The Chinese University of Hong Kong
RMSC 4002: Financial Data Analytics with Machine Learning

Optimizing Stock Portfolio Performance through Principal Component Analysis-Based Stock Selection Method and Multiple Portfolio Weighting Scheme

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Abstract

Using S&P500 constituents as our investigation, we select around 10 - 30 stocks based on a discarding variables method in a Principal Component Analysis. Then, we use them to construct long-only equal risk contribution and equal weighting portfolios. Then, we compare the stock portfolios' performance and index by different risk measures, including Value-at Risk and Expected Shortfall. Our long-only portfolios are able to outperform the S&P index in a bear market. Furthermore, we extend our portfolio to integrate long/short strategies in order to be more aggressive in some market conditions, enabling investors to exploit more opportunities in the markets.

1 Introduction

S&P 500 is a well-diversified index built from around 500 stocks. A natural question that comes to our mind is, can we select a few stocks from it and build a diverse enough portfolio that is able to mimic the index or even outperform it? This paper uses Principal Component Analysis (PCA) to discard stocks and select only a few that best represent the index. Then, we build equal risk contribution and equal weighting portfolios and see how their performance differs from the index.

First, we construct a timeline with 3 different time points $t = a, b, c$ where $a < b < c$. Assuming it's currently time $t = b$, we want to use the S&P 500 constituents Ω as of $t = b$ and collect their historical price from $t = a$ to $t = b$. Then, we perform PCA on the correlation matrix of returns to select a subset of stocks $S \in \Omega$. Next, we build equal risk contribution and equal weighting portfolios. Before investing, we estimate risk by calculating both portfolio's VaR using historical data from $t = a$ to $t = b$. Finally, we trade these 2 portfolios from $t = b$ to $t = c$ and see how their performances differ from the S&P 500 index. We will also use the returns from $t = b$ to $t = c$ to backtest the VaR we have estimated.

Here, we define $t = a$ to $t = b$ as a two-year historical period and $t = b$ to $t = c$ as a one-month trading period. In total, we have 82 periods chosen for testing, as shown at the table below.

Periods	Historical Period (2 Years)	Trading Period (1 Month)
Period 1	Jan 2015 - Dec 2016	Jan 2017 - Feb 2017
Period 2	Feb 2015 - Jan 2017	Feb 2017 - Mar 2017
...	...	...
Period 81	Aug 2021 - Jul 2023	Aug 2023 - Sep 2023
Period 82	Sep 2021 - Aug 2023	Sep 2023 - Oct 2023

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2 Data Preparation

2.1 Data Collection

We get historical S&P 500 constituents from Bloomberg Terminal and historical adjusted close price data from Yahoo Finance using `get.hist.quote()` function in R. A general function to get the price data is written in the `collect_clean_data()` function. Note that some of the stock data are unavailable in Yahoo Finance. The reason for referring to historical constituents but not recent ones for selecting stocks is to avoid survivorship bias. However, we simplified our data collection process by only collecting 7 lists of historical constituents, which are constituents as of 31/12/2016, 31/12/2017, ..., 31/12/2022, and each of them will be used throughout the following trading years, which are 2017, 2018, ..., 2023 respectively.

2.2 Data Cleaning

We clean price data using the `subset_and_clean()` function to remove the stocks if no price data is recorded over the whole time horizon. We also use this function to subset price data for different trading periods.

2.3 Data Transformation

Since we will perform PCA on the correlation matrix of returns of stocks, we use the `sret()` function to get the simple returns of all stocks, where we define simple return at time t as

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

, where P_t is the adjusted close price of the stock at time t.

3 Stock Selection

3.1 Principal Component Analysis

Let's say we have p number of variables. A common usage of PCA is to select only m PCs when $m \ll p$ to reduce the overall dimension to account for most of the variation of the dataset. However, in this paper, we do not choose a subset of m PCs to explain the variation of stock returns. Instead, we choose a subset of m stocks that can best explain the variation of all constituents' returns. Even though this is achievable using multiple regression analysis, we decided to attempt using PCA.

3.2 Discarding Stocks in a Principal Component Analysis

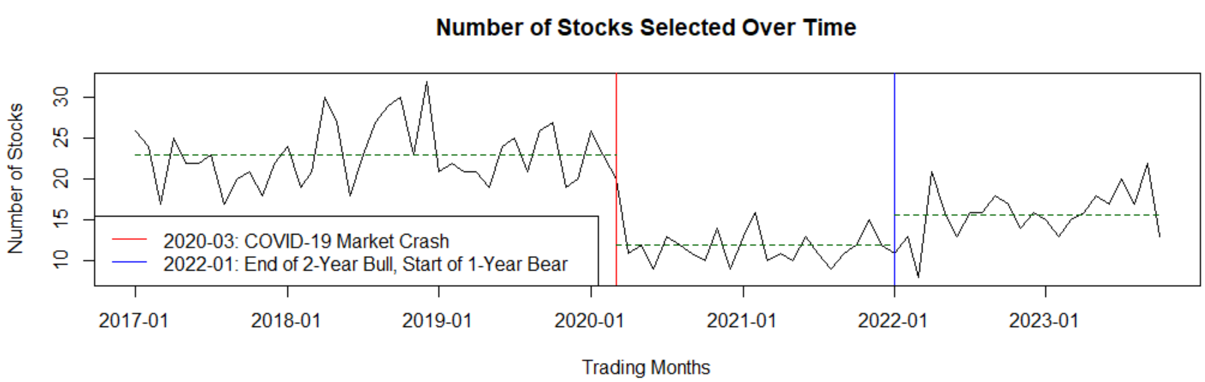
[1] discussed several ways to discard variables in a PCA, and we used its B2 method. As shown below, we followed the procedure stated in [2] and [3]:

First, we associate one stock with the highest coefficient in absolute value with each of the last m_1 principal components that have eigenvalues less than ℓ, then delete those m_1 stocks. A new PCA is performed on the remaining stocks. Again, associate one stock with the highest coefficient in absolute value with each of the last m_2 principal components that have eigenvalues less than ℓ, then delete those m_2 stocks. Repeat the procedure until the eigenvalue of the last PC is higher than s, where ℓ is the deletion criteria, and s is the stopping criteria.

The intuition of this method [2] is PCs that have small eigenvalues corresponding to near-constant relationships among the stocks. Little information would be lost if a stock with the highest loading in absolute value is deleted. Here, we set $\ell = 1$ based on Kaiser's rule [4], which means we are retaining factors that can explain more variance than a single variable. We also set $s = 0.5$ as an optimized parameter by trial and error, where it can give optimal returns and reasonable numbers of stocks selected.

3.3 Stocks Selected

The number of stocks selected varies over time. We want to investigate if the market condition will affect how many stocks PCA are going to select.

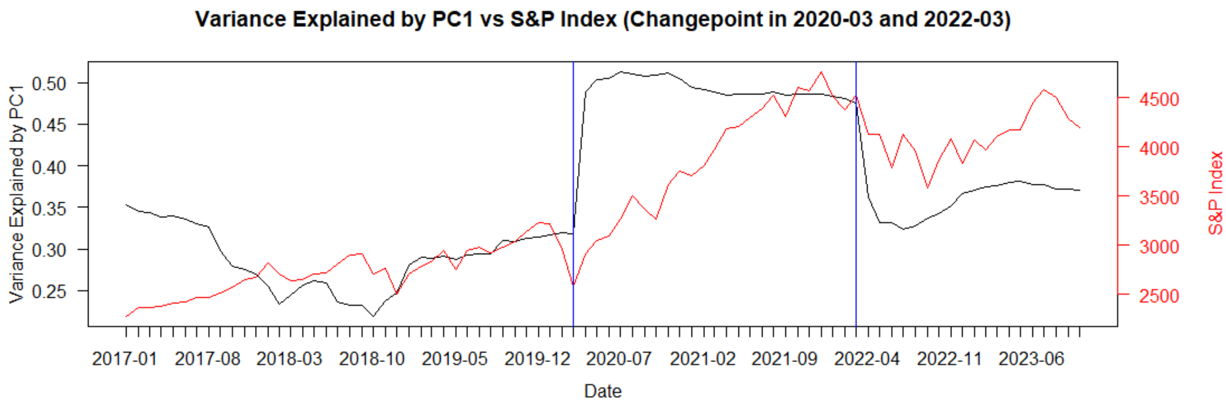


The plot shows a change in the mean of selected stocks whenever a big event happens in the market. The mean number of stocks selected was 22.95 before the COVID-19 market crash. Then, there was a two-year bull rally, and the mean changed to 11.96. When the trend reversed in 2022, the mean changed to 15.68.

We are also interested in the frequency of stocks chosen and their relationship with their period geometric mean daily return. An ideal result is that this method could select good stocks more often than bad ones. The box plots of each year are shown in Appendix 1. Unfortunately, we are not able to identify their relationships clearly.

3.4 Interpretation of Principal Component One

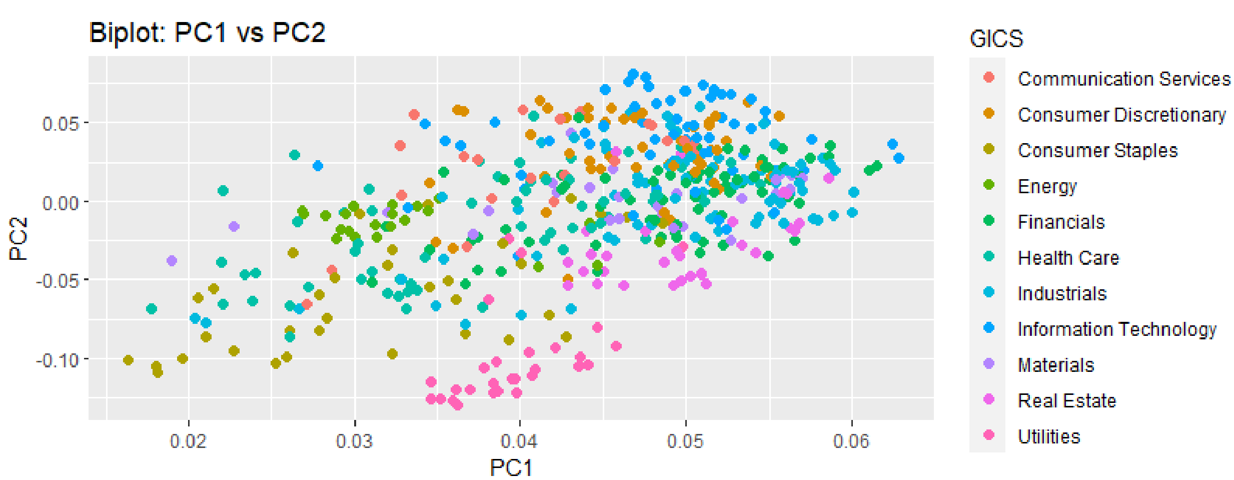
A detailed interpretation of Principal Component One (PC1) can be found in chapter 8 of [3]. First, the paper discussed using PC1 as a measure of the level of systemic risk. Note that systemic risk is the ratio of systematic risk to idiosyncratic risk. The idea is if a set of stocks are highly correlated, the PC1 will have high eigenvalues, which implies capturing more variation. and can be used to measure systemic risk. Below is the relationship between the variance explained by PC1 and the S&P index.



We can see that the variance explained by PC1 is indeed a good measure of the level of systemic risk. Two of the big change points are at the COVID-19 market crash in 2020-03 and at the end of the bull market since that crash in 2022-03. Therefore, we can conclude that during the recovery from the COVID-19 market crash, the systematic risk of the market is high. Besides, the paper discussed that the eigenvalue of PC1 is correlated with the diversification ratio [5] and KMO measure of sampling adequacy, which are possible extensions of our project.

3.5 Interpretation of Principal Component Two

A detailed interpretation of Principal Component Two (PC2) can be found in chapter 9 of [3]. The paper stated that PC2 showed a grouping of industries. We get the Global Industry Classification Standard (GICS) data from Wikipedia [6] and just perform PCA on the most recent two years' data, and the biplot below verified that stocks from the same industries will cluster together.



4 Long-only Portfolio Construction

4.1 Long-only Portfolio Overview

In the construction of our long-only portfolio strategies, we primarily utilize equal-weight and equal-risk contribution (ERC) portfolios. The selection of stocks is guided by the Principal Component Analysis (PCA). To optimize our portfolio, we reselect stocks and rebalance the portfolio on a monthly basis. This approach has proven to be more effective than maintaining a constant stock constituent throughout the year, as evidenced by our empirical analysis. For performance evaluation and comparison, we use the buy-and-hold S&P500 as our benchmark.

4.2 Equal-Risk Contributions (ERC) Portfolio

The objective of an ERC portfolio is to assign weights to each asset in a way that equalizes the risk associated with each weight. In practical terms, we consider the risk of each constituent in our portfolio to be a function of its weight as discussed in [7] and [8]. This allows us to apply an optimization method to solve the following mathematical problems:

Porfolio's Risk Equation: $\sigma_p = \sqrt{\sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_{ij}}$

Marginal Contribution to Risk $= w_i \frac{\partial \sigma_p}{\partial w_i}$

For an Equal Risk Portfolio, we wants to find w such that: $w_i \frac{\partial \sigma_p}{\partial w_i} = w_j \frac{\partial \sigma_p}{\partial w_j}$ for all securities i, j

4.3 Motivation behind ERC and Equal-Weight Porfolio

Based on the discussion in [7] published by Graham Capital, The primary objective of employing both Equal-Risk Contribution (ERC) and Equal-Weight portfolio strategies following stock selection is diversification. While an equal-weight strategy offers a simple and intuitive approach to diversify a portfolio, the ERC strategy incorporates an essential metric, which is risk, to determine portfolio weights. In certain scenarios, an equal-weight strategy might lead to excessive or insufficient risk exposure to the securities we hold. The ERC strategy addresses this issue, leading to improved diversification and less concentrated risks. Another advantage of the ERC strategy is that it eliminates the need to estimate expected returns, which can be challenging. Instead, we can rely on robust volatility models, such as the GARCH(1,1) model, to provide a more accurate estimate of our securities' standard deviation. Furthermore, several studies suggest that an ERC portfolio can enhance investment performance by improving risk-adjusted returns, reducing maximum drawdowns within a specific trading period, and minimizing undesirable risk exposure.

4.4 Implementation of both ERC and Equal-Weight Portfolio

1. First, try to use DCC-GARCH(1,1) model to fit the past 252 days return data. We use 252 days data so that we are able to capture long-enough historical behavior of the stocks' volatility, while still making it relevant to trade it for a relatively shorter period of 1 month.
2. Suppose we are standing at time t, we then forecast the covariance matrix Σ_{t+1}.
3. By using the forecasted covariance matrix, we calculate the weights for the ERC portfolio by essentially solving the following objective function:

$$\min_w \sum_{i,j} \left[w_i (\Sigma w)_i - w_j (\Sigma w)_j \right]^2$$

   Subject to $\sum_i w_i = 1$.
4. For the equal-weight portfolio, we simply put $\frac{1}{n}$ on each constituent where n is the total number of stocks selected.

5. Then, we obtain both weights for equal-weight and ERC portfolio for our portfolio initiation at the beginning of the month.
6. We let the stock moves without any intervention throughout the month and perform rebalancing at the beginning of next month in order to minimize the transaction cost of this strategy.

All these steps are implemented under our `portfolio_construction()` function, which also output all the necessary graphs, trading results, and Value at Risk for our analysis.

4.5 Trading Performance Analysis Metrics

By taking reference to Investopedia [9], we will consider four metrics to analyze our annual trading performance from 2017 to 2023:

- **Maximum Drawdown (MDD)**: This is the greatest observed loss from the highest (peak) to the lowest point (trough) of a portfolio, before a new high (peak) is achieved.

$$MDD = \frac{\text{Trough Value} - \text{Peak Value}}{\text{Peak Value}}$$

- **Sharpe Ratio**: This measures risk-adjusted performance and is calculated as the ratio of a portfolio's excess returns to its volatility.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

- **Calmar Ratio**: This is calculated as the ratio of a portfolio's excess returns to its Maximum Drawdown (MDD).

$$\text{Calmar Ratio} = \frac{R_p - R_f}{\text{MDD}}$$

- **Sortino Ratio**: Similar to Sharpe Ratio, but it now only considers the standard deviation of the downside risk, rather than the total risk (both upside and downside).

$$\text{Sortino Ratio} = \frac{R_p - R_f}{\sigma_d}$$

Sharpe ratio, Calmar ratio, and Sortino ratio are the three metrics used to assess the risk-adjusted return of a portfolio. Hence, higher results are desirable to indicate better performance.

5 Annual Performance Review: Index VS Long-only Portfolio

5.1 Annual Performance Data

In this section, ERC stands for Equal Risk Contribution (Red line), EWS stands for Equal-weight strategies (Blue line), and B&H stands for Buy and Hold S&P500 (Green line). The graphs are illustrated in appendix.

Year	Trading Period (1 year)	Total Return	Sharpe Ratio	Calmar Ratio	Sortino Ratio	Max Drawdown
Year 1	2017-01-01 to 2017-12-31	ERC: +8.87% EWS: +13.34% B&H: +14.7%	ERC: 1.07 EWS: 1.44 B&H: 2.30	ERC: 1.44 EWS: 2.36 B&H: 5.14	ERC: 0.10 EWS: 0.13 B&H: 0.21	ERC: -6.21% EWS: -5.7% B&H: -2.89%
Year 2	2018-01-01 to 2018-12-31	ERC: -5.98% EWS: -7.02% B&H: -7.20%	ERC: -0.40 EWS: -0.43 B&H: -0.43	ERC: -0.36 EWS: -0.35 B&H: -0.33	ERC: -0.03 EWS: -0.03 B&H: -0.03	ERC: -16.71% EWS: -20.29% B&H: -21.76%
Year 3	2019-01-01 to 2019-12-31	ERC: +19.93% EWS: +20.28% B&H: +30.10%	ERC: 1.72 EWS: 1.51 B&H: 2.48	ERC: 1.94 EWS: 1.52 B&H: 4.60	ERC: 0.15 EWS: 0.13 B&H: 0.20	ERC: -10.35% EWS: -13.36% B&H: -6.57%
Year 4	2020-01-01 to 2020-12-31	ERC: +3.99% EWS: +7.24% B&H: +11.80%	ERC: 0.14 EWS: 0.25 B&H: 0.35	ERC: 0.12 EWS: 0.22 B&H: 0.32	ERC: 0.02 EWS: 0.03 B&H: 0.04	ERC: -33.27% EWS: -33.40% B&H: -36.83%
Year 5	2021-01-01 to 2021-12-31	ERC: +16.39% EWS: +9.21% B&H: +21.48%	ERC: 1.31 EWS: 0.71 B&H: 1.72	ERC: 1.57 EWS: 0.70 B&H: 3.34	ERC: 0.12 EWS: 0.07 B&H: 0.15	ERC: -10.51% EWS: -13.26% B&H: -6.45%
Year 6	2022-01-01 to 2022-12-31	ERC: -12.67% EWS: -13.13% B&H: -21.85%	ERC: -0.69 EWS: -0.67 B&H: -0.92	ERC: -0.63 EWS: -0.67 B&H: -0.80	ERC: -0.05 EWS: -0.05 B&H: -0.08	ERC: -20.19% EWS: -19.81% B&H: -27.56%
Year 7	2023-01-01 to 2023-08-31	ERC: -5.91% EWS: -22.97% B&H: +6.89%	ERC: -0.56 EWS: -1.15 B&H: 0.63	ERC: -0.60 EWS: -0.88 B&H: 0.83	ERC: -0.04 EWS: -0.11 B&H: 0.06	ERC: -11.88% EWS: -30.87% B&H: -10.21%

Table 1: Annual Trading Performance of Long-Only Portfolio from 2017 to 2023

5.2 Our Portfolio Tends to Outperform in The Bear Market

Referring to Appendix 2, we observe bear markets occurring in 2018, 2020, and 2022. In 2018, as part of efforts to reduce the US trade deficit, the US Government imposed tariffs on imported goods from its trading partners, not just China [10]. This policy inadvertently hurt the US domestic economy due to increased uncertainty and disruptions for US companies that rely on these imported raw materials for their production inputs. Ultimately, not only did foreign exporters bear the cost of tariffs, but also US companies that depend on imported goods. Coupled with four interest rate hikes throughout the year, markets became pessimistic, driving the S&P500 down by the end of the year.

In 2020, the Covid-19 pandemic severely impacted the markets, leading to a more than 30% decrease in the S&P500 in Q1 2020. However, the market rebounded quickly following aggressive quantitative easing in 2020, which led interest rates to reach near zero. Consequently, when the economy heated up in 2022, the Fed was forced to aggressively hike interest rates, leading to another bear market that year [11].

Our stock selection with the Equal Risk Contribution (ERC) and equal-weight portfolio (EWS) strategies tends to outperform in bear markets due to their defensive nature. In 2018, the ERC Portfolio had the highest total return of -5.98% and the best maximum drawdown of -16.71% compared to the Buy and Hold (B&H) S&P500 benchmark and the EWS, although their risk-adjusted measures (Sharpe, Calmar, and Sortino Ratios) were relatively similar. In 2020, the ERC and EWS had similar maximum drawdowns of around 33%, which was again better than the S&P500 benchmark. However, this time the total return of the S&P500 was the highest due to a strong market rebound after Q1 2020. In this case, the S&P500 performed the best, while the EWS performed better than the ERC Portfolio. Lastly, in 2022, the ERC and EWS had similar maximum drawdowns and total returns, as well as similar risk-adjusted measures, which were again much better than the S&P500.

Based on these three different periods, the ERC appears to be the most conservative strategy with the best ability to protect our performance during the bear market, having the safest maximum drawdown and highest total return. However, the ERC did not perform well when rebounds occurred following the bear market, as indicated by the worst total return and lowest risk-adjusted measures in 2020, although it still had the safest maximum drawdown in that trading period. The EWS sits in the middle, although it still leans towards being conservative. It performed better than the S&P500 in the bear markets, with the ability to still capture the effect of stock rebounds in 2020. Lastly, the S&P500 tends to be riskier with more extreme downside potential in any bear market, although bearing more risks also enabled this portfolio to recover more quickly after the index level bottomed out in 2020. However, in uncertain periods, investors might prefer to choose less risky investment options, such as the ERC, and switch to a riskier one once a rebound signal is confirmed.

5.3 Our Portfolio Tends to Underperform in The Bull Market

As observed of graphs in Appendix, bull markets occurred in 2019, 2020 (after Q1), and 2021, with performances in 2017 and 2023 are also generally in uptrends. The events of 2020 and 2021 were largely influenced by the recovery from the Covid-19 pandemic. In 2019, the Federal Reserve cut interest rates three times in the second half of the year [11]. Despite bearish predictions throughout 2023 due to aggressive rate hikes, the overall trend of the S&P500 was going up, primarily driven by AI companies such as NVIDIA and META [12].

