Statistical Network Data Analysis in Economics

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Abstract

Networks are ubiquitous in economic research on organizations, trade, and many other topics. However, while economic theory extensively considers networks, no general framework for their empirical modeling has yet emerged. We thus introduce two different statistical models for this purpose – the Exponential Random Graph Model (ERGM) and the Additive and Multiplicative Effects network model (AME). Both model classes can account for network effects, such as reciprocity and triadic closure, but differ in how they do so. The ERGM allows one to explicitly specify and test the influence of particular network structures, making it a natural choice if one is substantively interested in estimating endogenous network effects. In contrast, AME captures these effects by introducing actor-specific latent variables which affect their propensity to form ties. This makes the model a good choice if the researcher is interested in capturing the effect of exogenous covariates on tie formation without having a specific idea of the endogenous dependence structures at play. We introduce the two model classes and apply them to networks stemming from international arms trade and foreign exchange activity. Moreover, we provide full replication materials to facilitate the adoption of these methods in empirical economic research.

Keywords – Inferential Network Analysis, Network Data, Endogeneity, Arms Trade, Foreign Exchange Networks
JEL classification – C20, C49, F14, F31, L14

1 Introduction

The study of networks has become an important topic in economics. Thanks in large part to the seminal work of Jackson (2008), research in the field has surged in recent years. Within the broader context of the study of complex and interdependent systems (see e.g. Flaschel et al., 1997, 2007, 2018), networks can be defined as interconnected structures which can naturally be represented though graphs. Networks have been extensively considered from a theoretical perspective, with the primary goal of understanding how economic behavior is shaped by network patterns of interaction (Jackson et al., 2017). Research in this direction on, e.g., organizations as networks, diffusion in networks, network experiments, or network games, is surveyed in Bramoullé et al. (2016), Jackson (2014), and Jackson et al. (2017). These theoretical advances find application in many different fields, in which network structures naturally arise from, e.g., national and international trade, commercial agreements, firms' organization, and collaboration activity. However, they have not yet been accompanied by a corresponding shift in the standard methods used to empirically validate them. Some recent contributions (see e.g. Chaney, 2014; Morales et al., 2019) develop estimators tailored specifically to their network-based theoretical models, but more generally applicable modeling frameworks for the analysis

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of real-world network data have not yet emerged. Statistical methods specifically designed to empirically test theories where interdependencies arise from network structures, such as the Exponential Random Graph Model (ERGM), exist but are not yet widely used by economists. Jackson (2014), for instance, discusses ERGMs but argues that they "suffer from proven computational problems" (2014, p.76). Jackson et al. (2017) further explain that "it is practically impossible to estimate the likelihood of a given network at even a moderately large scale", concluding that with ERGMs, "there is an important computational hurdle that must be overcome in working with data" (2017, p.85).

Contrasting this assessment, we argue that recent work in the realm of empirical network analysis provides robust and scalable methods with readily available implementations in the statistical software package `R` (R Core Team, 2021). Computational issues thus do not represent an insurmountable barrier to employ robust inferential network methods anymore. In this paper, we demonstrate the effectiveness and usability of some of those methods by applying them to real economic data. We specifically focus on Exponential Random Graph Models (ERGM) (Robins et al., 2007) and Additive and Multiplicative Effect (AME) network models (Hoff, 2021), respectively implemented in the `R` packages `statnet` (Handcock et al., 2008) and `amen` (Hoff, 2015). We find these two model classes to be among the most promising ones for applications in the economic sciences, as they are well suited for answering two broad categories of research questions. The ERGM is an ideal fit if, based on economic theory, the researcher envisages a particular dependence structure for the existence of ties in the network at hand and wants to test whether their theory is corroborated by empirical data. On the other hand, AME, and more generally continuous latent variable models, are a good choice when the researcher is interested in capturing the effect of exogenous variables on tie formation without having prior knowledge on which endogenous network dependencies are prevalent. In this case, AME offers the possibility to estimate the effect of both nodal and pairwise covariates of interest while simultaneously controlling for network effects, which may induce bias if ignored. In addition, the estimated latent structure can provide the researcher with insight on the underlying network mechanisms for which they are controlling.

Our goal is to explicitly discuss these models in terms of their value for economic research. Due to its focus on economic questions, this paper differs from similar surveys in Physics (Newman, 2003), Statistics (Goldenberg et al., 2010), or Political Science (Cranmer et al., 2017). By discussing two broadly applicable modeling frameworks, i.e. ERGM and AME, it also departs from the existing overviews of network approaches in economics that we are aware of (see Bramoullé et al., 2016; Jackson, 2014; Jackson et al., 2017). Rather than providing a comprehensive overview, this paper showcases the practical application of ERGM and AME models, demonstrating their capabilities by applying them to two substantively relevant economic networks. We first use the ERGM to model the international trade of major conventional weapons, where a directed tie exists if one country transfers arms to another. In line with Chaney (2014), network effects such as directed triadic closure are of explicit theoretical interest in this application, and the ERGM allows for their proper specification and testing. We then make use of the AME model to study a historical network of global foreign exchange activity, where a directed edge is present if one country's national currency is actively traded within the other country. AME allows us to estimate how relevant country features, such as per-capita gdp and the gold standard, and pairwise covariates, such as the distance between two countries and their reciprocal trade volume, influence tie formation, while controlling for network effects to provide unbiased estimates. In addition to a step-by-step analysis and interpre-

tation of these application cases, we provide full replication code in the Supplementary Material and in our GitHub repository[1], allowing for seamless reproducibility. This is done to demonstrate the "off-the-shelf" applicability of these methods, and offer applied researchers a headstart in employing them to study substantive economic problems.

2   Economic Networks

2.1   Related literature

Even though network structures naturally arise in many aspects of economics and are subject of important economic research, much of the previous literature has ignored the implied interdependencies, instead opting for regression models assuming independence conditional on the covariates (e.g. Anderson and Van Wincoop, 2003, Rose, 2004, Lewer and Van den Berg, 2008). This assumption is often unreasonable in practice. It would, for example, imply that Germany imposing economic sanctions on Russia is independent of Italy imposing sanctions on Russia, and, in the directed case, even of Russia imposing them on Germany itself. While no standard framework for the modeling of empirical network data has emerged in economics so far, a number of contributions in – or adjacent to – the field do make use of statistical network models. We shortly survey these works here to show that the models we present are indeed suitable for the analysis of economic data. Possibly the most obvious kind of economic network is the international trade network (see Chaney, 2014) and many of these studies accordingly seek to model the formation of trade ties. In this vein, two early studies (Ward and Hoff, 2007; Ward et al., 2013) apply latent position models to show that, beyond what a standard gravity model can capture, trade exhibits a latent network structure (see also Fagiolo, 2010; Dueñas and Fagiolo, 2013). More recently, numerous contributions have used the ERGM to explicitly theorize and understand network interdependence in the general trade (Herman, 2022; Liu et al., 2022; Smith and Sarabi, 2022) as well as the trade in arms (Thurner et al., 2019; Lebacher et al., 2021), patents (He et al., 2019), and services (Feng et al., 2021).

That being said, empirical research on economic networks is not limited to international trade. Smith et al. (2019) use multilevel ERGMs to study a production network consisting of ownership ties between firms at the micro-level and trade ties between countries at the macro-level, while Mundt (2021) explores the European Union's sector-level production network via ERGMs as well as an alternative network model, the stochastic agent-oriented model (SAOM). Fritz et al. (2022) deploy ERGMs to investigate patent collaboration networks. Studies on foreign direct investments document network influences using latent position models (Cao and Ward, 2014) and seek to model them via extensions of the ERGM (Schoeneman et al., 2022). Finally, economists also study networks of interstate alliances and armed conflict (see e.g. Jackson and Nei, 2015; König et al., 2017), both of which have been modeled via ERGMs (Cranmer et al., 2012; Campbell et al., 2018) and AME (Dorff et al., 2020; Minhas et al., 2021). This short survey indicates that both ERGM and AME can be used to answer questions which are of substantive interest to economists. To facilitate their wider acceptance as complementary options for answering different questions when facing economic network data, we present applied introductions to these two models.

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[1]https://github.com/gdenicola/statistical-network-analysis-in-economics

2.2 Setup

Before introducing models for networks in which dependencies between ties are expected, we introduce the mathematical framework for networks, as well as the necessary notation. Let $\boldsymbol{y} = (y_{ij})_{i,j=1,...,n}$ be the adjacency matrix representing the observed binary network comprising n fixed and known agents (nodes). In this context, $y_{ij} = 1$ indicates an edge directed from agent i to agent j, while $y_{ij} = 0$ translates to no edge. Since self-loops are not admitted for most studied networks, the diagonal of $\boldsymbol{y}$ is left unspecified or set to zero. Depending on the application, the direction of an edge can carry additional information. If it does, we call the network directed. In this article, we mainly focus on this type of networks. Also note that all matrix-valued objects are written in bold font for consistency. In addition to the network connections, we often observe covariate information on the agents, which can be at the level of single agents (e.g. the gdp of a country) or at the pairwise level (e.g. the distance between two countries). We denote covariates by $\boldsymbol{x}_1, ..., \boldsymbol{x}_p$, and our goal is to specify a statistical model for $\boldsymbol{Y}$, that is the random variable corresponding to $\boldsymbol{y}$, conditional on $\boldsymbol{x}_1, ..., \boldsymbol{x}_p$. A natural way to do this is to specify a probability distribution over the space of all possible networks, which we define by the set $\mathcal{Y}$. Two main characteristics differentiate our modeling endeavor from classical regression techniques, such as Probit or logistic regression models. First, for most applications, we only observe one realization $\boldsymbol{y}$ from $\boldsymbol{Y}$, rendering the estimation of the parameters to characterize this distribution particularly challenging. Second, the entries of $\boldsymbol{Y}$ are generally co-dependent; thus, most conditional dependence assumptions inherent to common regression models are violated. Generally, we term mechanisms that induce direct dependence between edges to be endogenous, while all effects external to the modeled network, such as covariates, are called exogenous.

3 The Exponential Random Graph Model

The ERGM is one of the most popular models for analyzing network data. First introduced by Holland and Leinhardt (1981) as a model class that builds on the platform of exponential families, it was later extended with respect to fitting algorithms and more complex dependence structures (Lusher et al., 2012; Robins et al., 2007). We next introduce the model step-by-step to highlight its ability to generalize approaches building on conditional dependence assumptions.

