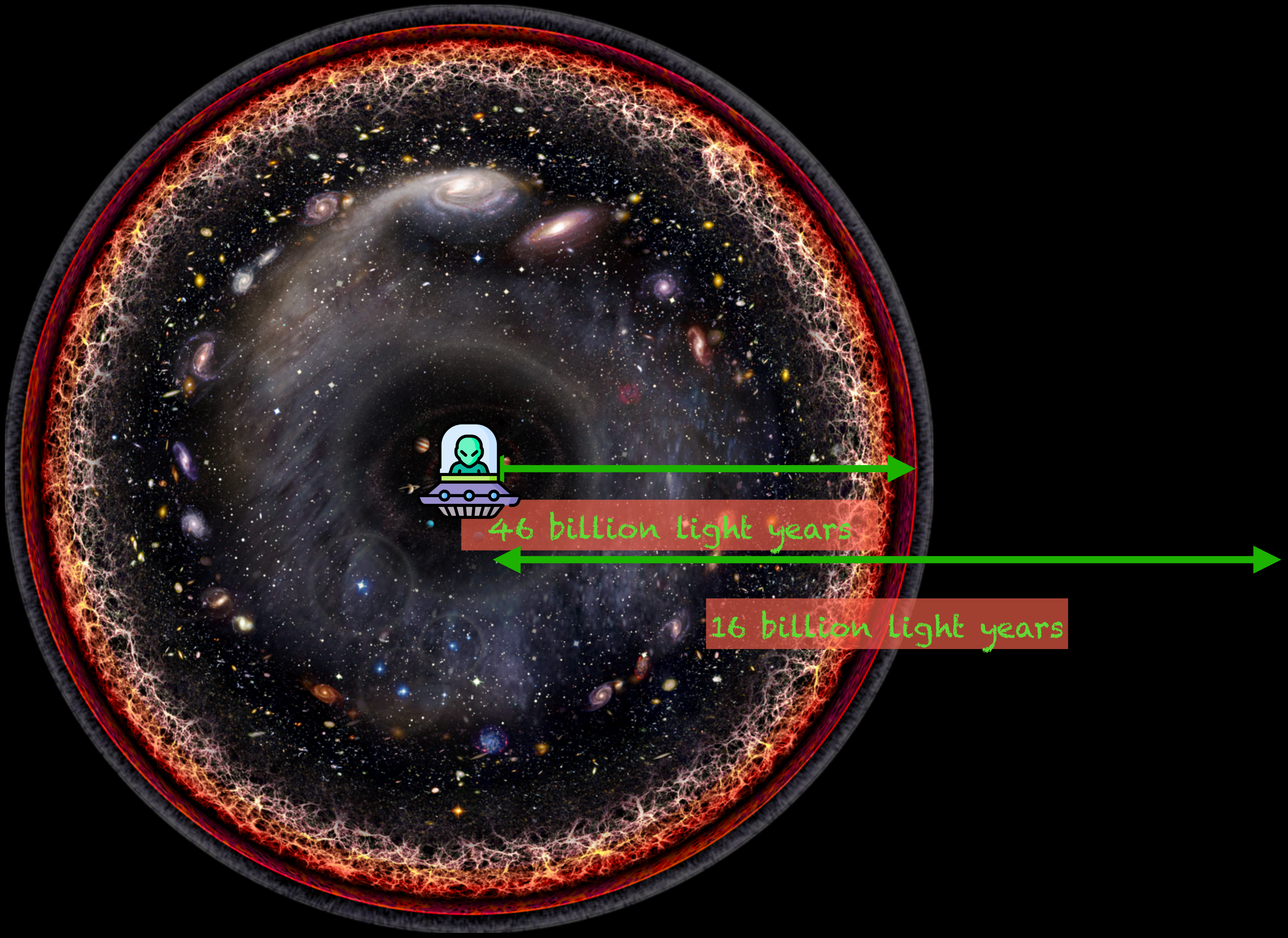

46 billion light years
16 billion light years

Dark Energy


16 billion light years
EVENT HORIZON

With infinite time

And as fast as light

. . . in a simple way

Theory of Relativity

Metric = Distance

$$ds^2 = c^2dt^2 - dr^2$$

Minkowski Metric

