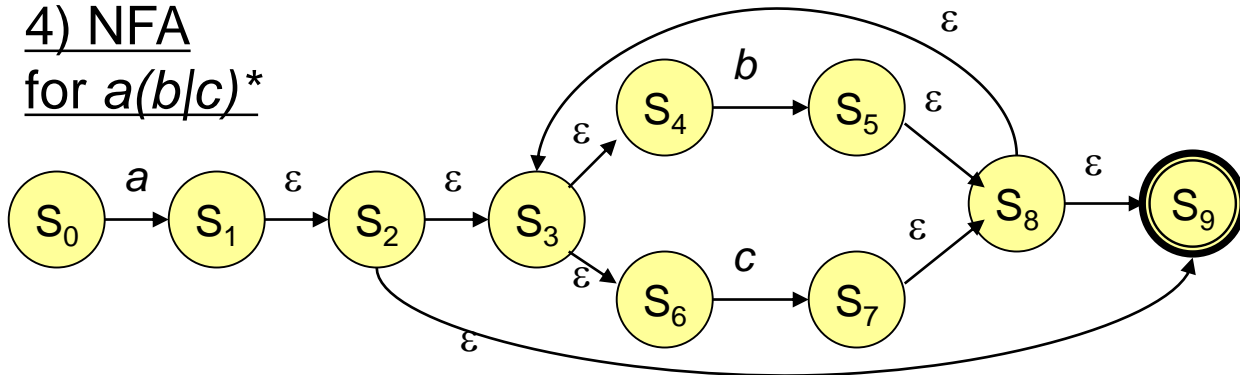


2) NFA for  $b/c$

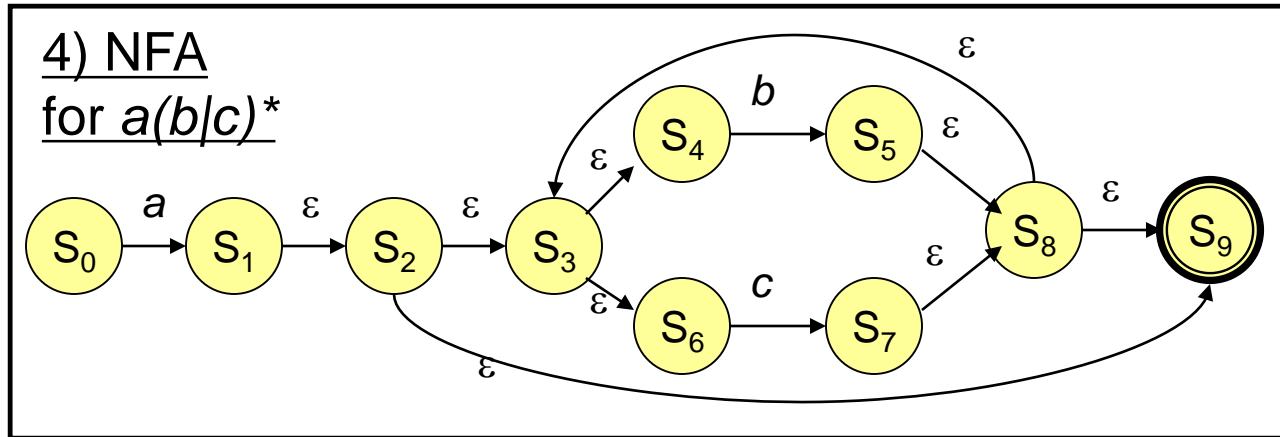


3) NFA for  $(b/c)^*$

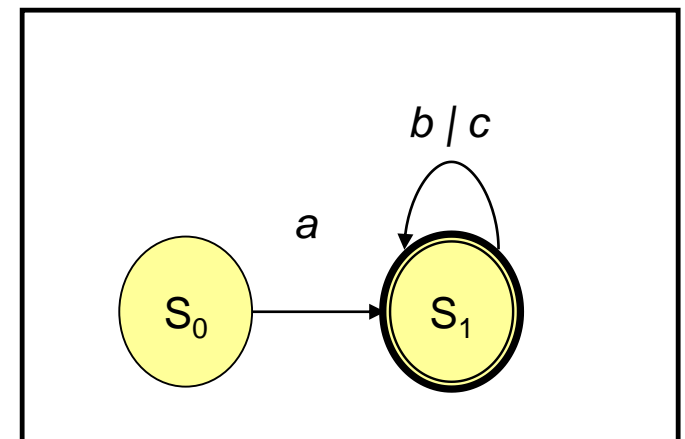
4) NFA  
for  $a(b/c)^*$



# Example: Construct the NFA of $a(b/c)^*$



Of course, a human would design a simpler one...  
But, we can automate production of the complex one...



# Challenges in Scanning

- How do we determine which lexemes are associated with each token?
- When there are multiple ways we could scan the input, how do we know which one to pick?
- How do we address these concerns efficiently?

# DFA's

- The automata we've seen so far have all been NFAs.
- A DFA is like an NFA, but with tighter restrictions:
  - Every state must have exactly one transition defined for every letter.
  - $\epsilon$ -moves are not allowed.

# Speeding up Matching

- In the worst-case, an NFA with  $n$  states takes time  $O(mn^2)$  to match a string of length  $m$ .
- DFAs, on the other hand, take only  $O(m)$ .
- There is another straightforward algorithm (Subset Construction) to convert NFAs to DFAs.