Our portfolio underperformed the benchmark in all these bull markets. This is because in bull markets, stock prices are generally increasing. By taking on more risk, investors can realize higher total returns during a bull run. However, predicting the timing and duration of a bull market rally is challenging, leading investors to face trade-offs when deciding their portfolio risk exposure.

Based on the performance during these uptrend years, the Buy and Hold S&P500 strategy consistently emerged as the most optimal option, delivering the highest total return and highest risk-adjusted return. From 2019, 2021, and also in 2023, we observed that the Equal Risk Contribution (ERC) portfolio had better risk-adjusted returns across all three ratios compared to its equal-weight counterparts, although the total return was not necessarily higher. Moreover, the ERC proved to be safer in terms of maximum drawdowns compared to the equal-weight strategy (EWS). In contrast, in 2020, the EWS performed better than the ERC in terms of total return and risk-adjusted return due to the stronger-than-expected post-Covid recovery momentum. As we know, the ERC tends to overweight less risky stocks and underweight riskier stocks. Although the EWS also has this tendency, it is less pronounced compared to the ERC, causing the ERC to fail to exploit the market opportunity during the recovery period.

In summary, we still rank the ERC as the most conservative strategy, followed by the EWS in the middle. The S&P500 is definitely a more aggressive strategy and hence performs better in the bull market.

6 Performance Review (2022): Index VS Long-only Portfolio

Since our portfolio shows the best in 2022, let's further review the performance of our strategies in Trading period 2022.

Periods	**Historical Period (2 Years)**	**Trading Period (1 Month)**
Period 61	Jan 2020 - Dec 2021	Jan 2022 - Feb 2022
Period 62	Feb 2020 - Jan 2022	Feb 2022 - Mar 2022
Period 63	Mar 2020 - Feb 2022	Mar 2022 - Apr 2022
Period 64	Apr 2020 - Mar 2022	Apr 2022 - May 2022
Period 65	May 2020 - Apr 2022	May 2022 - Jun 2022
Period 66	Jun 2020 - May 2022	Jun 2022 - Jul 2022
Period 67	Jul 2020 - Jun 2022	Jul 2022 - Aug 2022
Period 68	Aug 2020 - Jul 2022	Aug 2022 - Sep 2022
Period 69	Sep 2020 - Aug 2022	Sep 2022 - Oct 2022
Period 70	Oct 2020 - Sep 2022	Oct 2022 - Nov 2022
Period 71	Nov 2020 - Oct 2022	Nov 2022 - Dec 2022
Period 72	Dec 2020 - Nov 2022	Dec 2022 - Jan 2023

Firstly, we evaluated daily P&L performance in both historical and trading period, where

$$\text{Daily P\&L} = \text{Portfolio}_t - \text{Portfolio}_{t-1}$$

Secondly, we conducted Value at Risk analysis of P&L in historical period and validated models' accuracy.

Thirdly, we conducted Normality test on its Return distribution in historical period, where

$$\text{Daily Return} = \frac{\text{Portfolio}_t - \text{Portfolio}_{t-1}}{\text{Portfolio}_{t-1}}$$

6.1 Daily P&L performance in Historical and Trading Period

Let's review the P&L and Historical VaR performance of our strategies in historical period (period 61 to period 72).

Here is the criteria to determine a better strategy 1 than strategy 2:

$$\begin{cases} \text{Mean}_1 \gg \text{Mean}_2 \text{ ; or/and} \\ \sigma_1 \ll \sigma_2 \text{ ; or/and} \\ \text{10th percentile}_1 \gg \text{10th percentile}_2 \text{ ; or/and} \\ \text{5th percentile}_1 \gg \text{5th percentile}_2 \end{cases}$$

In historical period (2020-2022 which covers bullish market mostly), Both ERC and Equal-Weight Portfolio (EWS) do not perform better than Buy and Hold Strategy (B&H). Only 1 out of 12 historical periods (8.33%) fulfill the criteria of better performance for our trading strategies than Buy and Hold Strategy. In fact, B&H Strategy performed better most of these periods.

However, in trading period (2022 which covers bearish market mostly), both ERC and Equal-Weight Portfolio perform better than Buy and Hold Strategy. 10 out of 12 periods (83.33%) fulfill the criteria of better performance with higher mean, less volatility in our trading strategies than Buy and Hold Strategy. 6 out of 12 periods (50%) shows ERC outperforms Equal-Weight Portfolio, In the remaining 6 periods, They exhibit similar result or Equal-Weight Portfolio outperforms ERC. Thus both of trading strategies outperform in 2022 trading period, meanwhile ERC does not significantly show a better result than Equal-Weight Strategy.

Periods	Historical Period (2 Years)	Min($)	Max($)	Mean($)	Volatility($)	10th perc.($)	5th perc.($)
Period 61	Jan 2020 - Dec 2021	ERC: -37.67	ERC: +32.76	ERC: +0.34	**ERC: 5.44**	**ERC: -5.18**	**ERC: -7.12**
		EWS: -53.38	EWS: +46.42	EWS: +0.48	EWS: 7.71	EWS: -7.33	EWS: -10.10
		B&H: -99.73	B&H: +70.72	**B&H: +0.92**	B&H: 15.25	B&H: -15.13	B&H: -24.71
Period 62	Feb 2020 - Jan 2022	ERC: -40.11	ERC: +30.93	ERC: +0.16	**ERC: 6.35**	**ERC: -6.52**	**ERC: -9.60**
		EWS: -51.05	EWS: +39.37	EWS: +0.20	EWS: 8.07	EWS: -8.30	EWS: -12.22
		B&H: -94.14	B&H: +66.76	**B&H: +0.73**	B&H: 14.68	B&H: -14.88	B&H: -24.80
Period 63	Mar 2020 - Feb 2022	ERC: -51.09	ERC: +52.91	ERC: +0.10	ERC: 7.39	ERC: -6.86	ERC: -10.80
		EWS: -45.47	EWS: +47.09	EWS: +0.09	**EWS: 6.58**	**EWS: -6.11**	**EWS: -9.61**
		B&H: -95.22	B&H: +67.52	**B&H: +0.75**	B&H: 14.90	B&H: -15.19	B&H: -25.01
Period 64	Apr 2020 - Mar 2022	ERC: -69.57	ERC: +64.17	ERC: +1.26	ERC: 16.62	ERC: -19.30	**ERC: -28.11**
		EWS: -90.42	EWS: +83.97	**EWS: +1.50**	EWS: 19.66	EWS: -20.77	EWS: -32.63
		B&H: -72.52	B&H: +67.51	B&H: -1.57	**B&H: 16.07**	**B&H: -18.49**	B&H: -28.12
Period 65	May 2020 - Apr 2022	ERC: -36.98	ERC: +39.51	ERC: +0.41	ERC: 6.29	ERC: -6.70	ERC: -9.24
		EWS: -28.92	EWS: +30.89	EWS: +0.32	**EWS: 4.92**	**EWS: -5.24**	**EWS: -7.23**
		B&H: -57.53	B&H: +32.79	**B&H: +0.79**	B&H: 12.79	B&H: -15.43	B&H: -23.00
Period 66	Jun 2020 - May 2022	ERC: -85.70	ERC: +95.06	ERC: +0.05	ERC: 13.10	**ERC: -13.22**	**ERC: -19.93**
		EWS: -116.35	EWS: +129.05	EWS: +0.07	EWS: 17.79	EWS: -17.94	EWS: -27.05
		B&H: -53.00	B&H: +35.14	**B&H: +0.60**	**B&H: 12.45**	B&H: -14.75	B&H: -22.30
Period 67	Jul 2020 - Jun 2022	ERC: -67.33	ERC: +76.76	ERC: +0.40	ERC: 15.00	ERC: -16.13	ERC: -24.04
		EWS: -91.35	EWS: +102.78	**EWS: +0.54**	EWS: 19.67	EWS: -21.89	EWS: -32.61
		B&H: -42.14	B&H: +31.81	B&H: +0.34	**B&H: 11.51**	**B&H: -13.77**	**B&H: -21.57**
Period 68	Aug 2020 - Jul 2022	ERC: -120.51	ERC: +55.84	ERC: +0.51	ERC: 19.43	ERC: -23.07	ERC: -30.68
		EWS: -200.45	EWS: +92.88	**EWS: +0.84**	EWS: 32.32	EWS: -38.38	EWS: -51.03
		B&H: -43.03	B&H: +32.48	B&H: +0.43	**B&H: 11.95**	B&H: **-14.07**	**B&H: -22.07**
Period 69	Sep 2020 - Aug 2022	ERC: -20.82	ERC: +26.21	**ERC: +0.00**	**ERC: 3.72**	**ERC: -4.14**	**ERC: -6.03**
		EWS: -25.29	EWS: +31.83	**EWS: +0.00**	EWS: 4.52	EWS: -5.03	EWS: -7.32
		B&H: -38.60	B&H: +29.14	B&H: -0.20	B&H: 10.95	B&H: -12.93	B&H: -20.03
Period 70	Oct 2020 - Sep 2022	ERC: -103.72	ERC: +129.24	ERC: -0.17	ERC: 16.95	ERC: -18.08	ERC: -24.04
		EWS: -130.44	EWS: +162.53	EWS: -0.21	EWS: 21.32	EWS: -22.74	EWS: -30.23
		B&H: -39.16	B&H: +27.48	**B&H: +0.09**	**B&H: 10.42**	**B&H: -12.19**	**B&H: -18.81**
Period 71	Nov 2020 - Oct 2022	ERC: -112.99	ERC: +131.29	ERC: +0.42	ERC: 21.02	ERC: -22.83	ERC: -29.20
		EWS: -153.02	EWS: +177.81	**EWS: +0.57**	EWS: 28.46	EWS: -30.92	EWS: -39.55
		B&H: -42.10	B&H: +29.54	B&H: +0.26	**B&H: 11.43**	**B&H: -13.09**	**B&H: -20.30**
Period 72	Dec 2020 - Nov 2022	ERC: -47.66	ERC: +93.70	ERC: +0.73	ERC: 16.03	ERC: -17.83	ERC: -23.89
		EWS: -67.80	EWS: +133.31	**EWS: +1.03**	EWS: 22.81	EWS: -25.36	EWS: -33.99
		B&H: -40.26	B&H: +47.08	B&H: +0.19	**B&H: 11.20**	**B&H: -12.55**	**B&H: -19.64**

Table 2: daily P&L performance for ERC, EWS and B&H in Historical Periods 61-72

Periods	Trading Period (1 Month)	Min($)	Max($)	Mean($)	Volatility($)	10th perc.($)	5th perc.($)
Period 61	Jan 2022 - Feb 2022	ERC: -19.29	ERC: +16.77	**ERC: -2.31**	**ERC: 10.03**	ERC: -17.02	ERC: -19.28
		EWS: -17.91	EWS: +19.67	EWS: -2.57	EWS: 10.67	**EWS: -14.18**	**EWS: -15.95**
		B&H: -19.38	B&H: +21.96	B&H: -2.93	B&H: 10.95	B&H: -17.70	B&H: -17.95
Period 62	Feb 2022 - Mar 2022	ERC: -16.62	ERC: +18.06	ERC: -2.17	**ERC: 10.06**	ERC: -14.92	**ERC: -15.52**
		EWS: -21.42	EWS: +17.27	EWS: -2.37	EWS: 10.59	**EWS: -14.21**	EWS: -17.56
		B&H: -23.18	B&H: +19.87	**B&H: -1.88**	B&H: 12.93	B&H: -18.08	B&H: -19.98
Period 63	Mar 2022 - Apr 2022	ERC: -21.44	ERC: +15.66	ERC: +2.57	**ERC: 9.17**	ERC: -5.07	**ERC: -8.08**
		EWS: -21.71	EWS: +15.97	**EWS: +2.76**	EWS: 9.53	**EWS: -4.89**	EWS: -8.51
		B&H: -26.87	B&H: +67.52	B&H: +2.05	B&H: 12.85	B&H: -11.64	B&H: -14.80
Period 64	Apr 2022 - May 2022	ERC: -24.96	ERC: +13.67	**ERC: -3.33**	**ERC: 11.46**	**ERC: -20.44**	**ERC: -22.47**
		EWS: -24.54	EWS: +11.84	EWS: -3.85	EWS: 11.70	EWS: -22.23	EWS: -23.74
		B&H: -32.61	B&H: +21.70	B&H: -4.34	B&H: 13.95	B&H: -25.37	B&H: -25.90
Period 65	May 2022 - Jun 2022	ERC: -30.79	ERC: +18.03	ERC: +0.36	**ERC: 13.33**	**ERC: -20.80**	**ERC: -24.79**
		EWS: -31.97	EWS: +20.43	**EWS: +0.52**	EWS: 14.45	EWS: -23.64	EWS: -25.94
		B&H: -34.42	B&H: +25.99	B&H: -0.23	B&H: 16.80	B&H: -27.53	B&H: -31.95
Period 66	Jun 2022 - Jul 2022	ERC: -30.39	ERC: +18.94	ERC: -1.72	ERC: 12.48	ERC: -18.76	ERC: -18.92
		EWS: -28.75	EWS: +19.30	**EWS: -1.69**	**EWS: 12.06**	**EWS: -17.63**	**EWS: -18.04**
		B&H: -31.76	B&H: +24.36	B&H: -3.16	B&H: 15.16	B&H: -24.56	B&H: -25.87
Period 67	Jul 2022 - Aug 2022	ERC: -9.08	ERC: +16.86	ERC: +0.43	**ERC: 7.38**	ERC: -8.07	ERC: -8.71
		EWS: -7.31	EWS: +17.89	EWS: +1.25	EWS: 7.79	**EWS: -7.01**	**EWS: -7.06**
		B&H: -9.52	B&H: +21.99	**B&H: +3.17**	B&H: 9.77	B&H: -7.91	B&H: -9.35
Period 68	Aug 2022 - Sep 2022	ERC: -22.09	ERC: +17.19	ERC: -1.12	**ERC: 8.13**	**ERC: -9.20**	**ERC: -12.98**
		EWS: -23.09	EWS: +17.14	**EWS: -0.92**	EWS: 8.64	EWS: -10.15	EWS: -14.09
		B&H: -29.48	B&H: +18.29	B&H: -1.48	B&H: 10.61	B&H: -11.06	B&H: -18.12
Period 69	Sep 2022 - Oct 2022	ERC: -31.59	ERC: +30.57	**ERC: -2.36**	ERC: 12.66	ERC: -14.97	**ERC: -15.14**
		EWS: -33.04	EWS: +34.52	EWS: -2.52	EWS: 13.91	EWS: -15.90	EWS: -16.91
		B&H: -36.92	B&H: +14.91	B&H: -3.77	**B&H: 12.22**	**B&H: -13.71**	B&H: -16.32
Period 70	Oct 2022 - Nov 2022	ERC: -15.14	ERC: +19.75	**ERC: +2.67**	**ERC: 10.20**	ERC: -9.39	**ERC: -10.02**
		EWS: -16.61	EWS: +20.24	EWS: +2.60	EWS: 10.78	EWS: -10.58	EWS: -10.78
		B&H: -21.24	B&H: +22.78	B&H: +1.87	B&H: 10.42	**B&H: -7.85**	B&H: -17.59
Period 71	Nov 2022 - Dec 2022	ERC: -24.23	ERC: +27.13	**ERC: +2.49**	**ERC: 10.83**	**ERC: -9.88**	**ERC: -11.43**
		EWS: -25.86	EWS: +30.53	EWS: +2.35	EWS: 12.45	EWS: -13.73	EWS: -14.66
		B&H: -19.61	B&H: +42.26	B&H: +2.17	B&H: 13.66	B&H: -12.64	B&H: -16.17
Period 72	Dec 2022 - Dec 2022	ERC: -16.08	ERC: +10.36	**ERC: -2.15**	**ERC: 6.84**	ERC: -10.79	**ERC: -13.76**
		EWS: -16.86	EWS: +9.93	EWS: -2.47	EWS: 6.94	**EWS: -10.43**	EWS: -14.39
		B&H: -20.27	B&H: +13.45	B&H: -2.30	B&H: 8.99	B&H: -11.72	B&H: -14.83

Table 3: daily P&L performance for ERC, EWS and B&H in Trading Periods 61-72

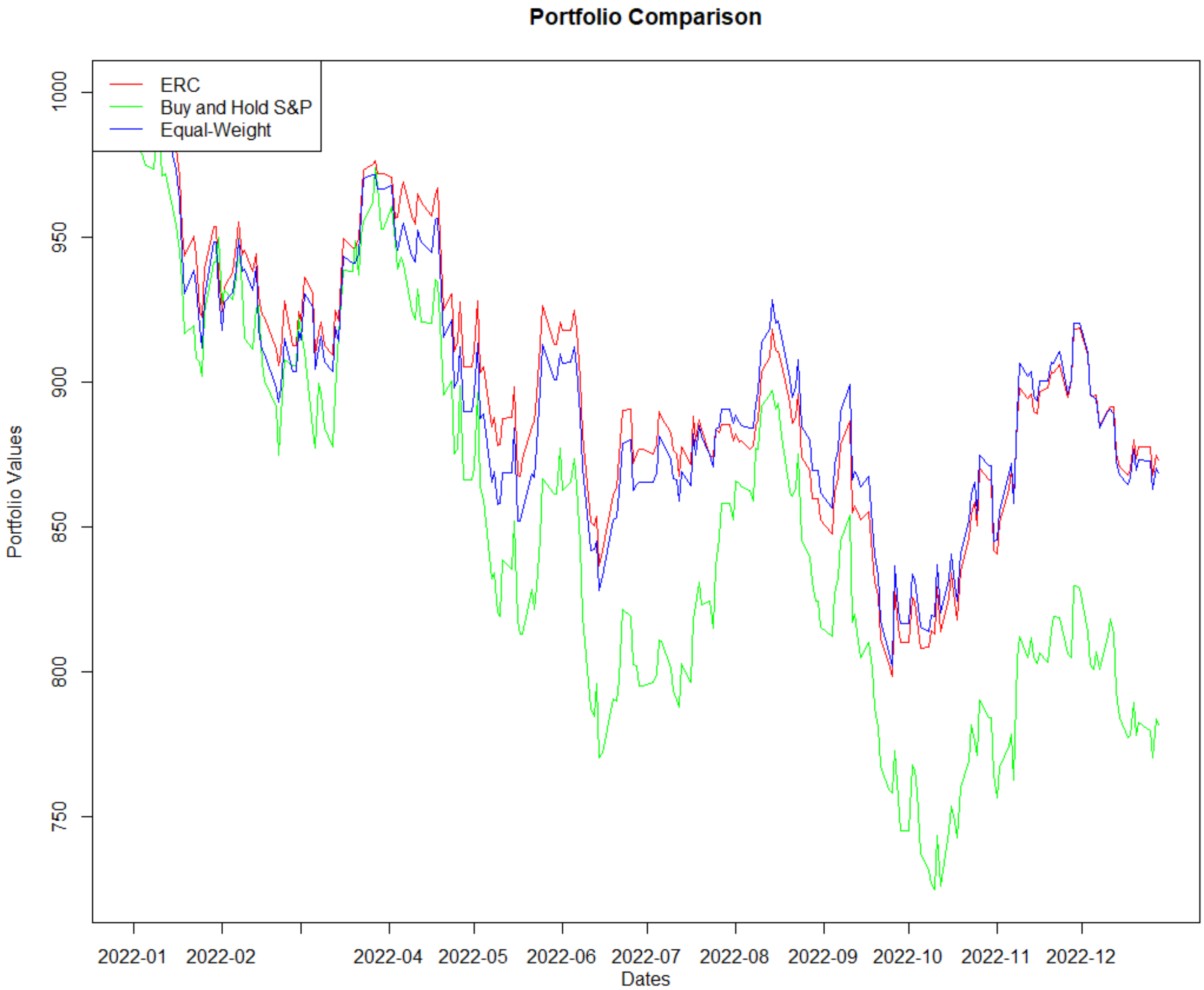


Figure 1: Portfolio value for ERC, EWS and B&H in Trading Periods 61-72 (2022)

6.2 1-day VaR analysis and VaR Backtesting in Historical Period

We also evaluated the daily potential losses (given certain confidence level) of different trading strategies in historical period by Value-at Risk (VaR) analysis. We estimated both 97.5%VaR (most bank requirement) and 99%VaR (Basel III requirement) by 3 methods: Historical Simulation, Normality assumption and Student-t distribution assumption.

In Historical Simulation of 1-day VaR, we are constructing an empirical distribution of daily P&L by using historical period data. Suppose today is day T and v_i is the stock price on i-th scenario. Then stock price tomorrow T+1 is estimated as following:

$$\widetilde{v_i} = v_T \times \frac{v_i}{v_{i-1}}$$

where it assumes the PNL distribution will repeat in the next period. The 1-day (1-α)×100% VaR is calculated by α×100% quantile of the simulated P&L distribution.

In Parametric approach, we assume the daily P&L of each stock $u = (u_1, ..., u_n)^T$ with n number of stocks follows multivariate normal distribution:

$$u = \begin{pmatrix} u_1 \\ u_2 \\ .. \\ u_n \end{pmatrix} \sim N_n(E(u), \Sigma)$$

where the mean and variance of portfolio P&L with its weight $w = (w_1, ..., w_n)^T$ is:

$$u_p = w^T E(u); \quad \sigma_p^2 = w^T \Sigma w$$

In Normality assumption of 1-day VaR, the (1-α)×100% VaR is:

$$VaR_{1-\alpha} = -u_p + z_{1-\alpha} \times \sigma_p; \quad \text{where } z \sim N_n(0, \Sigma)$$

In Student-t distribution assumption, the 1-day (1-α)×100% VaR with d.f. $v = 6/K + 4$ is:

$$VaR_{1-\alpha} = -u_p + t_{v,1-\alpha} \times \sigma_p \times \sqrt{\frac{v}{v-2}}; \quad \text{where } v > 2 \textit{ and } f(t_v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{t^2}{v}\right)^{-\frac{v+1}{2}}$$

In table 4, we calculated 1-day VaR in each model and counted the number of exceptional days in historical period such that daily P&L is larger than its 1-day VaR. Generally, We observed that ERC and Equal-Weight Portfolio contain less extreme days than Buy and Hold Strategy in every models.