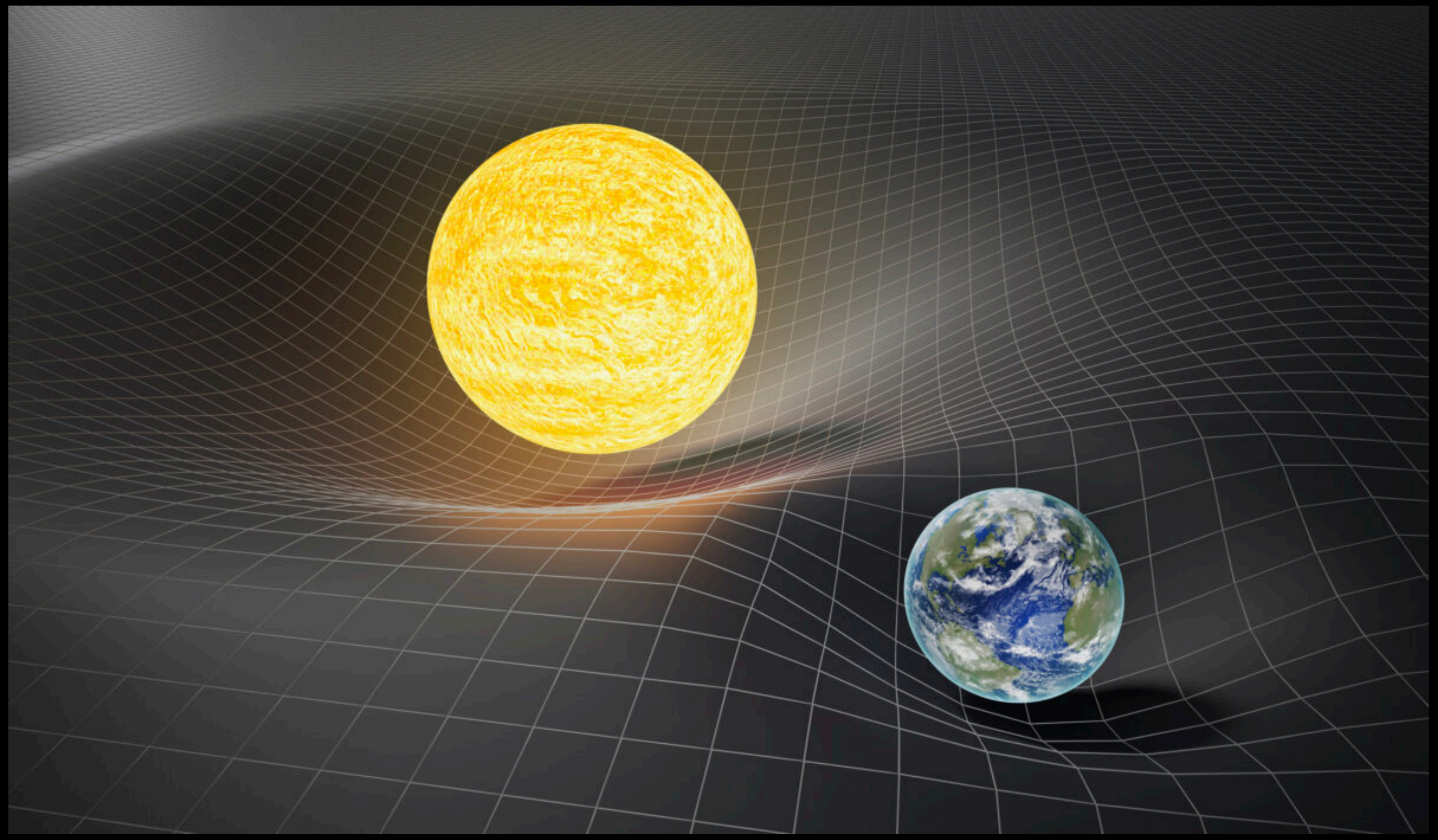
time-part

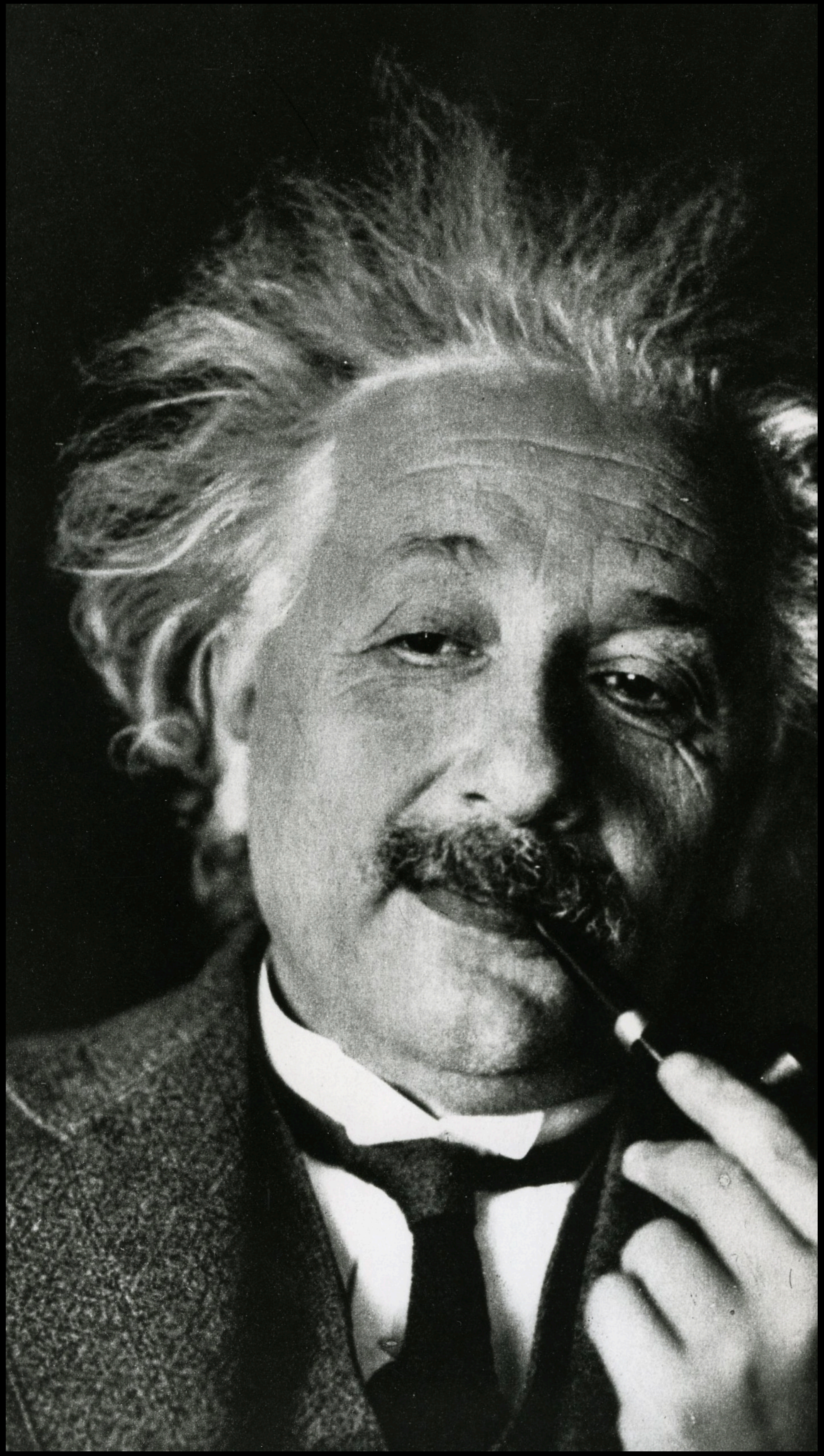
space-part

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 \sin^2\theta \, d\varphi^2 - r^2 d\theta^2$$

Schwarzschild Metric

Describes the spacetime around spherical massive objects.