# NFA to DFA: two key functions

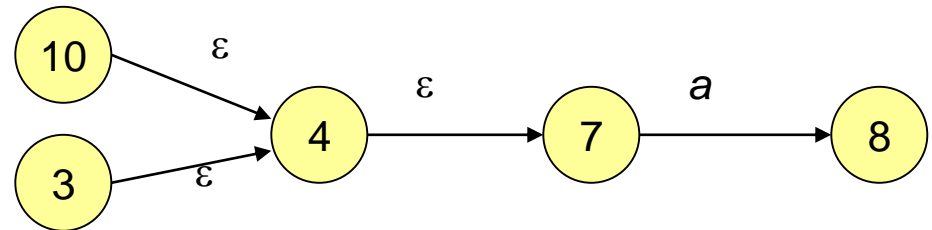
- **move( $s_i, a$ ):** the (union of the) set of states to which there is a transition on input symbol **a** from state  $s_i$
- **$\epsilon$ -closure( $s_i$ ):** the (union of the) set of states reachable by  $\epsilon$  from  $s_i$ .



# NFA to DFA: two key functions

## ■ Example:

- $\varepsilon\text{-closure}(3)=\{3,4,7\}$
- $\varepsilon\text{-closure}(10)=\{4,7,10\}$ ;
- $\text{move}(7,a)=8$ ;



- The Algorithm starts with the  $\varepsilon$ -closure of  $s_0$  from NFA.
- Do for each unmarked state until there are no unmarked states:
  - for each symbol take their  $\varepsilon\text{-closure}(\text{move}(\text{state}, \text{symbol}))$

# NFA to DFA: Algorithm

Initially,  $\epsilon$ -closure the start state

**while** there is an unmarked state T in Dstates

mark T

**for each** input symbol a

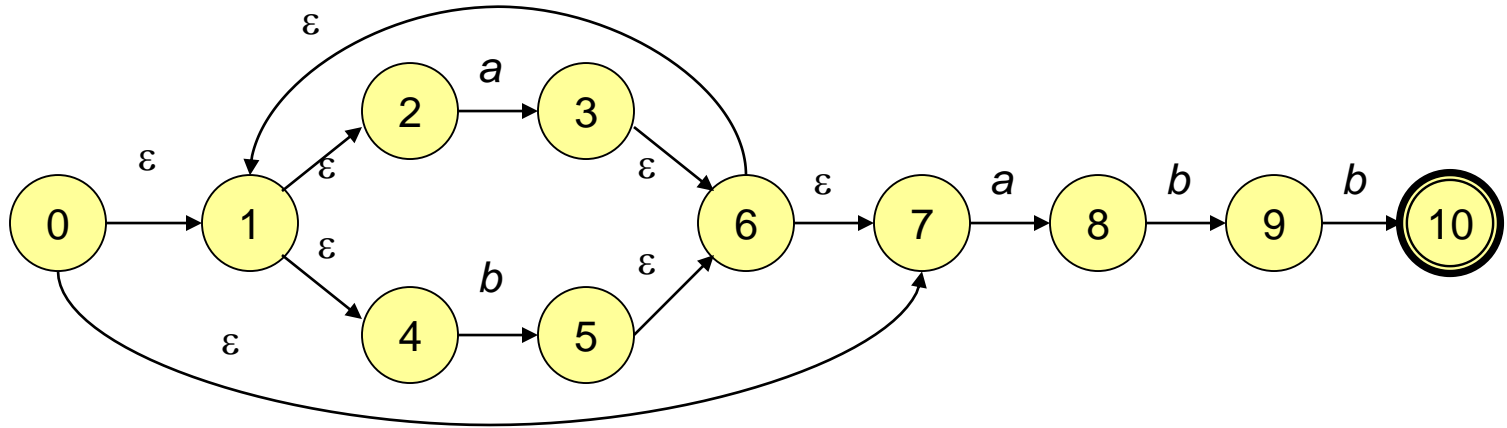
U:= $\epsilon$ -closure(move(T,a))

**if** U is not in Dstates then add U as unmarked to Dstates

Dtable[T,a]:=U

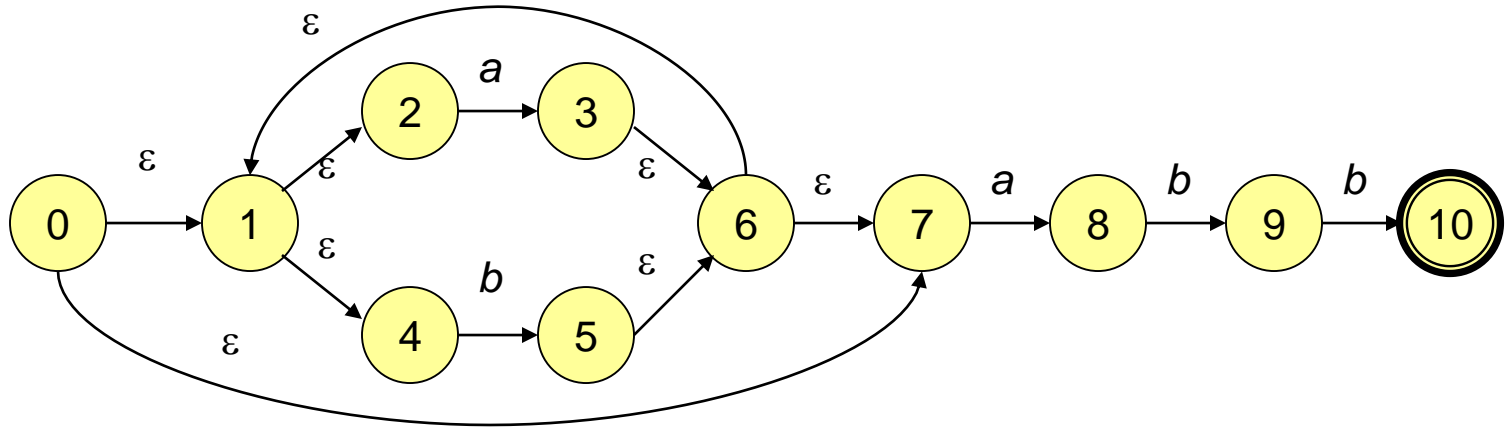
- Dstates (set of states for DFA) and Dtable form the DFA.
- Each state of DFA corresponds to a set of NFA states that NFA could be in after reading some sequences of input symbols.