We also conducted VaR Backtesting, if the VaR model fulfills the following criterias for validation, it suggests that the estimation of 1-day VaR in that period is accurate:

$$\begin{cases} \text{For 97.5\% VaR: Number of exceptional days} \approx 504 \times 2.5\% = 12 \text{ (}\pm 3 \text{ marginal days)} \\ \text{For 99\% VaR: Number of exceptional days} \approx 504 \times 1\% = 5 \text{ (}\pm 2 \text{ marginal days)} \end{cases}$$

In table 5 we found that in historical simulation, there are only 0-3 out of 12 periods (0-25%) shows accurate estimation of both 97.5%VaR and 99%VaR in three strategies, implying this might not be a good model to estimate VaR. In Normality assumption, 8 out of 12 periods (67%) shows accurate estimation of 97.5%VaR in ERC and Equal-Weight Strategies, but only 5 out of 12 periods (42%) for 99%VaR estimation, even worse, 0-1 out of 12 periods (0-8%) for 97.5%VaR and 99%VaR respectively in Buy and Hold strategy. It shows that normality assumption of VaR model may not be able to capture the extreme loss in tail end. Even though t-distribution assumption of VaR model works the best, with 9-10 out of 12 periods (75-83%) in ERC and Equal-Weight Strategy, only 1 out of 12 periods (8%) shows good estimate of 97.5%VaR in Buy and Hold Strategy, showing that after taking account of kurtosis of t-distribution, it may still underestimate the tail risk of s&p performance in extreme market conditions during 2020-2022.

The possible reason that all three VaR models do not perform well especially in buying and holding S&P is that they may not be able to capture extreme and volatile movement of the stock during 2020 (bearish: COVID), 2021 (bullish: Economic recovery) and 2022 (bearish: Interest rate hike), which create a fatter tails in P&L distribution, and it may also exhibit negative skewness in loss distribution, where both models are not able to capture these features, which furthers prove ERC and Equal-Weight Strategies are less volatile and can be used as a conservative investment strategies in stumbling market conditions.

Periods	**Historical Period**	**Hist_sim_VaR($)**		**Norm_VaR($)**		**t_VaR($)**	
		97.5% (exc. day)	**99% (exc. day)**	**97.5% (exc. day)**	**99% (exc. day)**	**97.5% (exc. day)**	**99% (exc. day)**
Period 61	Jan 2020 - Dec 2021	ERC: 25.08 (1) EWS: 35.54 (1) B&H: 71.18 (2)	ERC: 33.90 (1) EWS: 48.04 (1) B&H: 111.25 (0)	ERC: 10.33 (11) EWS: 14.64 (11) B&H: 28.98 (18)	ERC: 12.33 (6) EWS: 17.46 (6) B&H: 34.56 (10)	ERC: 10.35 (11) EWS: 14.66 (11) B&H: 29.45 (17)	ERC: 14.08 (4) ERC: 19.95 (4) B&H: 38.84 (7)
Period 62	Feb 2020 - Jan 2022	ERC: 22.10 (4) EWS: 28.12 (4) B&H: 63.71 (3)	ERC: 29.01 (2) EWS: 36.92 (2) B&H: 99.53 (0)	ERC: 12.28 (9) EWS: 15.63 (9) B&H: 28.04 (16)	ERC: 14.60 (5) EWS: 18.59 (5) B&H: 33.42 (9)	ERC: 12.48 (9) EWS: 15.88 (9) B&H: 28.50 (15)	ERC: 16.38 (5) ERC:20.85 (5) B&H: 37.53 (7)
Period 63	Mar 2020 - Feb 2022	ERC: 26.48 (3) EWS: 23.56 (3) B&H: 60.27 (3)	ERC: 36.18 (1) EWS: 32.20 (1) B&H: 97.42 (0)	ERC: 14.39 (8) EWS: 12.80 (8) B&H: 28.41 (14)	ERC: 17.10 (6) EWS: 15.21 (6) B&H: 33.86 (8)	ERC: 14.41 (8) EWS: 18.82 (8) B&H: 28.88 (14)	ERC: 19.48 (4) ERC: 17.34 (4) B&H: 38.03 (6)
Period 64	Apr 2020 - Mar 2022	ERC: 72.20 (0) EWS: 90.76 (0) B&H: 57.84 (2)	ERC: 89.34 (0) EWS: 111.16 (0) B&H: 73.08 (0)	ERC: 31.31 (19) EWS: 37.04 (18) B&H: 29.92 (18)	ERC: 37.39 (11) EWS: 44.24 (10) B&H: 35.81 (10)	ERC: 31.82 (16) EWS: 37.65 (17) B&H: 30.42 (17)	ERC: 42.05 (7) ERC: 49.75 (4) B&H: 40.31 (7)
Period 65	May 2020 - Apr 2022	ERC: 20.13 (2) EWS: 15.74 (2) B&H: 40.33 (2)	ERC: 25.76 (1) EWS: 20.14 (1) B&H: 44.45 (2)	ERC: 11.91 (14) EWS: 9.31 (14) B&H: 23.28 (22)	ERC: 14.22 (9) EWS: 11.12 (9) B&H: 28.97 (13)	ERC: 12.11 (13) EWS: 9.47 (13) B&H: 24.68 (22)	ERC: 15.98 (8) ERC: 12.49 (8) B&H: 32.55 (9)
Period 66	Jun 2020 - May 2022	ERC: 31.37 (8) EWS: 42.59 (8) B&H: 37.50 (4)	ERC: 45.22 (2) EWS: 61.39 (2) B&H: 43.85 (2)	ERC: 25.62 (15) EWS: 34.79 (15) B&H: 23.80 (23)	ERC: 30.42 (9) EWS: 41.30 (9) B&H: 28.36 (12)	ERC: 25.67 (15) EWS: 34.85 (15) B&H: 24.19 (20)	ERC: 34.66 (5) ERC: 47.05 (5) B&H: 31.84 (10)
Period 67	Jul 2020 - Jun 2022	ERC: 41.61 (6) EWS: 56.44 (6) B&H: 31.31 (8)	ERC: 47.26 (4) EWS: 64.11 (4) B&H: 36.38 (4)	ERC: 28.02 (17) EWS: 38.01 (17) B&H: 22.21 (23)	ERC: 33.33 (11) EWS: 45.22 (11) B&H: 26.43 (14)	ERC: 28.47 (17) EWS: 38.63 (17) B&H: 22.57 (22)	ERC: 37.39 (10) ERC: 50.73 (10) B&H: 29.65 (12)
Period 68	Aug 2020 - Jul 2022	ERC: 58.28 (4) EWS: 96.94 (4) B&H: 34.90 (4)	ERC: 70.47 (3) EWS: 117.21 (3) B&H: 40.58 (1)	ERC: 37.58 (15) EWS: 62.51 (15) B&H: 22.99 (23)	ERC: 44.70 (10) EWS: 74.35 (10) B&H: 27.37 (14)	ERC: 38.19 (15) EWS: 63.52 (15) B&H: 23.36 (21)	ERC: 50.15 (7) EWS: 83.41 (7) B&H: 30.72 (11)
Period 69	Sep 2020 - Aug 2022	ERC: 9.46 (5) EWS: 11.49 (5) B&H: 30.52 (6)	ERC: 11.72 (1) EWS: 14.23 (1) B&H: 43.28 (0)	ERC: 7.29 (13) EWS: 8.86 (13) B&H: 21.27 (21)	ERC: 8.66 (7) EWS: 10.52 (7) B&H: 25.28 (14)	ERC: 7.41 (13) EWS: 9.00 (13) B&H: 21.69 (21)	ERC: 9.70 (5) EWS: 11.78 (5) B&H: 27.91 (12)
Period 70	Oct 2020 - Sep 2022	ERC: 47.73 (4) EWS: 60.02 (4) B&H: 25.61 (13)	ERC: 58.57 (3) EWS: 73.66 (3) B&H: 37.07 (1)	ERC: 33.39 (11) EWS: 41.98 (11) B&H: 20.33 (20)	ERC: 39.59 (9) EWS: 49.80 (9) B&H: 24.15 (14)	ERC: 33.44 (11) EWS: 42.06 (11) B&H: 20.66 (19)	ERC: 45.08 (6) EWS: 56.69 (6) B&H: 27.07 (9)
Period 71	Nov 2020 - Oct 2022	ERC: 58.52 (2) EWS: 79.26 (2) B&H: 30.30 (7)	ERC: 88.34 (1) EWS: 119.64 (1) B&H: 43.81 (0)	ERC: 40.77 (11) EWS: 55.22 (11) B&H: 22.14 (19)	ERC: 48.47 (7) EWS: 65.64 (7) B&H: 26.32 (13)	ERC: 41.43 (11) EWS: 56.10 (11) B&H: 22.57 (18)	ERC: 54.46 (3) ERC: 73.62 (3) B&H: 29.06 (9)
Period 72	Dec 2020 - Nov 2022	ERC: 49.50 (0) EWS: 70.43 (0) B&H: 31.11 (6)	ERC: 62.24 (0) EWS: 88.54 (0) B&H: 44.15 (0)	ERC: 30.70 (16) EWS: 43.67 (16) B&H: 21.79 (18)	ERC: 36.57 (8) EWS: 52.03 (8) B&H: 25.89 (12)	ERC: 31.20 (15) EWS: 44.38 (15) B&H: 22.13 (17)	ERC: 41.06 (3) ERC: 58.42 (3) B&H: 29.03 (7)

Table 4: 1-day VaR analysis and number of exceptional days in Historical Periods 61-72

Hist_sim_VaR		**Norm_VaR**		**t_VaR**	
97.5%	**99%**	**97.5%**	**99%**	**97.5%**	**99%**
ERC: $\frac{0}{12}$ (0%) EWS: $\frac{0}{12}$ (0%) B&H: $\frac{0}{12}$ (0%)	ERC: $\frac{3}{12}$ (25%) EWS: $\frac{3}{12}$ (25%) B&H: $\frac{1}{12}$ (8%)	ERC: $\frac{8}{12}$ (67%) EWS: $\frac{8}{12}$ (67%) B&H: $\frac{1}{12}$ (8%)	ERC: $\frac{5}{12}$ (42%) EWS: $\frac{5}{12}$ (42%) B&H: $\frac{0}{12}$ (0%)	ERC: $\frac{9}{12}$ (75%) EWS: $\frac{9}{12}$ (75%) B&H: $\frac{1}{12}$ (8%)	ERC: $\frac{10}{12}$ (83%) EWS: $\frac{10}{12}$ (83%) B&H: $\frac{5}{12}$ (42%)

Table 5: Proportion of periods (out of 12 periods) showing accurate estimation of 1-day VaR in each model

6.3 Normality test of Return distribution in Historical Period

In theory, the normality of the returns for the stock price is one of the most important assumptions in finance. It is assumed the dynamics of stock price are following Geometric Brownian Motion (GBM), where the return are independent and normally distributed with mean μ and volatility σ.

$$u_i = \frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

However, in reality it is generally not the case. Return of stocks often exhibits fat tails with frequent extreme events occurance, as well as asymmetric skewness and kurtosis, is which different from normal distribution. In this section, we conduct Normality test of return distribution of different trading strategies to see whether it aligns with the normal assumption. We generally use 5 criterias to determine the return distribution is NOT normally distributed:

$$\begin{cases} KS_{p.value} < \alpha = 0.05; \text{or/and} \\ KS_{stat} \gg 0; \text{or/and} \\ JB_{p.value} < \alpha = 0.05; \text{or/and} \\ JB_{stat} \gg 0; \text{or/and} \\ \text{Fat tail shown in normal QQ-plot} \end{cases}$$

In Kolmogorov–Smirnov test Test, the test calculates maximum distance between cdf of theoretical distribution (Normal) $F(z)$ and cdf of observed distribution $S_N(z)$:

$$D_N = sup\big[F(z) - S_N(z)\big]$$

KS_{stat} should be far away from zero if u_i does not follow theoretical (Normal) distribution
The critical value at α in [0, 1] is:

$$c(\alpha) = \sqrt{\frac{-0.5ln(\frac{\alpha}{2})}{N}}$$

If $KS_{p.value} < \alpha = 0.05$, the null hypothesis can be rejected (u_i does not follow theoretical distribution).

In Jarque-Bera Test, the test is based on two properties of a normal distribution:

$$S = Skewness = 0 \ \& \ K = Kurtosis = 3$$

The JB test statistic is given by:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right)$$

JB_{stat} should be far away from zero if u_i does not follow Normal distribution
If $JB_{p.value} < \alpha = 0.05$, the null hypothesis can be rejected (u_i does not follow Normal distribution).

In Table 6 Most KS & JB P-value are smaller than critical value (= 0.05) for all three strategies, which means the null hypothesis of stock return exhibiting normal distribution can be rejected. Most JB Statistics are far from 0, implying the return distribution exhibits skewness and kurtosis. However, most KS Statistics are close to 0. We believe that while KS test can be sensitive to differences in the central part of the distribution, it may not be as effective in detecting deviations in fat tails with low proportion of the distribution. So even when it exhibits similar shape, low KS statistics does not necessary mean it follows normal distribution as extreme tails exist.

In Figure 1 we took period 67 as an example. We found that Both ERC (red line), Equal-Weight portfolio (blue line) and Buy and Hold strategy (green line) exhibit similar bell shape in the density curve, while the lines for ERC and Equal-Weight overlapped, showing almost same return distribution.

In QQ plot, we found normal QQ plot for both ERC (leftmost), Equal-Weight portfolio (middle) and Buy and Hold strategy (rightmost) contain fat tails and they both show better fit to the reference line in QQ-t plot, which further prove their non-normality property.

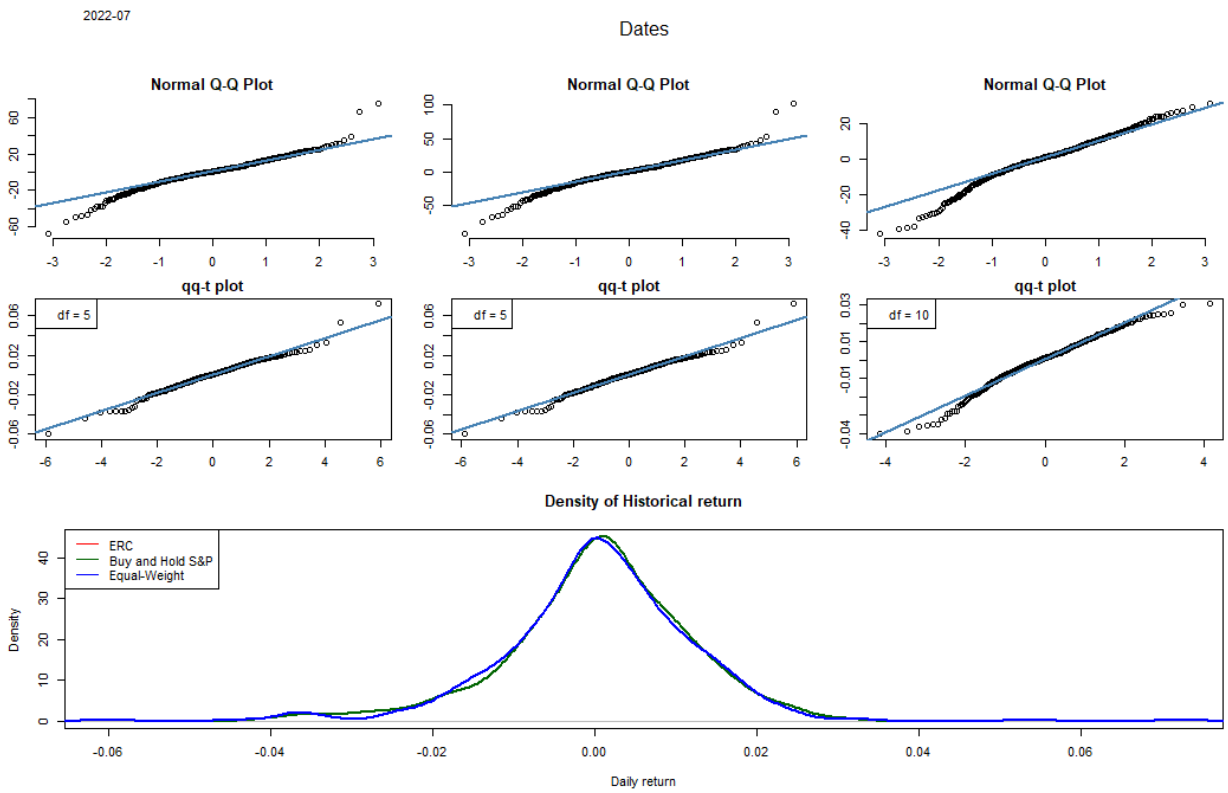


Figure 2: Q-Q plots (Left: ERC, Middle: EWS, Right: B&H) and Density of Return Distribution in historical period of Period 67

Periods	Historical Period	KS.stat	KS.p.value	JB.stat	JB.p.value	Fat tails in QQ-Plot?	t-Dist fit better?
Period 61	Jan 2020 - Dec 2021	ERC: 0.11 EWS: 0.11 B&H: 0.15	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: 1796.76 EWS: 1796.76 B&H: 3975.02	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 62	Feb 2020 - Jan 2022	ERC: 0.13 EWS: 0.13 B&H: 0.14	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: 1301.16 EWS: 1301.16 B&H: 3708.22	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 63	Mar 2020 - Feb 2022	ERC: 0.12 EWS: 0.12 B&H: 0.14	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: 4687.38 EWS: 4687.48 B&H: 4008.76	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 64	Apr 2020 - Mar 2022	ERC: 0.10 EWS: 0.09 B&H: 0.10	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: 43.27 EWS: 62.61 B&H: 346.21	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 65	May 2020 - Apr 2022	ERC: 0.09 EWS: 0.09 B&H: 0.09	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: 1696.96 EWS: 1969.96 B&H: 150.18	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 66	Jun 2020 - May 2022	ERC: 0.08 EWS: 0.08 B&H: 0.09	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: 3994.34 EWS: 3994.34 B&H: 159.32	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 67	Jul 2020 - Jun 2022	ERC: 0.08 EWS: 0.08 B&H: 0.08	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: 483.49 EWS: 483.49 B&H: 56.02	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 68	Aug 2020 - Jul 2022	ERC: 0.07 EWS: 0.07 B&H: 0.08	ERC: 0.01 EWS: 0.01 B&H: 0.00	ERC: 238.65 EWS: 238.65 B&H: 47.22	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 69	Sep 2020 - Aug 2022	ERC: 0.07 EWS: 0.07 B&H: 0.07	ERC: 0.01 EWS: 0.01 B&H: 0.01	ERC: 2825.44 EWS: 2825.44 B&H: 39.12	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 70	Oct 2020 - Sep 2022	ERC: 0.07 EWS: 0.07 B&H: 0.07	ERC: 0.01 EWS: 0.01 B&H: 0.03	ERC: 2097.46 EWS: 2097.46 B&H: 45.25	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 71	Nov 2020 - Oct 2022	ERC: 0.07 EWS: 0.07 B&H: 0.06	ERC: 0.02 EWS: 0.02 B&H: 0.03	ERC: 1648.66 EWS: 1648.66 B&H: 31.90	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes
Period 72	Dec 2020 - Nov 2022	ERC: 0.06 EWS: 0.06 B&H: 0.07	ERC: 0.09 EWS: 0.09 B&H: 0.03	ERC: 508.02 EWS: 508.02 B&H: 52.65	ERC: 0.00 EWS: 0.00 B&H: 0.00	ERC: Yes EWS: Yes B&H: Yes	ERC: Yes EWS: Yes B&H: Yes

Table 6: Return distribution in Historical Periods 61-72

7 Extending to Long/Short Portfolio Strategy

7.1 The weakness of long-only portfolio based on our stock selection criteria

By using Principal Component Analysis (PCA) as our stock selection criterion, we have successfully reduced the number of traded stocks in each period significantly. This approach proves to be effective for active investment management. However, it has been observed that our portfolio outperforms the S&P 500 benchmark only during bear markets, while it underperforms during bull markets.

One reason for this, as explained in the referenced source [1], is that PCA selection tends to include both the best and worst stocks in our portfolio. Given that we assign equal weight and risk to our portfolio, we tend to overweight some underperforming stocks and underweight some high-performing stocks, thereby exposing ourselves to undue risk.

Additionally, our portfolio tends to overweight low-risk stocks and underweight high-risk stocks. This makes our portfolio overly defensive, preparing excessively for worst-case scenarios. In a bull market, where almost all stocks are rising and those with higher risk tend to outperform, this defensive strategy causes our portfolio to underperform.

7.2 Motivation to Implement Long/Short Portfolio

In response to the challenges identified with our portfolio, we are inspired to incorporate a long/short strategy into our portfolio. This approach will enable us to short-sell any underperforming stocks identified through our PCA criteria. We will establish specific criteria or rules to determine which stocks to short.

Our goal is to strike a balance between maintaining a relatively conservative portfolio weighting and implementing sufficiently aggressive trading rules. We aim to avoid excessive exposure to any particular stocks, focusing instead on enhancing our portfolio by making a more informed directional trading decision. Moreover, this strategy allows us to respond more effectively to recent market conditions by shorting stocks when necessary. This proactive approach aims to optimize our portfolio performance across varying market conditions.

7.3 Long/Short Portfolio Construction: Ideas and Assumption

In this section, we will utilize both equal-weight and the first principal component (PC1) of the selected stocks to establish the initial weights of the portfolio we aim to construct. We opt not to use the Equal Risk Contribution (ERC) portfolio as we intend to adopt a less conservative approach, and hence we use equal-weight and PC1.

The rationale behind choosing PC1 is our desire to gain exposure to the primary/leading risk factor in our portfolio, which we assume to be more aggressive. On the other hand, equal-weighting offers a simply and effective way to diversify the portfolio without being overly conservative.

As for our short-selling strategy, we operate under the assumption that a minimum initial margin, equivalent to the total short-selling exposures, will need to be posted in our trading account. Consequently, when we decide to short-sell stocks, we simply assign a negative sign to the corresponding stock's predetermined weight. As a result, the sum of all stock weights will not equal one due to the margin requirement in our assumption.

7.4 Determining the Short-Selling Signal

To identify which stocks to short, we first need to classify which stocks are considered 'bad'. According to the same source used to determine Kelly's formula in the tutorial notes [13], we can define a 'goodness index' (α_{it}) for each stock i at the starting trading date t. The formula is as follows:

$$\alpha_{it} = \frac{\mu_{it}}{{\sigma_{it}}^2}$$

In this case, μ_{it} represents the average daily return, which can be estimated by taking the average of 1-month or 3-month returns, while ${\sigma_{it}}^2$ represents the daily volatility, which can be estimated using the GARCH(1,1) model for each stock i.

Based on the paper, we can classify a stock as 'bad' if $\alpha_{it} < 0$. However, we decide to use a different trading rule compared to Kelly's formula as we want to use the index as a short-selling signal. We view α_{it} as the risk-adjusted measure of a particular stock, and when it turns negative, it is simply because the past average return is negative.