3.1 Accounting for dependence in networks

We begin with the simplest possible stochastic network model, the Erdös-Rényi-Gilbert model (Erdös and Rényi, 1959; Gilbert, 1959), where all edges are assumed to be independent with the same probability of being observed. In stochastic terms, each observed tie is then a realization of a binomial random variable with success probability $\boldsymbol{\pi}$, which yields

$$\mathbb{P}_{\boldsymbol{\pi}}(\boldsymbol{Y} = \boldsymbol{y}) = \prod_{i=1}^{n} \prod_{j \neq i} \boldsymbol{\pi}^{y_{ij}} (1 - \boldsymbol{\pi})^{1 - y_{ij}} \tag{1}$$

for the probability to observe $\boldsymbol{y}$. Evidently, model (1) with equal probability for all possible ties is too restrictive for application to real world problems. In the next step, we, therefore, additionally incorporate x_{ij} by letting $\boldsymbol{\pi}$ vary over those covariates, leading

to edge-specific probabilities π_{ij}. Following the common practice in logistic regression, we parameterize the log-odds by $\log\left(\frac{\pi_{ij}}{1-\pi_{ij}}\right) = \boldsymbol{\theta}^\top x_{ij}$, where x_{ij} is a vector of exogenous statistics with the first entry set to 1 to incorporate an intercept, and get

$$\mathbb{P}_\theta(\boldsymbol{Y} = \boldsymbol{y}) = \prod_{i=1}^n \prod_{j \neq i} \left(\frac{\exp\{\boldsymbol{\theta}^\top x_{ij}\}}{1 + \exp\{\boldsymbol{\theta}^\top x_{ij}\}} \right)^{y_{ij}} \left(\frac{1}{1 + \exp\{\boldsymbol{\theta}^\top x_{ij}\}} \right)^{1-y_{ij}}. \tag{2}$$

From (2), the analogy to standard logistic regression being a special case of generalized linear models (Nelder and Wedderburn, 1972) becomes apparent. The joint distribution of $\boldsymbol{Y}$ can be formulated in exponential family form, yielding

$$\mathbb{P}_\theta(\boldsymbol{Y} = \boldsymbol{y}|x) = \frac{\exp\{\boldsymbol{\theta}^\top s(\boldsymbol{y})\}}{\kappa(\boldsymbol{\theta})}, \tag{3}$$

where $s(\boldsymbol{y}) = (s_1(\boldsymbol{y}), ..., s_p(\boldsymbol{y}))$, $s_q(\boldsymbol{y}) = \sum_{i=1}^1 \sum_{j \neq i} y_{ij} x_{ij,q} \ \forall \ q = 1, ..., p$, with $x_{ij,q}$ as $q-th$ entry in x_{ij} and $\kappa(\boldsymbol{\theta}) = \prod_{i=1}^n \prod_{j \neq i}(1 + \exp\{\boldsymbol{\theta}^\top x_{ij}\})$. In the jargon of exponential families, we term $s(\boldsymbol{y})$ sufficient statistics.

Newcomb (1979) observed that many observed networks exhibit complicated relational mechanisms, including reciprocity, which we can account for by extending the set of sufficient statistics. Under reciprocity, an edge Y_{ji} influences the probability of its reciprocal edge Y_{ij} to occur. Analyzing social networks, we would expect that the probability of agent i nominating agent j to be a friend is higher if agent j has nominated agent i as a friend. Holland and Leinhardt (1981) extended model (1) to such settings with the so-called p_1 model. To represent reciprocity, we assume dyads, each of them defined by (Y_{ij}, Y_{ji}), to be independent of one another, which again yields an exponential family distribution similar to (3) with sufficient statistics that count the number of mutual ties $(s_{\text{Mut}}(\boldsymbol{y}) = \sum_{i<j} y_{ij} y_{ji})$, of edges $(s_{\text{Edges}}(\boldsymbol{y}) = \sum_{i=1}^n \sum_{j \neq i} y_{ij})$, and the in- and out-degree statistics for all degrees observed in the networks[2]. Agents' in- and out-degrees are their number of incoming and outgoing edges and relate to their relative position in the network (Wasserman and Faust, 1994).

Next to reciprocity, another important endogenous network mechanism is transitivity, originating in the structural balance theory of Heider (1946) and adapted to binary networks by Davis (1970). Transitivity affects the clustering in the network, implying that a two-path between agents i and j, i.e. $y_{ih} = y_{hj} = 1$ for some other agent h, affects the edge probability of Y_{ij}. Put differently, Y_{ij} and Y_{kh} are assumed to be independent iff $i, j \neq k$ and $i, j \neq h$. Frank and Strauss (1986) proposed the Markov model to capture such dependencies. For this model, the sufficient statistics are star-statistics, which are counts of sub-structures in the network where one agent has (incoming and outgoing) edges to between 0 and $n-1$ other agents, and counts of triangular structures. We may define different types of triangular structures depicted in Figure 1 for directed networks.

3.2 Extension to general dependencies

Starting from the Erdös-Rényi-Gilbert model, which is a special case of a generalized linear model, we have consecutively allowed more complicated dependencies between edges, resulting in the Markov graphs of Frank and Strauss (1986). Over this course, we showed that each model can be stated in the exponential family form, characterized by a particular set of sufficient statistics. We now make this more explicit to allow for more

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[2]In the Supplementary Material we provide more details on this derivation.
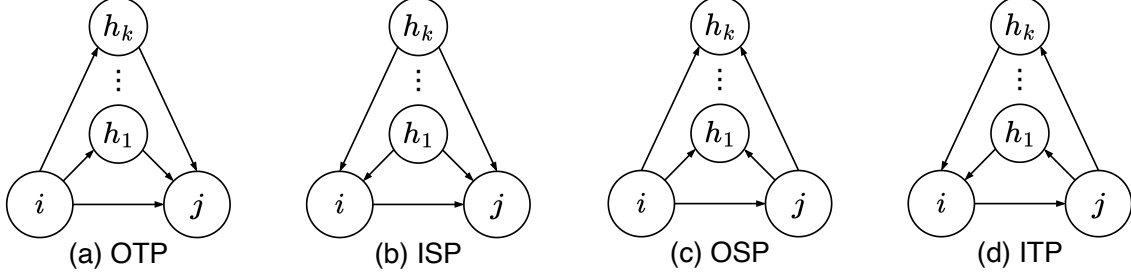
Figure 1: Illustration of the triangular statistics for k partners. Circles represent agents, black lines represent edges between them. The names follow `statnet` nomenclature: OTP = "Outgoing Two-Path", ISP = "Incoming Shared Partner", OSP = "Outgoing Shared Partner", and ITP = "Incoming Two-path".

general dependence structures, and specify a probabilistic model for $\boldsymbol{Y}$ directly through the sufficient statistics[3]. Wasserman and Pattison (1996) introduced this model as

$$\mathbb{P}_{\boldsymbol{\theta}}(\boldsymbol{Y} = \boldsymbol{y}) = \frac{\exp\{\boldsymbol{\theta}^{\top} s(\boldsymbol{y})\}}{\kappa(\boldsymbol{\theta})}, \tag{4}$$

where $\boldsymbol{\theta}$ is a p-dimensional vector of parameters to be estimated, $s(\boldsymbol{y})$ is a function calculating the vector of p sufficient statistics for network $\boldsymbol{y}$, and $\kappa(\boldsymbol{\theta}) = \sum_{\tilde{\boldsymbol{y}} \in \mathcal{Y}} \exp\{\boldsymbol{\theta}^{\top} s(\tilde{\boldsymbol{y}})\}$ is a normalizing constant to ensure that (4) sums up to one over all $\boldsymbol{y} \in \mathcal{Y}$. To estimate $\boldsymbol{\theta}$, Handcock (2003) adapted the Monte Carlo Maximum Likelihood technique of Geyer and Thompson (1992), approximating the logarithmic likelihood ratio of $\boldsymbol{\theta}$ and a fixed $\boldsymbol{\theta}_0$ via Monte Carlo quadrature (see Hunter et al., 2012 for an in-depth discussion).

A problem often encountered when fitting model (4) to networks is degeneracy (Handcock, 2003; Schweinberger, 2011). Degenerate models are characterized by probability distributions that put most probability mass either on the empty or on the full network, i.e., where either all or no ties are observed. To detect this behavior, one can use a goodness-of-fit procedure where observed network statistics are compared to statistics of networks simulated under the estimated model (Hunter et al., 2008). To address it, Snijders et al. (2006) and Hunter and Handcock (2006) propose weighted statistics that, in many cases, have better empirical behavior. Degeneracy commonly affects model specifications encompassing statistics for triad counts and multiple degree statistics. For in-degree statistics, we would thus incorporate the statistic

$$GWIDEG(\boldsymbol{y}, \boldsymbol{\alpha}) = \exp\{\boldsymbol{\alpha}\} \sum_{k=1}^{n-1} \left(1 - (1 - \exp\{-\boldsymbol{\alpha}\})^{k}\right) IDEG_{k}(\boldsymbol{y}), \tag{5}$$

where $IDEG_{k}(\boldsymbol{y})$ is the number of agents in the studied network with degree k and $\boldsymbol{\alpha}$ is a fixed decay parameter. One can substitute $IDEG_{k}(\boldsymbol{y})$ in (5) with the number of agents with a specific out-degree to capture the out-degree distribution. We term these statistics geometrically weighted since the weights in (5) are a geometric series[4]. A positive

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[3]Alternatively, (4) can also be derived as the equilibrium distribution of a strategic game where players myopically reassess and update their links to optimize their utility in the network (see Mele, 2017; Boucher and Mourifié, 2017).

[4]Geometrically weighted statistics require setting the decay parameter $\boldsymbol{\alpha}$. We set $\boldsymbol{\alpha} = \log(2)$, though it can also be estimated as an additional parameter given sufficient data (Hunter and Handcock, 2006).

estimate implies that an edge from a low-degree agent is more likely than an edge from a high-degree agent, resulting in a decentralized network. If, on the other hand, the corresponding coefficient is negative, one may interpret it as an indicator for a centralized network.

