Supplementary material
Statistical modelling of COVID-19 data: Putting Generalised Additive Models to work

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1 Analysing the association between infections from different age groups

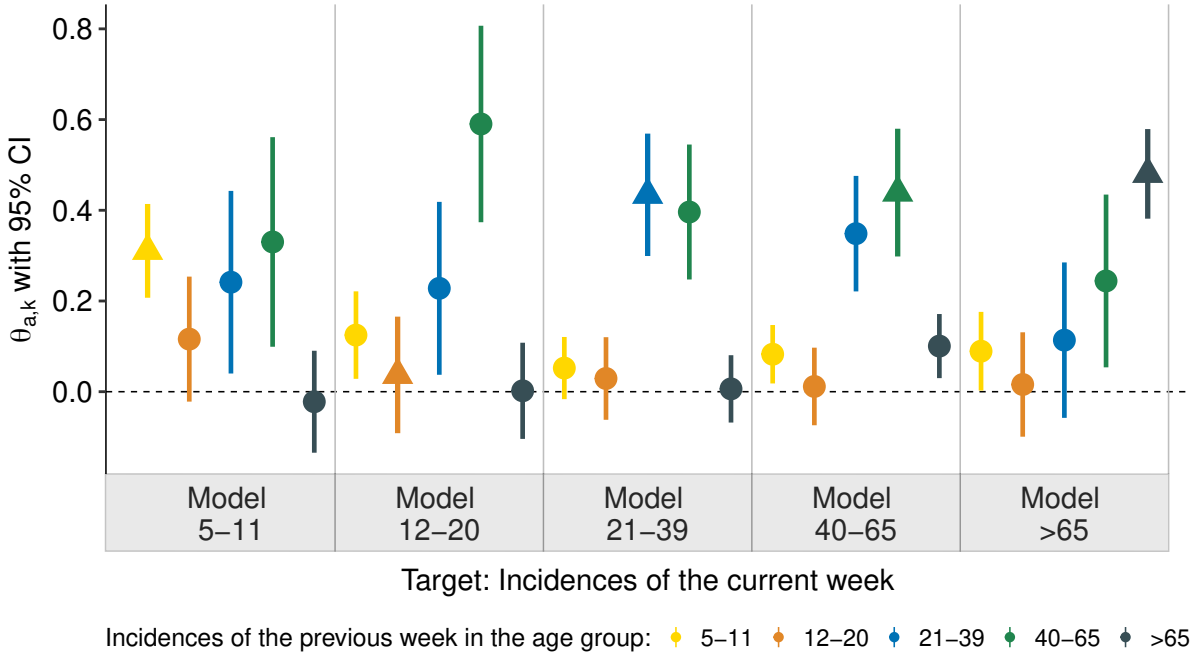

Figure 1: Association of previous week's incidences in different age groups (colour-coded marked) with the current-week incidences for calendar weeks 4-7 2021 stratified by age group.

To prove the robustness of the findings shown in Section 3.3 from the main article, we repeat the same analysis with data between 24.1.2021 and 29.2.2021 (calendar weeks 4 to 7). As can be seen in Figure 1, the estimated coefficients are similar to the ones reported in the principal analysis of Section 3 and result in analogous interpretations.

2 Modelling hospitalisations accounting for reporting delay

2.1 Nowcasting model

In the following, we visualise all estimated effects of the nowcasting model described in Section 4.1 of the main paper. We observe the strongest association from the time delay showing a steep decrease especially in the first days after admission to hospital (Figure 3). Comparing the effects for both age groups, only minor differences can be noticed. The piece-wise linear time effect (Figure 2) shows a positive trend over the considered period with a lower slope in the last four weeks. Differences in weekdays are visible for both, the admission date to hospital as well as the date on which a hospitalised case is reported (Figure 4). For the latter, however, it should be noticed that new data versions of the central Bavarian register are provided from Monday to Friday only and hospitalisations newly reported on Mondays are distributed over Saturday, Sunday and Monday in a preprocessing step.

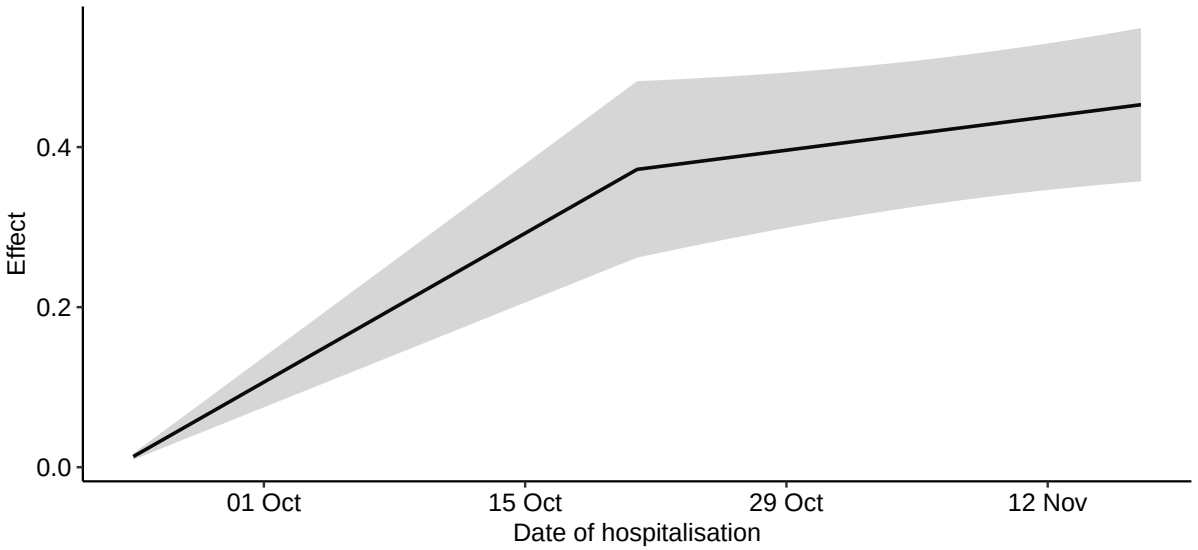

Figure 2: Estimated temporal effect within the time period between September, 24th and November 18th, 2021, accompanied by 95% confidence intervals.