# NFA to DFA: Example



- $A = \epsilon\text{-closure}(0) = \{0, 1, 2, 4, 7\}$
- for each input symbol ( $a$  and  $b$ ):
  - $B = \epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{3, 8\}) = \{1, 2, 3, 4, 6, 7, 8\}$
  - $C = \epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{5\}) = \{1, 2, 4, 5, 6, 7\}$
  - $\text{Dtable}[A, a] = B; \text{Dtable}[A, b] = C$

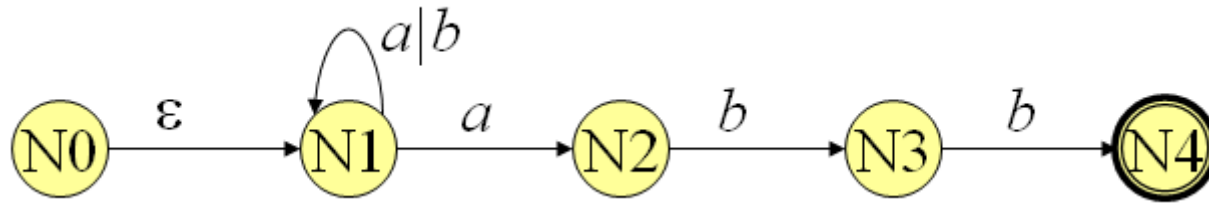
# NFA to DFA: Example



- $A = \{0, 1, 2, 4, 7\}$ ,
- $Dtable[A, a] = \mathbf{B}$ ;  $Dtable[A, b] = \mathbf{C}$ 
  - $B = \epsilon\text{-closure}(\{3, 8\}) = \{1, 2, 3, 4, 6, 7, 8\}$  ,  $C = \epsilon\text{-closure}(\{5\}) = \{1, 2, 4, 5, 6, 7\}$
- $Dtable[B, a] = B$ ;  $Dtable[B, b] = \mathbf{D}$ ;  $Dtable[C, a] = B$ ;  $Dtable[C, b] = C$ ;
  - $D = \epsilon\text{-closure}(\{5, 9\}) = \{1, 2, 4, 5, 6, 7, 9\}$ ;
- $Dtable[D, a] = B$ ;  $Dtable[D, b] = \mathbf{E}$ ;  $Dtable[E, a] = B$ ;  $Dtable[E, b] = C$ ;
  - $E = \epsilon\text{-closure}(\{5, 10\}) = \{1, 2, 4, 5, 6, 7, 10\}$ ;

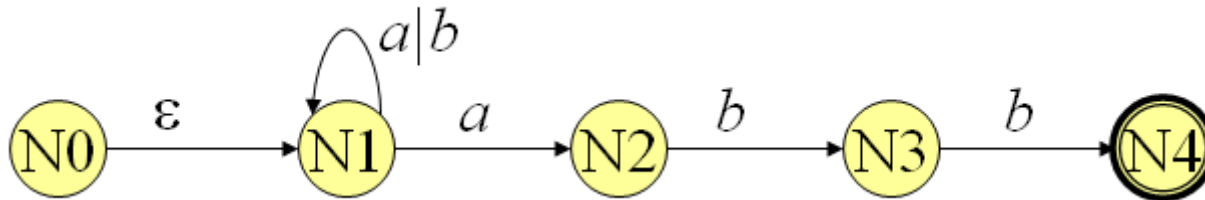
# NFA to DFA: Example

## (Another NFA for the same RE)



# NFA to DFA: Example

## (Another NFA for the same RE)



Iteration	State	Contains	$\epsilon$ -closure(move(s,a))	$\epsilon$ -closure(move(s,b))
0	A	N0,N1	N1,N2	N1
1	B	N1,N2	N1,N2	N1,N3
	C	N1	N1,N2	N1
2	D	N1,N3	N1,N2	N1,N4
3	E	N1,N4	N1,N2	N1

- iteration 3 adds nothing new, so the algorithm stops.
- state E contains N4 (final state)

# Outline

- Definition
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- Toward Automation