To capture clustering, we have to define the distribution of edgewise-shared partners. This distribution is defined as the relative frequency of edges in the network with a specific number of k shared partners that we denote by $ESP(\boldsymbol{y}))$ for $k \in \{1, ..., n-2\}$. As shown in Figure 1, one can propose various versions of shared partners in directed networks, depending on the direction of edges between the three involved agents. Similarly as for degree statistics, we can state the geometrically weighted statistic for the outgoing two-path (OTP) as

$$GWOTP(\boldsymbol{y}, \boldsymbol{\alpha}) = \exp\{\boldsymbol{\alpha}\} \sum_{k=1}^{n-2} \left(1 - (1 - \exp\{-\boldsymbol{\alpha}\})^k\right) OTP_k(\boldsymbol{y}). \qquad (6)$$

In this case, a positive coefficient indicates that the corresponding relation with third actors increases the probability of observing an event between two agents.

In summary, the ERGM allows to account for network dependencies via explicitly specifying them in $s(\boldsymbol{y})$. A large variety of potential network statistics, such as those given in (5) and (6), can be included in $s(\boldsymbol{y})$, enabling to test for their influence in the formation of the observed network. By allowing for this explicit inclusion and testing of network statistics, the ERGM requires researchers to at least have an implicit theory regarding what types of network dependence should exist in the network they study. Without such theory to guide the selection of network statistics, the range of potential network dependencies, and corresponding statistics, is virtually endless[5]. As a result, the ERGM is best suited for research questions that explicitly concern interdependencies within the network. If these interdependencies are, instead, only a potential source of bias the researcher wants to control for, the AME model introduced in Section 4 might be a better fit.

3.3 Application to the international arms trade network

We next make use of the ERGM to analyze the international arms transfer network. Recent studies on the trade in Major Conventional Weapons (MCW), such as fighter aircraft or tanks, not only emphasize its networked nature, but also argue that it is of substantive theoretical interest (Thurner et al., 2019; Fritz et al., 2021). In line with Chaney (2014), triadic trade structures are held to reveal information regarding the participants' economic and security interests. Explicitly modeling these structures allows us to test hypotheses regarding their effects on further arms transfers. Accordingly, we seek to model the network of international arms transfers in the year 2018, where countries are nodes and a directed edge indicates MCW being delivered from country i to country j. Our interest here mainly lies in uncovering the network's endogenous mechanisms. MCW trade data come from SIPRI (2021), and the resulting network is depicted in Figure 2.

As discussed above, the ERGM allows us to use both exogenous (monadic or dyadic) attributes as well as endogenous structures to model the network of interest. Here, we select both types of covariates, based on existing studies on the arms trade (Thurner et al., 2019; Fritz et al., 2021). In addition to an edges term, which corresponds to

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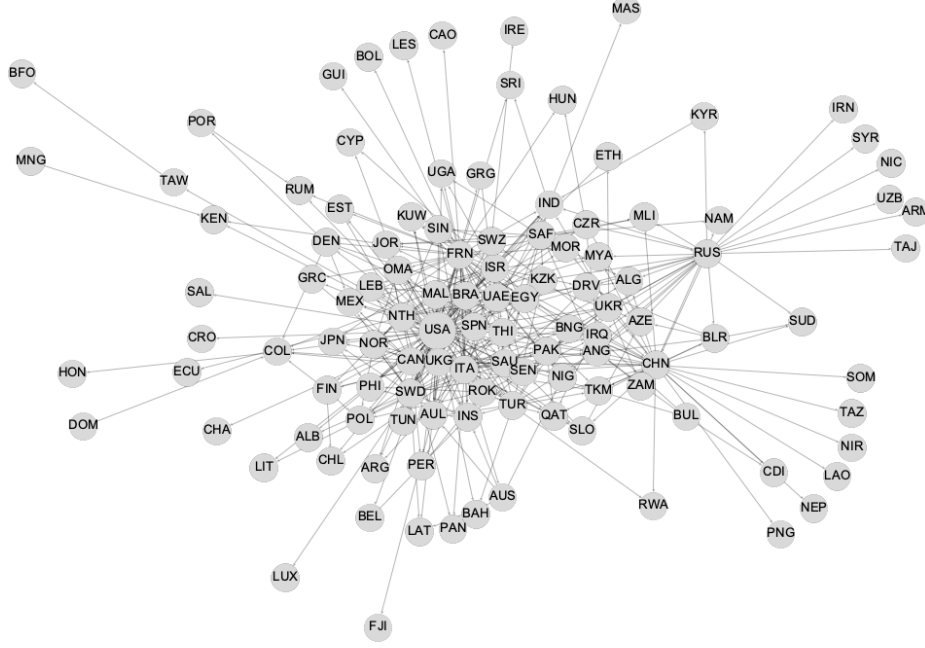[5]For a survey of possible endogenous terms, see Morris et al. (2008).

Figure 2: Illustration of the international arms trade network in 2018. Countries are labeled by their ISO 3166-1 codes, and a directed edge from node i to node j indicates major conventional weapons being delivered from country i to country j.

the intercept in standard regression models, we include importers' and exporters' logged GDP, whether they share a defense pact, their absolute difference in polity scores (a type of democracy index), as well as their geographical distance[6]. As the time between MCW being ordered and delivered is often substantial, we lag these exogenous covariates by three years. More importantly for the purpose of demonstrating how to model network data with the ERGM, we specify five endogenous network terms. In- and out-degree measure, respectively, importers' and exporters' trade activity, and thus capture whether highly active importers and exporters are particularly attractive trading partners, or instead less likely to form additional trade ties. Moreover, we specify a reciprocity term to capture whether countries trade MCW uni- or bidirectionally. We further include two types of triadic structures which respectively indicate transitivity and a shared supplier between countries i and j. The transitivity term counts how often country i exports arms to j while i exports to k which in turn exports to j, thus capturing i's tendency to directly trade with j if they engage in indirect trade (OTP, see Figure 1a). In contrast, the shared supplier term counts how often country i sends arms to j while both of them import weapons from a shared supplier k (ISP, see Figure 1b). Given the issue of degeneracy discussed above, we use geometrically weighted versions of all endogenous statistics except reciprocity. Finally, we include a repetition term capturing whether arms transfer dyads observed in 2018 had already occurred in any of the three previous years.

Results of this ERGM, as well as, for comparison's sake, a logistic regression that includes the same exogenous covariates but does not capture any of the endogenous network structures, are presented in Table 1. These results can be compared directly, as, just like in a logistic regression model, coefficients in the ERGM indicate the additive change in the log odds of a tie occurring in association with a unit change in the respective variable. From the table, we can see how the two models differ both in their in-sample prediction

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[6]Data for these covariates come from the `peacesciencer` package (Miller, 2022).

	ERGM	Logit
Intercept	−15.356 (2.017)***	−28.197 (1.731)***
Repetition	3.254 (0.141)***	3.957 (0.141)***
Distance	−0.081 (0.087)	−0.239 (0.088)**
Abs. Diff. Polity	−0.001 (0.010)	−0.003 (0.012)
Alliance	0.350 (0.207)	0.209 (0.207)
log-GDP (Sender)	0.300 (0.050)***	0.588 (0.045)***
log-GDP (Receiver)	0.166 (0.049)***	0.355 (0.039)***
Mutual	−0.311 (0.438)	
GWIDEG	−1.478 (0.296)***	
GWODEG	−2.848 (0.296)***	
GWOTP	−0.146 (0.104)	
GWISP	0.210 (0.083)*	
AIC	1769.718	1891.984
BIC	1866.053	1948.179
Log Likelihood	−872.859	−938.992

***$p < 0.001$; **$p < 0.01$; *$p < 0.05$

Table 1: Estimated coefficients of the ERGM and the corresponding logistical regression model for the international arms trade network in 2018.

performance, as captured by AIC and BIC[7], as well as in the substantive effects they identify for the exogenous covariates. For instance, the effect of the geographical distance between sender and receiver is three times as large in the logistic regression as in the ERGM and, while statistically significant in the former, indistinguishable from zero in the latter. Similarly, the repetition coefficient is positive and statistically significant in both models, but differs substantially in its size. An arms transfer edge having occurred at least once in 2015-17 increases the log odds of it occurring also in 2018 by the additive factor $\exp\{3.96\} = 52.46$ in the Logit but by only $\exp\{3.26\} = 26.05$ in the ERGM. Second, three of the endogenous statistics included in the ERGM exhibit statistically significant effects on the probability of arms being traded. The results for in- and out-degree replicate the finding by Thurner et al. (2019), showing that highly active importers and exporters are less likely to form additional trade ties. In the ERGM, coefficients can also be interpreted at the global level, in addition to the edge-level interpretation given above. The shared supplier term having a positive coefficient thus indicates, at the edge level, that an exporter is more likely to transfer weapons to a potential receiver if both of them import arms from the same source. Globally, it means that the observed network exhibits more shared supplier configurations – where country i sends weapon to j while both receive arms from k – than would be expected in a random network of the same size. On the whole, the results presented in Table 1 offer an example for the striking differences that modeling network structures (instead of assuming them away) can make.

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[7]As shown in the Supplementary Material, the ERGM also outperforms the Logit model when assessing their respective areas under the receiver-operator and precision-recall curves.

4 The Additive and Multiplicative Effect Network Model

4.1 Latent variable network models

Another way to account for network dependencies is by making use of latent variables. Models within this class assume that latent variables Z_i are associated with each node i. Depending on the type of model, these latent variables can either be discrete (e.g. indicating group memberships for each node) or continuous, and affect the connection probability in different ways (Matias and Robin, 2014). An early (but still popular) approach in this direction is the stochastic blockmodel, which assumes that each agent possesses a latent, categorical class (or group membership). Nodes within each class are assumed to be stochastically equivalent in their connectivity behavior, meaning that the probability of two nodes to connect depends solely on their group memberships (Holland et al., 1983; De Nicola et al., 2022). This family of models is attractive due to its simplicity in detecting and describing subgroups of nodes in networks. In many applications, however, discrete groupings fail to adequately represent the observed data, as agents behave more heterogeneously. Moving from discrete to continuous latent variable network models, another prominent approach is the latent distance model. We here postulate that agents are positioned in a latent Euclidean "social space", and that the closer they are within it, the more likely they are to form ties (Hoff et al., 2002). More precisely, the classical latent distance model specifies the probability of observing an edge between nodes i and j, conditional on Z, through

$$\mathbb{P}_\theta(Y_{ij} = 1|Z) = \frac{\exp\{\boldsymbol{\theta}^\top x_{ij} - \|z_i - z_j\|\}}{1 + \exp\{\boldsymbol{\theta}^\top x_{ij} - \|z_i - z_j\|\}}, \tag{7}$$

where $Z = (z_1, ..., z_n)$ denotes the latent positions of the nodes in the d-dimensional space, and $\boldsymbol{\theta}$ is the coefficient vector for the covariates x_{ij}. The latent positions Z are assumed to originate independently from a spherical Gaussian distribution, i.e., $Z \sim N_d(0, \tau^2 \boldsymbol{I}_d)$, where $\boldsymbol{I}_d$ indicates a d-dimensional matrix where only the diagonal is filled with ones.