Therefore, we can intuitively deduce that the more negative the value of α_{it}, the more negative the average return and/or the smaller the volatility. Given that the average return for these 'bad' stocks is always negative, the smaller volatility means that the stock will likely continue in the downtrend. The level of stock's volatility will directly influence the likelihood of significant/dramatic movement in the stock's return. When volatility is low, the probability of dramatic change in the stock's return is also low. Conversely, when volatility is high, there is a greater chance for substantial price fluctuations.

Hence, based on this logic, we rank the goodness index α_{it} for each stock i from highest to lowest, and then we only short stocks of which the $\alpha_{it} < 0$ and α_{it} is in the bottom ε of the overall rank. We decide to choose ε to be either 20% or 40% because we still want to keep the majority of our portfolio in the long position and we want to see the effectiveness of different choices of ε for our portfolio return.

Since we have two different choices for our μ_{it} and ε, we define three different options that can be used as our short-selling signal:

- μ_{it} is estimated using 3-months average daily return and $\varepsilon = 20\%$
- μ_{it} is estimated using 1-month average daily return and $\varepsilon = 20\%$
- μ_{it} is estimated using 1-month average daily return and $\varepsilon = 40\%$

7.5 Implementation of Long/Short Portfolio with Equal-Weight and PC1 Weight

1. First, we use the GARCH(1,1) model to fit the past 252 days return data for each individual stock i. Then, we can use the last day fitted value as the estimated ${\sigma_{it}}^2$. Using the same reason as before, we use 252 days data so that we are able to capture long-enough historical behavior of the stocks' volatility, while still making it relevant to trade it for a relatively shorter period of 1 month.
2. Now, we estimate μ_{it} using either 1-month or 3-months average return.
3. Using the estimated μ_{it} and ${\sigma_{it}}^2$, we then calculate α_{it}.
4. For the equal-weight portfolio, we simply put $1/n$ on each constituent where n is the total number of stocks selected. On the other hand, for the PC1 portfolio, we will perform PCA on the selected stocks, and we extract the absolute value of the PC1 loading and standardize it such that the sum of all the weights is 1.
5. Now, we will short-sell stocks that have $\alpha_{it} < 0$ and are in the bottom 20% or 40% based on the ranked α_{it}. Then, we assign a negative sign to the weights of the corresponding stocks that need to be short-sold.
6. We let the stock move without any intervention throughout the month and perform rebalancing at the beginning of the next month in order to minimize the transaction cost of this strategy.

All these steps are implemented under our `portfolio_construction2()` function, which also output all the necessary graphs and trading results.

8 Annual Performance Review: Index VS Long/Short Portfolio

8.1 Annual Performance Data

In this section, we define 4 different portfolios. Our main strategies are performing PCA after stock selection and using PC1 for long-short portfolio weight (PCLS: Black Line), as well as using equal-weight long/short (EWLS: Purple Line) portfolio. Buy-and-hold S&P500 (B&H: Green Line) serves as our benchmark. We also consider another portfolio for comparison: we apply PCA to the correlation matrix of the selected stocks (2-years return) and then uses PC1 to construct long-only portfolio (PCS: Orange Line). Since we apply three different combinations of μ_{it} and ε, we will only report selected parameters that yield the best trading performance in the following table: (The graphs are illustrated in appendix)

Year	**Trading Period (1 year)**	**Selected Parameter**	**Total Return**	**Sharpe Ratio**	**Calmar Ratio**	**Sortino Ratio**	**Max Drawdown**
Year 1	2017-01-01 to 2017-12-31	μ_{it} use 3-months return $\varepsilon = 0.2$	PCS: +20.57% EWLS: +22.25% B&H: +14.72% PCLS: +23.02%	PCS: 1.94 EWLS: 3.06 B&H: 2.30 PCLS: 3.05	PCS: 3.62 EWLS: 7.99 B&H: 5.14 PCLS: 8.15	PCS: 0.17 EWLS: 0.28 B&H: 0.21 PCLS: 0.28	PCS: -5.74% EWLS: -2.81% B&H: -2.89% PCLS: -2.85%
Year 2	2018-01-01 to 2018-12-31	μ_{it} use 1-month return $\varepsilon = 0.4$	PCS: -9.08% EWLS: +2.13% B&H: -7.20% PCLS: +1.68%	PCS: -0.57 EWLS: 0.28 B&H: -0.43 PCLS: 0.21	PCS: -0.35 EWLS: 0.44 B&H: -0.33 PCLS: 0.32	PCS: -0.04 EWLS: 0.03 B&H: -0.03 PCLS: 0.02	PCS: -26.30% EWLS: -4.89% B&H: -21.76% PCLS: -5.33%
Year 3	2019-01-01 to 2019-12-31	μ_{it} use 1-month return $\varepsilon = 0.2$	PCS: +27.48% EWLS: +20.98% B&H: +30.10% PCLS: +22.92%	PCS: 2.16 EWLS: 1.93 B&H: 2.48 PCLS: 2.33	PCS: 2.73 EWLS: 3.33 B&H: 4.60 PCLS: 3.76	PCS: 0.18 EWLS: 0.17 B&H: 0.20 PCLS: 0.21	PCS: -10.09% EWLS: -6.32% B&H: -6.57% PCLS: -6.13%
Year 4	2020-01-01 to 2020-12-31	μ_{it} use 1-month return $\varepsilon = 0.2$	PCS: +4.32% EWLS: +1.68% B&H: +11.80% PCLS: -0.82%	PCS: 0.16 EWLS: 0.10 B&H: 0.35 PCLS: -0.05	PCS: 0.16 EWLS: 0.11 B&H: 0.32 PCLS: -0.05	PCS: 0.03 EWLS: 0.02 B&H: 0.04 PCLS: 0.00	PCS: -26.36% EWLS: -15.60% B&H: -36.83% PCLS: -16.92%
Year 5	2021-01-01 to 2021-12-31	μ_{it} use 1-month return $\varepsilon = 0.2$	PCS: +27.81% EWLS: +10.94% B&H: +21.48% PCLS: +10.57%	PCS: 2.11 EWLS: 0.85 B&H: 1.72 PCLS: 0.79	PCS: 3.81 EWLS: 1.15 B&H: 3.34 PCLS: 1.10	PCS: 0.20 EWLS: 0.09 B&H: 0.15 PCLS: 0.08	PCS: -7.33% EWLS: -9.55% B&H: -6.45% PCLS: -9.68%
Year 6	2022-01-01 to 2022-12-31	μ_{it} use 1-month return $\varepsilon = 0.4$	PCS: -24.71% EWLS: +3.32% B&H: -21.85% PCLS: +5.73%	PCS: -1.25 EWLS: 0.30 B&H: -0.92 PCLS: 0.53	PCS: -0.92 EWLS: 0.30 B&H: -0.80 PCLS: 0.68	PCS: -0.11 EWLS: 0.03 B&H: -0.08 PCLS: 0.05	PCS: -27.11% EWLS: -10.98% B&H: -27.56% PCLS: -8.55%
Year 7	2023-01-01 to 2023-08-31	μ_{it} use 1-month return $\varepsilon = 0.2$	PCS: -31.06% EWLS: -7.95% B&H: +6.89% PCLS: -5.36%	PCS: -1.67 EWLS: -0.46 B&H: 0.63 PCLS: -0.38	PCS: -1.04 EWLS: -0.56 B&H: 0.83 PCLS: -0.59	PCS: -0.16 EWLS: -0.04 B&H: 0.06 PCLS: -0.03	PCS: -34.90% EWLS: -17.18% B&H: -10.21% PCLS: -11.02%

Table 7: Annual Trading Performance of Long/Short Portfolio from 2017 to 2023

8.2 Comparison of all trading strategies in Trading period 2022 (Period 61-72)

Below shows P&L result and the Portfolio value graph. In the graph we can find that PCLS (Black) and EWLS (Purple) show upward trend during bearish market in mid-late 2022, while other strategies shows huge downward trend in same period. Meanwhile in the P&L table, it shows both of their volatility and extreme value of losses (5th and 10th perc) improve significantly than other strategies all 12 periods, showing that implementation of selling strategies not only takes profit from bad market, it also further reduces fluctuation of losses.

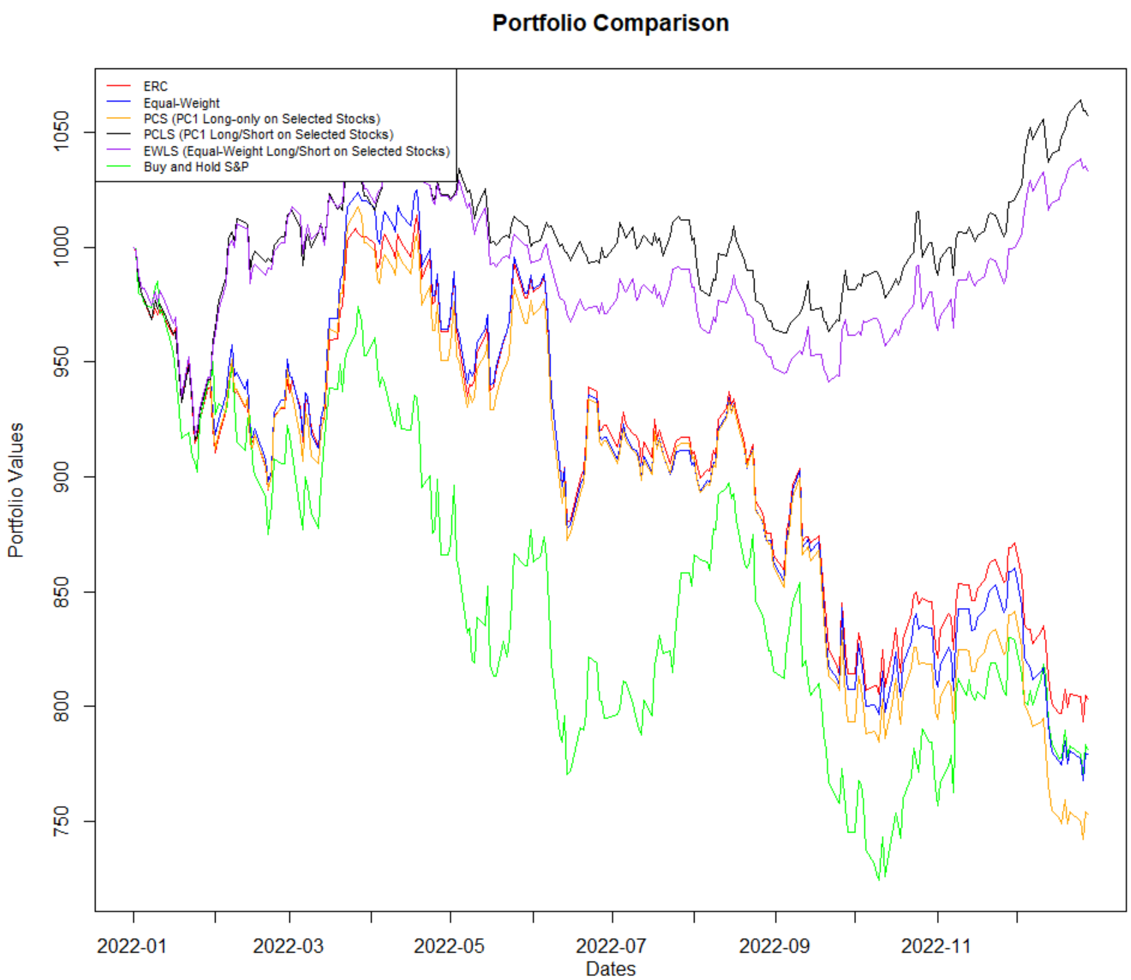


Figure 3: Portfolio value for all strategy in Trading Periods 61-72 (2022)

Periods	Trading Period (1 Month)	Min($)	Max($)	Mean($)	Volatility($)	10th perc.($)	5th perc.($)
Period 61	Jan 2022 - Feb 2022	ERC: -19.29	ERC: +16.77	ERC: -2.31	ERC: 10.03	ERC: -17.02	ERC: -19.28
		EWS: -17.91	EWS: +19.67	EWS: -2.57	EWS: 10.67	EWS: -14.18	EWS: -15.95
		PCLS: -16.11	PCLS: +13.71	PCLS: -2.36	**PCLS: 8.28**	PCLS: -12.85	**PCLS: -13.10**
		EWLS: -16.70	EWLS: +14.22	**EWLS: -2.23**	EWLS: 8.29	**EWLS: -12.47**	EWLS: -13.70
		PCS: -17.91	PCS: +16.22	PCS: -2.89	PCS: 10.04	PCS: -15.73	PCS: -17.10
		B&H: -19.38	B&H: +21.96	B&H: -2.93	B&H: 10.95	B&H: -17.70	B&H: -17.95
Period 62	Feb 2022 - Mar 2022	ERC: -16.62	ERC: +18.06	ERC: -2.17	ERC: 10.06	ERC: -14.92	ERC: -15.52
		EWS: -21.42	EWS: +17.27	EWS: -2.37	EWS: 10.59	EWS: -14.21	EWS: -17.56
		PCLS: -11.94	PCLS: +18.97	PCLS: +1.06	PCLS: 8.95	PCLS: -10.63	PCLS: -11.21
		EWLS: -12.13	EWLS: +18.86	**EWLS: +1.38**	**EWLS: 8.89**	**EWLS: -8.90**	**EWLS: -11.04**
		PCS: -20.74	PCS: +25.12	PCS: -0.63	PCS: 11.91	PCS: -14.71	PCS: -19.16
		B&H: -23.18	B&H: +19.87	B&H: -1.88	B&H: 12.93	B&H: -18.08	B&H: -19.98
Period 63	Mar 2022 - Apr 2022	ERC: -21.44	ERC: +15.66	ERC: +2.57	ERC: 9.17	ERC: -5.07	**ERC: -8.08**
		EWS: -21.71	EWS: +15.97	EWS: +2.76	EWS: 9.53	**EWS: -4.89**	EWS: -8.51
		PCLS: -15.25	PCLS: +14.23	PCLS: +0.23	**PCLS: 7.76**	PCLS: -9.19	PCLS: -14.52
		EWLS: -15.86	EWLS: +15.93	EWLS: +0.71	EWLS: 8.11	EWLS: -8.83	EWLS: -13.67
		PCS: -18.64	PCS: +21.44	**PCS: +3.13**	PCS: 12.03	PCS: -11.33	PCS: -15.55
		B&H: -26.87	B&H: +67.52	B&H: +2.05	B&H: 12.85	B&H: -11.64	B&H: -14.80
Period 64	Apr 2022 - May 2022	ERC: -24.96	ERC: +13.67	ERC: -3.33	ERC: 11.46	ERC: -20.44	ERC: -22.47
		EWS: -24.54	EWS: +11.84	EWS: -3.85	EWS: 11.70	EWS: -22.23	EWS: -23.74
		PCLS: -24.17	PCLS: +9.80	**PCLS: -0.90**	**PCLS: 8.52**	**PCLS: -9.62**	PCLS: -13.39
		EWLS: -25.09	EWLS: +10.26	EWLS: -1.01	EWLS: 8.77	EWLS: -10.23	**EWLS: -12.47**
		PCS: -24.19	PCS: +12.62	PCS: -2.58	PCS: 10.63	PCS: -19.39	PCS: -19.97
		B&H: -32.61	B&H: +21.70	B&H: -4.34	B&H: 13.95	B&H: -25.37	B&H: -25.90
Period 65	May 2022 - Jun 2022	ERC: -30.79	ERC: +18.03	ERC: +0.36	ERC: 13.33	ERC: -20.80	ERC: -24.79
		EWS: -31.97	EWS: +20.43	EWS: +0.52	EWS: 14.45	EWS: -23.64	EWS: -25.94
		PCLS: -18.90	PCLS: +9.36	PCLS: -0.40	**PCLS: 8.00**	**PCLS: -12.26**	**PCLS: -14.59**
		EWLS: -20.61	EWLS: +9.96	EWLS: -0.54	EWLS: 8.47	EWLS: -12.99	EWLS: -15.63
		PCS: -30.10	PCS: +19.43	**PCS: +0.77**	PCS: 13.03	PCS: -20.39	PCS: -23.58
		B&H: -34.42	B&H: +25.99	B&H: -0.23	B&H: 16.80	B&H: -27.53	B&H: -31.95
Period 66	Jun 2022 - Jul 2022	ERC: -30.39	ERC: +18.94	ERC: -1.72	ERC: 12.48	ERC: -18.76	ERC: -18.92
		EWS: -28.75	EWS: +19.30	**EWS: -1.69**	EWS: 12.06	EWS: -17.63	EWS: -18.04
		PCLS: -19.34	PCLS: +10.83	PCLS: -1.94	PCLS: 6.99	**PCLS: -8.64**	**PCLS: -12.27**
		EWLS: -23.35	EWLS: +11.92	EWLS: -2.58	**EWLS: 5.01**	EWLS: -10.44	EWLS: -15.03
		PCS: -28.61	PCS: +23.61	PCS: -2.43	PCS: 14.08	PCS: -23.78	PCS: -25.57
		B&H: -31.76	B&H: +24.36	B&H: -3.16	B&H: 15.16	B&H: -24.56	B&H: -25.87
Period 67	Jul 2022 - Aug 2022	ERC: -9.08	ERC: +16.86	ERC: +0.43	ERC: 7.38	ERC: -8.07	ERC: -8.71
		EWS: -7.31	EWS: +17.89	EWS: +1.25	EWS: 7.79	**EWS: -7.01**	**EWS: -7.06**
		PCLS: -12.20	PCLS: +6.58	PCLS: -1.10	**PCLS: 5.31**	PCLS: -7.18	PCLS: -9.31
		EWLS: -12.58	EWLS: +6.49	EWLS: -1.31	EWLS: 5.37	EWLS: -7.49	EWLS: -10.35
		PCS: -9.65	PCS: +18.60	PCS: -0.04	PCS: 7.52	PCS: -8.26	PCS: -8.39
		B&H: -9.52	B&H: +21.99	**B&H: +3.17**	B&H: 9.77	B&H: -7.91	B&H: -9.35
Period 68	Aug 2022 - Sep 2022	ERC: -22.09	ERC: +17.19	ERC: -1.12	ERC: 8.13	ERC: -9.20	ERC: -12.98
		EWS: -23.09	EWS: +17.14	EWS: -0.92	EWS: 8.64	EWS: -10.15	EWS: -14.09
		PCLS: -11.65	PCLS: +12.84	**PCLS: -0.23**	PCLS: 5.17	**PCLS: -6.48**	**PCLS: -8.20**
		EWLS: -12.55	EWLS: +9.24	EWLS: -0.42	**EWLS: 5.15**	EWLS: -7.47	EWLS: -8.64
		PCS: -26.19	PCS: +15.34	PCS: -1.91	PCS: 8.63	PCS: -9.19	PCS: -13.34
		B&H: -29.48	B&H: +18.29	B&H: -1.48	B&H: 10.61	B&H: -11.06	B&H: -18.12
Period 69	Sep 2022 - Oct 2022	ERC: -31.59	ERC: +30.57	ERC: -2.36	ERC: 12.66	ERC: -14.97	**ERC: -15.14**
		EWS: -33.04	EWS: +34.52	EWS: -2.52	EWS: 13.91	EWS: -15.90	EWS: -16.91
		PCLS: -25.21	PCLS: +20.40	PCLS: -2.00	**PCLS: 10.14**	PCLS: -12.76	PCLS: -17.83
		EWLS: -22.51	EWLS: +26.70	**EWLS: -1.67**	EWLS: 10.53	**EWLS: -12.04**	EWLS: -18.51
		PCS: -33.47	PCS: +26.18	PCS: -3.69	PCS: 14.60	PCS: -15.51	PCS: -29.87
		B&H: -36.92	B&H: +14.91	B&H: -3.77	B&H: 12.22	B&H: -13.71	B&H: -16.32
Period 70	Oct 2022 - Nov 2022	ERC: -15.14	ERC: +19.75	**ERC: +2.67**	ERC: 10.20	ERC: -9.39	ERC: -10.02
		EWS: -16.61	EWS: +20.24	EWS: +2.60	EWS: 10.78	EWS: -10.58	EWS: -10.78
		PCLS: -7.18	PCLS: +13.58	PCLS: +1.52	**PCLS: 5.67**	PCLS: -4.61	PCLS: -6.06
		EWLS: -7.68	EWLS: +12.38	EWLS: +1.43	EWLS: 5.46	**EWLS: -4.45**	**EWLS: -4.93**
		PCS: -17.09	PCS: +19.81	PCS: +1.19	PCS: 10.63	PCS: -11.81	PCS: -12.03
		B&H: -21.24	B&H: +22.78	B&H: +1.87	B&H: 10.42	B&H: -7.85	B&H: -17.59
Period 71	Nov 2022 - Dec 2022	ERC: -24.23	ERC: +27.13	**ERC: +2.49**	ERC: 10.83	ERC: -9.88	ERC: -11.43
		EWS: -25.86	EWS: +30.53	EWS: +2.35	EWS: 12.45	EWS: -13.73	EWS: -14.66
		PCLS: -8.59	PCLS: +12.28	PCLS: -0.21	**PCLS: 5.08**	PCLS: -6.19	PCLS: -6.07
		EWLS: -8.54	EWLS: +12.76	EWLS: -0.08	EWLS: 5.24	**EWLS: -5.87**	**EWLS: -5.91**
		PCS: -20.97	PCS: +20.52	PCS: +1.02	PCS: 9.72	PCS: -11.00	PCS: -15.40
		B&H: -19.61	B&H: +42.26	B&H: +2.17	B&H: 13.66	B&H: -12.64	B&H: -16.17
Period 72	Dec 2022 - Dec 2022	ERC: -16.08	ERC: +10.36	ERC: -2.15	ERC: 6.84	ERC: -10.79	ERC: -13.76
		EWS: -16.86	EWS: +9.93	EWS: -2.47	EWS: 6.94	EWS: -10.43	EWS: -14.39
		PCLS: -16.85	PCLS: +9.35	PCLS: -1.72	PCLS: 7.13	PCLS: -9.95	PCLS: -12.18
		EWLS: -15.06	EWLS: +9.12	**EWLS: -1.66**	**EWLS: 6.53**	**EWLS: -9.25**	**EWLS: -10.28**
		PCS: -24.32	PCS: +12.20	PCS: -4.14	PCS: 8.85	PCS: -14.84	PCS: -15.94
		B&H: -20.27	B&H: +13.45	B&H: -2.30	B&H: 8.99	B&H: -11.72	B&H: -14.83

Table 8: daily P&L performance for all 4 strategy in Trading Periods 61-72

8.3 Performance Analysis: Improvement from the Previous Strategies

As the graphs illustrated above section and in Appendix, the long/short portfolios (both PCLS and EWLS) notably outperformed other portfolios in 2017, 2018, and 2022. In the upward trending market of 2017, these portfolios excelled in terms of total return, risk-adjusted return metrics, and maximum drawdown, even achieving a Sharpe ratio above 3, which is considered highly favorable. Although the PCS portfolio also surpassed the benchmark, its volatility resulted in the lowest Sharpe ratio.