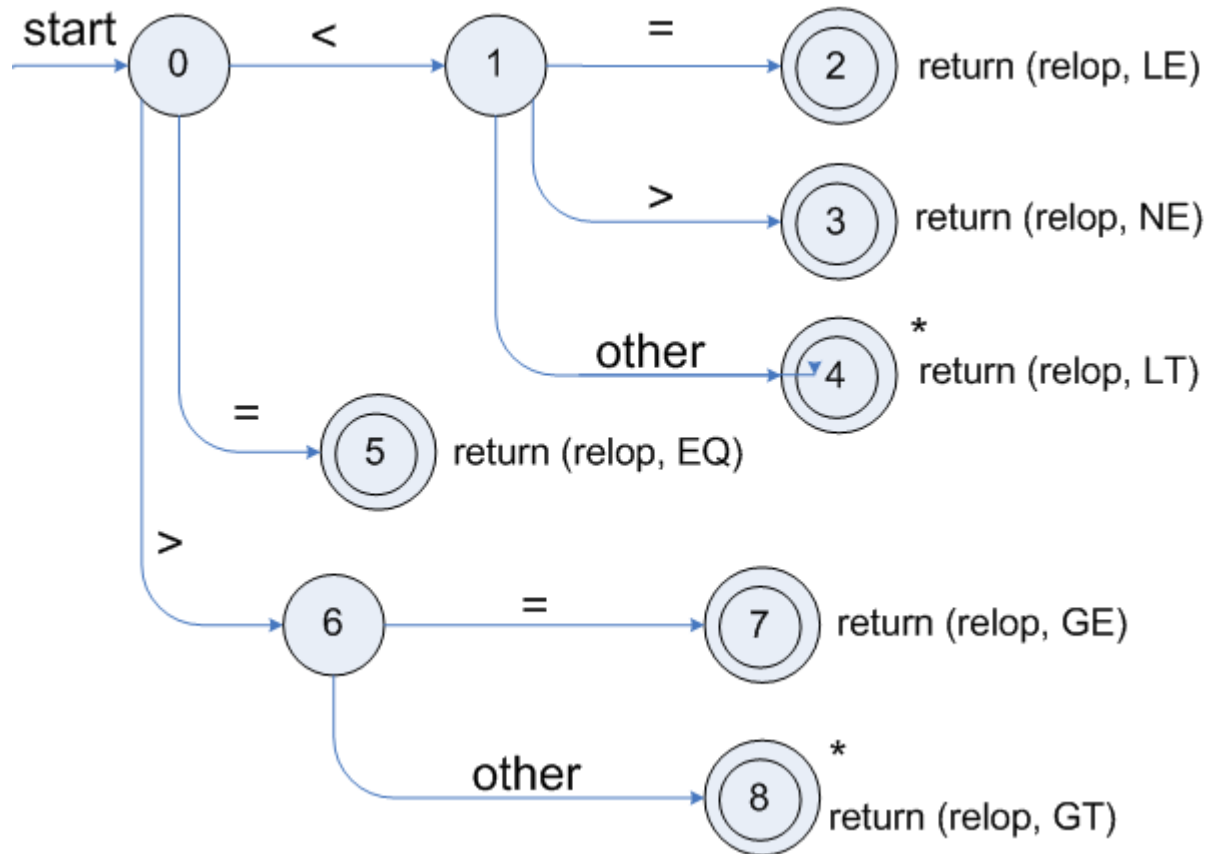
# Relop

Regular Expression	Token	Attribute-Value
<	relop	LT
<=	relop	LE
=	relop	EQ
<>	relop	NE
>	relop	GT
>=	relop	GE