Latent distance models are particularly attractive for social networks in which triadic closure plays a major role, and where nodes with similar characteristics tend to form connections with each other (i.e. homophilic networks, see Rivera et al., 2010). It is also possible to add nodal random effects to the model, to control for agent-specific heterogeneity in the propensity to form edges (Krivitsky et al., 2009). The model then becomes

$$\mathbb{P}_\theta(Y_{ij} = 1|Z, a, b) = \frac{\exp\{\boldsymbol{\theta}^\top x_{ij} - \|z_i - z_j\| + a_i + b_j\}}{1 + \exp\{\boldsymbol{\theta}^\top x_{ij} - \|z_i - z_j\| + a_i + b_j\}}, \tag{8}$$

where $a = (a_1, ..., a_n)$ and $b = (b_1, ..., b_n)$ are node-specific sender and receiver effects that account for individual agents' propensity to form ties, with $a \sim N_n(0, \tau_a^2 \boldsymbol{I}_n)$ and $b \sim N_n(0, \tau_b^2 \boldsymbol{I}_n)$.

Despite its advantages and easy interpretation, a Euclidean latent space is unable to effectively approximate the behavior of networks where nodes that are similar in terms of connectivity behavior are not necessarily more likely to form ties (Hoff, 2008), such as, e.g., many networks of amorous relationships (Ghani et al., 1997; Bearman et al., 2004). This is often the case in economics, where real-world networks can exhibit varying degrees and combinations of stochastic equivalence, triadic closure and homophily. Moreover, it is often *a priori* unclear which of these mechanisms are at play in a given observed network.

In this context, agent-specific multiplicative random effects instead of the additive latent positions allow for simultaneously representing all these patterns (Hoff, 2005). Further developments of this innovation have led to the modern specification of the Additive and Multiplicative Effects network model (AME, Hoff, 2011), which, from a matrix representation perspective, generalizes both the stochastic blockmodel and the latent space model (Hoff, 2021).

4.2   AME: Motivation and framework

The AME approach can be motivated by considering that network data often exhibit first-, second-, and third-order interdependence. *First-order effects* capture agent-specific heterogeneity in sending (or receiving) ties within a network. For example, in the case of companies and legal disputes, first-order effects can be viewed as the propensity of each firm to initiate (or be hit by) legal disputes. *Second-order effects*, i.e., reciprocity, describe the statistical dependency of the directed relationship between two agents in the network. In the previous example, this effect can be described as the correlation between (a) company *i* initiating a legal dispute against company *j* and (b) *j* doing the same towards *i*. Of course, second-order effects only occur in directed networks. *Third-order effects* are described as the dependency within triads, defined as the connections between three agents and relate to the triangular statistics previously illustrated in Figure 1. How likely is it that "a friend of a friend is also my friend"? Or, returning to the previous example: given that *i* has legal disputes with *j* and *k*, how likely are disputes to occur between *j* and *k*?

The AME network model is designed to simultaneously capture these three orders of dependencies. More specifically, it extends the classical (generalized) linear model framework by incorporating extra terms into the systematic component to account for them. In the case of binary network data, we can make use of the Probit AME model. As is well known, the classical Probit regression model can be motivated through a latent variable representation in which y_{ij} is the binary indicator that some latent normal random variable, say $L_{ij} \sim \mathcal{N}(\boldsymbol{\theta}^\top x_{ij}, \sigma^2)$, is greater than zero (Albert and Chib, 1993). But an ordinary Probit regression model assumes that L_{ij}, and thus the binary indicators (edges) y_{ij}, are independent, which is generally inappropriate for network data. In contrast, the AME Probit model specifies the probability of a tie y_{ij} from agent *i* to agent *j*, conditional on a set of latent variables W, as

$$\mathbb{P}(Y_{ij} = 1|W) = \boldsymbol{\Phi}(\boldsymbol{\theta}^\top x_{ij} + e_{ij}), \tag{9}$$

where $\boldsymbol{\Phi}$ is the cumulative distribution function of the standard normal distribution, $\boldsymbol{\theta}^\top x_{ij}$ accommodates the inclusion of dyadic, sender, and receiver covariates, and e_{ij} can be viewed as a structured residual, containing the latent terms in W to account for the network dependencies described above. In the directed case, e_{ij} is composed as

$$e_{ij} = a_i + b_j + u_i v_j + \varepsilon_{ij}. \tag{10}$$

In this context, a_i and b_j are zero-mean additive effects for sender *i* and receiver *j* accounting for first-order dependencies, jointly specified as

$$(a_1, b_1), ..., (a_n, b_n) \overset{\text{i.i.d.}}{\sim} N_2(0, \Sigma_1), \quad \text{with} \quad \Sigma_1 = \begin{pmatrix} \sigma_a & \sigma_{ab} \\ \sigma_{ab} & \sigma_b \end{pmatrix}. \tag{11}$$

The parameters $\boldsymbol{\sigma}_a$ and $\boldsymbol{\sigma}_b$ measure the variance of the additive sender and receiver effects, respectively, while $\boldsymbol{\sigma}_{ab}$ relates to the covariance between sender and receiver effects for the same node. Going back to (10), $\boldsymbol{\varepsilon}_{ij}$ is a zero-mean residual term which accounts for second order dependencies, i.e. reciprocity. More specifically, it holds that

$$\{(\boldsymbol{\varepsilon}_{ij}, \boldsymbol{\varepsilon}_{ji}) : i < j\} \overset{\text{i.i.d.}}{\sim} N_2(0, \Sigma_2), \quad \text{with} \quad \Sigma_2 = \boldsymbol{\sigma}^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \tag{12}$$

where $\boldsymbol{\sigma}^2$ denotes the error variance and $\boldsymbol{\rho}$ determines the correlation between $\boldsymbol{\varepsilon}_{ij}$ and $\boldsymbol{\varepsilon}_{ji}$, thus quantifying the tendency towards reciprocity. Finally, $\boldsymbol{u}_i$ and $\boldsymbol{v}_j$ in (10) are d-dimensional multiplicative sender and receiver effect vectors that account for third-order dependencies, and for which $(u_1, v_1), ..., (u_n, v_n) \sim \mathcal{N}_{2d}(0, \Sigma_3)$ holds.

As noted above, AME is able to represent a wide variety of network structures, generalizing several other latent variable model classes. This generality comes at the price of a high level of complexity for the estimated latent structure. This can make the model class a sub-optimal choice if one wants to interpret the latent structure with respect to, e.g., clustering. On the other hand, its flexibility makes it an ideal fit when the underlying network dependencies are unknown, and the researchers' interest mainly lies in evaluating and interpreting the effect of dyadic and nodal covariates on tie formation while controlling for network effects. This strength has led to AME being used for several applications of this type (Koster, 2018; Minhas et al., 2019, 2021; Dorff et al., 2020). We next showcase the AME framework by applying it to the world foreign exchange activity network as of 1900, originally introduced and studied by Flandreau and Jobst (2005, 2009).

4.3 Application to the global foreign exchange activity network

In 1900, every financial center featured a foreign exchange market were bankers bought and sold foreign currency against the domestic one. Foreign exchange market activity was monitored in local bulletins, which allowed Flandreau and Jobst (2005) to collect a global dataset with all currencies used in the world during that time. In the resulting network structure, depicted in Figure 3, countries are nodes, and a (directed) edge from country i to country j occurs if the currency of country j was actively traded in at least one financial center within country i. In Figure 3, we observe that the most actively traded currencies at the time belonged to large European economies, such as Great Britain, France and Germany. To determine the drivers of currency adoption, Flandreau and Jobst (2009) model this network as a function of several covariates by employing ordinary binary regression. As we show, it is possible to use AME to pursue the same goal while taking network dependencies into account.

We specify the AME model as in (9), using directed edges y_{ij} as response variable. The nodal covariates we use, sourced from and described in detail in the replication materials of Flandreau and Jobst (2009), are (log)per-capita GDP, democracy index score, coverage of foreign currencies traded in the country, and an indicator of whether the country's currency was on the gold standard. We also include, as dyadic covariates, the distance between two countries as well as their total trade volume. As specified in (10), the structured residual term e_{ij} comprises additive effects a_i and b_j for each node, which capture the country-specific propensities to send and receive ties, respectively. Multiplicative effects u_i and v_j are included to account for third order dependencies. We here set the dimensionality of the multiplicative effects to two, which we assume to be sufficient given the relatively small size of the network.

Figure 3: Illustration of the global foreign exchange activity network in 1900. Countries are labeled by their ISO 3166-1 codes, and an edge from node i to node j indicates active trading of the currency from country j within a financial center of country i.