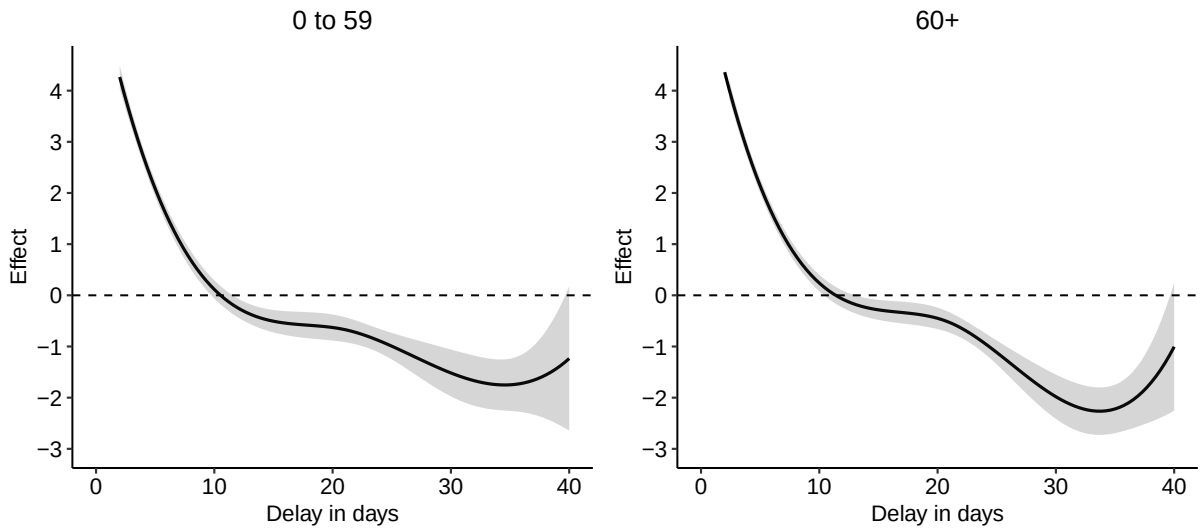

Figure 3: Estimated smooth delay effects between admission to hospital and its reporting for age groups 0-59 (left) and 60+ (right) accompanied by 95% confidence bounds.

2.2 Hospitalisation model

In addition to the results described in Section 4.3, Figures 5 and 6 visualise additional effects in the hospitalisation model for which we controlled for. The smooth time indicates steady positive effects comparable to the development of the nowcasted seven day sum of hospitalisation counts illustrated in Figure 5. The weekday effects reveal a clear pattern over the week, whereas there are rather small differences from Monday to Friday and considerably less hospitalisations occur at the weekend.

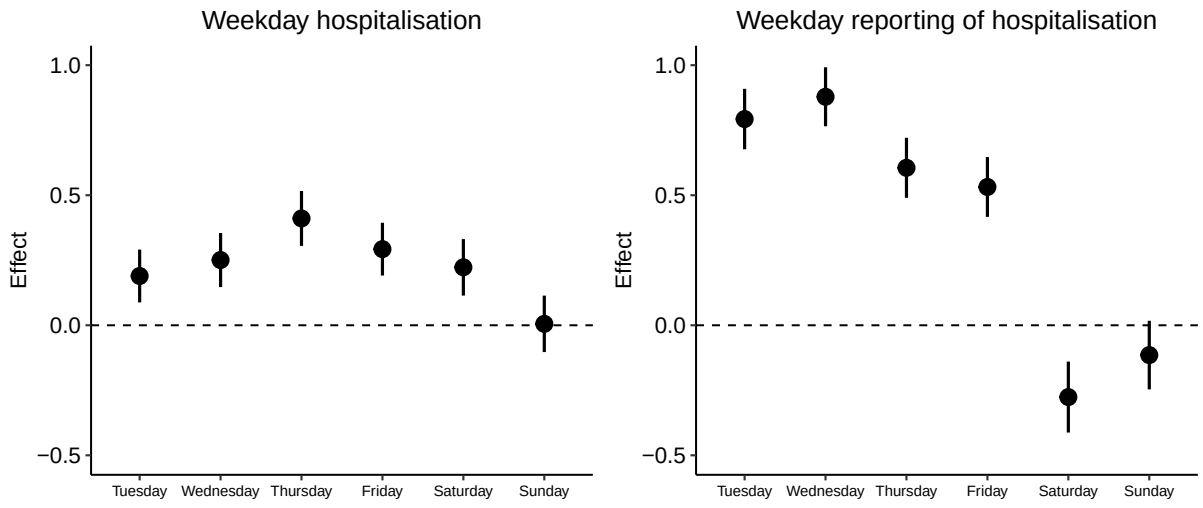

Figure 4: Estimated linear weekday effects regarding date of hospitalisation (left) and reporting date of hospitalisation (right) accompanied by 95% confidence intervals. The reference category is taken to be Monday, respectively.

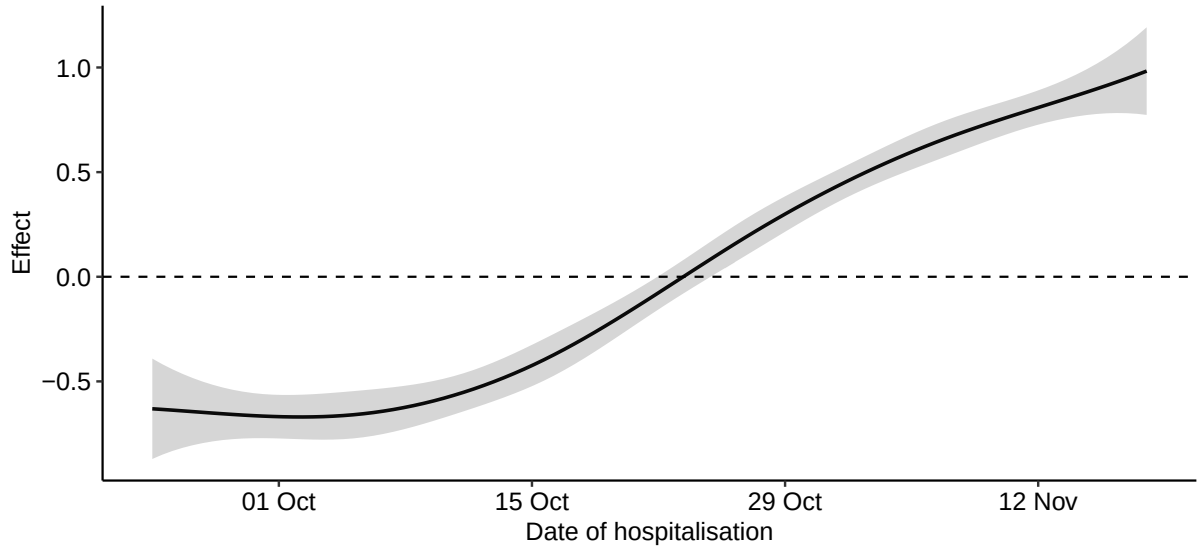

Figure 5: Estimated smooth temporal effect on hospitalisation counts within the time period between September, 24th and November 18th, 2021, accompanied by 95% confidence bounds.