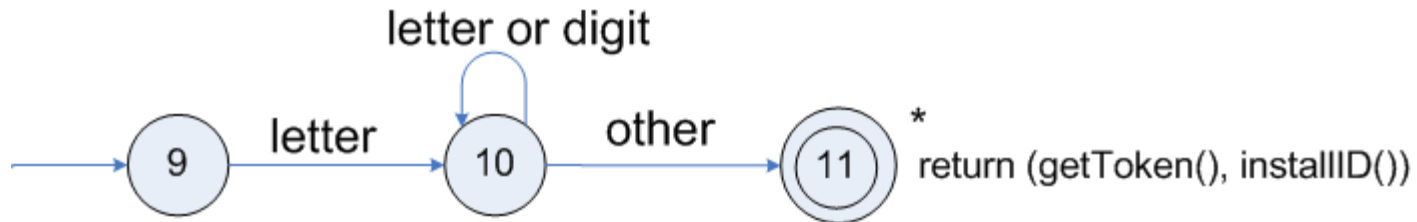
# Transition diagrams

## ■ Transition diagram for relop



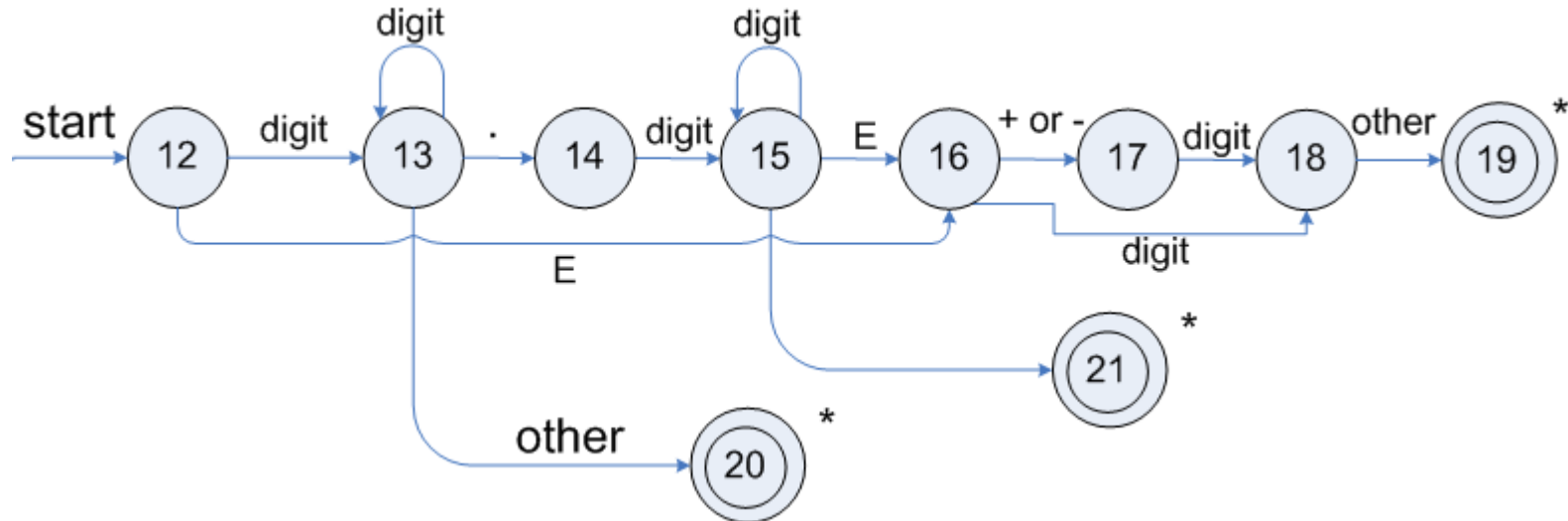
# Transition diagrams

- Transition diagram for reserved words and identifiers



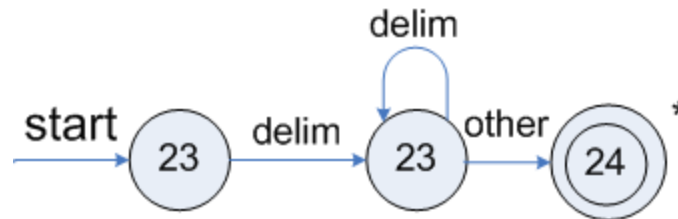
# Transition diagrams

## ■ Transition diagram for unsigned numbers



# Transition diagrams

## ■ Transition diagram for whitespace



# Python Blocks

- Scoping handled by whitespace:

```
if w == z:
    a = b
    c = d
else:
    e = f
g = h
```

- What does that mean for the scanner?

# Whitespace Tokens

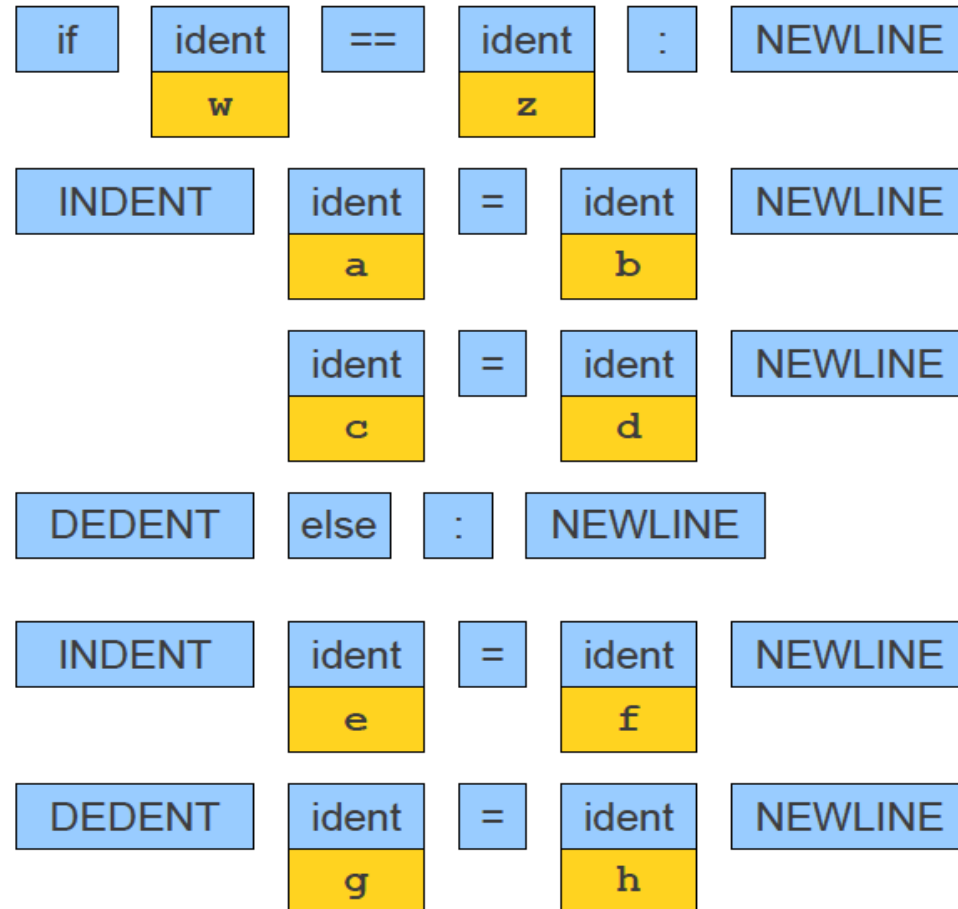
- Special tokens inserted to indicate changes in levels of indentation.
- **NEWLINE** marks the end of a line.
- **INDENT** indicates an increase in indentation.
- **DEDENT** indicates a decrease in indentation.
- Note that **INDENT** and **DEDENT** encode change in indentation, not the total amount of indentation.

