Brief Article

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Variable 1 (U) $U\ :\ Type.$

Definition 1 (Ensemble) $Ensemble\ :=\ U\ \Rightarrow\ Prop.$

Definition 2 (In) $In\,(A : Ensemble)\,(x : U)\ :=\ A\,x.$

Notation 1 ("x) $"x\ \in\ A"\ :=\ (In\,A\,x)\,(at\,level\,10).$

Definition 3 (Included) $Included\,(B\,C\ :\ Ensemble)\ :\ Prop\ :=\ \forall\,x\ :\ U,\,x \in B\ \Rightarrow\ x\ \in\ C.$

Notation 2 ("A) $"A\ \subseteq\ B"\ :=\ (Included\,A\,B)(at\,level\,10).$

Definition 4 (Union) $Union\,(B\,C\ :\ Ensemble)\ :\ Ensemble\ :=\ fun\,x\ :\ U\ =>\ (x \in B)\ \vee\ (x \in C).$

Notation 3 ("A) $"A\ \cup\ B"\ :=\ (Union\,A\,B)(at\,level\,8).$

Definition 5 (Intersection) $Intersection\,(B\,C : Ensemble)\ :\ Ensemble\ :=\ fun\,x : U\ =>\ (x \in B)\ \wedge (x \in C).$

Notation 4 ("A) $"A\ \cap\ B"\ :=\ (Intersection\,A\,B)\,(at\,level\,10).$

Lemma 1 (a) $a\,(A\,B\,C : Ensemble) : ((A{\cap}(B{\cup}C))\ \subseteq\ ((A{\cap}B){\cup}(A{\cap}C))).$

Proof: Using the definition Included, our conclusion becomes

$$\forall x : U, (x \in (A \cap (B \cup C))) \Rightarrow (x \in ((A \cap B) \cup (A \cap C))).$$

In order to show $\forall x : U, (x \in (A \cap (B \cup C))) \Rightarrow (x \in ((A \cap B) \cup (A \cap C)))$ we pick an arbitrary

$$x$$

and show

$$(x \in (A \cap (B \cup C))) \Rightarrow (x \in ((A \cap B) \cup (A \cap C))).$$

We will assume

$$Hyp : x \in (A \cap (B \cup C))$$

and show

$$x \in ((A \cap B) \cup (A \cap C)).$$

Using the definition (In, Union), our conclusion becomes

$$(x \in (A \cap B)) \vee (x \in (A \cap C)).$$

Using the definition of (In, (Intersection)),

$$Hyp$$

becomes

$$Hyp : (x \in A) \wedge (x \in (B \cup C))$$

Since we know $Hyp : (x \in A) \wedge (x \in (B \cup C))$ we also know

$$Hyp0 : x \in A$$

$$Hyp1 : x \in (B \cup C).$$

Using the definition of (In,Union),

$$Hyp1$$

becomes

$$Hyp1 : (x \in B) \vee (x \in C)$$

Since we know $Hyp1 : (x \in B) \vee (x \in C)$ we can consider two cases:

Case 1

$$Hyp : x \in B$$

We will prove the left hand side of $(x \in (A \cap B)) \vee (x \in (A \cap C))$. That is we need to prove

$$x \in (A \cap B).$$

Using the definition (In, Intersection), our conclusion becomes

$$(x \in A) \wedge (x \in B).$$

In order to prove $(x \in A) \wedge (x \in B)$ will first prove

$$x \in A$$

and then

$$x \in B.$$

First we show

$$x \in A.$$

$x \in A$ follows trivially from the assumptions.

Next we show

$$x \in B.$$

$x \in B$ follows trivially from the assumptions.

Since we showed

$$x \in A$$

and

$$x \in B$$

we also have $(x \in A) \wedge (x \in B)$.

Therefore we have showed

$$(x \in A) \wedge (x \in B)$$

and so $x \in (A \cap B)$.

We have proved

$$x \in (A \cap B)$$

and so $(x \in (A \cap B)) \vee (x \in (A \cap C))$ follows.

Case 2

$$Hyp2 : x \in C$$

We will prove the right hand side of $(x \in (A \cap B)) \vee (x \in (A \cap C))$. That is we need to prove

$$x \in (A \cap C).$$

Using the definition (In, Intersection), our conclusion becomes

$$(x \in A) \wedge (x \in C).$$

In order to prove $(x \in A) \wedge (x \in C)$ will first prove

$$x \in A$$

and then

$$x \in C.$$

First we show

$$x \in A.$$

$x \in A$ follows trivially from the assumptions.

Next we show

$$x \in C.$$

$x \in C$ follows trivially from the assumptions.

Since we showed

$$x \in A$$

and

$$x \in C$$

we also have $(x \in A) \wedge (x \in C)$.

Therefore we have showed

$$(x \in A) \wedge (x \in C)$$

and so $x \in (A \cap C)$.

We are done with

$$x \in (A \cap C)$$

and so $(x \in (A \cap B)) \vee (x \in (A \cap C))$ follows.

Since we proved both cases, we are done with $(x \in (A \cap B)) \vee (x \in (A \cap C))$

We are done with $(x \in (A \cap B)) \vee (x \in (A \cap C))$

Therefore we have showed

$$(x \in (A \cap B)) \vee (x \in (A \cap C))$$

and so $x \in ((A \cap B) \cup (A \cap C))$.

We have showed that if

$$Hyp : x \in (A \cap (B \cup C))$$

then

$$x \in ((A \cap B) \cup (A \cap C))$$

a proof of $(x \in (A \cap (B \cup C))) \Rightarrow (x \in ((A \cap B) \cup (A \cap C)))$.

Since

$$x$$

was arbitrary this shows $\forall x : U, (x \in (A \cap (B \cup C))) \Rightarrow (x \in ((A \cap B) \cup (A \cap C)))$.

Therefore we have showed

$$\forall x : U, (x \in (A \cap (B \cup C))) \Rightarrow (x \in ((A \cap B) \cup (A \cap C)))$$

and so $(A \cap (B \cup C)) \subseteq ((A \cap B) \cup (A \cap C))$.This is done