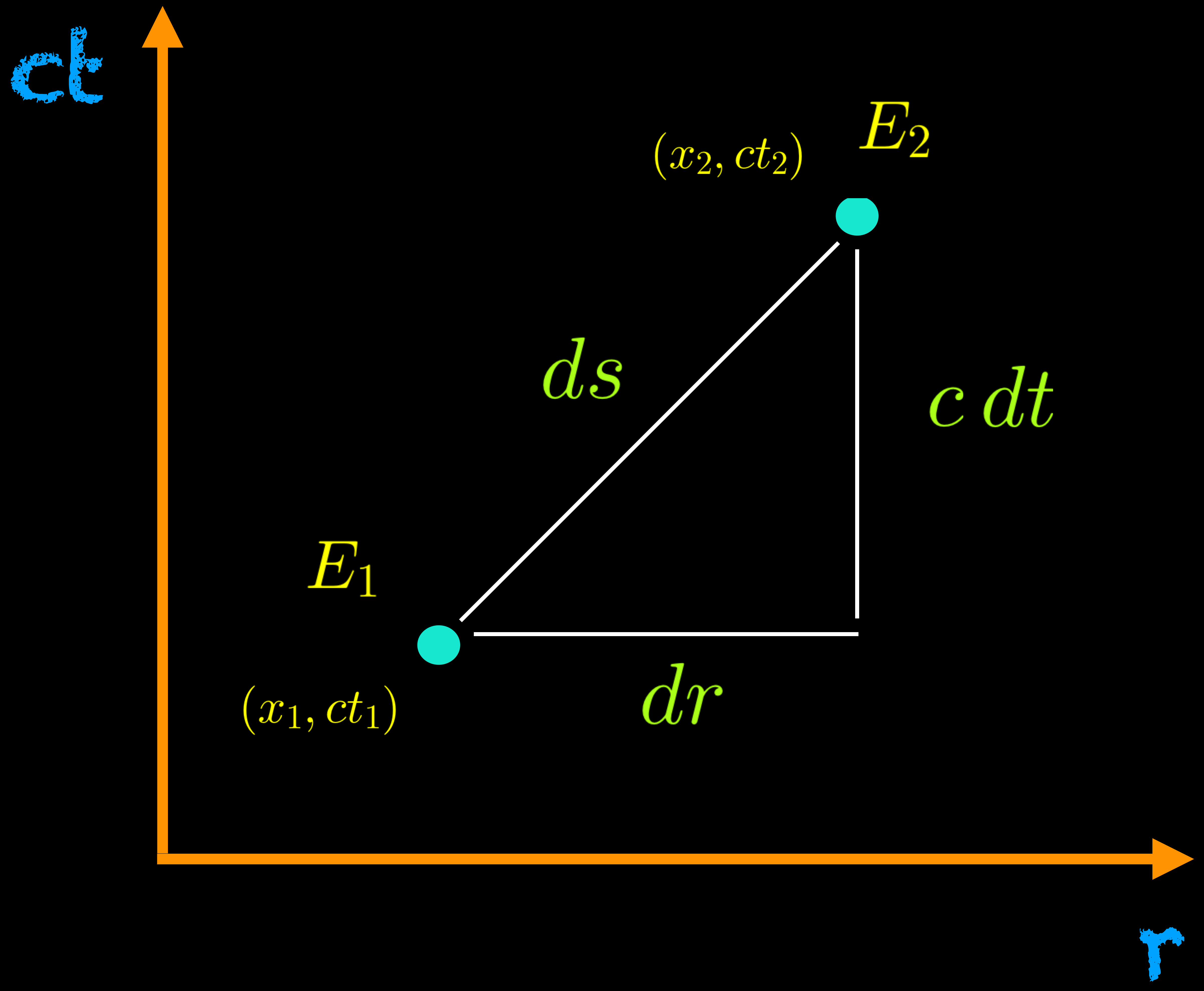




Minkowski Metric

$$ds^2 = c^2dt^2 - dr^2$$

Static and non-changing



Metrics

. . . scale factor

$$ds^2 = c^2 dt^2 \; a^2 (t)^2$$

Friedman-Lemaitre-Robertson-Walker Metric
Without Curvature

$$dr = c\frac{dt}{a(t)}$$

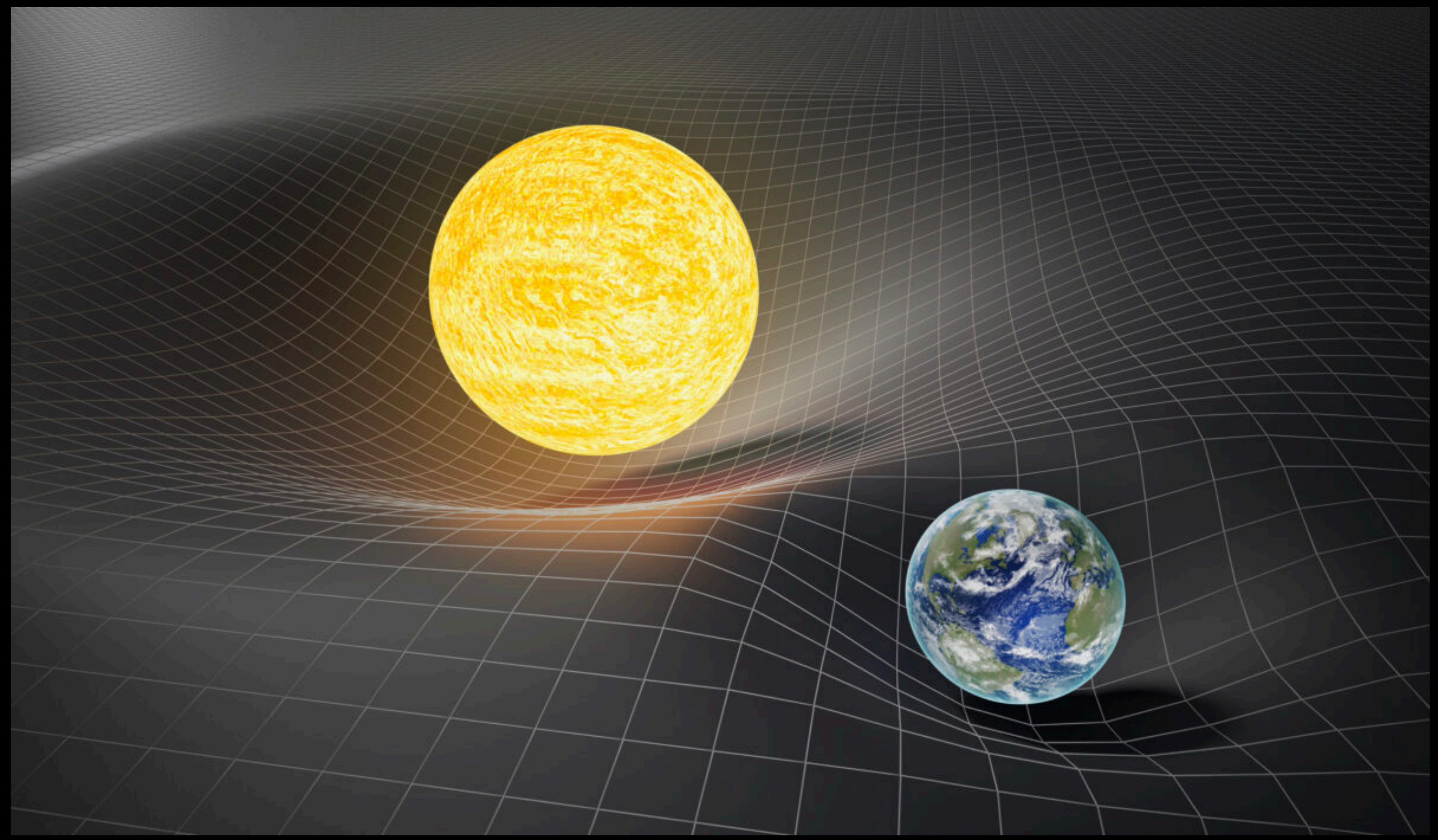