To estimate the AME model, we make use of the R package amen (Hoff, 2015). The results of this analysis, as well as, for comparison's sake, a Probit regression including the same covariates but ignoring network dependencies, are displayed in Table 2. Additional model diagnostics and goodness of fit measures, together with the estimated variance and covariance parameters, are provided in the Supplementary Material. The estimated coefficients (for both models) can be interpreted as in standard Probit regression: For the nodal covariate per-capita GDP, for example, a unit increase in the log-per-capita GDP for country i corresponds to a decrease of 0.453 in the linear predictor, therefore negatively influencing the expected probability of the country to *send* a tie. The same unit increase in the log-per-capita-gdp for country i corresponds to an increase of 0.426 in the linear predictor, and has therefore a positive impact on the expected probability of that country to *receive* a tie. In the case of a dyadic covariate, such as distance, a unit increase in distance between two countries leads to a decrease of 1.019 in the linear predictor, resulting in a decrease in the expected probability of the two countries to form a tie in either direction. Overall, we find that the principal drivers of the formation of a tie between i and j are the magnitude of the foreign exchange coverage of the two countries involved, the distance between them, and their reciprocal trade volume. These results correspond the thesis of Kindleberger (1967) and to Flandreau and Jobst (2009), who suggest that the most important determinants of international adoption for a currency are size and convenience of use. At the same time, we note that, as in the ERGM example, the results of the Probit and AME model differ in several regards. In particular, several effects are statistically significant in the Probit but not significant in the AME model. Indeed, unacknowledged network dependence can cause downward bias in the estimation of standard errors, leading to spurious associations (Lee and Ogburn, 2021). This finding once again highlights how accounting for network dependencies can make a difference when it comes to the substantive results.

		AME	Classical Probit
	Intercept	−4.845 (5.310)	−3.211 (1.580)*
Sender	Gold standard	−0.629 (0.397)	−0.354 (0.155)*
	log-GDP per-capita	−0.453 (0.419)	−0.259 (0.152)
	Democracy index	−0.033 (0.064)	−0.025 (0.026)
	Currency coverage	1.418 (0.405)***	0.470 (0.137)***
Receiver	Gold standard	−0.599 (0.667)	−0.468 (0.191)*
	log-GDP per-capita	0.426 (0.703)	0.240 (0.159)
	Democracy index	0.121 (0.102)	0.066 (0.019)***
	Currency coverage	2.734 (0.691)***	1.363 (0.181)***
Dyadic	Distance	−1.019 (0.151)***	−0.471 (0.064)***
	log-trade volume	0.488 (0.081)***	0.346 (0.036)***

$^{***}p < 0.001;\ ^{**}p < 0.01;\ ^{*}p < 0.05$

Table 2: Estimated coefficients for the AME model and the corresponding Probit model for the global foreign exchange activity network in 1900.

5 Conclusion

Complex dependencies are ubiquitous in the economic sciences (Chiarella et al., 2005; Flaschel et al., 2008), and many economic interactions can be naturally perceived as networks. This area of research has thus received considerable interest in recent years. However, this attention has not yet been accompanied by a corresponding general take-up of empirical research methods tailored towards networks. Instead, researchers either develop their own estimators to reproduce the features of their theoretical network models, or use standard regression methods that assume conditional independence of edges in the network. Against this background, this paper seeks to provide a hands-on introduction to two statistical models which account for network dependencies, the Exponential Random Graph Model (ERGM) and the Additive and Multiplicative Effects network model (AME). These two classes serve different purposes: While the ERGM is most appropriate when explicitly interested in testing the effects of endogenous network structure, the AME model allows one to control for network dependencies while substantively focusing on the effects of more classical, exogenous covariates. We present the statistical foundations of both models, and demonstrate their applicability to economic networks through examples in the international arms trade and foreign exchange, showing that modeling network dependencies can alter the substantive results one obtains. Importantly, we provide the full data and code necessary to replicate these example applications. We explicitly want to encourage readers to use these replication materials to get started with analyzing economic networks via ERGM and AME, beginning with the examples covered here to then transfer the code and methods to their own research.

We especially want to encourage doing so as not accounting for interdependence between observations when it exists can lead to biased estimates and spurious findings. Our two applications demonstrate that this bias can result in very different empirical results and thus affect substantive conclusions. It is thus vital to account for network structure

when studying interactions between economic agents such as individuals, firms, or countries, regardless of whether one is substantively interested in this structure. As shown by Lee and Ogburn (2021), our applications are just two examples of how unaccounted dependence in the observed data may lead to spurious findings.

At the same time, this paper can only serve as an introduction to statistical network data analysis in economics. We covered two general frameworks in this realm, but, in the interest of brevity, focused only on their simplest versions that apply to networks observed at only one time-point and with binary edges. However, both frameworks have been extended to cover more general settings. For the ERGM, there are extensions for longitudinal data (Hanneke et al., 2010), distinguishing between edge formation and continuation (Krivitsky and Handcock, 2014), as well as to settings where edges are not binary but instead count-valued or signed (Krivitsky, 2012; Fritz et al., 2022). As for AME, approaches for longitudinal networks are described by Minhas et al. (2016), while versions for undirected networks as well as for non-binary network data are presented by Hoff (2021). Both the ERGM and the AME frameworks are thus flexible enough to cover a wide array of potential economic interactions. We believe that increasingly adopting these methods will, in turn, aid our understanding of these interactions.

References

Albert, J. H. and S. Chib (1993). Bayesian analysis of binary and polychotomous response data. *Journal of the American Statistical Association 88*(422), 669–679.

Anderson, J. E. and E. Van Wincoop (2003). Gravity with gravitas: A solution to the border puzzle. *American Economic Review 93*(1), 170–192.

Bearman, P. S., J. Moody, and K. Stovel (2004). Chains of affection: The structure of adolescent romantic and sexual networks. *American journal of sociology 110*(1), 44–91.

Boucher, V. and I. Mourifié (2017). My friend far, far away: A random field approach to exponential random graph models. *Econometrics Journal 20*(3), S14–S46.

Bramoullé, Y., A. Galeotti, and B. W. Rogers (2016). *The Oxford Handbook of the Economics of Networks.* Oxford University Press.

Campbell, B., S. Cranmer, and B. Desmarais (2018). Triangulating war: Network structure and the democratic peace. *arXiv preprint.* arXiv:1809.04141.

Cao, X. and M. D. Ward (2014). Do democracies attract portfolio investment? Transnational portfolio investments modeled as dynamic network. *International Interactions 40*(2), 216–245.

Chaney, T. (2014). The network structure of international trade. *American Economic Review 104*(11), 3600–3634.

Chiarella, C., P. Flaschel, and R. Franke (2005). *Foundations for a disequilibrium theory of the business cycle: Qualitative analysis and quantitative assessment.* Cambridge University Press.

Cranmer, S. J., B. A. Desmarais, and J. H. Kirkland (2012). Toward a network theory of alliance formation. *International Interactions 38*(3), 295–324.

Cranmer, S. J., P. Leifeld, S. D. McClurg, and M. Rolfe (2017). Navigating the range of statistical tools for inferential network analysis. *American Journal of Political Science 61*(1), 237–251.

Davis, J. A. (1970). Clustering and hierarchy in interpersonal relations: Testing two graph theoretical models on 742 sociomatrices. *American Sociological Review 35*(5), 843–851.

De Nicola, G., B. Sischka, and G. Kauermann (2022). Mixture models and networks: The stochastic blockmodel. *Statistical Modelling 22*(1-2), 67–94.

Dorff, C., M. Gallop, and S. Minhas (2020). Networks of violence: Predicting conflict in Nigeria. *Journal of Politics 82*(2), 476–493.

Dueñas, M. and G. Fagiolo (2013). Modeling the international-trade network: A gravity approach. *Journal of Economic Interaction and Coordination 8*(1), 155–178.

Erdös, P. and A. Rényi (1959). On random graphs I. *Publicationes Mathematicae Debrecen 6*, 290.

Fagiolo, G. (2010). The international-trade network: Gravity equations and topological properties. *Journal of Economic Interaction and Coordination 5*(1), 1–25.

Feng, L., H. Xu, G. Wu, and W. Zhang (2021). Service trade network structure and its determinants in the belt and road based on the temporal exponential random graph model. *Pacific Economic Review 26*(5), 617–650.

Flandreau, M. and C. Jobst (2005). The ties that divide: A network analysis of the international monetary system, 1890–1910. *Journal of Economic History 65*(4), 977–1007.

Flandreau, M. and C. Jobst (2009). The empirics of international currencies: Network externalities, history and persistence. *The Economic Journal 119*(537), 643–664.

Flaschel, P., M. Charpe, G. Galanis, C. R. Proaño, and R. Veneziani (2018). Macroeconomic and stock market interactions with endogenous aggregate sentiment dynamics. *Journal of Economic Dynamics and Control 91*, 237–256.

Flaschel, P., R. Franke, and W. Semmler (1997). *Dynamic macroeconomics: Instability, fluctuation, and growth in monetary economies*. MIT Press.

Flaschel, P., G. Groh, C. Proaño, and W. Semmler (2008). *Topics in applied macrodynamic theory*, Volume 10. Springer Science & Business Media.

Flaschel, P., G. Kauermann, and W. Semmler (2007). Testing wage and price Phillips curves for the United States. *Metroeconomica 58*(4), 550–581.

Frank, O. and D. Strauss (1986). Markov graphs. *Journal of the American Statistical Association 81*(395), 832–842.

Fritz, C., G. De Nicola, S. Kevork, D. Harhoff, and G. Kauermann (2022). Modelling the large and dynamically growing bipartite network of german patents and inventors. *arXiv preprint*. arXiv:2201.09744.

Fritz, C., M. Mehrl, P. W. Thurner, and G. Kauermann (2022). Exponential random graph models for dynamic signed networks: An application to international relations. *arXiv preprint*. arXiv:2205.13411.

Fritz, C., P. W. Thurner, and G. Kauermann (2021). Separable and semiparametric network-based counting processes applied to the international combat aircraft trades. *Network Science 9*(3), 291–311.

Geyer, C. J. and E. A. Thompson (1992). Constrained Monte Carlo maximum likelihood for dependent data. *Journal of the Royal Statistical Society: Series B (Methodological) 54*(3), 657–683.

Ghani, A. C., J. Swinton, and G. P. Garnett (1997). The role of sexual partnership networks in the epidemiology of gonorrhea. *Sexually Transmitted Diseases 24*(1), 45–56.

Gilbert, E. N. (1959). Random graphs. *Annals of Mathematical Statistics 30*(4), 1141–1144.

Goldenberg, A., A. X. Zheng, S. E. Fienberg, and E. M. Airoldi (2010). A survey of statistical network models. *Foundations and Trends in Machine Learning 2*(2), 129–233.

Handcock, M. (2003). Assessing degeneracy in statistical models of social networks. *Working Paper no. 39, University of Washington*.

Handcock, M. S., D. R. Hunter, C. T. Butts, S. M. Goodreau, and M. Morris (2008). statnet: Software tools for the representation, visualization, analysis and simulation of network data. *Journal of statistical software 24*(1), 1548.