The bear markets of 2018 and 2022 were particularly beneficial for our Long/Short strategy, as there were sufficient enough underperforming stocks to short sell. This led our portfolio to yield positive total returns during these periods, indicating that the Long/Short strategy not only minimized losses but also generated substantial profits. Other metrics, such as risk-adjusted return ratios and maximum drawdown, further demonstrate the effectiveness of this strategy in bear markets.

However, this strategy fell short of the benchmark in the bullish years of 2019, 2020, 2021, and 2023. During these years, both PCLS and EWLS had lower total return and risk-adjusted returns compared to B&H. Yet, in 2019 and 2020, the maximum drawdown for PCLS and EWLS was better than other long-only portfolios. This was particularly evident during the extreme market downturn in Q1 2020, where the Long/Short portfolio was able to offset losses with gains from short-selling positions.

We did not extensively compare PCLS and EWLS to each other, as both portfolios had similar resulting weights, leading to their co-movement. The table below shows the PC1 loading and the resulting weights in the first trading period of 2017:

Stock	**PC1**	**Weights**
AYI	0.28811985	0.064991642
BMY	0.23913462	0.053941966
CHTR	0.25690790	0.057951114
CMG	0.15621908	0.035238580
COO	0.24658810	0.055623261
EBAY	0.25881502	0.058381305
GEN	0.26306807	0.059340672
HRB	0.20673913	0.046634467
HUM	0.19329264	0.043601321
ILMN	0.24327217	0.054875279
KR	0.21141234	0.047688608
NEM	0.02330244	0.005256367
NFLX	0.20830249	0.046987115
NVDA	0.22293205	0.050287128
PDCO	0.26155080	0.058998420
PRGO	0.19845513	0.044765832
SIG	0.22486509	0.050723166
TAP	0.21515985	0.048533942
URBN	0.15957341	0.035995221
VTR	0.15041430	0.033929186
WYNN	0.20505870	0.046255410

Table 9: PC1 Loading and Weights for each stock in January 2017

In summary, despite the significant improvements brought by integrating long/short strategies into our long-only strategy, we still underperformed in four trading periods. The primary issue remains our inability to allocate more weight to stocks with higher upside potential or risk, especially during the post-COVID recovery of 2020 and 2021 when almost all stocks rebounded. In 2023, the market movement was largely driven by AI-related companies, a factor that might be too idiosyncratic for our PCA-based stock selection method to capture, preventing us from selecting these exceptional performers. Nevertheless, the major improvement is our ability to profit during the bear markets of 2018 and 2022, and to be more defensive during the

2020 market downturn compared to using a long-only portfolio. Moreover, we managed to outperform the benchmark in the bullish year of 2017.

8.4 Analysis on Different choices of μ_{it} and ε

The performance of our long/short portfolio is undeniably influenced by the choices of μ_{it} and ε, which are crucial in determining the stocks we need to short-sell. We use both 1-month and 3-month daily average returns for μ_{it}, representing shorter and longer lookback periods respectively. The longer lookback period allows us to capture a longer trend, which is beneficial for investors aiming to hold stocks for a long horizon or those focusing more on the major trend rather than short-term fluctuations. However, a major disadvantage of using a longer lookback period to estimate μ_{it} is that it becomes less sensitive to the most recent stock price movements. Hence, it might fail to detect the start and end of bull or bear markets in a timely manner. Conversely, a shorter lookback period might be useful if we want to be more sensitive to the latest price movements, but it also might pose a risk if the recent trend is just short-term noise.

Moving to the choice of ε, we first restrict ε to be less than 0.5 to ensure the majority of our exposures are in long positions. When ε is low, we are simply less confident (or more conservative) with our bad stock signal and therefore only decide to short-sell if those stocks perform worse relative to other stocks and are ranked at the bottom of the stock list.

Based on our trading period from 2017 to 2023, the most commonly used parameter is μ_{it} using a 1-month average daily return and $\varepsilon = 20\%$ (2019-2021 and 2023). Moreover, the bear markets in 2018 and 2022 require us to use μ_{it} using a 1-month average daily return and $\varepsilon = 40\%$. The reason a 1-month lookback is performing well is that during these periods, there were quite a lot of fluctuations in the market, such as import tariffs and consecutive rate hikes in 2018, a rate cut in 2019, the Covid-19 Pandemic followed by quantitative easing in 2020, strong recovery in 2021, hawkish interest rates hike in 2022, and lastly the new megatrend of AI in 2023. Hence, being reactive is key to performing well in these market conditions. In the majority of years, $\varepsilon = 20\%$ is already good enough, while in the bear markets of 2018 and 2022, $\varepsilon = 40\%$ is more desirable as there are too many stocks performing poorly.

Interestingly, 2017 becomes the only year where using 3-months daily average return allows our long/short portfolio to outperform others. The possible reasons are that during that particular period, people had been optimistic for quite a long time as there was no significant crisis after the 2008 Global Financial Crisis. Although interest rates were starting to increase, the pace was still relatively slow and the economy remains resilient. Therefore, under the uncertainty of economic conditions over the last 5 years, $\varepsilon = 20\%$ and 1-month daily average return might be the optimal parameters to choose that work best in most scenarios.

9 Conclusion

In conclusion, our long-only portfolio's performance closely aligns with that of the S&P, albeit with lower volatility. Our PCA-based stock selection method, combined with both equal-weighted and ERC strategies, has proven its ability to outperform the S&P during bear markets. However, it tends to lag behind the S&P index during bull markets. This suggests that our portfolio may be particularly effective during periods of uncertainty when the market could potentially swing in either direction. This strategy allows us to capitalize on the upside potential during bullish trends, while also maintaining a defensive stance against unfavorable scenarios. Despite underperforming in certain periods, the portfolio's ability to profit from bear markets and provide a level of protection in volatile conditions underscores its value.

The integration of a long/short trading rule is one effort to enhance the portfolio's performance, ensuring it is not overly conservative. Additionally, we experimented with a new weighting method using PC1, which turned out to be similar to an equal-weights portfolio. This long/short strategy has notably improved our portfolio's performance, especially in bear markets where we are now able to realize profits.

Looking ahead, it would be beneficial to develop a strategy that can significantly improve performance in bull markets without sacrificing the ability to minimize risk/loss during market downturns. Implementing a more aggressive weighting in our portfolio could be one potential improvement, as we have not yet incorporated this feature into our project. For a detailed explanation of why weighting methods are important, please refer to the following section, which discusses the limitations of our project.

10 Limitations

10.1 Methods used to discard stocks

[1] discussed 4 PCA methods, namely B1, B2, B3, and B4, to discard stocks. In our work, we did not try all of the methods and did not do much parameter optimization. We only tried method B2 as the author mentioned that it is one of the most satisfactory ones on artificial data. We believe that an extension to our project is to try all methods to verify the result shown by the author, optimize different parameters, and see how their results differ.

10.2 Our portfolio has not included strategies that put more weights on the good-quality stocks

Our current portfolio construction strategies incorporate equal-weight, equal risk contribution, and first Principal Component (PC1) loading weighting methods. We've also expanded our approach to include a directional trading strategy, which allows us to take long or short positions. However, these strategies do not account for the potential outperformance of certain stocks. While we do perform initial stock selections, once a basket of stocks is determined, we do not integrate a specific view on which stocks are likely to outperform. This means we are unable to allocate more weight to stocks with higher prospects or upside potential.

Additionally, our monthly rebalancing setting inadvertently forces us to sell the winning stocks in our portfolio and realign it with our investment mandate. This could be a limiting factor, especially when compared to the S&P500, which naturally allows winning stocks to gain more weight in the overall portfolio. This characteristic of the S&P500 could explain its tendency to outperform in bull markets. In summary, our current strategies do not allow for the allocation of more weight to high-quality stocks, which could be a potential area for improvement.

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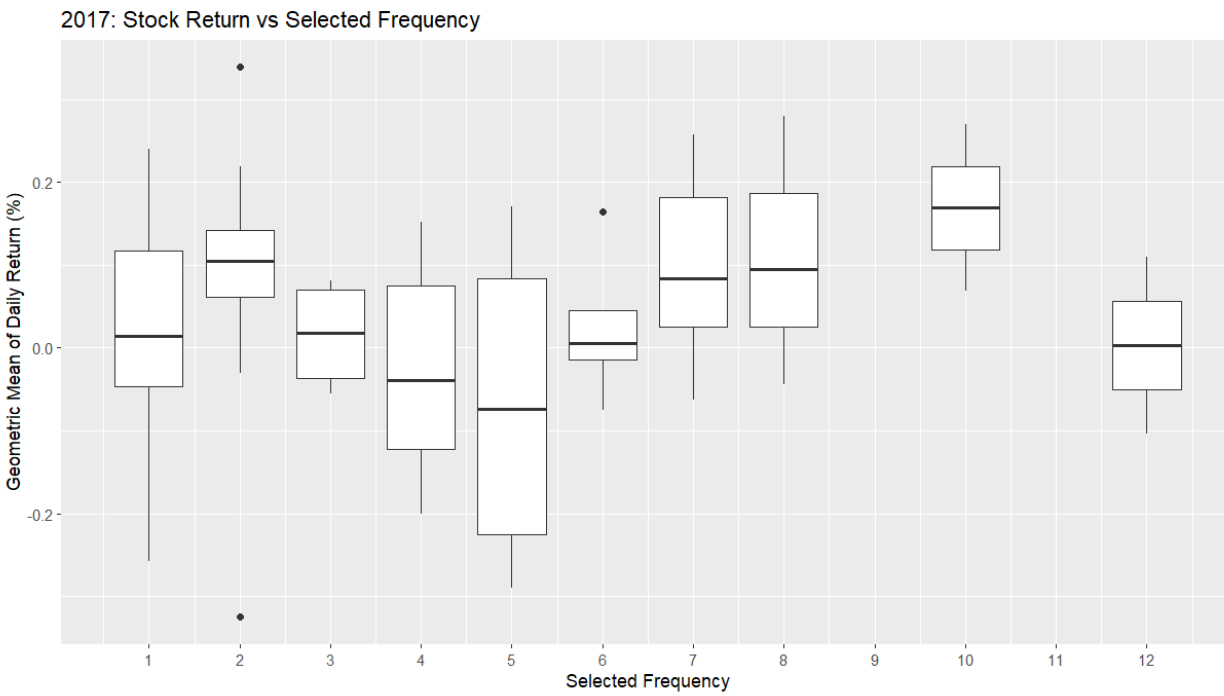
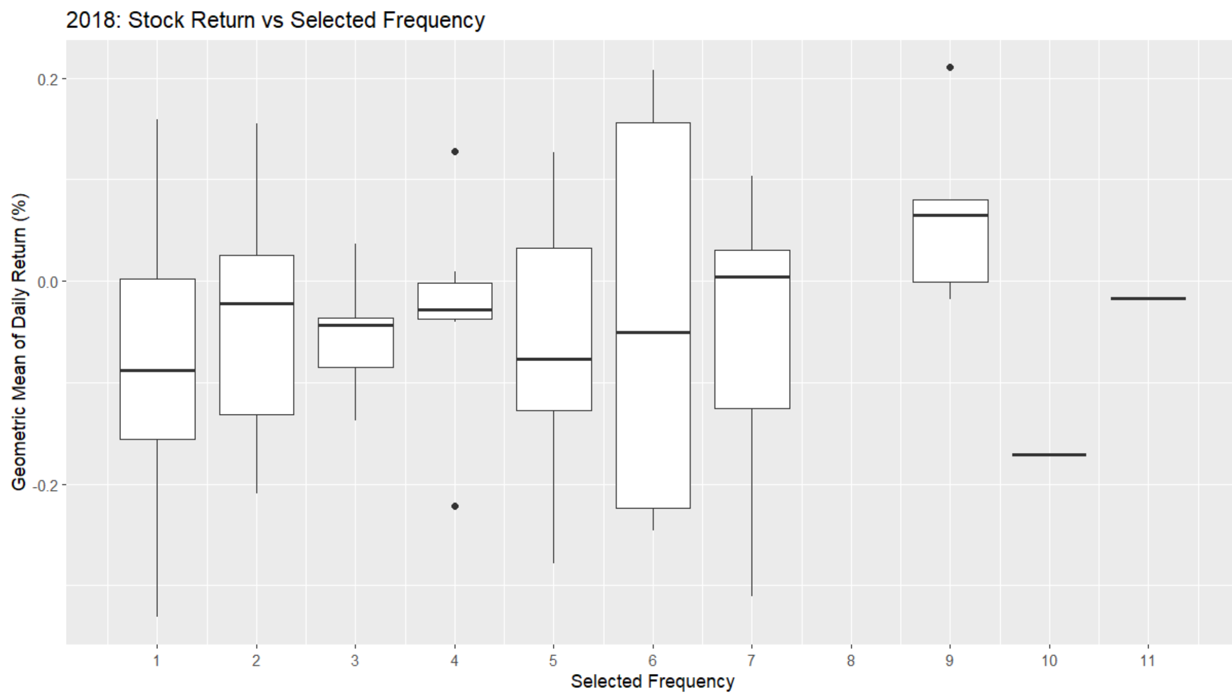
11 Appendix

11.1 Stocks Daily Return vs Selected Frequency





2018: Stock Return vs Selected Frequency
Geometric Mean of Daily Return (%)
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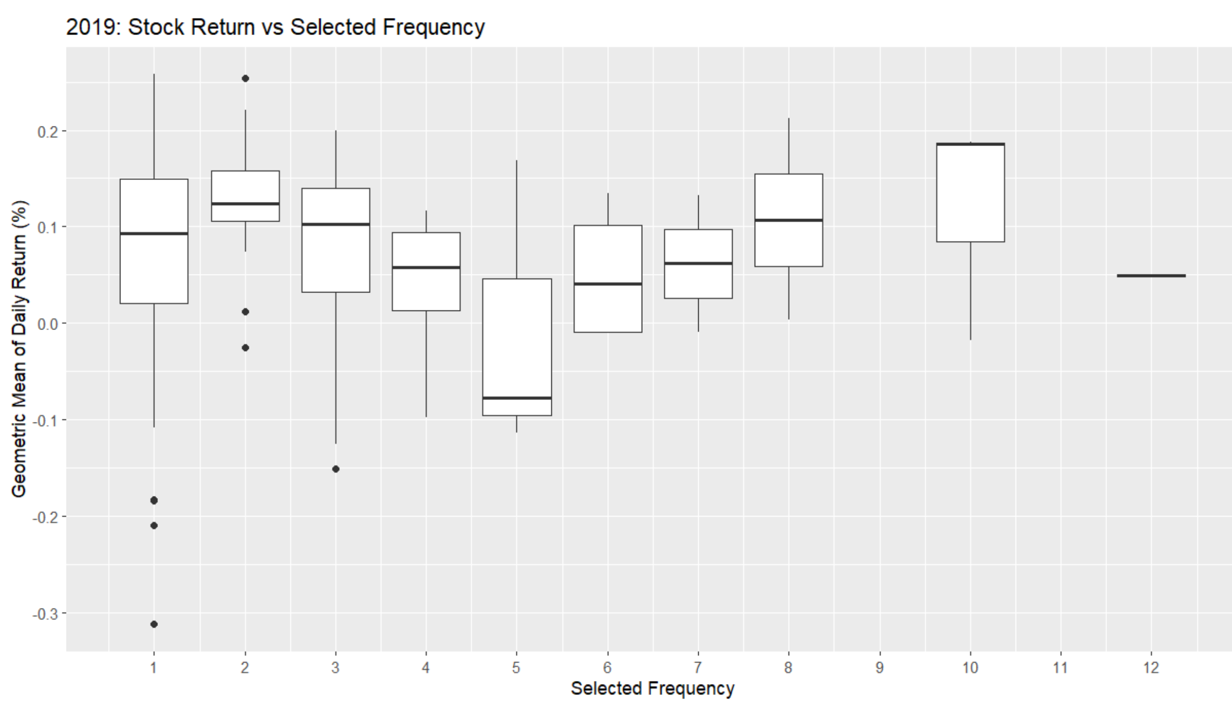
0.0
-0.2
1
2
3
4
5
6
7
8
9
10
11
Selected Frequency



2019: Stock Return vs Selected Frequency
Geometric Mean of Daily Return (%)
0.2
0.1
0.0
-0.1
-0.2
-0.3
1
2
3
4
5
6
7
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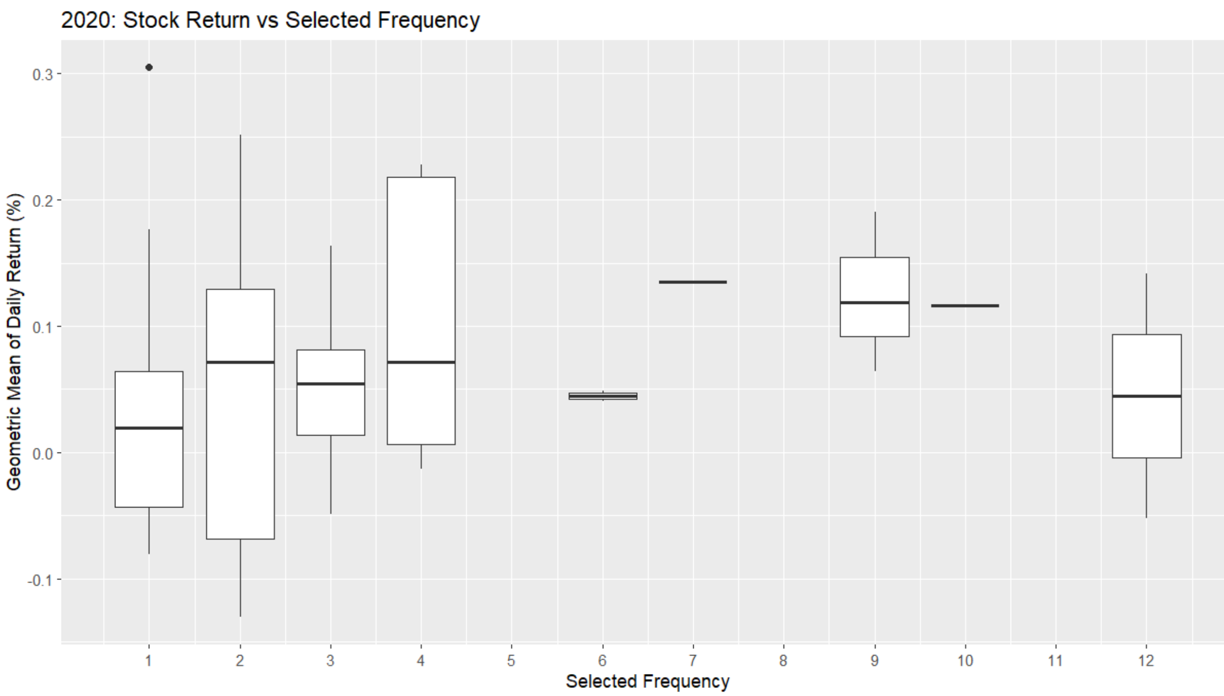
9
10
11
12
Selected Frequency


2020: Stock Return vs Selected Frequency
Geometric Mean of Daily Return (%)
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0.2
0.1
0.0
-0.1
1
2
3
4
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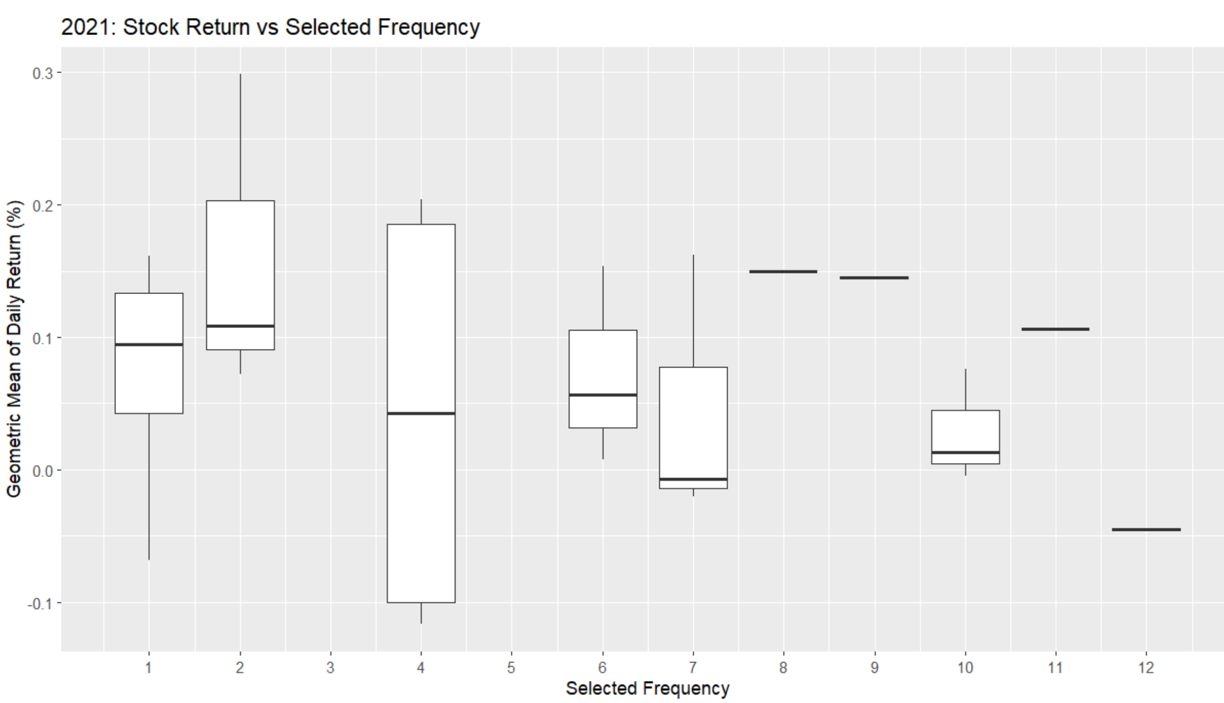
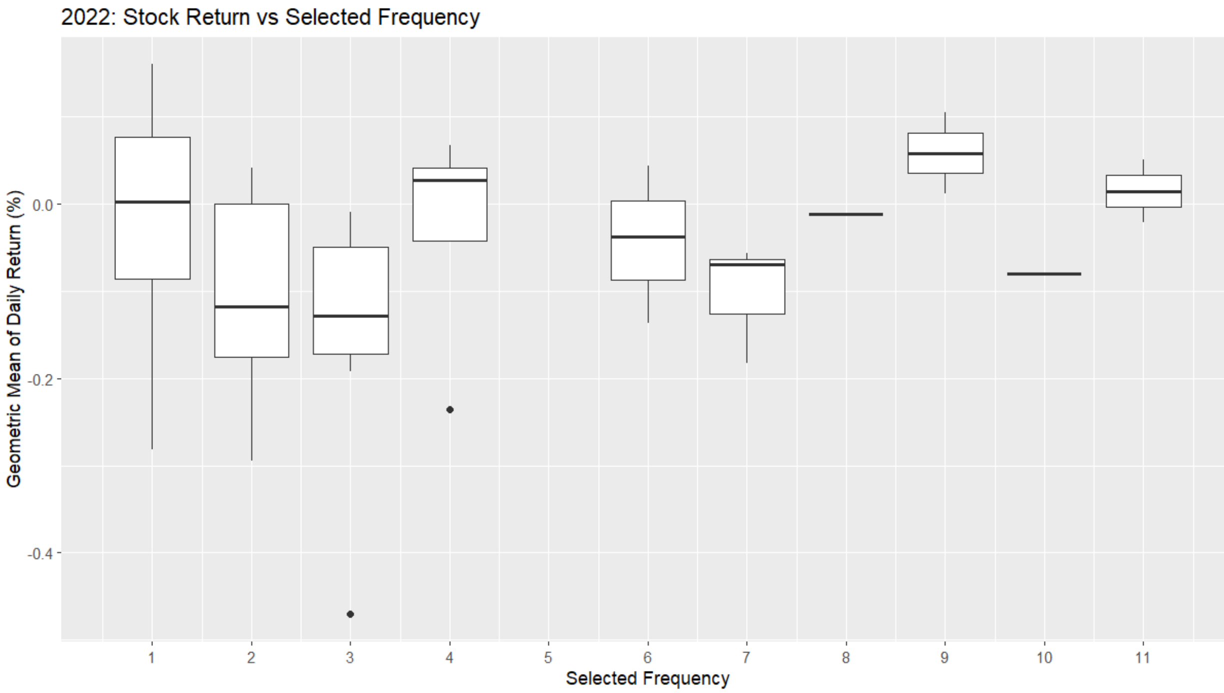
6
7
8
9
10
11
12
Selected Frequency