$$= c^2 dt^2 - a^2 (t$$

$$c^2 dt^2 = a^2 (t)$$

$$r_0 = c\int_{t_0}^{\infty} \frac{dt}{a(t)}$$

$$d_{EH} = c a(t_0) \int_{t_0}^{\infty} \frac{dt}{a(t)}$$

The Friedmann Equation



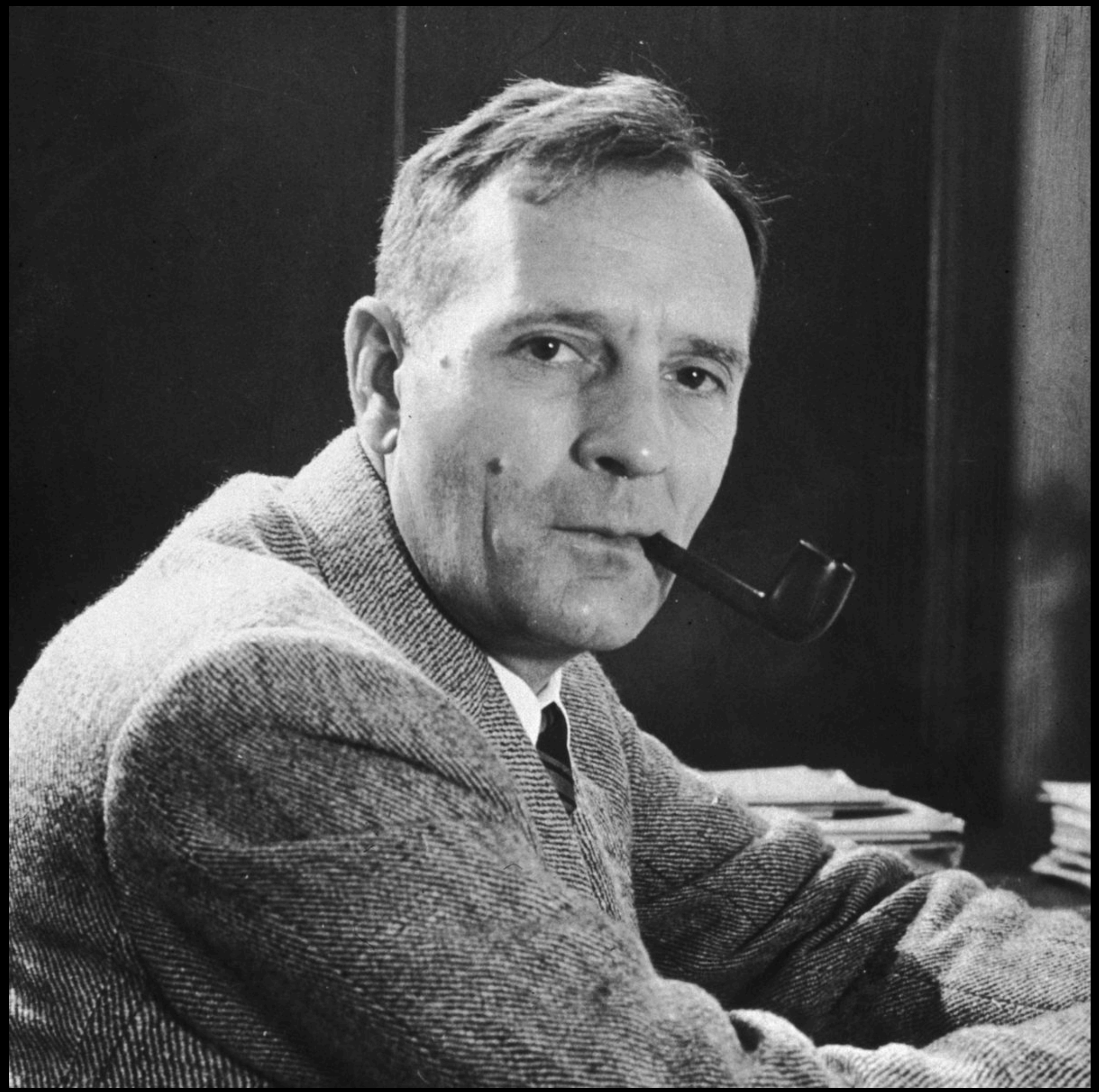
$$E = mc^2$$

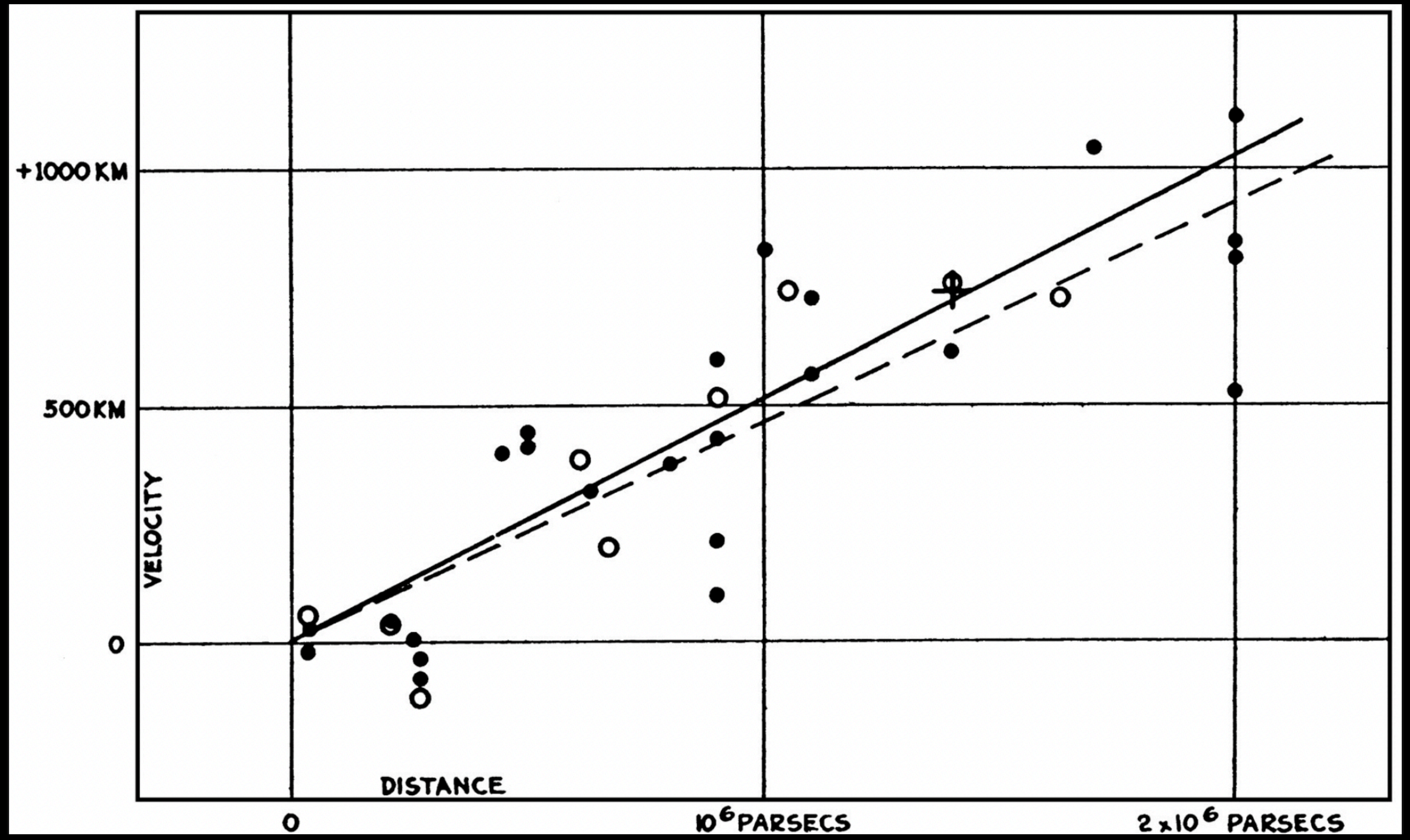
Alexander Friedmann
1922

