# First Logic set

# The Author

# August 14, 2017

**Lemma 1** Sow that the following are equivalent (here PQR are propositions)

1. 
$$(P \to Q) \land (P \to R)$$
 and  $P \to (Q \land R)$ 

2. 
$$(P \to Q) \lor (P \to R)$$
 and  $P \to (Q \lor R)$ 

3. 
$$R \to (Q \to R)$$
 and  $(P \land Q) \to R$ .

4. 
$$(P \to R) \land (Q \to R)$$
 and  $(P \lor Q) \to R$ 

5. 
$$(P \to R) \lor (Q \to R)$$
 and  $(P \land Q) \to R$ 

**Lemma 2** 1.  $(P \lor Q) \land \neg P \to Q$ 

2. 
$$\neg Q?(P \rightarrow Q) \rightarrow \neg P$$

3. 
$$C \wedge (A \rightarrow B) \wedge (C \rightarrow (A \rightarrow \neg B)) \rightarrow \neg A$$

4. 
$$(P \lor Q) \land (\neg P \lor R) \rightarrow Q \lor R$$
.

#### Lemma 3 Prove or disprove

- 1. Either sales or expenses will go up. If sales go up then the boss will be happy. If expenses go up then the boss will be unhappy. Therefore sales and expenses will not both go up.
- 2. If the tax rate and unemployment rate go up, then there will be a recession. If the GDP goes up then there will not be a recession, The GDP and taxes are both going up. Therefore the unemployment is not going up.
- 3. The warning light will come on if and only if the pressure is high and the relief valve is clogged. The relief valve is not clogged. therefore, the warning light will come on if and only if the pressure is too high.

Puzzle 1 What can you deduce from the following statements?

- 1. All babies are illogical.
- 2. Nobody is despised who can manage a crocodile.
- 3. Illogical persons are despised.

## Puzzle 2 What can you deduce from the following statements?

- 1. No ducks waltz.
- 2. No officers ever decline to waltz.
- 3. All my poultry are ducks.

### Puzzle 3 What can you deduce from the following statements?

- 1. Things sold in the street are of no great value.
- 2. Nothing but rubbish can be had for a song.
- 3. Eggs of the Great Auk are very valuable.
- 4. It is only what is sold in the street that is really rubbish.

# Puzzle 4 What can you deduce from the following statements?

- 1. All writers, who understand human nature, are clever.
- 2. No one is a true poet unless he can stir the hearts of men.
- 3. Shakespeare wrote ?Hamlet?.
- 4. No writer, who does not understand human nature, can stir the hearts of men.
- 5. None but a true poet could have written ?Hamlet?.