2021: Stock Return vs Selected Frequency
Geometric Mean of Daily Return (%)
0.3
0.2
0.1
0.0
-0.1
1
2
3
4
5
6
7
8
9
10
11
12
Selected Frequency


2022: Stock Return vs Selected Frequency
Geometric Mean of Daily Return (%)
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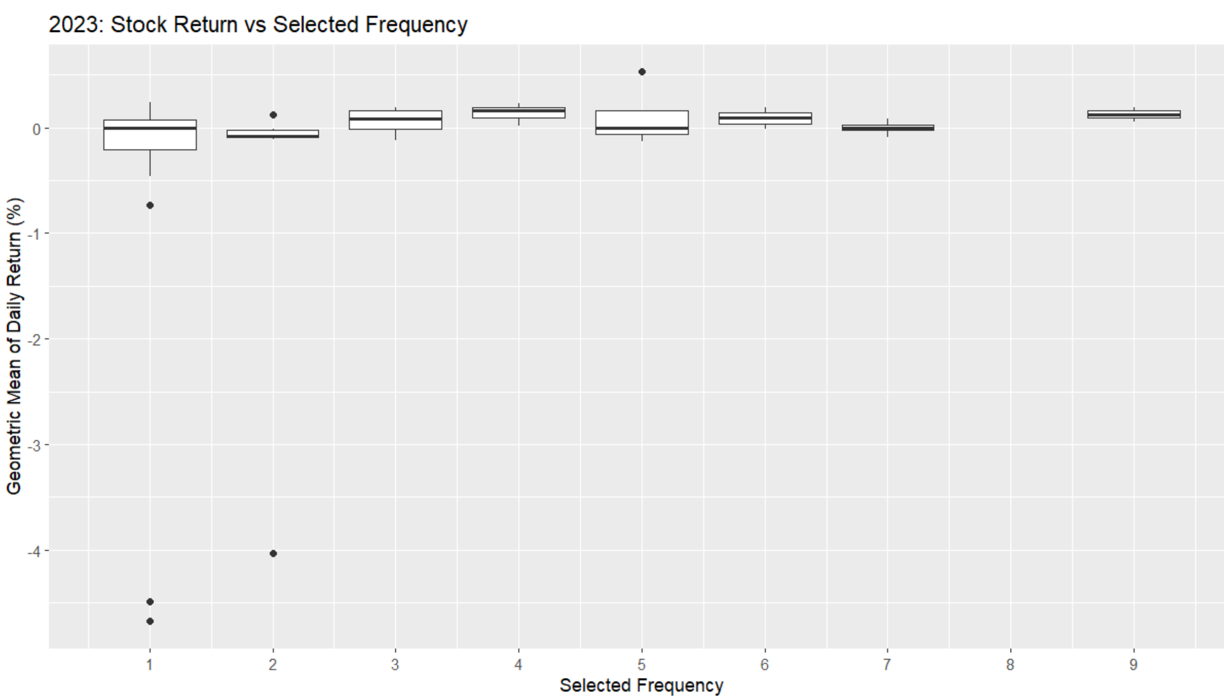
-0.4
1
2
3
4
5
6
7
8
9
10
11
Selected Frequency


2023: Stock Return vs Selected Frequency
Geometric Mean of Daily Return (%)
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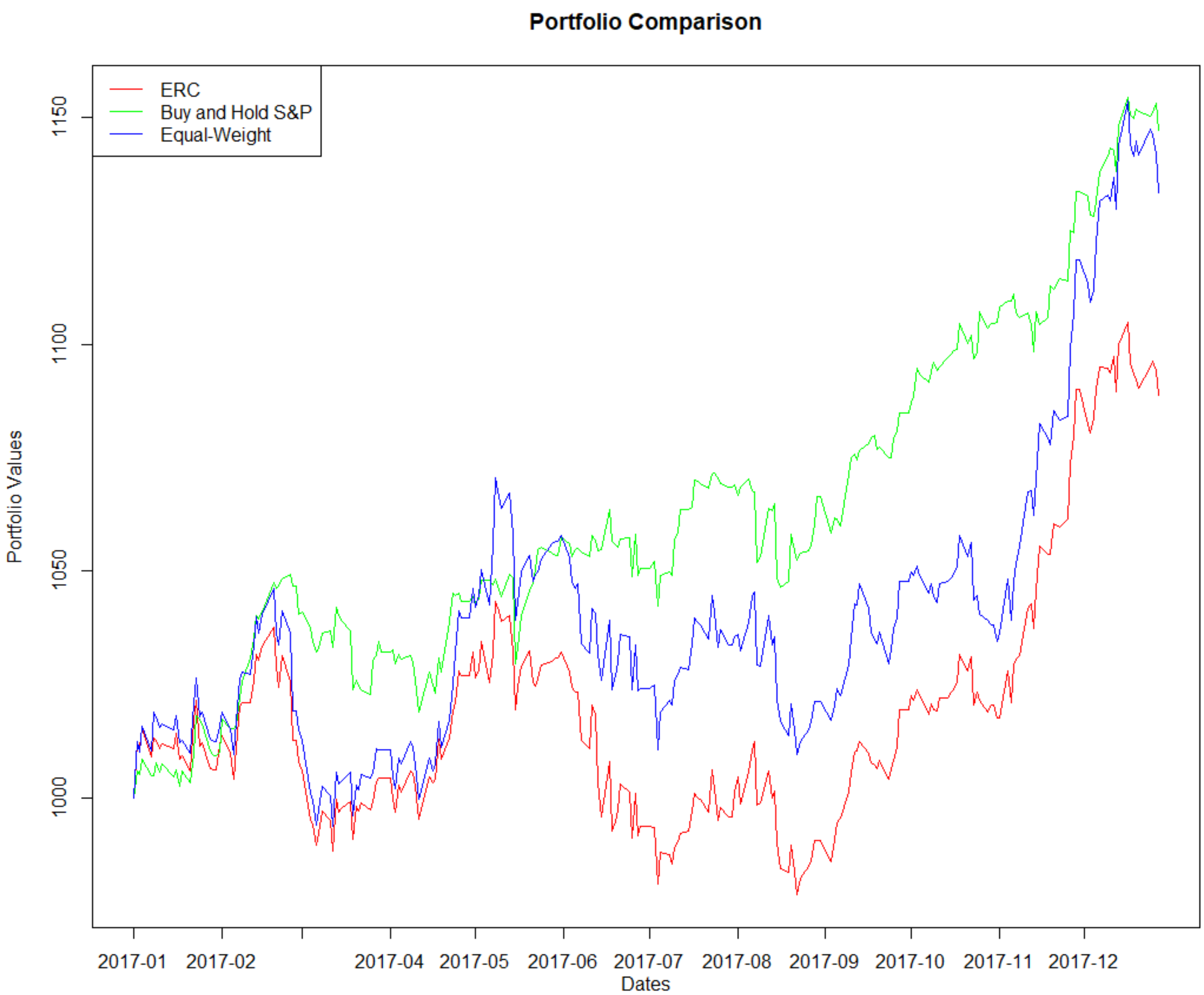
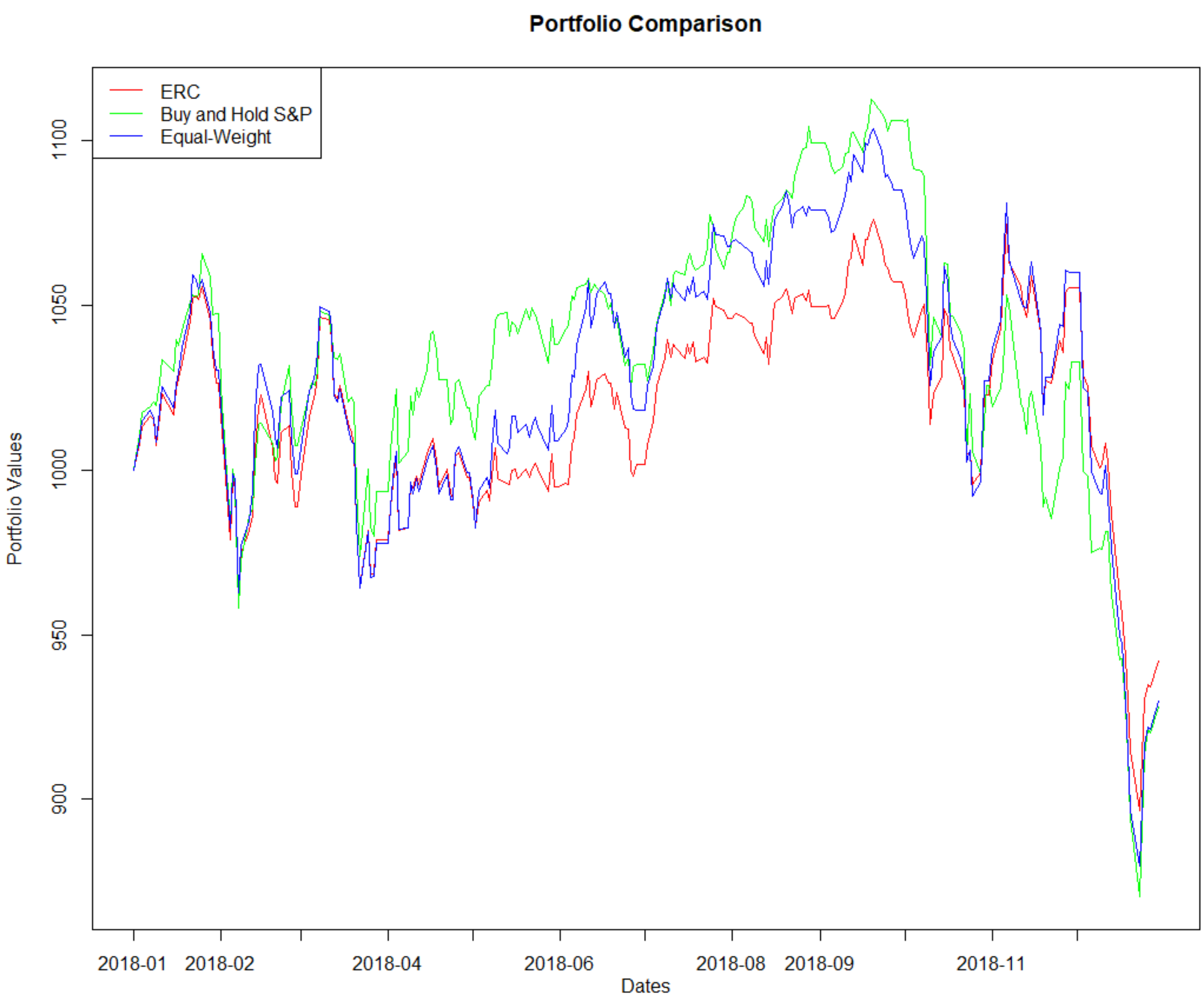
-1
-2
-3
-4
1
2
3
4
5
6
7
8
9
Selected Frequency