$$R_{\mu\nu} - \frac{1}{2} R \cdot g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\Downarrow$$

$$H^2(t) = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho(t) + \frac{\Lambda c^2}{3}$$





Hubble Plot 1929

Shows that the velocity of
Distant galaxies increases
With their distance.

$$H^2(t) = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho(t) + \frac{\Lambda c^2}{3}$$

Cosmological Constant

Time-Dependent Hubble Constant

Dark Energy Density

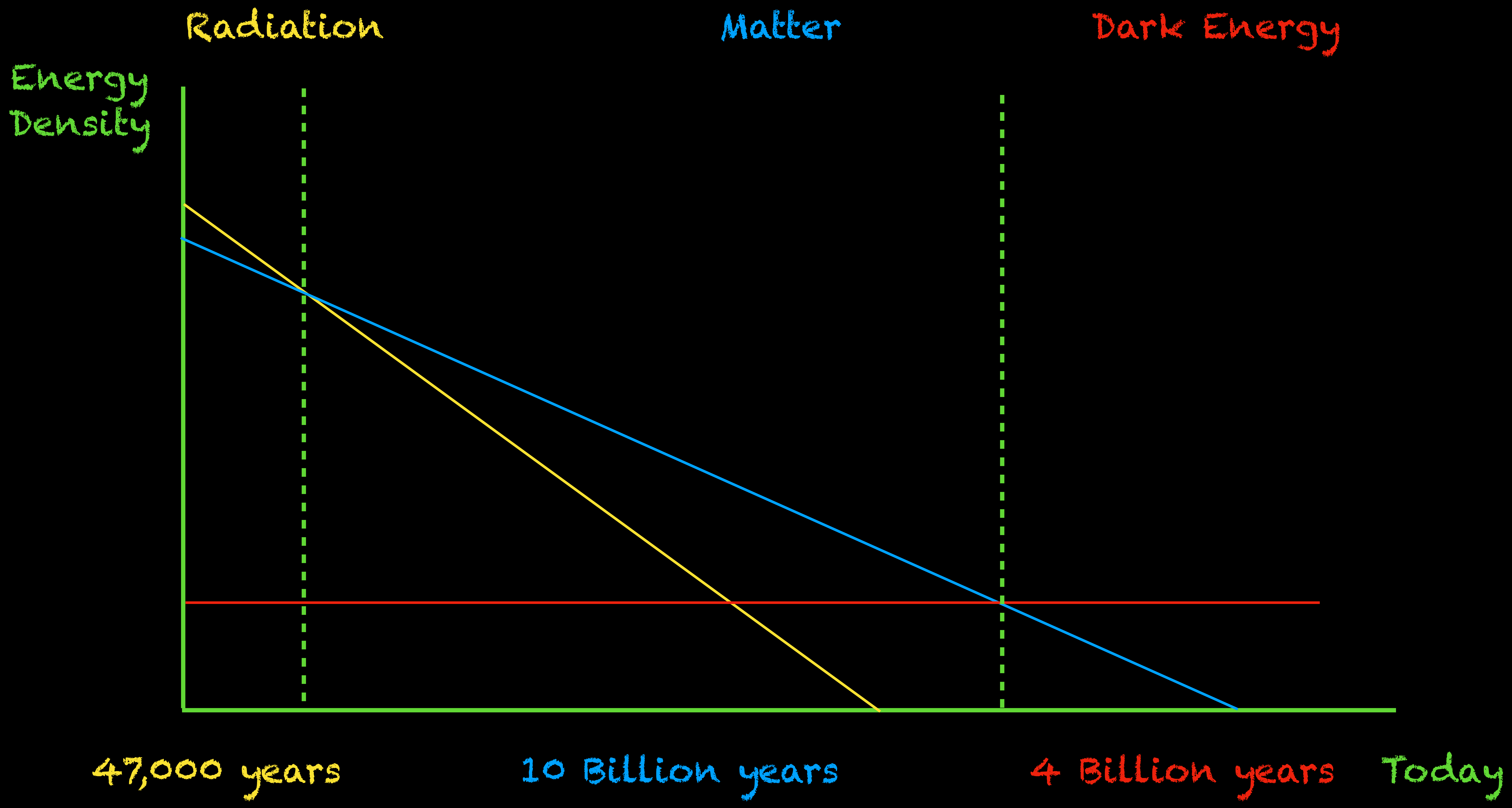
$$\left[\frac{\frac{\text{km}}{\text{s}}}{\text{Mpc}}\right] = \left[\frac{\text{Velocity}}{\text{Distance}}\right]$$