Hanneke, S., W. Fu, and E. P. Xing (2010). Discrete temporal models of social networks. *Electronic Journal of Statistics 4*, 585–605.

He, X., Y. Dong, Y. Wu, G. Jiang, and Y. Zheng (2019). Factors affecting evolution of the interprovincial technology patent trade networks in china based on exponential random graph models. *Physica A: Statistical Mechanics and its Applications 514*, 443–457.

Heider, F. (1946). Attitudes and cognitive organization. *Journal of Psychology 21*(1), 107–112.

Herman, P. R. (2022). Modeling complex network patterns in international trade. *Review of World Economics 158*, 127–79.

Hoff, P. (2008). Modeling homophily and stochastic equivalence in symmetric relational data. *Advances in Neural Information Processing Systems 20*, 657–664.

Hoff, P. (2021). Additive and multiplicative effects network models. *Statistical Science 36*(1), 34–50.

Hoff, P. D. (2005). Bilinear mixed-effects models for dyadic data. *Journal of the American Statistical Association 100*(469), 286–295.

Hoff, P. D. (2011). Hierarchical multilinear models for multiway data. *Computational Statistics & Data Analysis 55*(1), 530–543.

Hoff, P. D. (2015). Dyadic data analysis with amen. *arXiv preprint arXiv:1506.08237*.

Hoff, P. D., A. E. Raftery, and M. S. Handcock (2002). Latent space approaches to social network analysis. *Journal of the American Statistical Association 97*(460), 1090–1098.

Holland, P. W., K. B. Laskey, and S. Leinhardt (1983). Stochastic blockmodels: First steps. *Social networks 5*(2), 109–137.

Holland, P. W. and S. Leinhardt (1981). An exponential family of probability distributions for directed graphs. *Journal of the American Statistical Association 76*(373), 33–50.

Hunter, D. R., S. M. Goodreau, and M. S. Handcock (2008). Goodness of fit of social network models. *Journal of the American Statistical Association 103*(481), 248–258.

Hunter, D. R. and M. S. Handcock (2006). Inference in curved exponential family models for networks. *Journal of Computational and Graphical Statistics 15*(3), 565–583.

Hunter, D. R., P. N. Krivitsky, and M. Schweinberger (2012). Computational statistical methods for social network models. *Journal of Computational and Graphical Statistics 21*(4), 856–882.

Jackson, M. O. (2008). *Social and economic networks.* Princeton University Press.

Jackson, M. O. (2014). The past and future of network analysis in economics. In *The Oxford Handbook of the Economics of Networks.* Oxford University Press.

Jackson, M. O. and S. Nei (2015). Networks of military alliances, wars, and international trade. *Proceedings of the National Academy of Sciences 112*(50), 15277–15284.

Jackson, M. O., B. W. Rogers, and Y. Zenou (2017). The economic consequences of social-network structure. *Journal of Economic Literature 55*(1), 49–95.

Kindleberger, C. P. (1967). *The politics of international money and world language.* International Finance Section, Department of Economics, Princeton University.

Koster, J. (2018). Family ties: The multilevel effects of households and kinship on the networks of individuals. *Royal Society Open Science 5*(4), 172159.

Krivitsky, P. N. (2012). Exponential-family random graph models for valued networks. *Electronic journal of statistics 6*, 1100–1128.

Krivitsky, P. N. and M. S. Handcock (2014). A separable model for dynamic networks. *Journal of the Royal Statistical Society: Series B (Statistical Methodology) 76*(1), 29–46.

Krivitsky, P. N., M. S. Handcock, A. E. Raftery, and P. D. Hoff (2009). Representing degree distributions, clustering, and homophily in social networks with latent cluster random effects models. *Social networks 31*(3), 204–213.

König, M. D., D. Rohner, M. Thoenig, and F. Zilibotti (2017). Networks in conflict: Theory and evidence from the great war of Africa. *Econometrica 85*(4), 1093–1132.

Lebacher, M., P. W. Thurner, and G. Kauermann (2021). A dynamic separable network model with actor heterogeneity: An application to global weapons transfers. *Journal of the Royal Statistical Society: Series A (Statistics in Society) 184*(1), 201–226.

Lee, Y. and E. L. Ogburn (2021). Network dependence can lead to spurious associations and invalid inference. *Journal of the American Statistical Association 116*(535), 1060–1074.

Lewer, J. J. and H. Van den Berg (2008). A gravity model of immigration. *Economics Letters 99*(1), 164–167.

Liu, L., M. Shen, D. Sun, X. Yan, and S. Hu (2022). Preferential attachment, R&D expenditure and the evolution of international trade networks from the perspective of complex networks. *Physica A: Statistical Mechanics and its Applications 603*, 127579.

Lusher, D., J. Koskinen, and G. Robins (2012). *Exponential Random Graph Models for Social Networks.* Cambridge University Press.

Matias, C. and S. Robin (2014). Modeling heterogeneity in random graphs through latent space models: A selective review. *ESAIM: Proceedings and Surveys 47*, 55–74.

Mele, A. (2017). A structural model of dense network formation. *Econometrica 85*(3), 825–850.

Miller, S. V. (2022). {peacesciencer}: An R package for quantitative peace science research. *Conflict Management and Peace Science 39*(6), 755–779.

Minhas, S., C. Dorff, M. B. Gallop, M. Foster, H. Liu, J. Tellez, and M. D. Ward (2021). Taking dyads seriously. *Political Science Research and Methods* (First View), 1–19.

Minhas, S., P. D. Hoff, and M. D. Ward (2016). A new approach to analyzing coevolving longitudinal networks in international relations. *Journal of Peace Research 53*(3), 491–505.

Minhas, S., P. D. Hoff, and M. D. Ward (2019). Inferential approaches for network analysis: AMEN for latent factor models. *Political Analysis 27*(2), 208–222.

Morales, E., G. Sheu, and A. Zahler (2019). Extended gravity. *Review of Economic Studies 86*(6), 2668–2712.

Morris, M., M. S. Handcock, and D. R. Hunter (2008). Specification of exponential-family random graph models: Terms and computational aspects. *Journal of Statistical Software 24*(4), 1548.

Mundt, P. (2021). The formation of input–output architecture: Evidence from the European Union. *Journal of Economic Behavior & Organization 183*, 89–104.

Nelder, J. A. and R. W. M. Wedderburn (1972). Generalized linear models. *Journal of the Royal Statistical Society: Series A (General) 135*(3), 370–384.

Newcomb, T. M. (1979). Reciprocity of interpersonal attraction: A nonconfirmation of a plausible hypothesis. *Social Psychology Quarterly 42*(4), 299–306.

Newman, M. E. J. (2003). The structure and function of complex networks. *SIAM Review 45*(2), 167–256.

R Core Team (2021). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing.

Rivera, M. T., S. B. Soderstrom, and B. Uzzi (2010). Dynamics of dyads in social networks: Assortative, relational, and proximity mechanisms. *Annual Review of Sociology 36*, 91–115.

Robins, G., P. Pattison, Y. Kalish, and D. Lusher (2007). An introduction to exponential random graph (p*) models for social networks. *Social Networks 29*(2), 173–191.

Robins, G., T. Snijders, P. Wang, M. Handcock, and P. Pattison (2007). Recent developments in exponential random graph (p*) models for social networks. *Social Networks 29*(2), 192–215.

Rose, A. K. (2004). Do we really know that the WTO increases trade? *American Economic Review 94*(1), 98–114.

Schoeneman, J., B. Zhu, and B. A. Desmarais (2022). Complex dependence in foreign direct investment: Network theory and empirical analysis. *Political Science Research and Methods 10*(2), 243–259.

Schweinberger, M. (2011). Instability, sensitivity, and degeneracy of discrete exponential families. *Journal of the American Statistical Association 106*(496), 1361–1370.

SIPRI (2021). SIPRI Arms Transfers Database. *Stockholm International Peace Research Institute*. https://www.sipri.org/databases/armstransfers (accessed 10/20/2022).

Smith, M., S. Gorgoni, and B. Cronin (2019). International production and trade in a high-tech industry: A multilevel network analysis. *Social Networks 59*, 50–60.

Smith, M. and Y. Sarabi (2022). How does the behaviour of the core differ from the periphery? – An international trade network analysis. *Social Networks 70*, 1–15.

Snijders, T. A. B., P. E. Pattison, G. L. Robins, and M. S. Handcock (2006). New specifications for exponential random graph models. *Sociological Methodology 36*(1), 99–153.

Thurner, P. W., C. S. Schmid, S. J. Cranmer, and G. Kauermann (2019). Network interdependencies and the evolution of the international arms trade. *Journal of Conflict Resolution 63*(7), 1736–1764.

Ward, M. D., J. S. Ahlquist, and A. Rozenas (2013). Gravity's rainbow: A dynamic latent space model for the world trade network. *Network Science 1*(1), 95–118.

Ward, M. D. and P. D. Hoff (2007). Persistent patterns of international commerce. *Journal of Peace Research 44*(2), 157–175.

Wasserman, S. and K. Faust (1994). *Social Network Analysis: Methods and Applications.* Cambridge University Press.

Wasserman, S. and P. Pattison (1996). Logit models and logistic regressions for social networks: I. An introduction to Markov graphs and p*. *Psychometrika 61*(3), 401–425.

Supplementary Material: Statistical Network Data Analysis in Economics

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The following contains technical details and supplementary information to the manuscript. The full data and code to reproduce the analysis and to aid the reader in applying the presented models to their own data can be found in our Github repository, available at `https://github.com/gdenicola/statistical-network-analysis-in-economics`.