# Scanning Python

```
if w == z:  
    a = b  
    c = d  
else:  
    e = f  
g = h
```

# Scanning Python

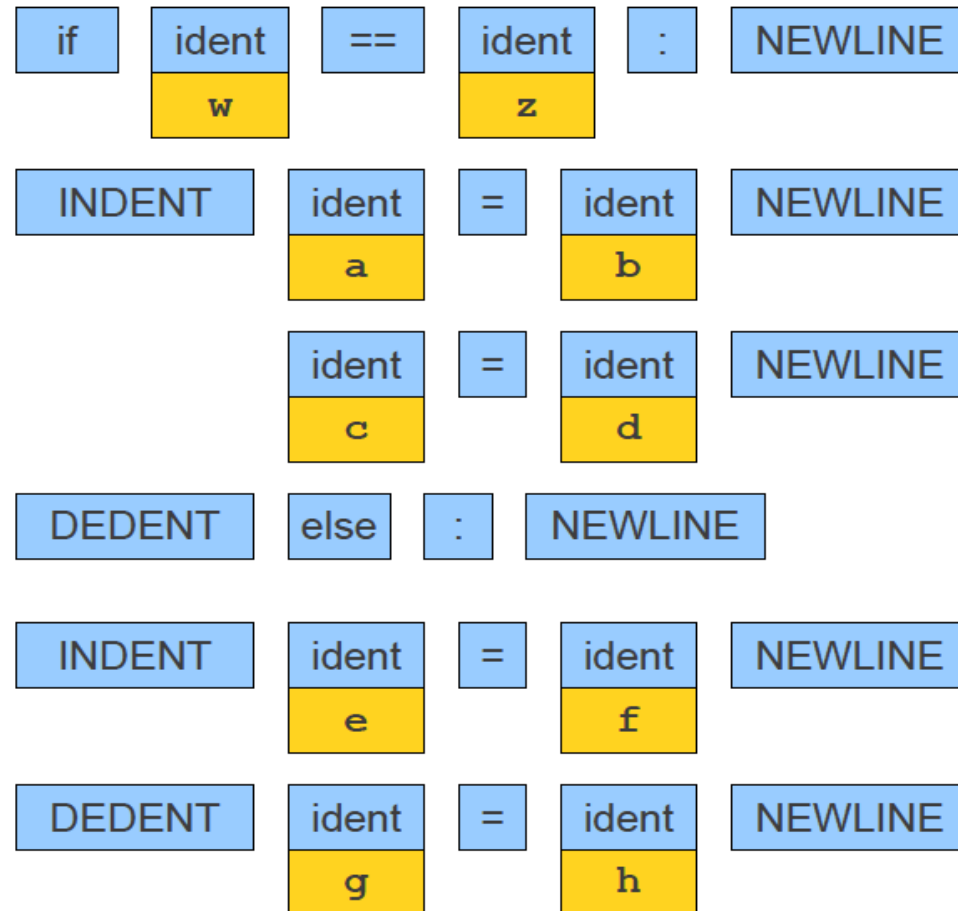
```
if w == z:  
    a = b  
    c = d  
else:  
    e = f  
g = h
```





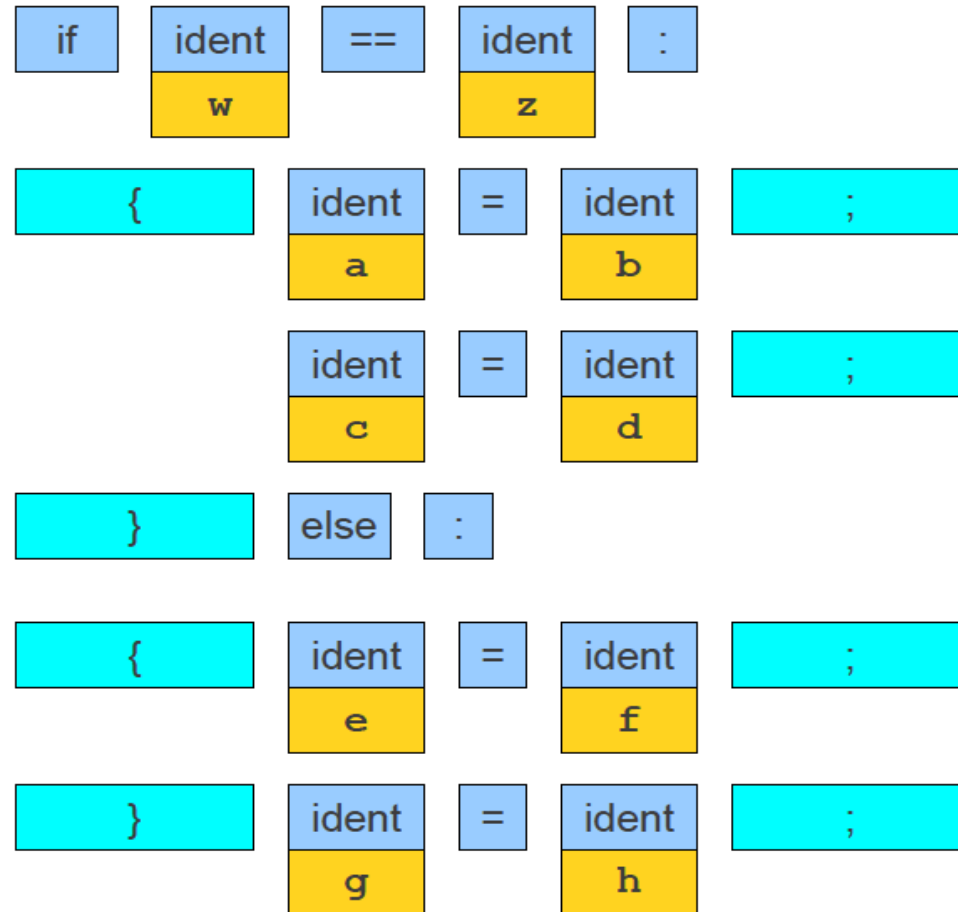
# Scanning Python

```
if w == z: {  
    a = b;  
    c = d;  
} else {  
    e = f;  
}  
g = h;
```