Radiation Density

Matter Density

$$H_0 = 70\ \frac{\text{km}/\text{s}}{\text{Mpc}}$$

$$\rho(t) = \rho_m(t) + \rho_r(t)$$


Radiation
Matter
Dark Energy
Energy Density
47,000 years
10 Billion years
4 Billion years
Today

$$H^2(t) = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho(t) + \frac{\Lambda c^2}{3}$$

$$\downarrow$$

$$\rho(t) = 0$$

$$\downarrow$$

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda c^2}{3}$$

Solving the Friedmann Equation

$$\frac{\dot{a}^2}{a^2} \Longrightarrow \dot{a}^2 = \frac{\Lambda c^2}{3} a^2$$

$$\dot{a} = \frac{da}{dt} \Longrightarrow \frac{da}{a} = \sqrt{\frac{\Lambda c^2}{3}}\, t$$

$$\int \frac{da}{a} = \sqrt{\frac{\Lambda c^2}{3}} \int dt$$

$$\int \frac{da}{a} = \sqrt{\frac{\Lambda c^2}{3}} \int dt$$

$$\ln|a(t)| = \ln(a(t)) = \sqrt{\frac{\Lambda c^2}{3}} t$$

$$\boxed{a(t) = \exp\left(\sqrt{\frac{\Lambda c^2}{3}} t\right)}$$

$$H^2(t) = \frac{\dot{a}^2(t)}{a^2(t)} \Rightarrow H_0 = \sqrt{\frac{\Lambda c^2}{3}}$$

$$d_{EH} = ca(t_0) \int_{t_0}^{\infty} \frac{dt}{a(t)}$$

$$= ca(t_0) \int_{t_0}^{\infty} \exp(-H_0 t)\, dt$$

$$d_{EH} = ca\left(t_0\right)\int_{t_0}^{\infty} \exp\left(-H_0 t\right) dt$$

$$= ca\left(t_0\right)\lim_{\lambda\to\infty}\int_{t_0}^{\lambda} \exp\left(-H_0 t\right) dt$$

$$= c\exp\left(H_0 t_0\right)\lim_{\lambda\to\infty}\left[\frac{1}{H_0}\exp\left(-H_0 t_0\right) - \frac{1}{H_0}\exp\left(-H_0 \lambda\right)\right]$$

$$= c\exp\left(H_0 t_0\right)\frac{1}{H_0}\exp\left(-H_0 t_0\right)$$

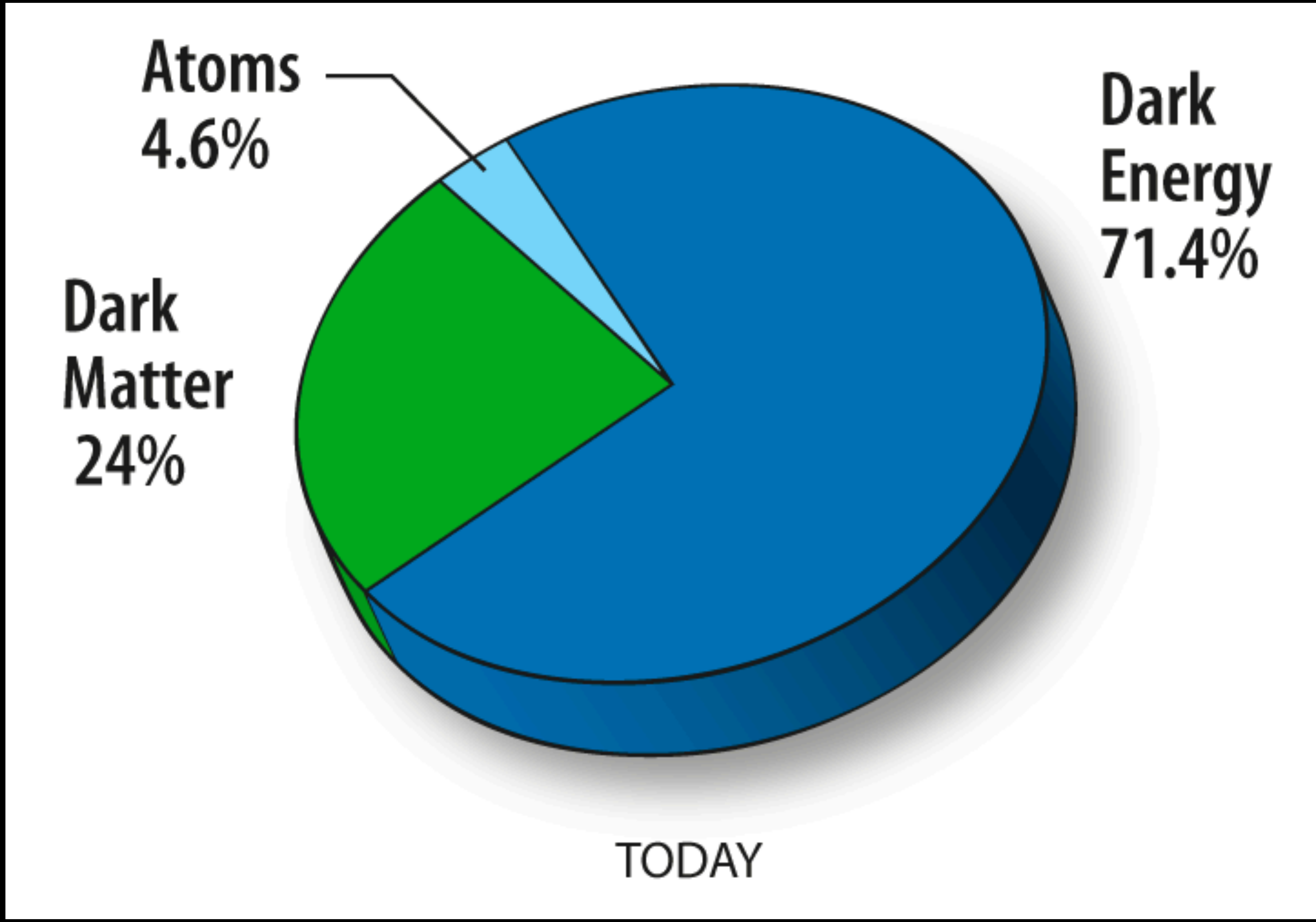
$$d_{EH} = c\exp\left(H_0 t_0\right)\frac{1}{H_0}\exp\left(-H_0 t_0\right)$$