11.2 Long-Only Portfolio Performance

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ERC
Buy and Hold S&P
Equal-Weight
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1000
950
900
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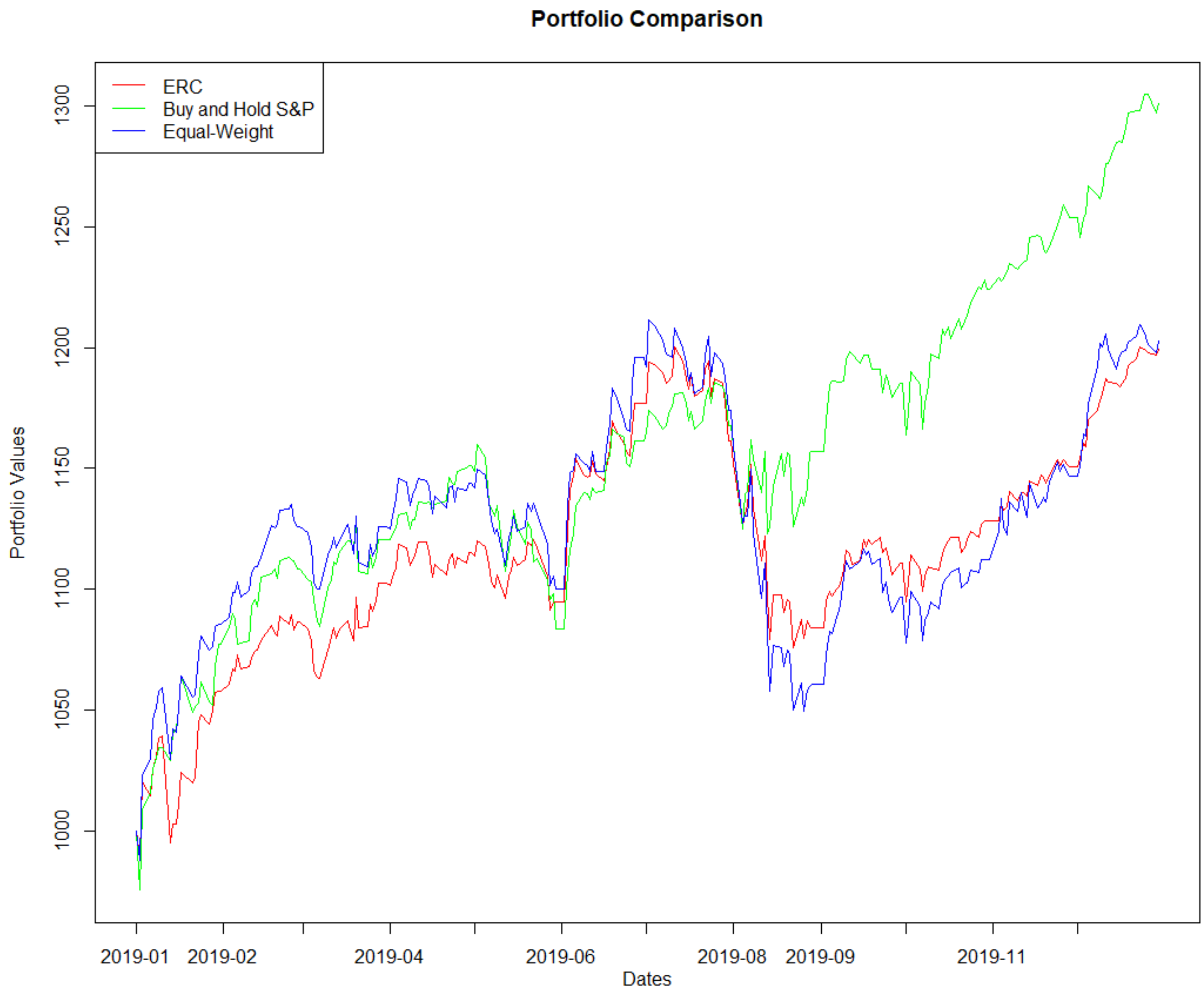
2018-02
2018-04
2018-06
2018-08
2018-09
2018-11
Dates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Buy and Hold S&P
Equal-Weight
Portfolio Values
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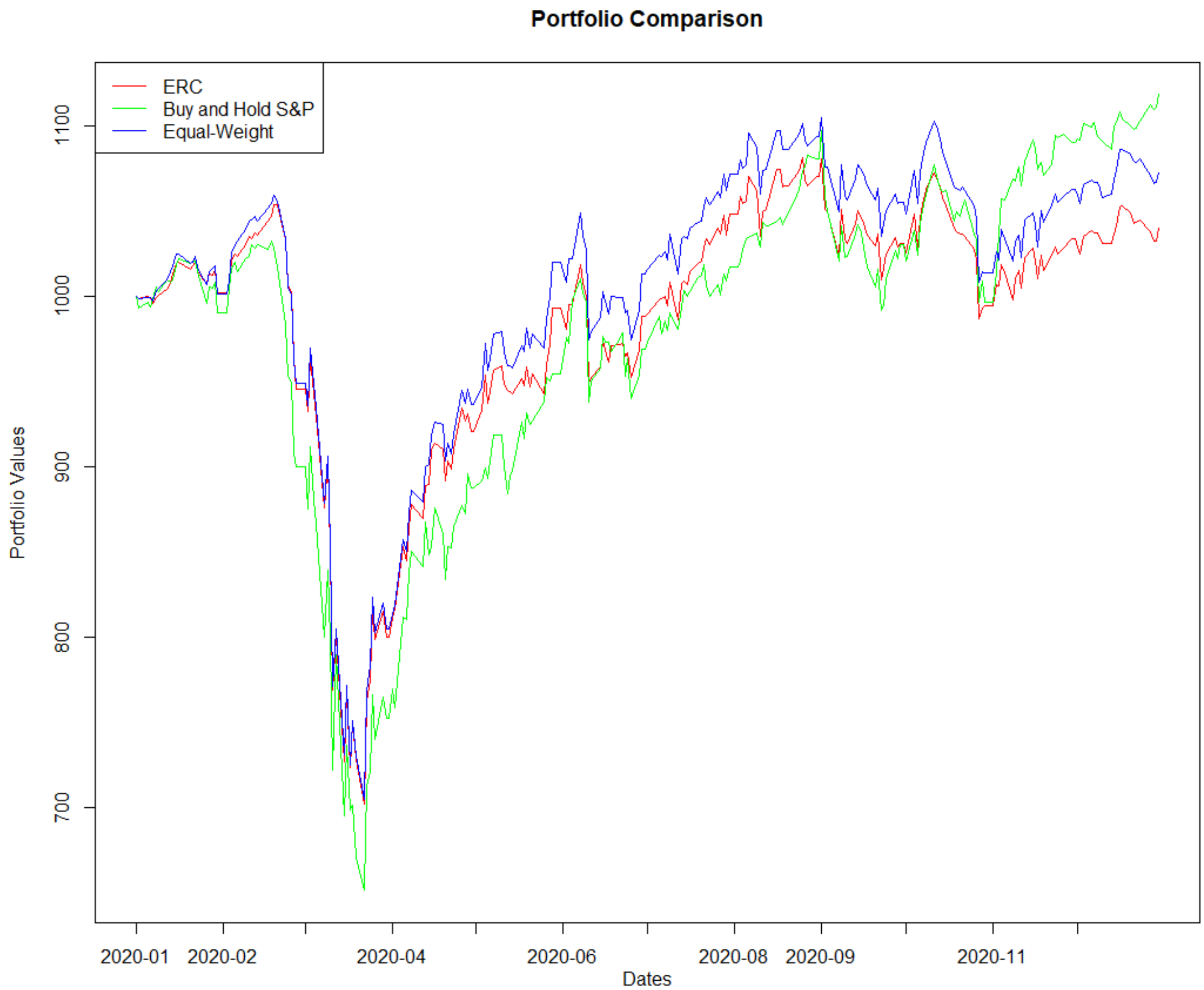
2019-09
2019-11
Dates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ERC
Buy and Hold S&P
Equal-Weight
Portfolio Values
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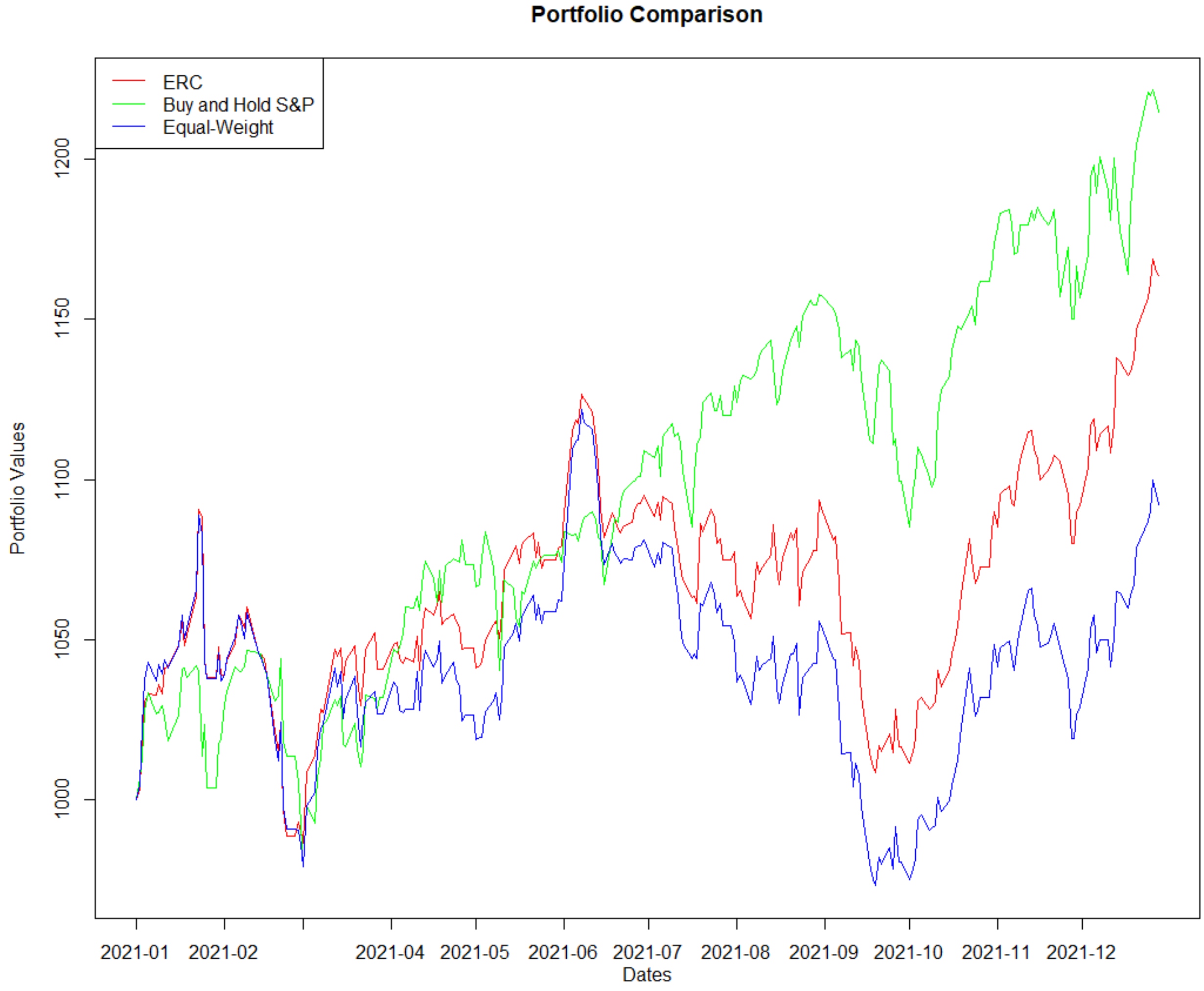
2020-09
2020-11
Dates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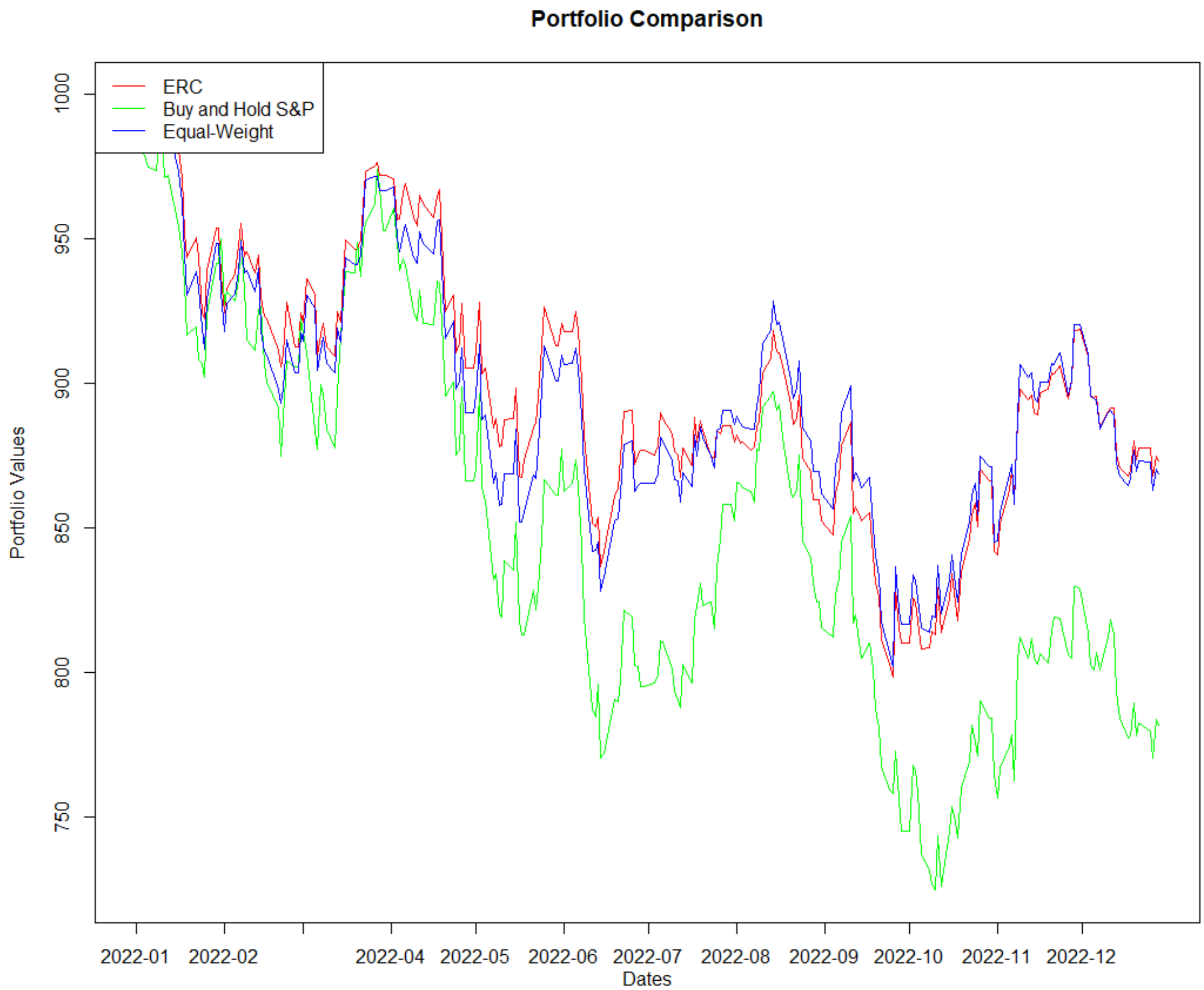
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Equal-Weight
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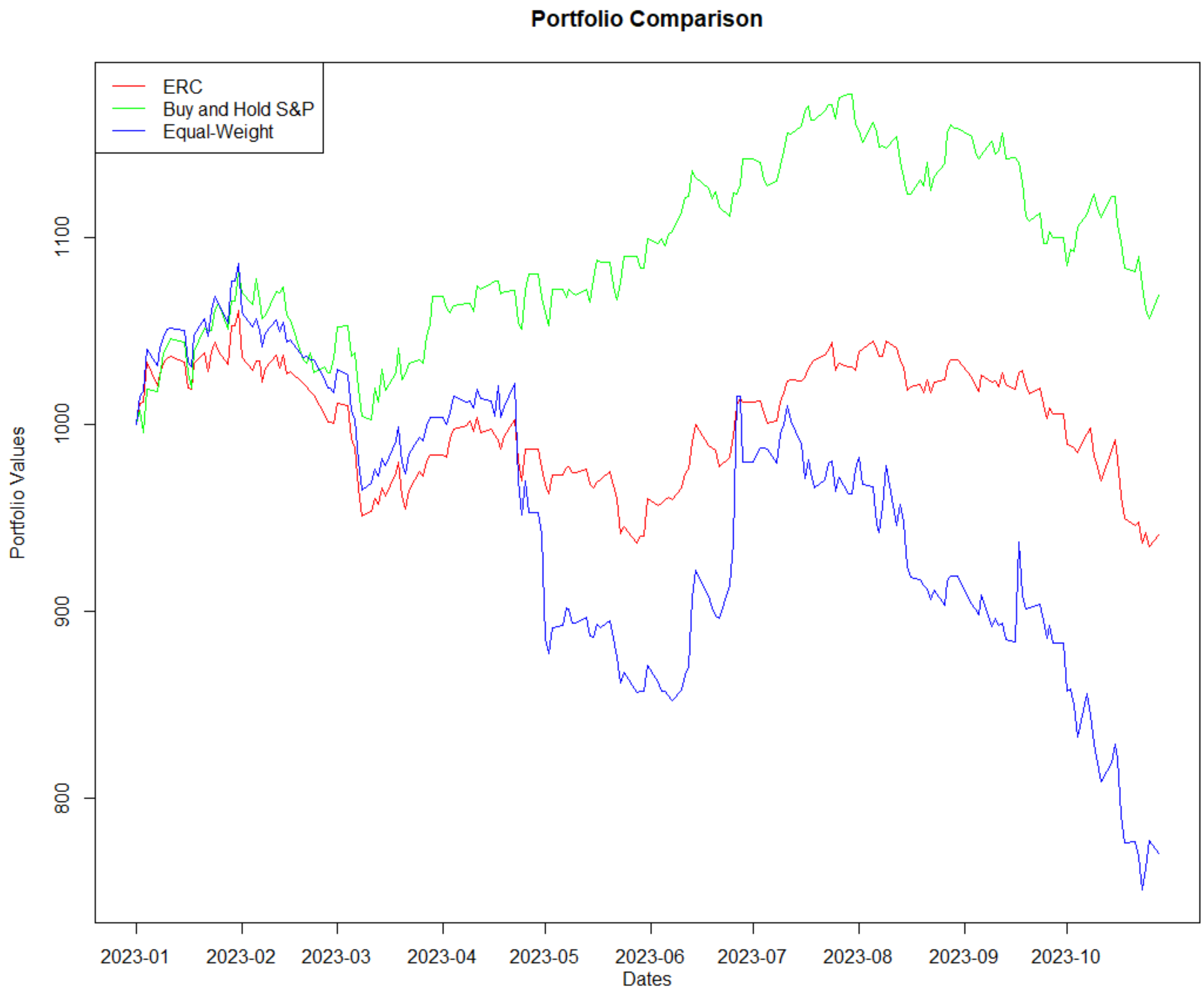
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Buy and Hold S&P
Equal-Weight
Portfolio Values
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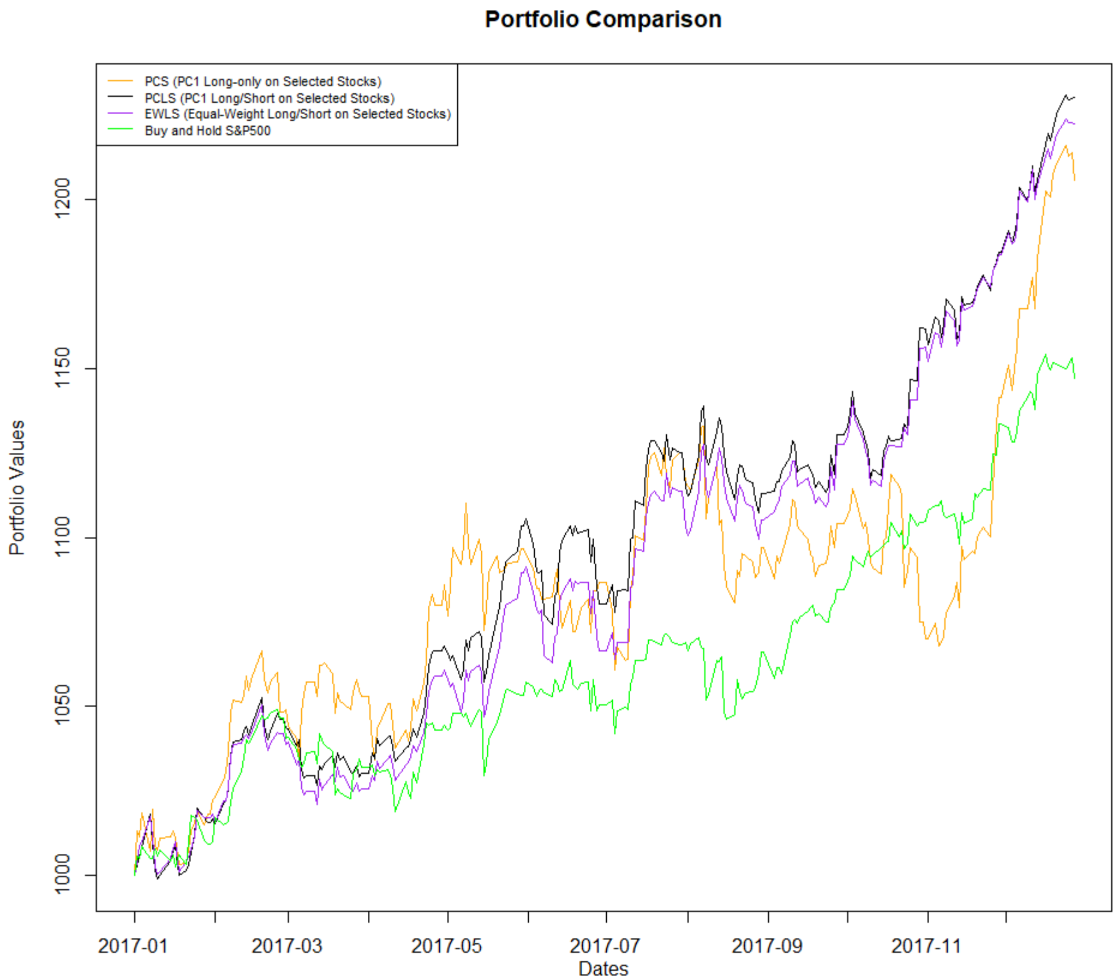
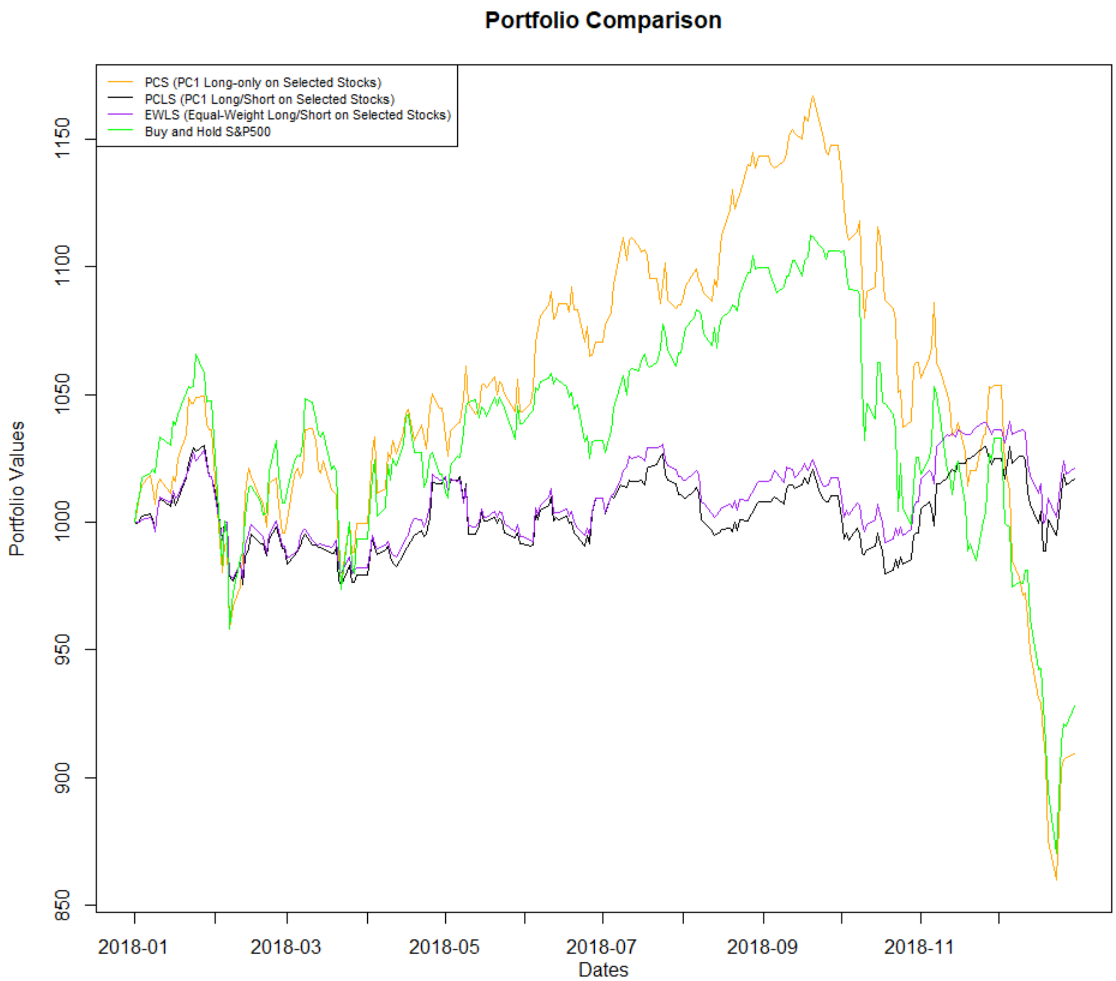
Dates