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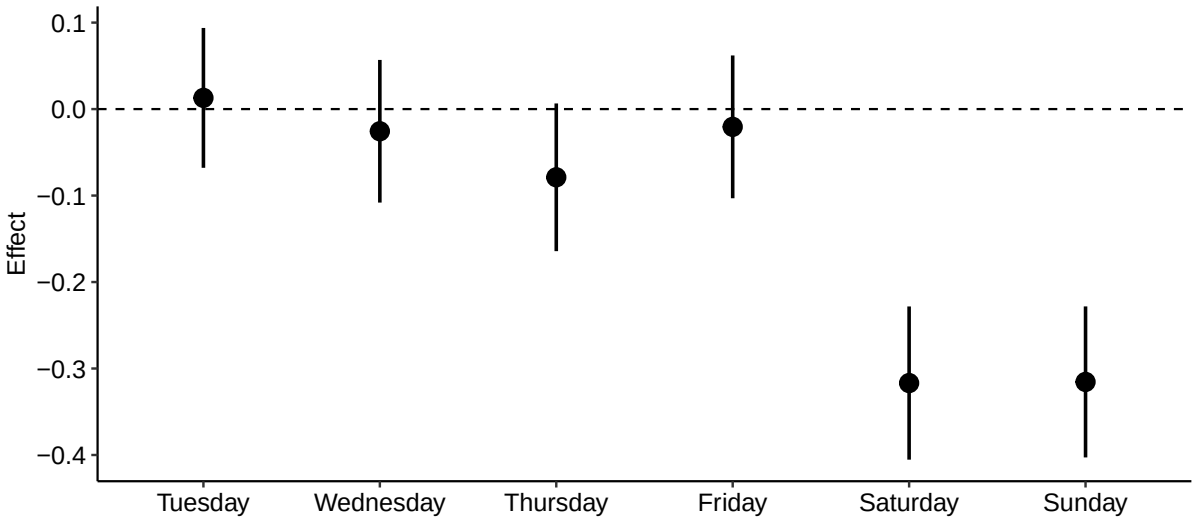

Figure 6: Estimated linear weekday effects accompanied by 95% confidence intervals. The reference category is taken to be Monday.

2.3 Missing hospitalisation dates

Modelling hospitalisations relies on the availability of hospital admission dates of COVID-19 patients. In cases where no dates are reported (about 9.6% of hospitalised cases reported after September 24th), the reporting date of the infection is used instead. This choice is justified by the seven-day sum of hospitalisations over time, calculated based on both types of dates, respectively. Figure 7 shows that there are no structural differences between both time series considering hospitalisations where both types of dates are available.

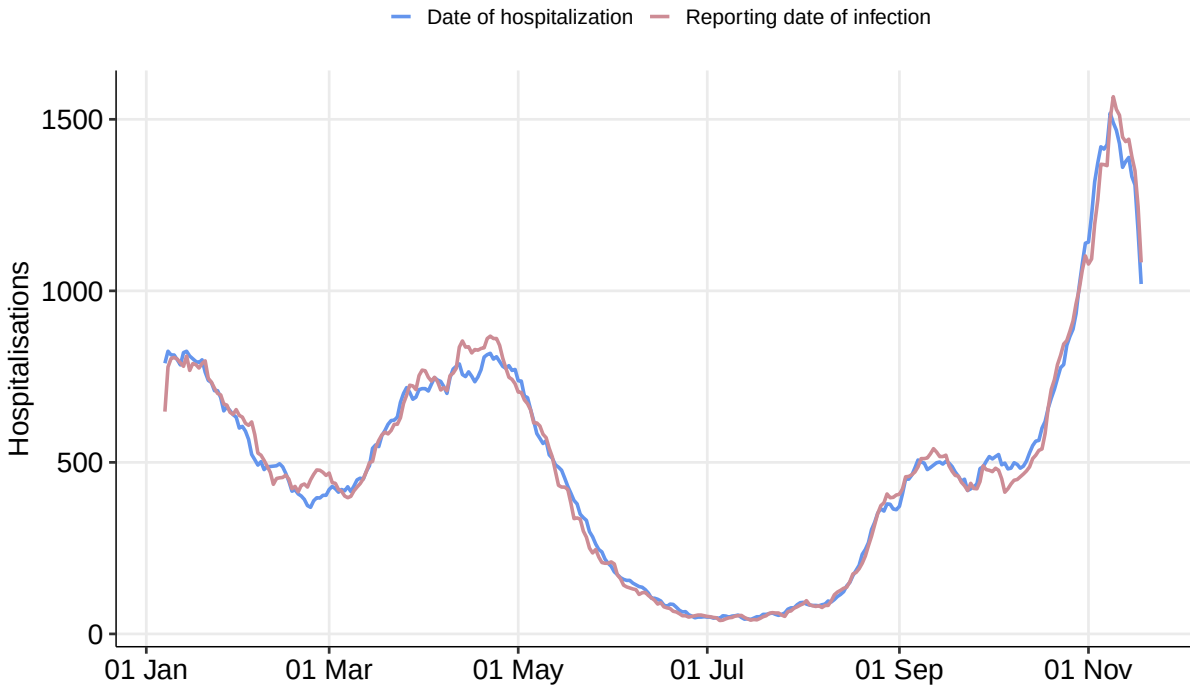

Figure 7: Comparison of the time series of the seven day sum of hospitalisations based on date of hospitalisation and reporting date of infection of hospitalised cases over the period from January 1st to November 18th, 2021. For this analysis only hospitalisations with known date of hospitalisation are included.

To further evaluate the robustness of our approach regarding missing dates of hospitalisation, we conducted a sensitivity analysis where the hospitalisation date was set seven days after the reporting date of the infection date of the infection. If the hospitalisation was reported earlier than seven days after the reporting date of the infection, the day before the report of the hospitalisation was considered as hospitalisation date. The results of this sensitivity analysis which show only minor differences compared to the analysis in the main part are given below.

