Number theory set

Prove the Lemmas bellow. Name them as before as oneone, twoone and so on. Please make sure you save the work as you go along and submit as many solutions as you want. I would prefer 10 flawed solutions to one perfect one. I want to see how you think about it, when do you guess, what searches you do, when do you make mistakes, how do you correct them and so on. Note that you will need to start with

```
Definition div a b:= exists c, b = a* c.
Notation "a | b" := (div a b)(at level 0).
Definition even a := exists c, a = 2* c.
Definition odd a := exists c, a = 2* c+1.
```

You will also get to use the tactics:

```
Rewrite hypothesis VAR using the definition of VAR.
Rewrite goal using the definition of VAR.
```

Another useful trick is to make use of the command SearchPattern. For example, the following

```
SearchPattern (_ * (_ + _) = _).
```

will find all usable theorems that look like that, it will respond with the message

```
Query commands should not be inserted in
scripts

Nat.mul_add_distr_l: ? n m p : nat, n * (m + p) = n * m + n * p
```

And therefore you can use the theorem Nat.mul_add_distr_l in your work, either with

```
  Apply result Nat.mul_add_distr_l.
```

or with

```
Rewrite the goal using  Nat.mul_add_distr_l.
```

Another example would be

```
SearchPattern (_ * _ = 0 -> _).
```

which gives:

```
Query commands should not be inserted in
scripts

mult_is_O: ? n m : nat, n * m = 0 ? n = 0 ? m = 0

Nat.eq_mul_0_l: ? n m : nat, n * m = 0 ? m ? 0 ? n = 0

Nat.eq_mul_0_r: ? n m : nat, n * m = 0 ? n ? 0 ? m = 0
========================
Nat.mul_eq_0_l: ? n m : nat, n * m = 0 ? m ? 0 ? n = 0
========================
Nat.mul_eq_0_r: ? n m : nat, n * m = 0 ? n ? 0 ? m = 0
```

Lemma 1 *Let* $a, b, c \in \mathbb{N}$

1. *if* $a|b$ *and* $b|c$, *then* $a|c$.
2. *If* $a|c$ *and* $b|d$, *then* $ab|cd$.
3. *If* $a|b$ *and* $a|c$ *then* $a|b+c$.

Lemma 2

1. *The sum of two odd numbers is even.*
2. *the sum of two consecutive numbers is odd*
3. *every number is either even or odd.*
4. *the product of two consecutive numbers is even.*

If you have time try those:

Lemma 3

1. $\forall n \in \mathbb{N}, 2|(n * n + n)$.
2. $\forall n \in \mathbb{N}$, *the sum of the first* n *numbers equals* $n(n-1)/2$.
3. $\forall n \in \mathbb{N}$, *he sum of the first* n *odd numbers equals* n^2.
4. $\forall n \in \mathbb{N}, 2|(3^n - 1)$.