S.1   Details on the p_1 Model

To represent reciprocity, we assume dyads, defined by (Y_{ij}, Y_{ji}), to be independent of one another. The resulting bivariate distribution of each dyad (Y_{ij}, Y_{ji}) comprises three parameters and a constraint:

$$m_{ij} = \mathbb{P}(Y_{ij} = Y_{ji} = 1)$$
$$a_{ij} = \mathbb{P}(Y_{ij} = 1, Y_{ji} = 0)$$
$$e_{ij} = \mathbb{P}(Y_{ij} = Y_{ji} = 0)$$
$$m_{ij} + a_{ij} + a_{ji} + e_{ij} = 1.$$

The joint distribution of the network is then given by:

$$\mathbb{P}_{\theta}(\boldsymbol{Y} = \boldsymbol{y}) = \prod_{i<j} m_{ij}^{y_{ij}y_{ji}} a_{ij}^{y_{ij}(1-y_{ji})} a_{ji}^{(1-y_{ij})y_{ji}} e_{ij}^{(1-y_{ij})(1-y_{ji})}$$

$$= \frac{\exp\left\{ \sum_{i<j} \rho_{ij} y_{ij} y_{ji} + \sum_{i \neq j} \theta_{ij} y_{ij} \right\}}{\kappa(\boldsymbol{\theta})}, \tag{1}$$

where $\rho_{ij} = \log\left(m_{ij}e_{ij}/a_{ij}^2\right)$ for $i < j$, $\theta_{ij} = \log\left(a_{ij}/n_{ij}\right)$ for $i \neq j$, and $\kappa(\boldsymbol{\theta})$ is the normalizing constant. To estimate the parameters in (1) we need to introduce some homogeneity assumption to avoid overparametrization. Following Holland and Leinhardt (1981), we assume

$$\rho_{ij} = \theta_{\text{Rep}} \ \forall \ i < j$$
$$\theta_{ij} = \theta_{\text{Edges}} + \theta_{\text{Out},i} + \theta_{\text{In},j} \ \forall \ i \neq j$$
$$\sum_{i=1}^{n} \theta_{\text{Out},i} = \sum_{i=1}^{n} \theta_{\text{In},i} = 0,$$

where θ_{Rep} is the global effect of reciprocity, θ_{Edges} quantifies the general sparsity in the network, and $\theta_{\text{Out},i}$ and $\theta_{\text{In},i}$ indicate the general tendency for all agents $i \in \{1, ..., n\}$ to form out- or in-going ties. Combining these homogeneity assumptions with (1) yields

$$\mathbb{P}_{\theta}(\boldsymbol{Y} = \boldsymbol{y}) \propto \exp\left\{ \theta_{\text{Rep}} s_{\text{Rep}}(\boldsymbol{y}) + \theta_{\text{Edges}} s_{\text{Edges}}(\boldsymbol{y}) + \right.$$

$$\left. \sum_{i=1}^{n} \theta_{\text{Out},i} s_{\text{Out},i}(\boldsymbol{y}) + \sum_{i=1}^{n} \theta_{\text{In},i} s_{\text{In},i}(\boldsymbol{y}) \right\}, \tag{2}$$

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where $s_{\mathrm{Rep}}(\boldsymbol{y}) = \sum_{i<j} y_{ij} y_{ji}$ is the number of reciprocal ties, $s_{\mathrm{Edges}}(\boldsymbol{y}) = \sum_{i=1}^{n} \sum_{j\neq i} y_{ij}$ the number of edges, and $s_{\mathrm{Out},i}(\boldsymbol{y})$ and $s_{\mathrm{In},i}(\boldsymbol{y})$ the number of out- and in-going ties of agent i.

Note that all these statistics are functions of the observed network and are the sufficient statistics, i.e. they contain all necessary information for determining all coefficients in (2). This sufficiency principle translates to $s_{\mathrm{Out},i}(\boldsymbol{y}) = s_{\mathrm{Out},j}(\boldsymbol{y}) \Rightarrow \theta_{\mathrm{Out},i} = \theta_{\mathrm{Out},j}$. This, in turn, allows us to write all agent-specific terms in (2) in terms of degree statistics:

$$\mathbb{P}_{\boldsymbol{\theta}}(\boldsymbol{Y} = \boldsymbol{y}) \propto \exp \Big\{ \theta_{\mathrm{Rep}} s_{\mathrm{Rep}}(\boldsymbol{y}) + \theta_{\mathrm{Edges}} s_{\mathrm{Edges}}(\boldsymbol{y}) +$$
$$\sum_{i=1}^{n} \theta_{\mathrm{Outdeg},i} s_{\mathrm{Outdeg},i}(\boldsymbol{y}) + \sum_{i=1}^{n} \theta_{\mathrm{Indeg},i} s_{\mathrm{Indeg},i}(\boldsymbol{y}) \Big\}, \qquad (3)$$

where $s_{\mathrm{Outdeg},i}(\boldsymbol{y})$ and $s_{\mathrm{Indeg},i}(\boldsymbol{y})$ are statistics counting the actors with out/in-degree i in $\boldsymbol{y}$.

S.2   ERGM Model Diagnostics and Goodness of Fit

To evaluate the goodness of fit of of an estimated ERGM, the standard approach is to compare the statistics observed in the real world network with the distribution of the same statistics calculated on networks simulated from the model (Hunter et al., 2008). The results of this comparison are depicted in Figure S.1, where we investigate the goodness of fit of the ERGM estimated on the 2018 international arms trade network. There, the black line in each subfigure represents the distribution of the respective network statistic observed in the real network. For instance, the top right figure indicates that approximately 10% of all nodes (countries) in the network had an indegree of 2. The boxplots then show the distribution of each value of a given statistic over the simulated networks. A good model will thus generally result in boxplots that include the observed values of the network statistics under consideration. In Figure S.1, this is almost always the case, though it is also visible that the real network included less countries with an in-degree of 0 but more with an in-degree of 1 than the large majority of networks simulated from the ERGM.

In Figure S.2, we further compare the performance of the fully specified ERGM against that of the logistic regression model including the same exogenous covariates as the ERGM, but of course omitting all endogenous network statistics. We already noted that the ERGM appears to do a better job at in-sample prediction than the Logit model given its lower AIC and BIC values. Figure S.2 documents both models' respective areas under the receiver-operator (ROC) and precision-recall curves (PR). Here, a higher value of each curve indicates better predictive performance, and again, the ERGM appears to outperform the logistic regression model for both metrics. Figure S.2 thus offers further evidence that in the case of the international arms trade, model performance is improved by accounting for endogenous network effects.
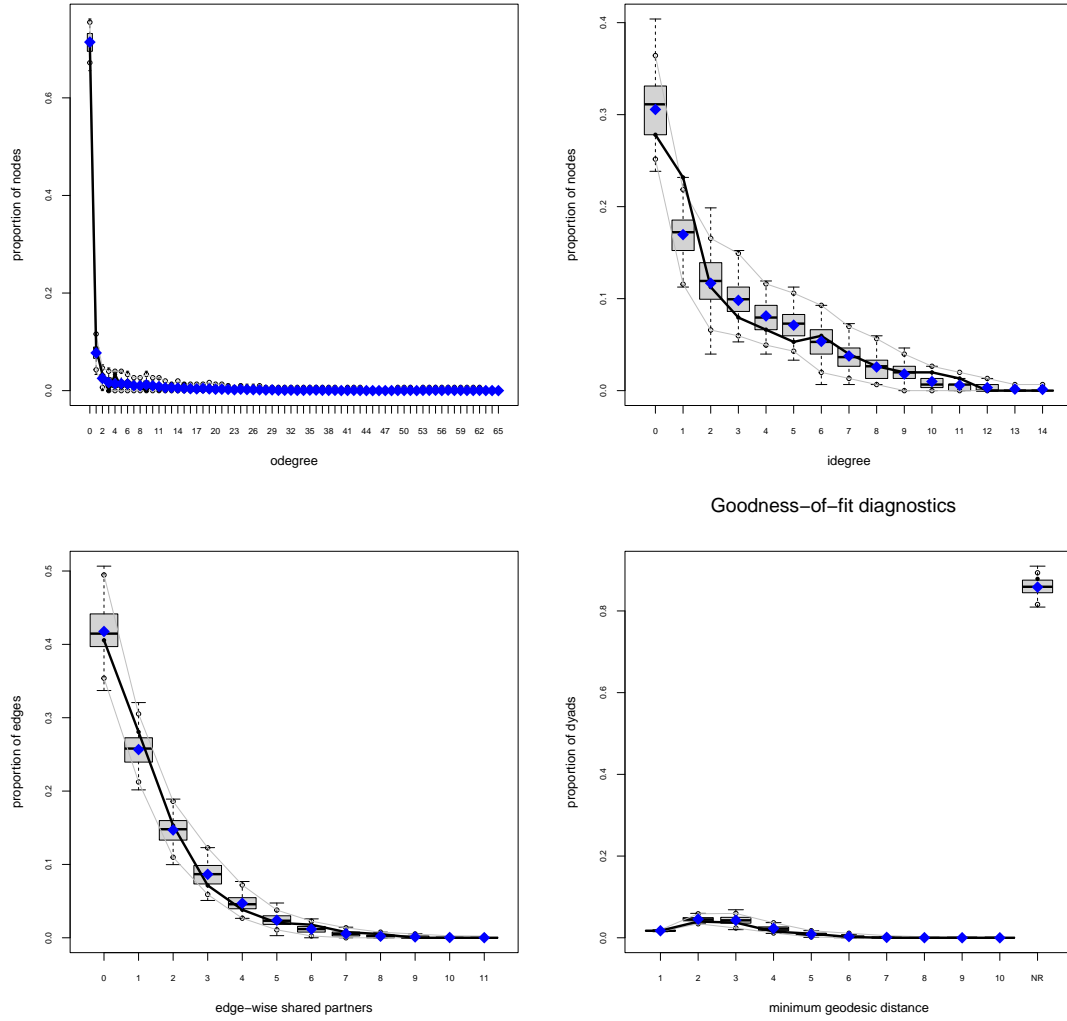
Figure S.1: Goodness of fit plots the international arms trade network in 2018. The metrics show a reasonably good fit for our model.
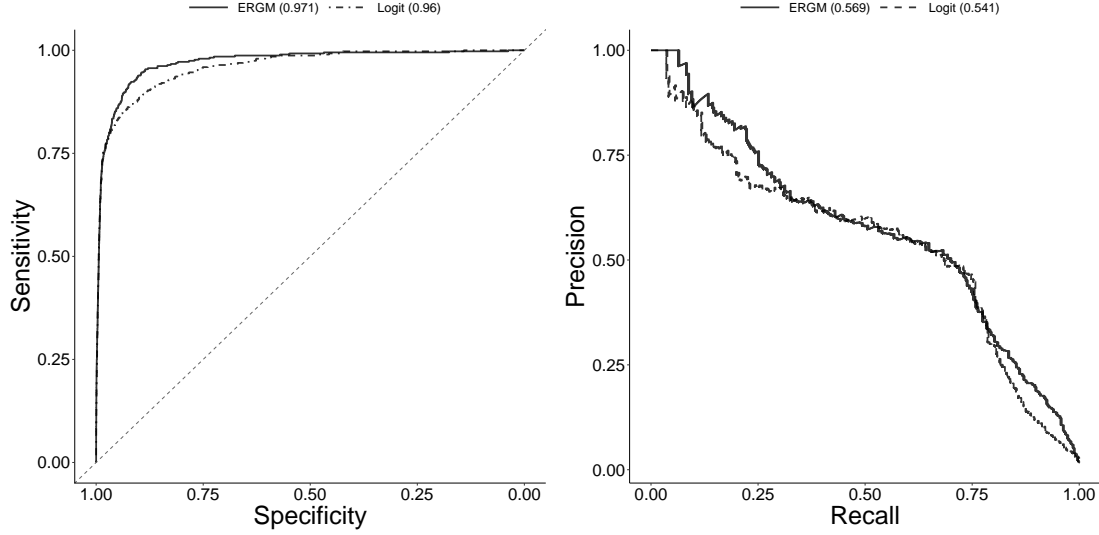
Figure S.2: In-sample prediction performance of the two models in Table 1. Left panel: ROC. Right panel: PR. ERGM outperforms Logit in both metrics.