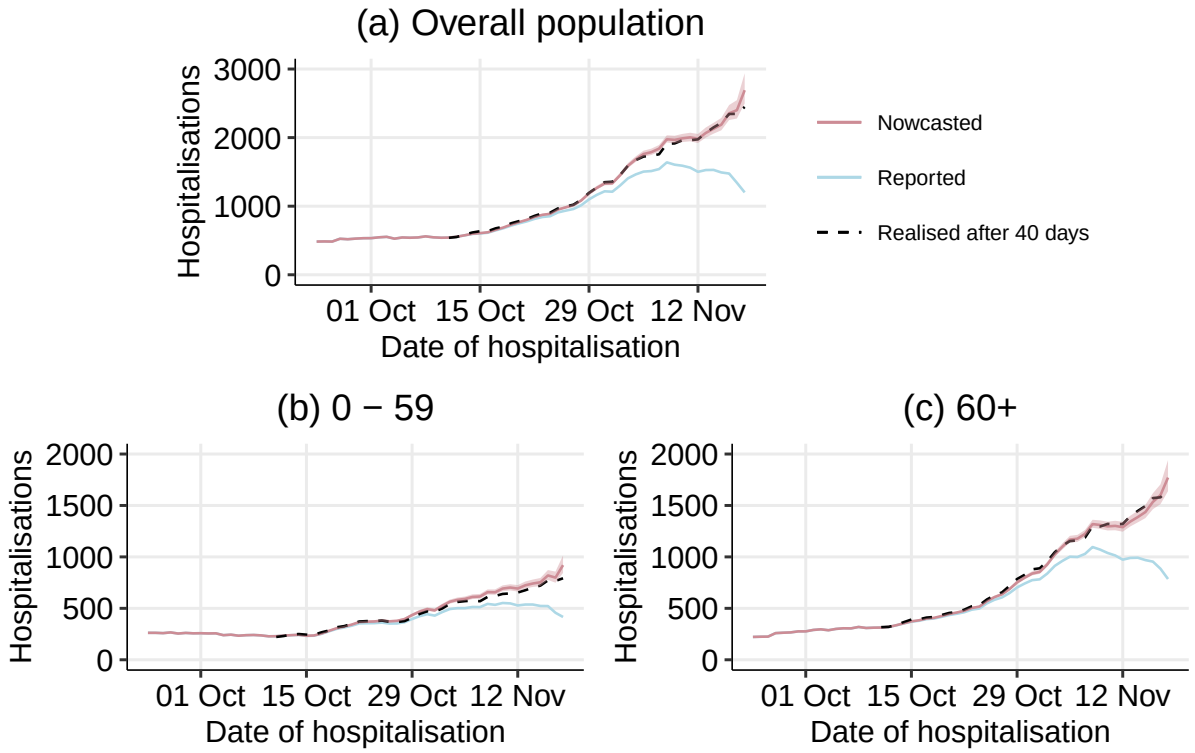

Figure 8: Comparison of nowcasted (red) and reported (blue) rolling weekly sums of hospitalisation counts between September 24th and November 18th, 2021, based on data reported as of November 19th, 2021. 95% confidence intervals of the nowcast estimates are indicated by the shaded areas. Results are displayed for the overall population (a) as well as for age groups 0-59 (b) and 60+ (c).

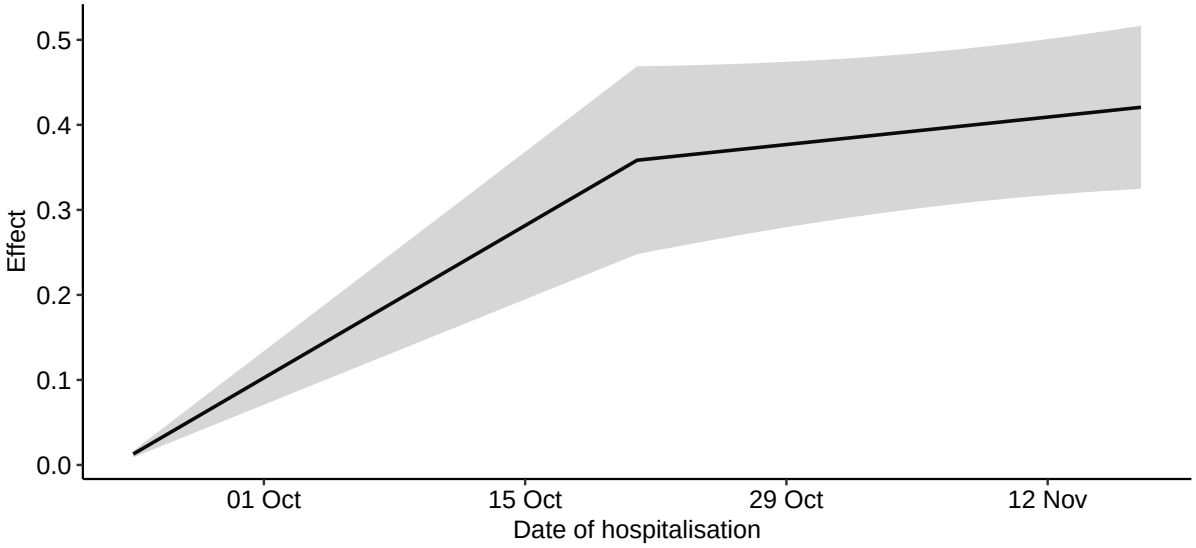

Figure 9: Estimated temporal effect within the time period between September, 24th and November 18th, 2021, accompanied by 95% confidence intervals.

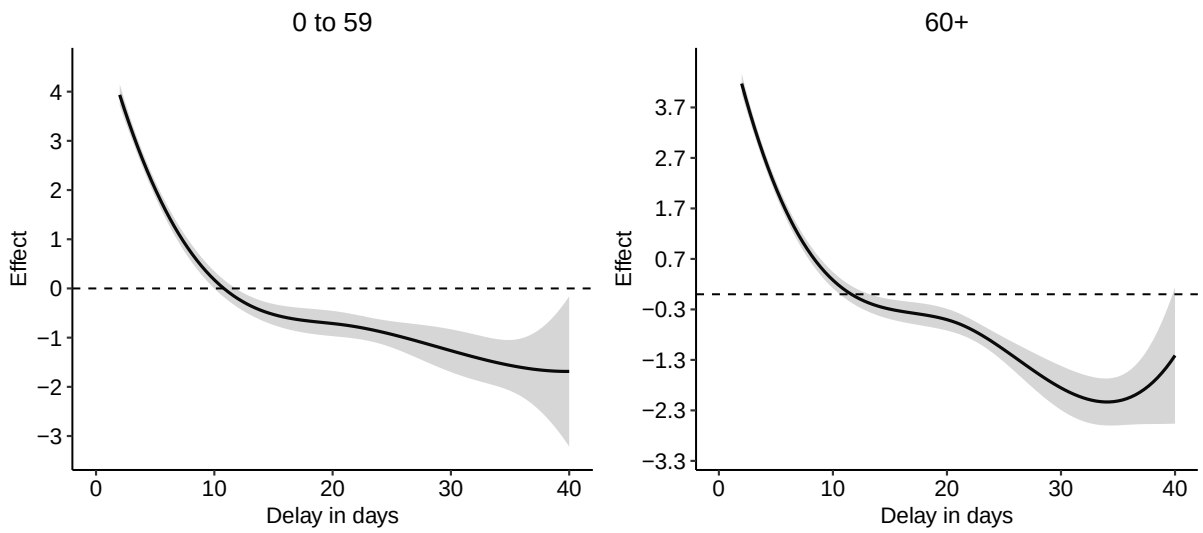

Figure 10: Estimated smooth delay effects between admission to hospital and its reporting for age groups 0-59 (left) and 60+ (right) accompanied by 95% confidence bounds.