$$= \frac{c}{H_0}$$

c = 300,000 km/s

H_0 = 70 $\frac{\text{km/s}}{\text{Mpc}}$

= 4285 Mpc = 13.9 billion light years


Atoms
4.6%
Dark
Energy
71.4%
Dark
Matter
24%
TODAY

$$H^2(t) = \frac{\dot{a}^2(t)}{a^2(t)} = \frac{\Lambda c^2}{3} \quad \Rightarrow H_0 = \sqrt{\frac{\Lambda c^2}{3}}$$

$$ds^2 = c^2 dt^2 - a^2(t)\, dr^2 = 0$$

$$cdt = adr \qquad dr = c\frac{dt}{a}$$

$$r_0 = c\int_{t_0}^{\infty} \frac{dt}{a(t)}$$

$$d_{EH} = ca(t_0)\int_{t_0}^{\infty} \frac{dt}{a(t)}$$

$$d_{EH} = ca(t_0)\int_{t_0}^{\infty} \exp(-H_0 t)\, dt$$

$$= ca(t_0)\lim_{\lambda\to\infty}\int_{t_0}^{\lambda} \exp(-H_0 t)\, dt$$

$$= c\exp(H_0 t_0)\lim_{\lambda\to\infty}\left[\frac{1}{H_0}\exp(-H_0 t_0) - \frac{1}{H_0}\exp(-H_0\lambda)\right]$$

$$= c\exp(H_0 t_0)\frac{1}{H_0}\exp(-H_0 t_0) \quad = \frac{c}{H_0}$$

$$\lim_{\lambda\to\infty}\exp(-H_0\lambda) = 0$$

$$d_{EH} = ca(t_0)\int_{t_0}^{\infty} \exp(-H_0 t)\, dt$$

$$H^2(t) = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho(t) + \frac{\Lambda c^2}{3}$$

$$\rho_\Lambda = \frac{\Lambda c^2}{3}$$

$$\rho(t) = \rho_m(t) + \rho_r(t)$$

$$\rho(t) = 0$$

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda c^2}{3}$$

$$\dot{a}^2 = \frac{\Lambda c^2}{3} a^2$$

$$\dot{a} = \frac{da}{dt} = \sqrt{\frac{\Lambda c^2}{3}} a$$

$$\frac{da}{a} = \sqrt{\frac{\Lambda c^2}{3}} t$$

$$\int \frac{da}{a} = \sqrt{\frac{\Lambda c^2}{3}} \int dt$$

$$\ln|a(t)| = \ln(a(t)) = \sqrt{\frac{\Lambda c^2}{3}} t$$

$$a(t) > 0$$

$$\Rightarrow a(t) = \exp\left(\sqrt{\frac{\Lambda c^2}{3}} t\right)$$

$$\dot{a} = \frac{da}{dt} = \sqrt{\frac{\Lambda c^2}{3}} a$$

$$ds^2 = c^2dt^2 - dr^2$$

$$ds^2 = c^2dt^2 - a^2(t)\,dr^2 = 0$$

$$c^2dt^2 = a^2(t)\,dr^2$$

$$cdt = a(t)\,dr$$

$$dr = c\frac{dt}{a(t)} \qquad r_0 = \int_0^{r_0} dr = c\int_0^{t_0} \frac{dt}{a(t)}$$

$$d = a(t_0)\,r_0 = c \cdot a(t_0) \cdot \int_0^{t_0} \frac{dt}{a(t)}$$

$$= c \cdot t_0^{\frac{2}{3}} \int_0^{t_0} t^{-\frac{2}{3}}dt \qquad = 3ct_0$$

$$\} = 3t_0^{1/3}$$

$$d = a(t_0)\,r_0 = c \cdot a(t_0) \cdot \int_0^{t_0} \frac{dt}{a(t)}$$