S.3 Variance and Covariance Parameters of the AME Model

In Section 4.3 of the paper, we showcased the AME model by fitting it to the historical network of global foreign exchange activity in 1900. In illustrating the results, we, for brevity, focused on the main effects of the covariates included, reported in Table 2. But the AME model also estimates several variance and covariance parameters, which also have a meaningful interpretation. The estimates for those parameters are reported in Table S.1. The first two parameters, σ_a^2 and σ_b^2, represent the estimated variances of the additive sender and receiver effects, respectively. We can see that receiver effects are much more variable then sender effects. This makes intuitive sense given that there are a few currencies which are traded by a large number of countries, while many currencies aren't traded at all outside of their origin countries. The skewness in the distribution of incoming ties thus induces a relatively large variance in the receiver effects.

The coefficient in the third row, σ_{ab}, measures the (global) correlation between sender and receiver effects of the same node. In this case, we can see that there is a slight negative correlation between the two, meaning that countries that trade many foreign currencies within their financial hub do not necessarily tend to have their home currency traded in many countries. Finally, the coefficient ρ indicates the covariance between the residuals on the same node-pair, ε_{ij} and ε_{ji}. This parameter quantifies the tendency towards reciprocity in the network. In this case, we can see that there is a slight positive tendency for ties to be reciprocated.

	Estimate	Standard deviation
σ_a^2	0.557	0.165
σ_b^2	1.863	0.860
σ_{ab}	-0.350	0.250
ρ	0.364	0.319

Table S.1: Estimated variance and covariance parameters for the AMEN model.

S.4   AMEN Model Diagnostics and Goodness of Fit

Similarly as for ERGM, the goodness of fit for AME models is evaluated by comparing the statistics observed in the real world network with the distribution of the same statistics calculated for networks simulated from the model. Figure S.3 depicts this comparison for first order effects (top panel) and for second- and third-order dependencies (bottom panel), as done by default in the `amen` R package (see Hoff, 2015 for details). All in all, we can see that the model does a reasonably good job in preserving the network statistics in question.

We can also compare the goodness of fit of the AME model with that of the Probit model including the same exogenous covariates, but of course omitting the latent variables. Figure S.4 depicts the same comparison of observed with simulated statistics just described, but for Probit instead of AME. From the plots therein we can see how the Probit model does a markedly worse job than the AME in reproducing first and third order dependencies, thus demonstrating an overall worse performance in capturing the mechanisms at play in the network.

In Figure S.5, also produced by default by the `amen` package, we can further check how the coefficients and their variance vary across the MCMC iterations. In general, the fit is considered to be acceptable if no visible trends emerge in the chains. If, to the contrary, trends in the estimates are visible, the researcher must consider running the chain for more iterations and/or using alternative model formulations. In the case of Figure S.5, visual inspection gives us confidence that the AME model has reasonably converged.
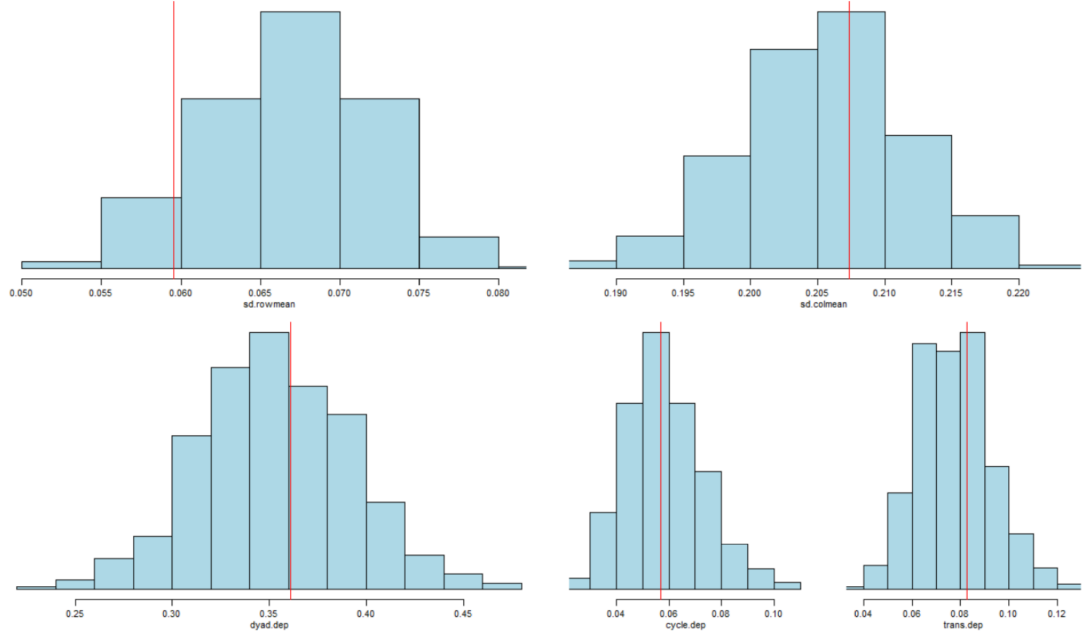
Figure S.3: Goodness of fit plots for the AME model applied to the global foreign exchange network in 1900. The model does an overall good job in replicating the observed network statistics.
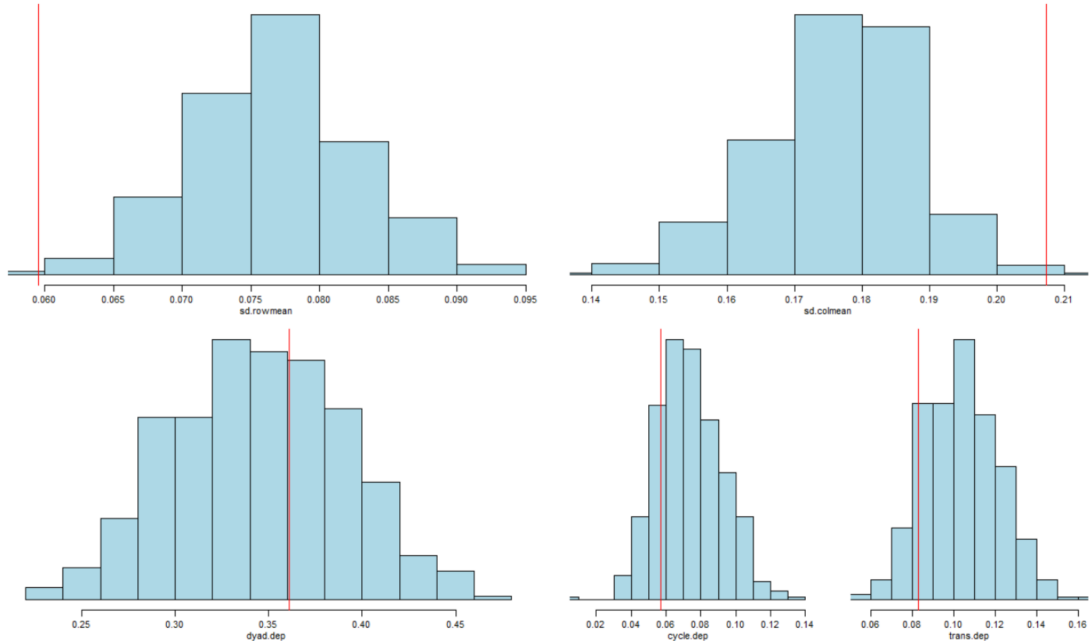

Figure S.4: Goodness of fit plots for the classical probit model applied to the global foreign exchange network in 1900. The model performs visibly worse than AME in replicating the observed network statistics.
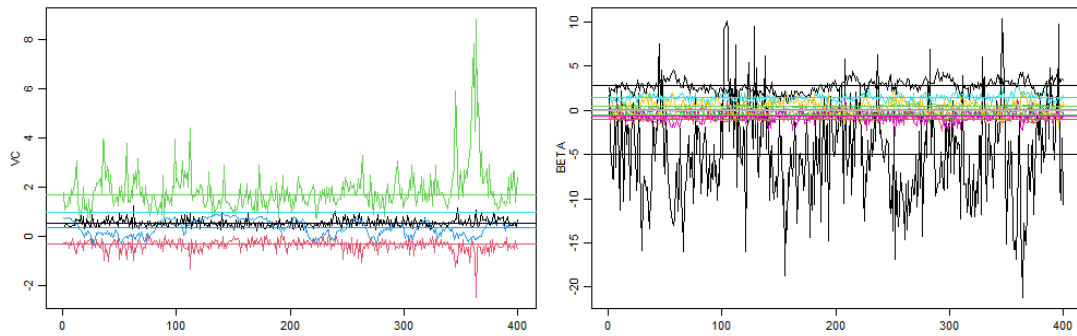
Figure S.5: MCMC diagnostics for the AME model applied to the global foreign exchange network in 1900. Visual inspection gives us confidence that the model has reasonably converged.