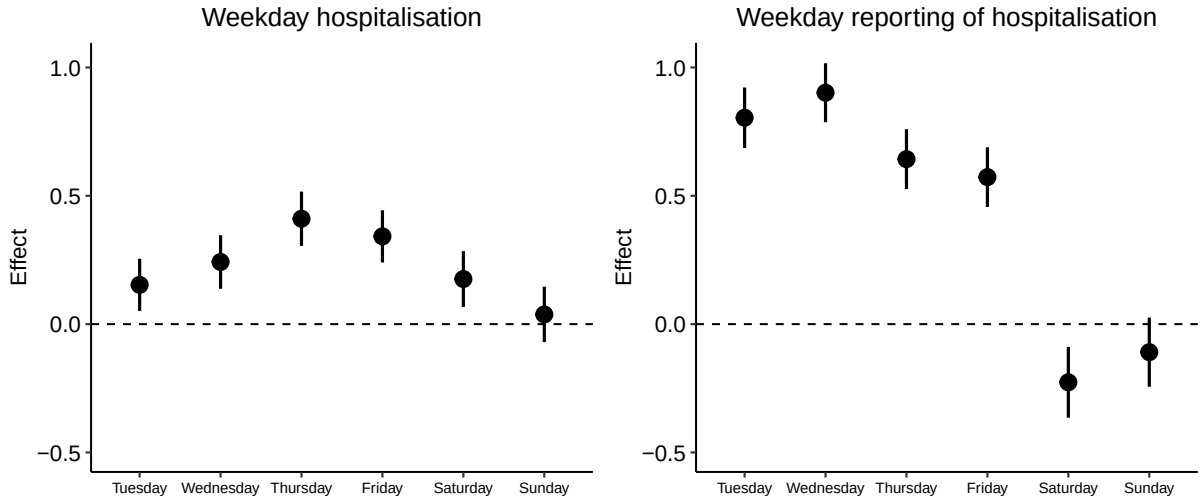

Figure 11: Estimated linear weekday effects regarding date of hospitalisation (left) and reporting date of hospitalisation (right) accompanied by 95% confidence intervals. The reference category is taken to be Monday, respectively.

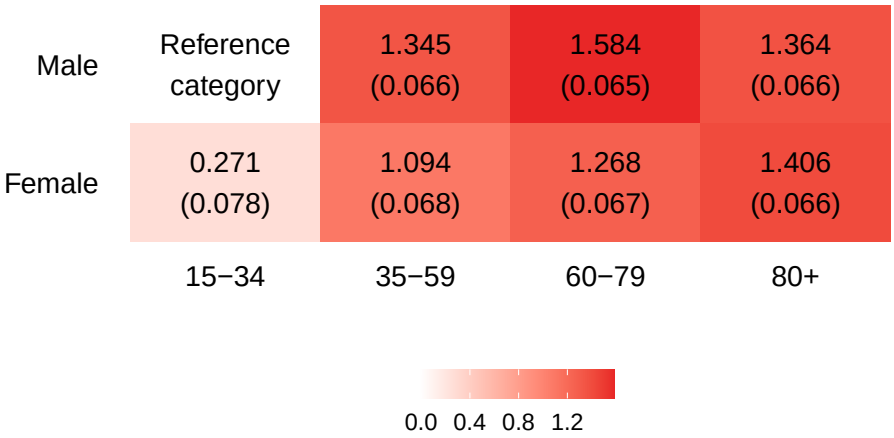

Figure 12: Estimated linear effects for different age and gender groups in the hospitalisation model, where males aged 15 to 34 are the reference category. Estimated standard deviations are written in brackets.

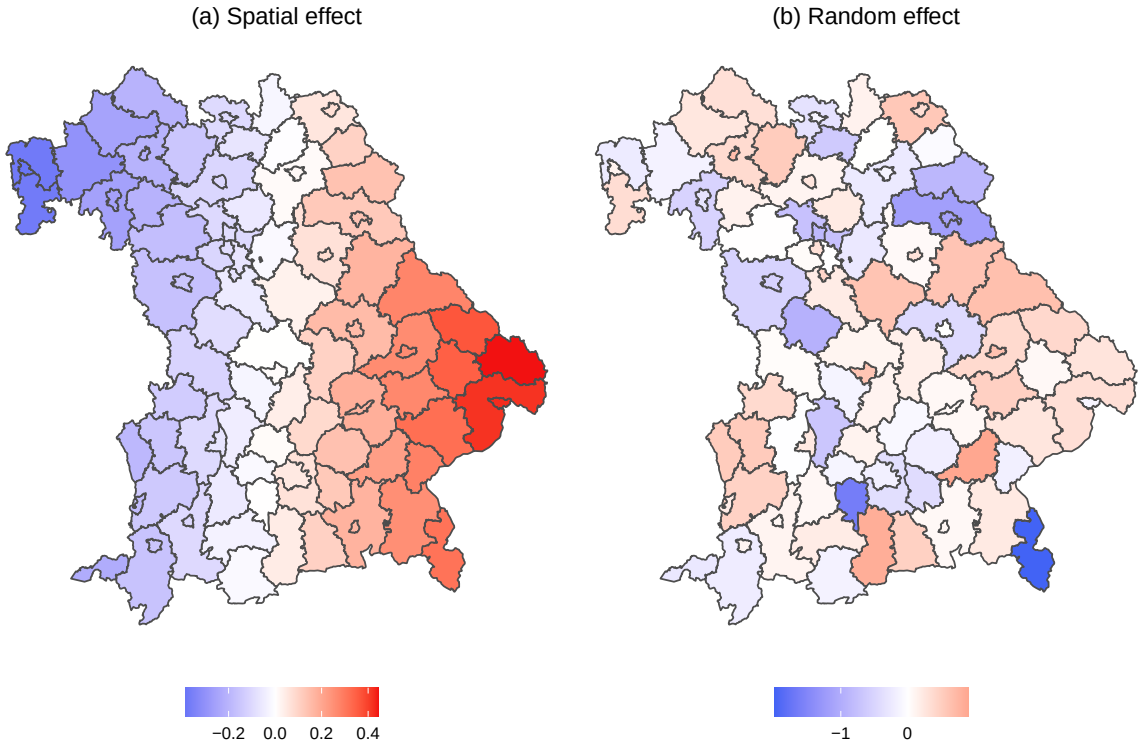

Figure 13: Estimated smooth spatial effect (a) and district-specific random effect (b) in the hospitalisation model.