# Scanning Python

```
if w == z: {  
    a = b;  
    c = d;  
} else {  
    e = f;  
}  
g = h;
```



# Where to INDENT/DEDENT?

- Scanner maintains a stack of line indentations keeping track of all indented contexts so far.
- Initially, this stack contains 0, since initially the contents of the file aren't indented.
- On a newline:
  - See how much whitespace is at the start of the line.
  - If this value exceeds the top of the stack:
    - Push the value onto the stack.
    - Emit an INDENT token.
- Otherwise, while the value is less than the top of the stack:
  - Pop the stack.
  - Emit a DEDENT token.

# General Practical Considerations

## ■ Poor language design may complicate lexical analysis:

- `if then then = else; else else = then` (PL/I)
- `DO5I=1,25` vs `DO5I=1.25`
  - (Fortran: urban legend has it that an error like this caused a crash of an early NASA mission)
- The development of a sound theoretical basis has influenced language design positively.

# General Practical Considerations

## ■ Template syntax in C++:

- `aaaa<mytype>`
- `aaaa<mytype<int>>`
  - (`>>` is an operator for writing to the output stream)
- The lexical analyser treats the `>>` operator as two consecutive `>` symbols. The confusion will be resolved by the parser (by matching the `<`, `>`)

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# Lexical errors

- Some errors are out of power of lexical analyzer to recognize
- It is able to recognize some errors when no pattern is found for tokens matches a character sequence

# Error recovery

- Panic mode: successive characters are ignored until we reach to a well formed token
- Delete one character from the remaining input
- Insert a missing character into the remaining input
- Replace a character by another character
- Transpose two adjacent characters
- Minimal Distance



# Outline

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# Implementation using DFA

- Option 1: Implement by hand using procedures
- Option 2: Use tool to generate table driven parser

# Implement Using Procedures

- One procedure for each token
- Each procedure reads one character
- Choices implemented using if and switch statements

# Implement Using Procedures

```
static char nextch;          // next unprocessed input character
void getch() { ... }        // advance to next input char

public Token getToken() {
    Token result;
    skipWhiteSpace();
    if (no more input) {
        result = new Token(Token.EOF); return result;
    }
    switch(nextch) {
        case '(': result = new Token(Token.LPAREN); getch(); return result;
        case ')': result = new Token(Token.RPAREN); getch(); return result;
        case ';': result = new Token(Token.SCOLON); getch(); return result;
        ...
        case '0': ... case '9':
        ...
        case 'a': ... case 'z':
        ...
    }
```

# Implement Using Procedures

## ■ Proc:

- Straightforward to write
- Fast

## ■ Cons

- A fair amount of tedious work
- May have subtle differences from the language specification

# Implementation Using Transition Table

- Rows: states of DFA
- Columns: input characters
- Entries: action
  - Go to next state
  - Accept token, go to start state
  - Error

# Implementation Using Transition Table

- An easy (computerized) implementation of a transition diagram is a **transition table**: a column for each input symbol and a row for each state. An entry is a set of states that can be reached from a state on some input symbol.

- E.g.:

state	'r'	digit
0	1	-
1	-	2
2 (final)	-	2

# Implementation Using Transition Table

- If we know the transition table and the final state(s) we can build directly a recognizer that detects acceptance:

```
char=input_char();  
state=0; // starting state  
while (char != EOF) {  
    state ← table(state,char);  
    if (state == '-') return failure;  
    word=word+char;  
    char=input_char();  
}  
if (state == FINAL) return acceptance; else return  
failure;
```



# Example

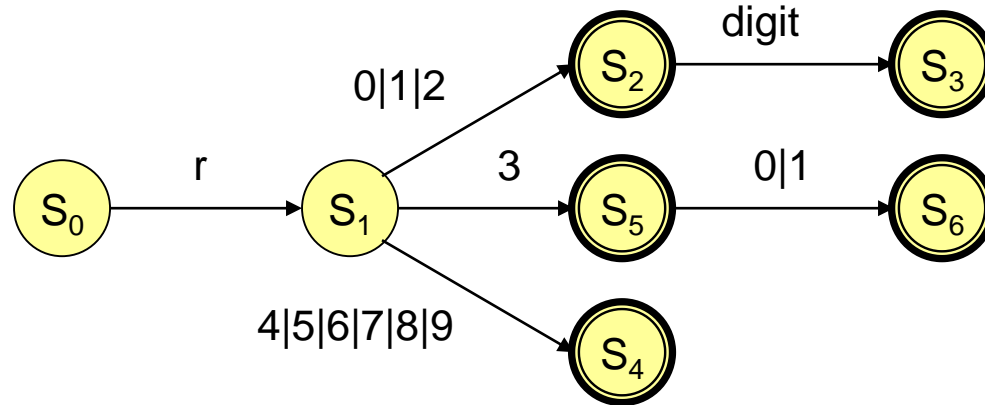
- What is this RE for?
- Produce the DFA for it:

*Register*  $\rightarrow r ((0/1/2) (Digit/\varepsilon) / (4/5/6/7/8/9) / (3/30/31))$

# Example

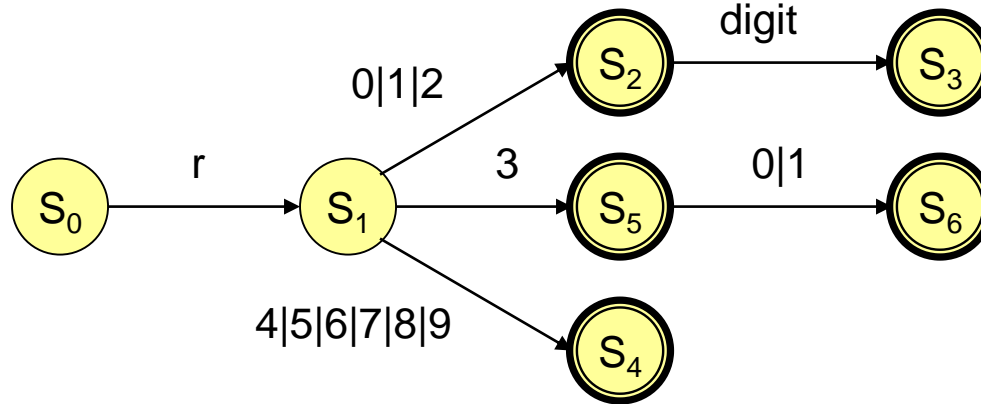
- What is this RE for?
- Produce the DFA for it:

*Register*  $\rightarrow r ((0/1/2) (Digit/\varepsilon) / (4/5/6/7/8/9) / (3/30/31))$



recognize  $r0$  through  $r31$

# Example



State	'r'	0,1	2	3	4,5,...,9
0	1	-	-	-	-
1	-	2	2	5	4
2 (final)	-	3	3	3	3
3 (final)	-	-	-	-	-
4 (final)	-	-	-	-	-
5 (final)	-	6	-	-	-
6 (final)	-	-	-	-	-

# Automatic Lexical Analyser Construction

- To convert a specification into code:
  - Write down the RE for the input language
  - Convert the RE to a NFA
  - Build the DFA that simulates the NFA
  - Shrink the DFA

# Lex/Flex: Generating Lexical Analysers

- Flex is a tool for generating scanners
  - Programs which recognized lexical patterns in text
- Lex input consists of 3 sections:
  - Regular expressions
  - Pairs of regular expressions and C code
  - Auxiliary C code

# Lex/Flex: Generating Lexical Analyzers

- When the lex input is compiled, it generates as output a C source file `lex.yy.c`
  - The source contains a routine `yylex()`
- After compiling the C file, the executable will start isolating tokens from the input according to the regular expressions
  - For each token, the associated code will be executed
  - The array `char yytext[]` contains the representation of a token

# Flex Example

```
%{
#define ERROR -1
int line_number=1;
%}
whitespace    [ \t]
letter        [a-zA-Z]
digit         [0-9]
integer       ({digit}+)
l_or_d        ({letter}|{digit})
identifier    ({letter}{l_or_d}*)
operator      [-+*/]
separator     [;,(){}]
%%
{integer}     {return 1;}
{identifier}  {return 2;}
{operator}|{separator} {return (int)yytext[0];}
{whitespace} {}
\n            {line_number++;}
.             {return ERROR;}
%%
int yywrap(void) {return 1;}
int main() {
    int token;
    yyin=fopen("myfile", "r");
    while ((token=yylex()) != 0)
        printf("%d %s \n", token, yytext);
    printf("lines %d \n", line_number);
}
```

Input file ("myfile")  
123+435+34=aaaa  
329\*45/a-34\*(45+23)\*\*3  
bye-bye

## Output:

```
1 123
43 +
1 435
43 +
1 34
-1 =
2 aaaa
1 329
42 *
1 45
47 /
2 a
45 -
1 34
42 *
40 (
1 45
43 +
1 23
41 )
42 *
42 *
1 3
2 bye
45 -
2 bye
lines 4
```

# Summary

- Lexical Analysis turns a stream of characters into a stream of tokens
  - A largely automatic process.
    - REs are powerful enough to specify scanners
    - DFAs have good properties for an implementation



# Summary

- Lexical analysis splits input text into tokens holding a lexeme and an attribute
- Lexemes are sets of strings often defined with regular expressions
- The generalized transition diagram is a **finite automaton**. It can be:
  - **Deterministic** (DFA)
  - **Non-Deterministic** (NFA)
- Regular expressions can be converted to NFAs and from there to DFAs

# Reading

- Aho2, Sections 2.2; 3.1-3.4; 3.5 (lex); 3.6-3.7; 3.9.6
- Aho1, pp. 25-29; 84-87; 92-105; 113-125; 141-144, 105-111 (lex)
- Hunter, Chapter 2 (too detailed); Sec. 3.1 -3.3 (too condensed)
- Grune 1.9; 2.1-2.5; 2.1.6.1-2.1.6.6; pp.86-96
- Cooper, Sections 2.1-2.3; 2.4-2.4.3; pp.55-72

Question?