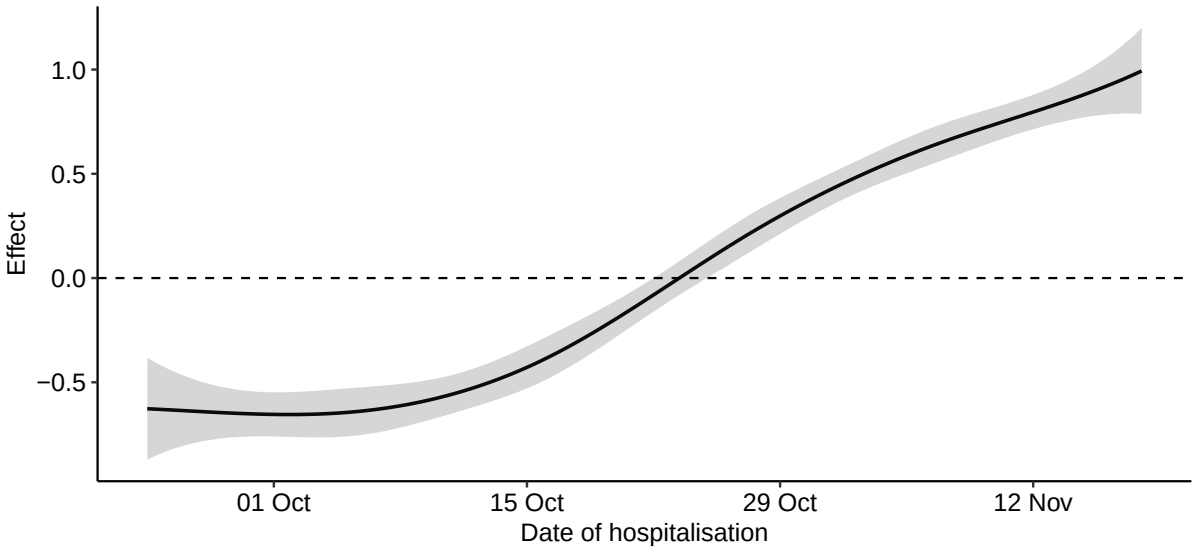

Figure 14: Estimated smooth temporal effect on hospitalisation counts within the time period between September, 24th and November 18th, 2021, accompanied by 95% confidence bounds.

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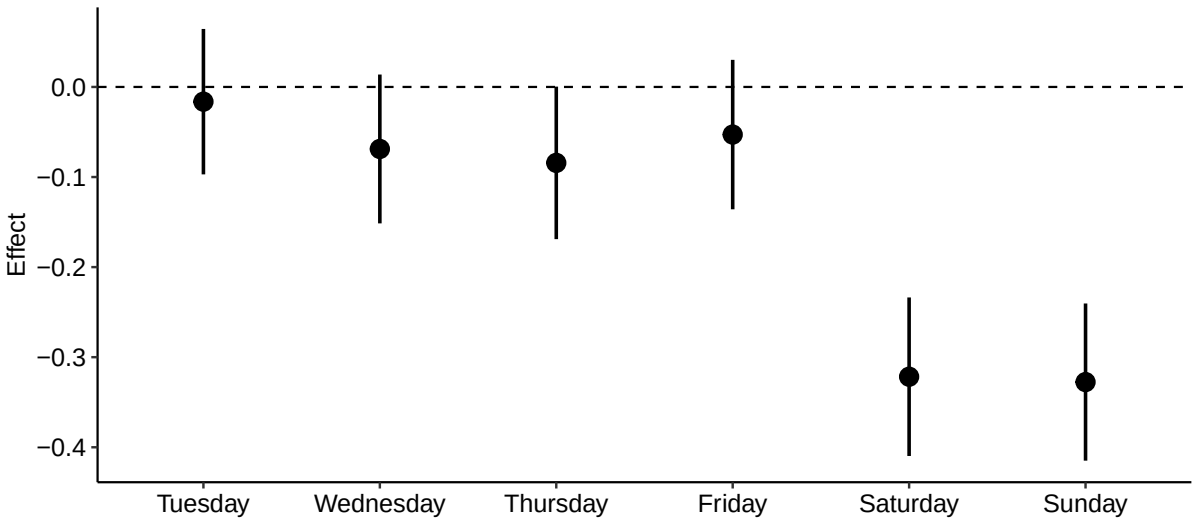

Figure 15: Estimated linear weekday effects accompanied by 95% confidence intervals. The reference category is taken to be Monday.

3 Modelling ICU occupancy

3.1 Estimates

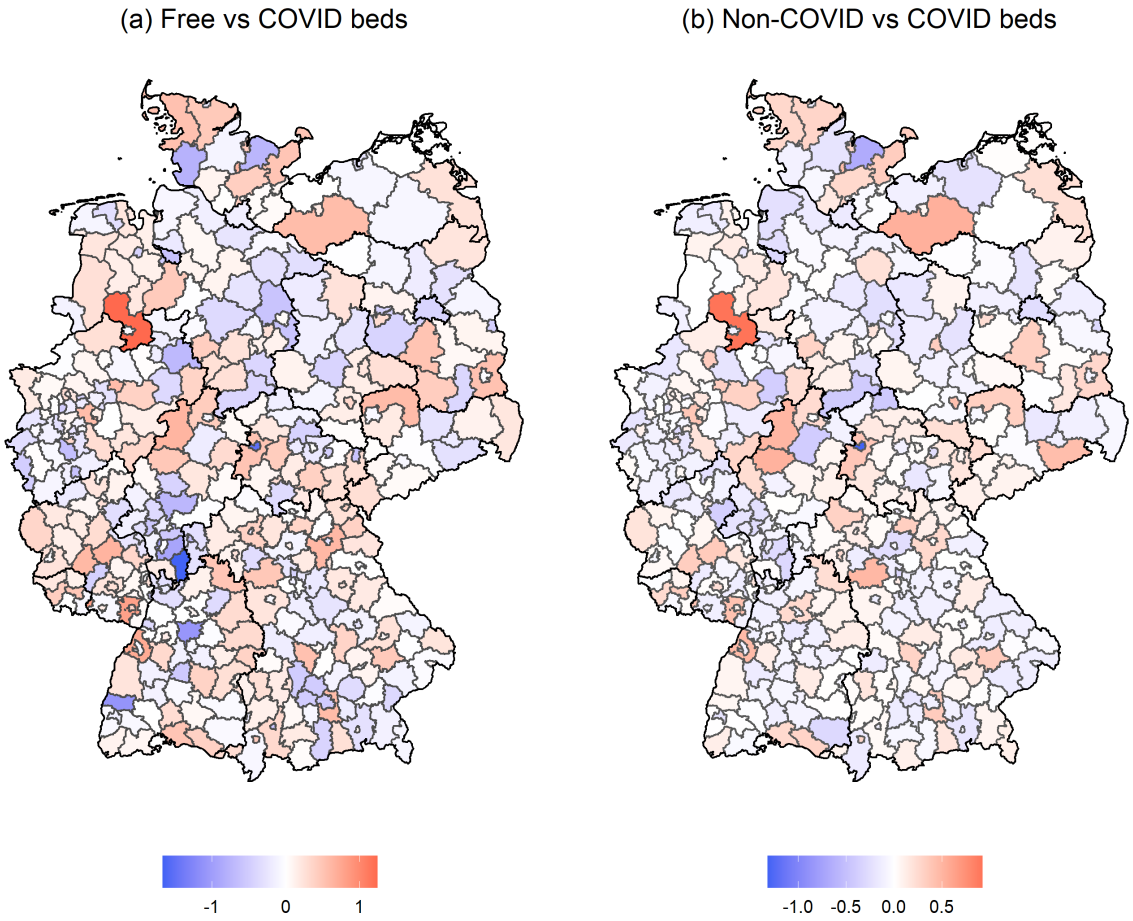

Figure 16: District specific random intercept in estimating the logarithmic odds of an ICU-bed not being occupied vs occupied by a patient infected with covid (a), and in estimating the logarithmic odds of an ICU bed being occupied by a patient not infected with COVID-19 vs a bed being occupied by a patient infected with COVID-19 (b)

Figures 16 and 17 depict the estimated district-specific random and smooth effects, respectively. Upon first inspection, there does not necessarily seem to be a clearly visible pattern in the estimated random effects, shown in Figure 16. However, on a closer look, districts that include towns or even cities seem to have a lower estimated effect, while purely rural areas seem to have a higher area-specific estimated effect on both pairwise comparisons of the ICU occupancy. The model used to analyse the number of free beds over the number of beds occupied by patients infected with COVID-19 estimates a variance of the district specific random effects of 0.137. Analogously, the model used to analyse the number of beds occupied by non-covid patients over the number of beds occupied by patients infected with COVID-19 estimates a variance of the district specific random effects of 0.051.

Besides the area-specific associations estimated by our model, there could also be a spatial association in the occupancy throughout Germany. The estimates are given in Figure 17. Generally, there seems to be a north-south divide, as the smoothed spatial

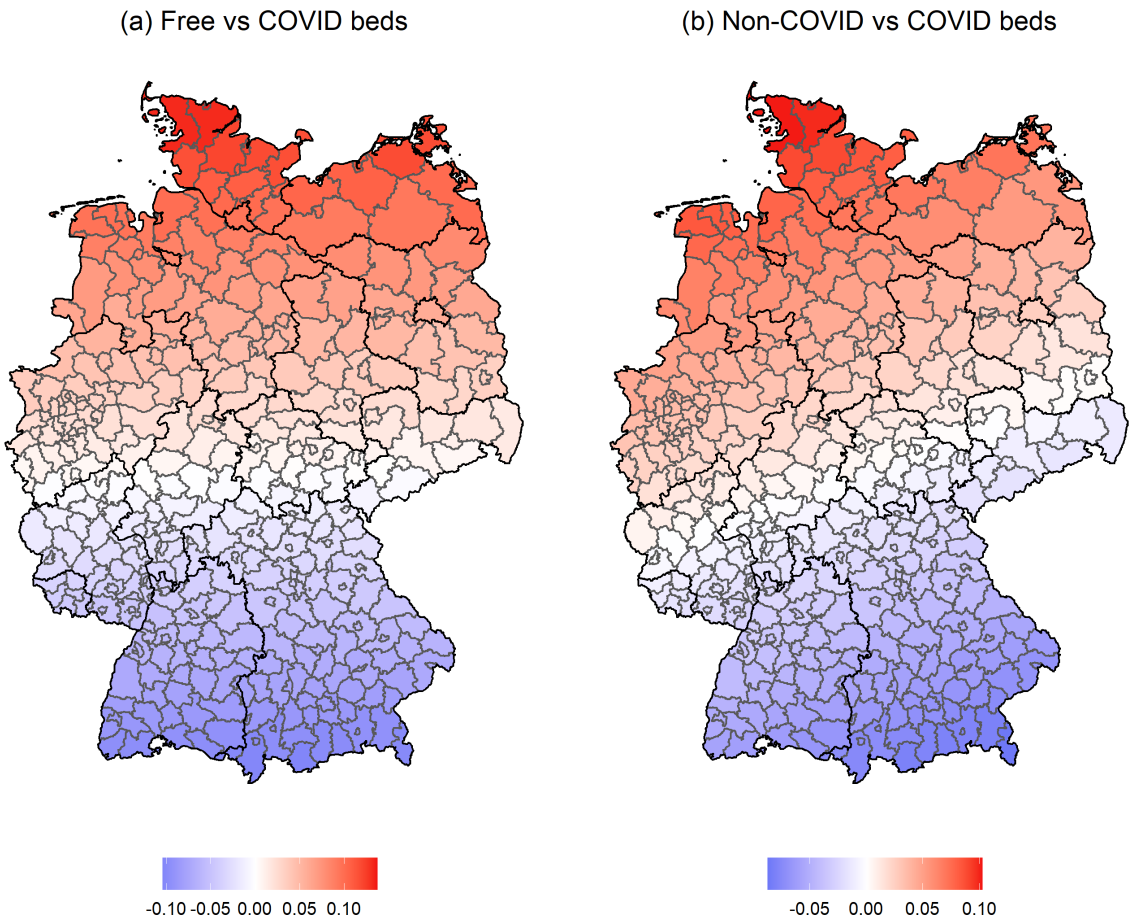

Figure 17: Estimated centred smooth spatial association with the logarithmic odds of an ICU-bed not being occupied vs occupied by a patient infected with covid (a), and with the logarithmic odds of an ICU bed being occupied by a patient not infected with COVID-19 vs a bed being occupied by a patient infected with COVID-19 (b)

effect appears to be higher in the north and lower in the south, where a belt from seemingly below Dresden stretching through Germany to what seems to be just below Bonn is the partition with an estimated centred effect of around zero.

3.2 Forecasting

Besides interpreting the coefficients, we can use the same model to predict the occupancy of beds in the next week. To show how this works in practice, we carry out one-step-ahead predictions and evaluate the results in a rolling window framework. To obtain forecasts for the occupancy in week w, we train our model with the prior 8 weeks, i.e., $w-9, ..., w-1$, and use the information on week w as a test set. Since we only incorporate covariables that are lagged by one week, this setting is equivalent to providing real forecasts for the week after the end of the observational period. We let w vary between the 27th of September in 2020 (calendar week 40 in 2020) and the 12th of September in 2021 (calendar week 37 in 2021). Within this time frame, we cover two critical infection waves as well as low-infection seasons. In this way, we can assess the behaviour of our predictions in many different realistic scenarios. To measure the goodness of the model, we rely on strictly proper scoring rules (Gneiting and Raftery, 2007) and use the logarithmic score

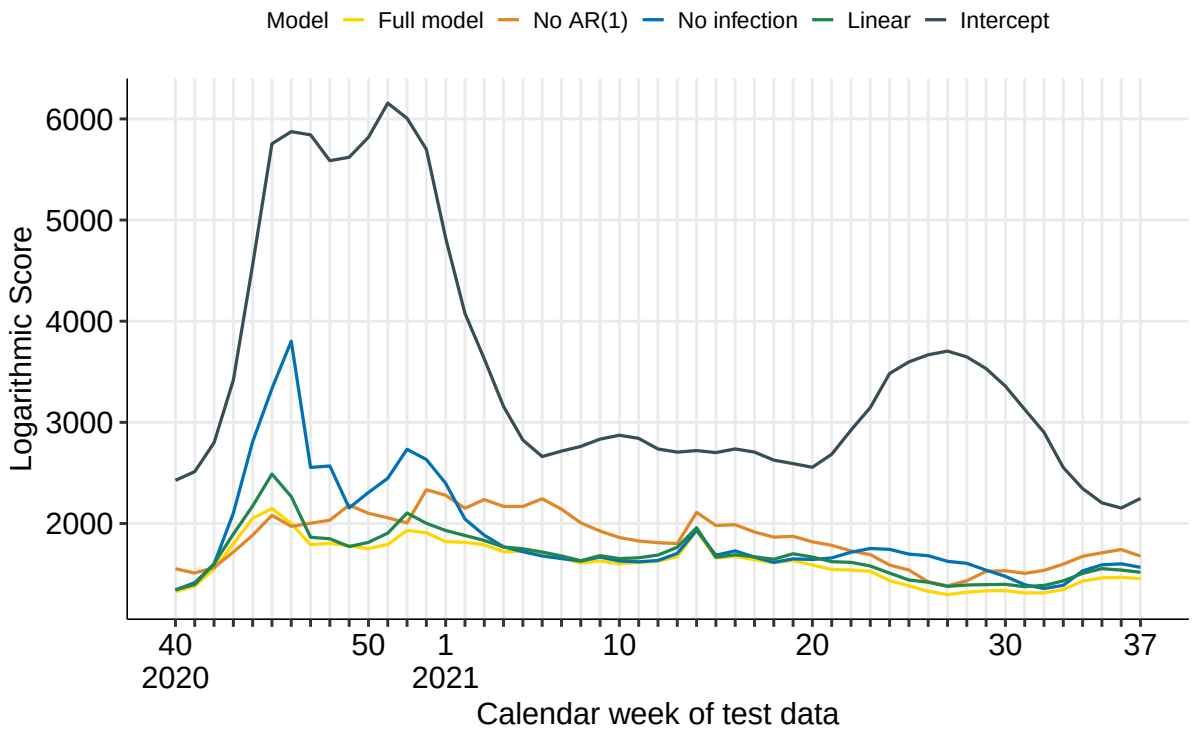

Figure 18: The logarithmic score of all candidate models from the one-week-ahead forecasts between October 2020 and September 2021.

Table 1: Averaged logarithmic score and p-values from pairwise permutation tests comparing all one-week-ahead forecasts to the full model.

Model	Omitted effects	Average score	p-value
Full model	-	1619.243	
No AR(1)	$\theta_{AR(1),j}$	1847.879	(<0.0001)
No infection	$\theta_{I,j}$	1876.082	(<0.0001)
Linear	$s_j(x_{r,\text{coord}}), u_{r,j}$	1689.651	(<0.0001)
Intercept	all but $\theta_{0,j}$	3502.312	(<0.0001)

as employed in Held et al. (2017) and defined by the logarithmic probability to observe the occupancy in week w under the multinomial model from (**??**) and the predicted $\pi_{w,r}$ from the trained model. We compare the performance of the full model as specified in (**??**) with simplified specifications of the model without the lagged infection effect, without random and spatial effects, without autoregressive effects, and only including an intercept term. To further differentiate between all models, we calculate average scores for each model and employ pairwise permutation tests to compare all sub-models against the predictions under the full model as proposed in Diebolt and Mariano (2002).

Figure 18 shows the results of our one-step-ahead forecasts for all proposed models. Overall, the average score indicates that the full model provides better performance than all other model specifications. Consistently low p-values from the permutation tests underscore this finding. We can use the intercept model to identify the high and low infection seasons regarding temporal discrepancies between the models. It is also apparent that the model performs poorly during the infection waves when leaving out the infection data, underlining that lagged infections are crucial during these periods. Not including

the autoregressive component in the model, on the other hand, seems to mainly impair the logarithmic score during low-infection periods such as in calendar weeks 1 to 25 in 2021. In comparison, the random and spatial effects hardly affect the model's predictive power. However, the logarithmic score is still significantly better when including them. In summary, we can deduce from Figure 18 that the full specification consistently provides the best forecasts for high and low infection periods.

References

Diebolt, F. X. and R. S. Mariano (2002). Comparing predictive accuracy. *Journal of Business & Economic Statistics 20*(1), 134–144.

Gneiting, T. and A. E. Raftery (2007). Strictly proper scoring rules, prediction, and estimation. *Journal of the American Statistical Association 102*(477), 359–378.

Held, L., S. Meyer, and J. Bracher (2017). Probabilistic forecasting in infectious disease epidemiology: The 13th Armitage lecture. *Statistics in Medicine 36*(22), 3443–3460.