11.3 Long/Short Portfolio Performance

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PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P500
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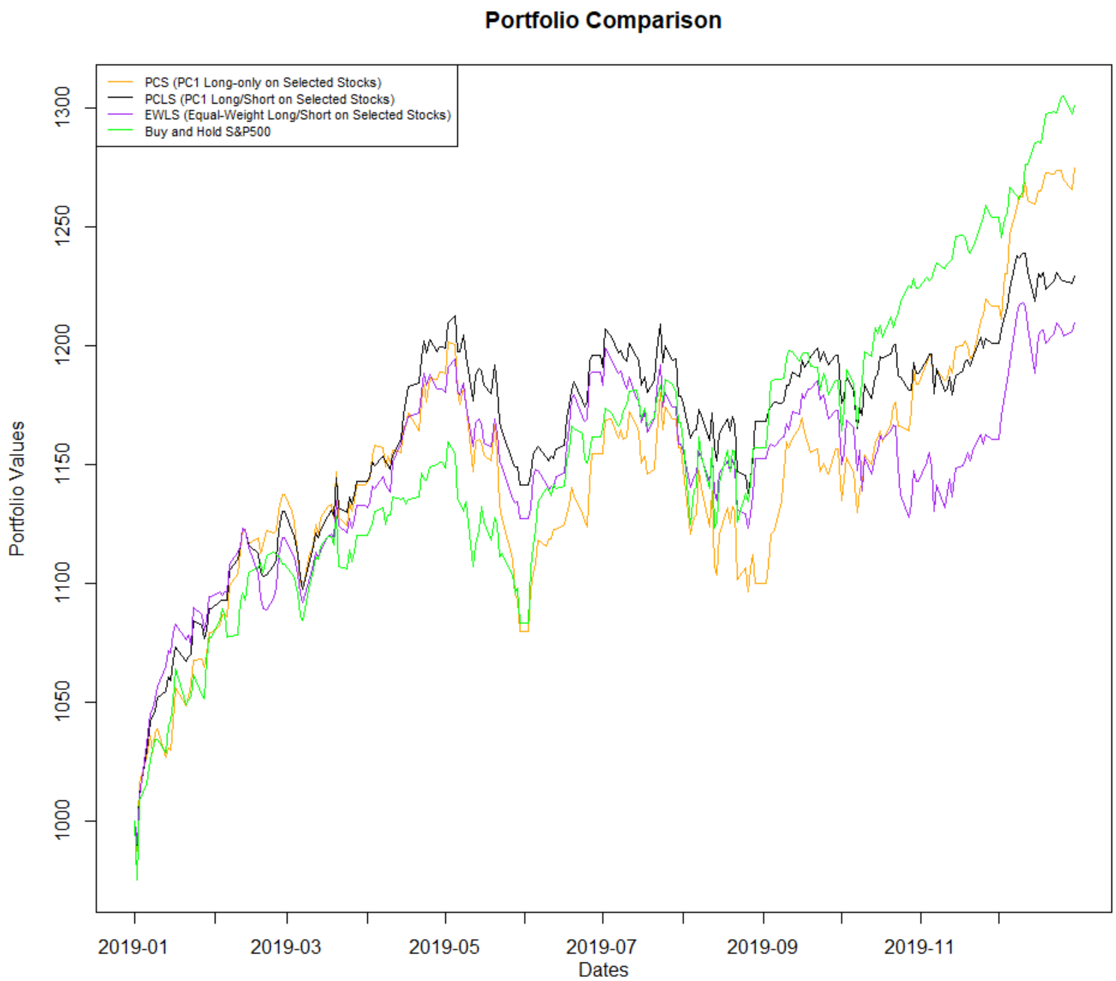
2018-09
2018-11
Dates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EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P500
Portfolio Values
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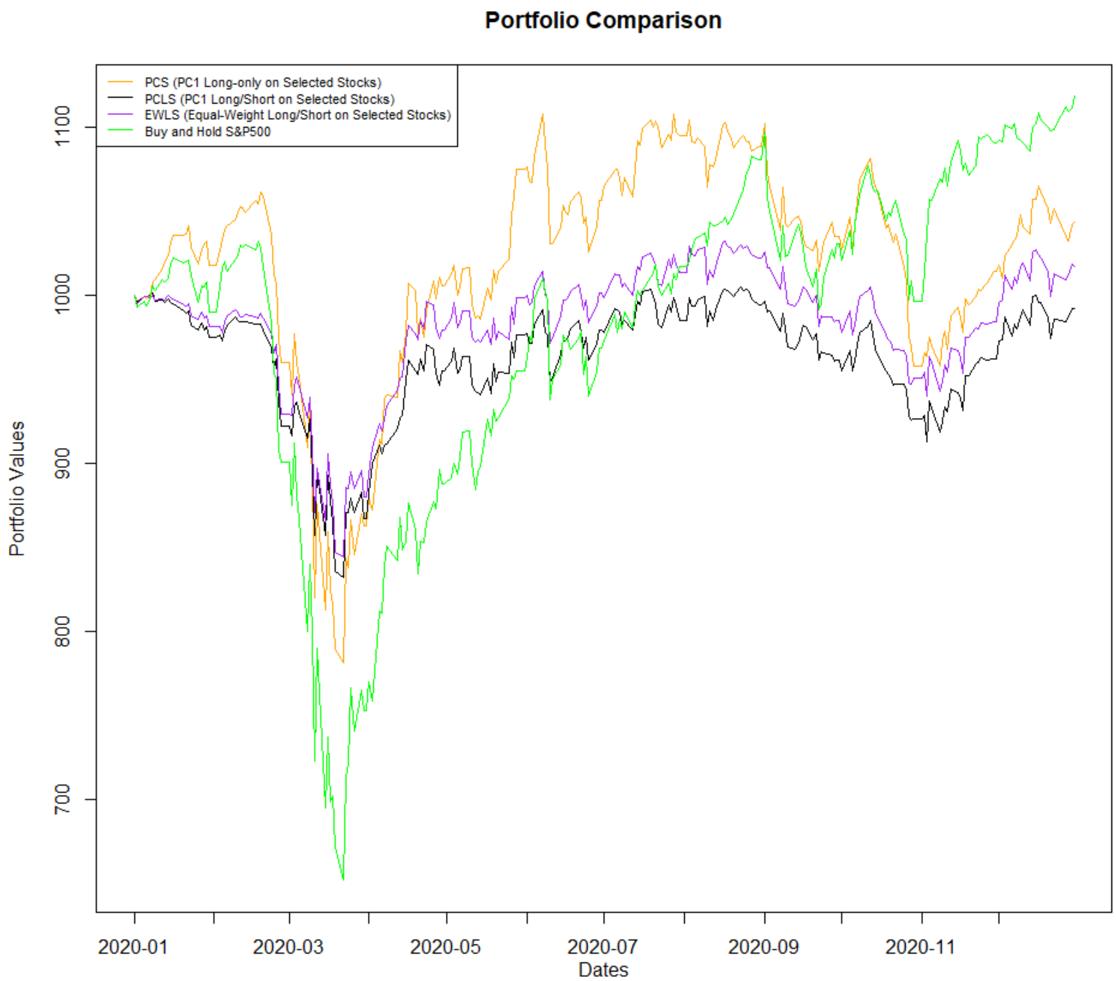
Dates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EWLS (Equal-Weight Long/Short on Selected Stocks)
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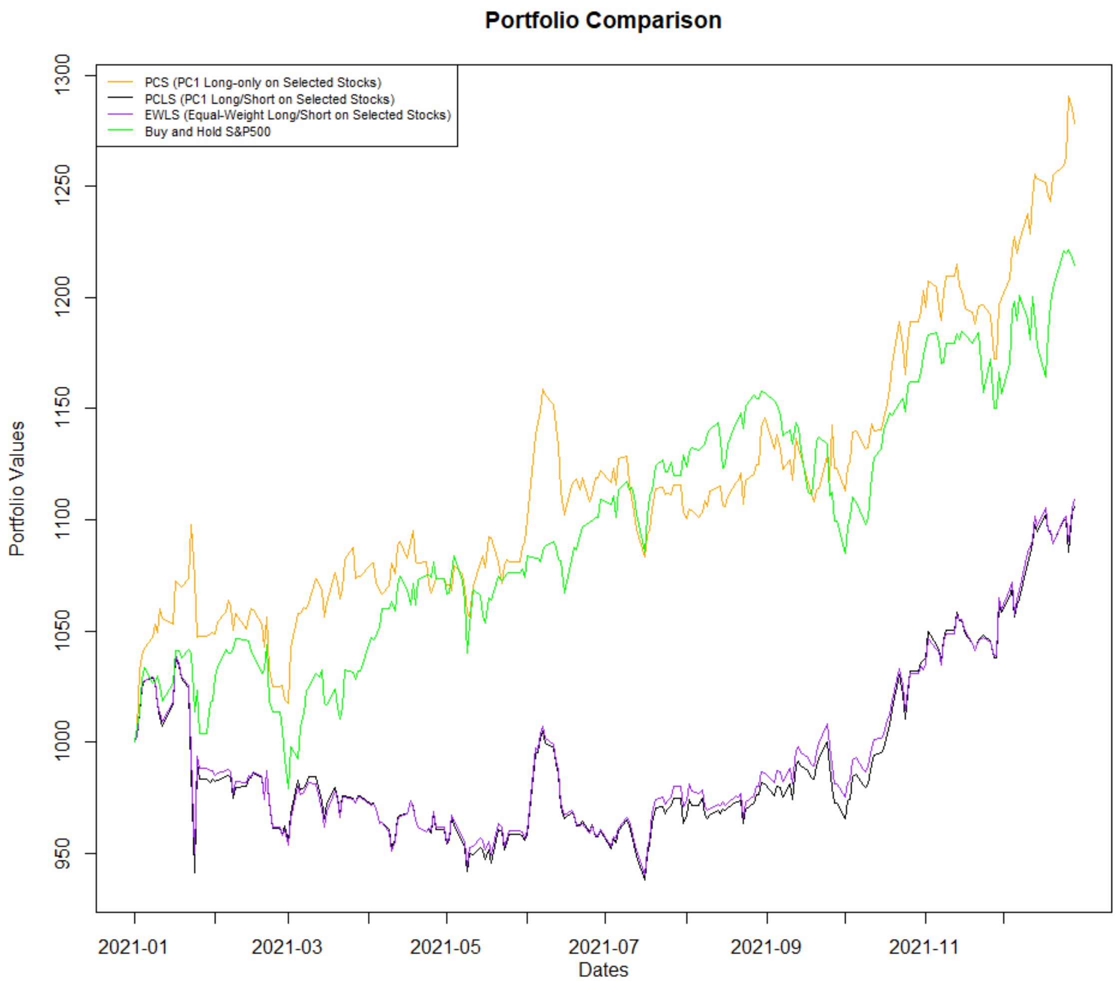
2020-09
2020-11
Dates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PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P500
Portfolio Values
950
1000
1050
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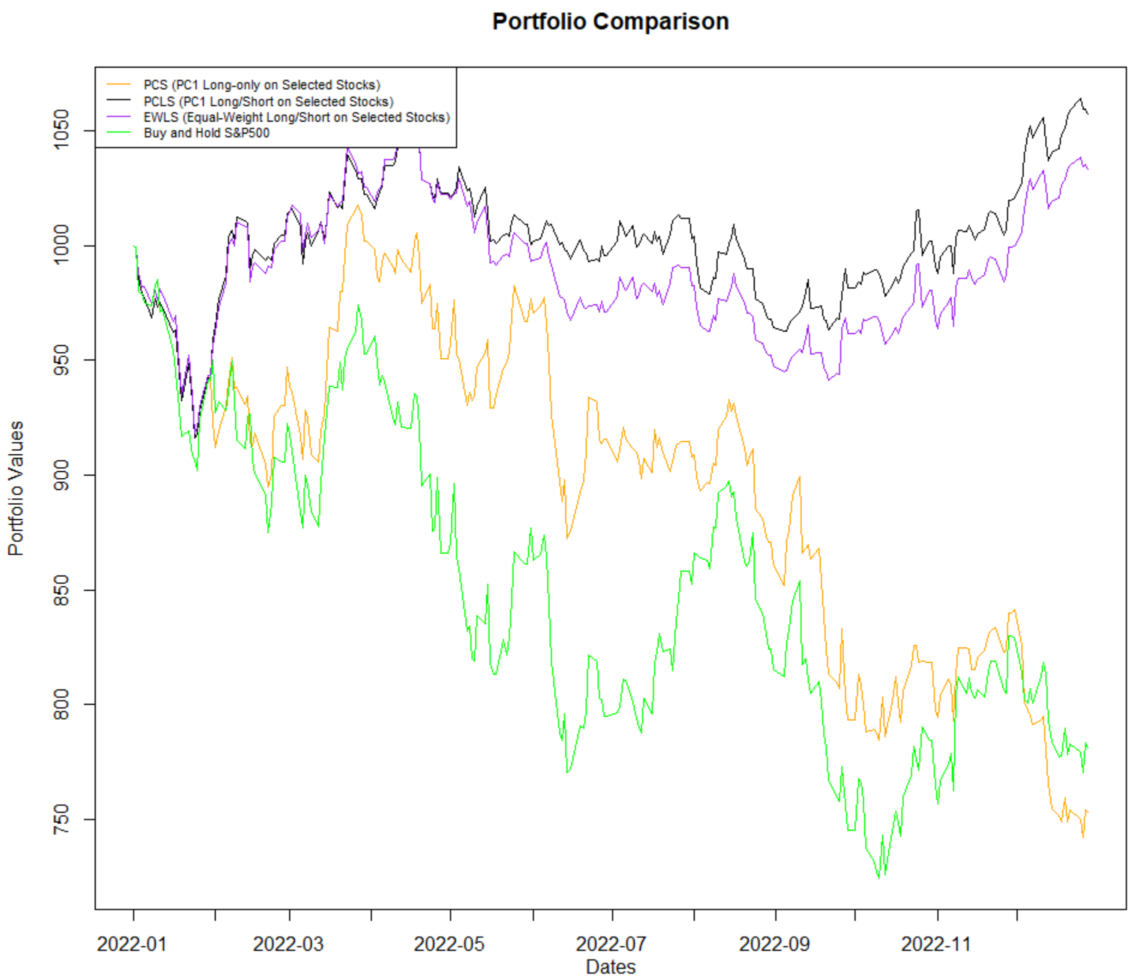
2021-09
2021-11
Dates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PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P500
Portfolio Values
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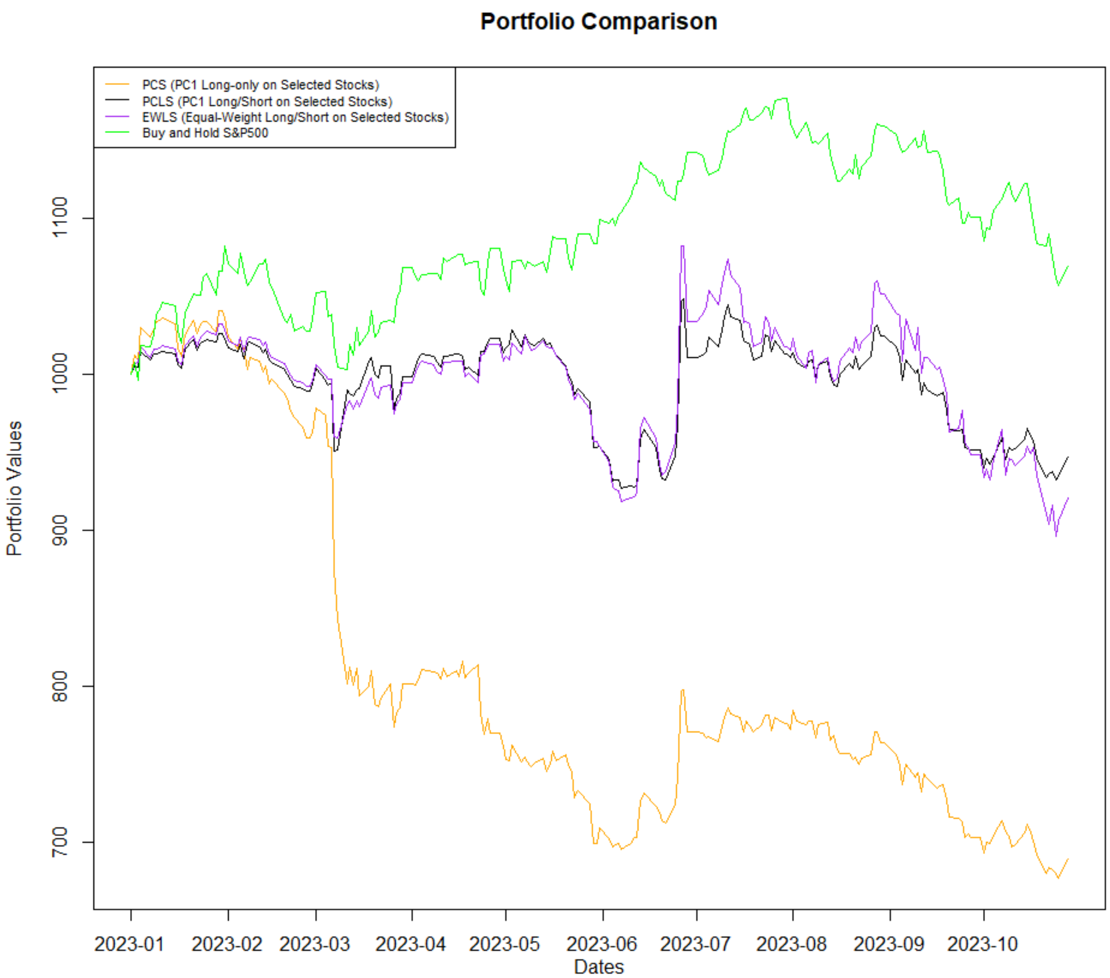
Dates


Portfolio Comparison
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PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P500
Portfolio Values
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2023-01
2023-02
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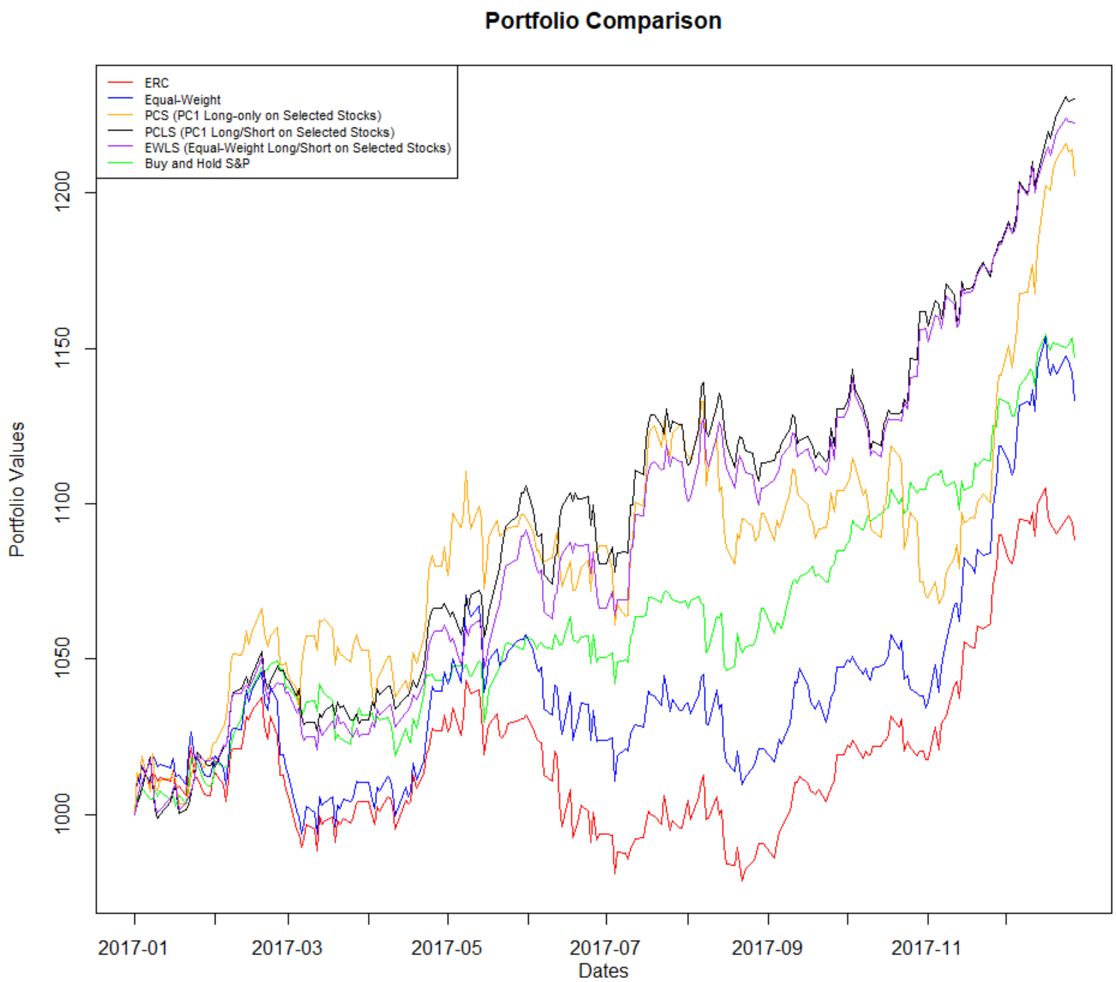
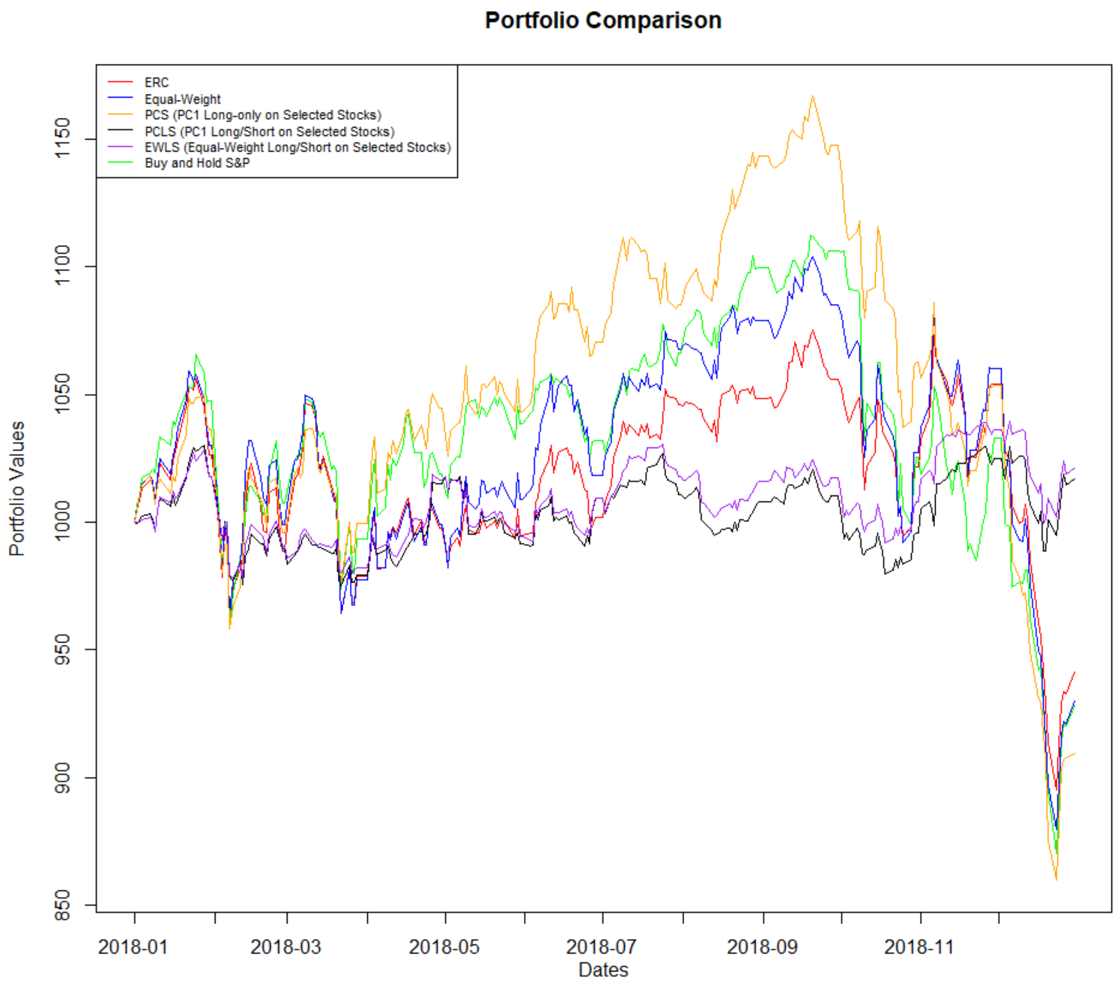
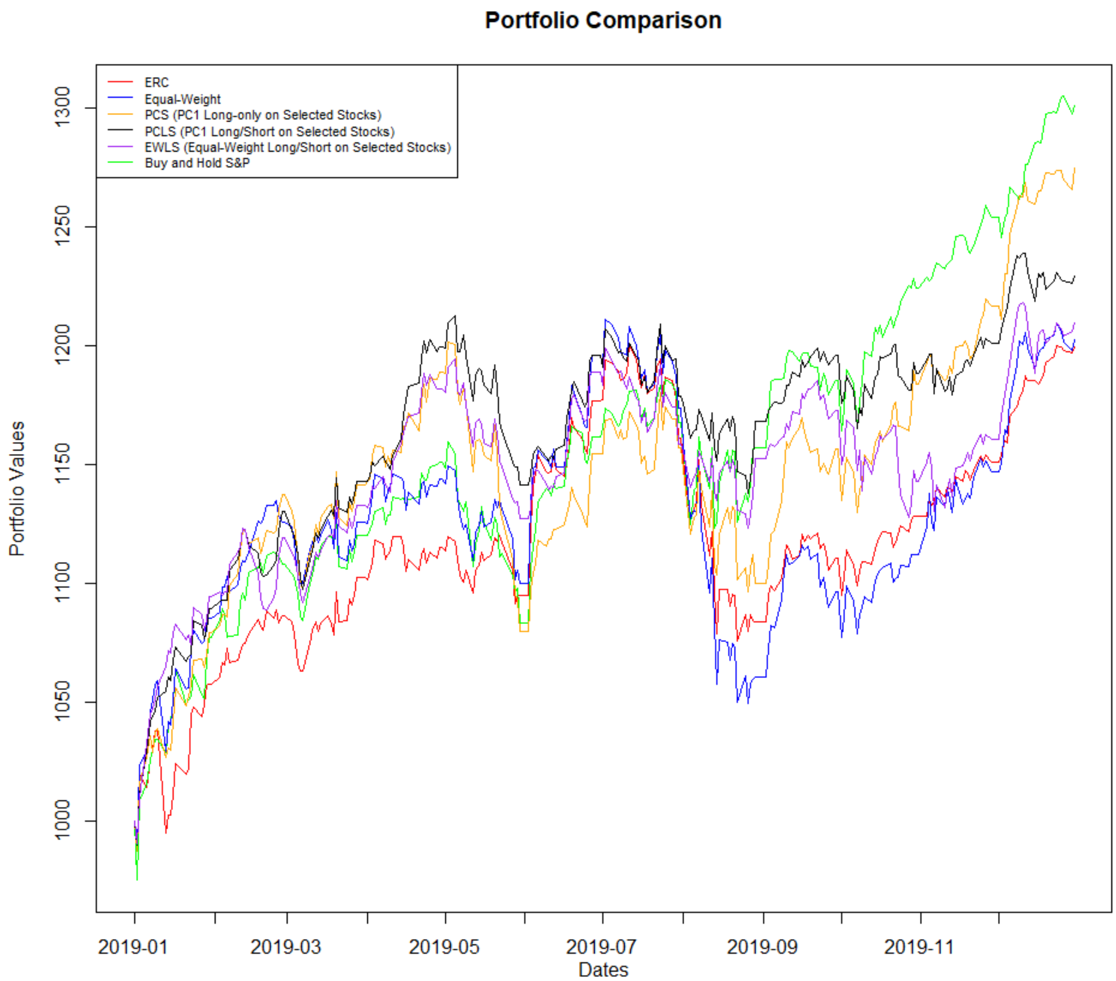
2023-10
Dates

11.4 All Portfolio Performance





Portfolio Comparison
ERC
Equal-Weight
PCS (PC1 Long-only on Selected Stocks)
PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P
Portfolio Values
850
900
950
1000
1050
1100
1150
2018-01
2018-03
2018-05
2018-07
2018-09
2018-11
Dates


Portfolio Comparison
ERC
Equal-Weight
PCS (PC1 Long-only on Selected Stocks)
PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P
Portfolio Values
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2019-01
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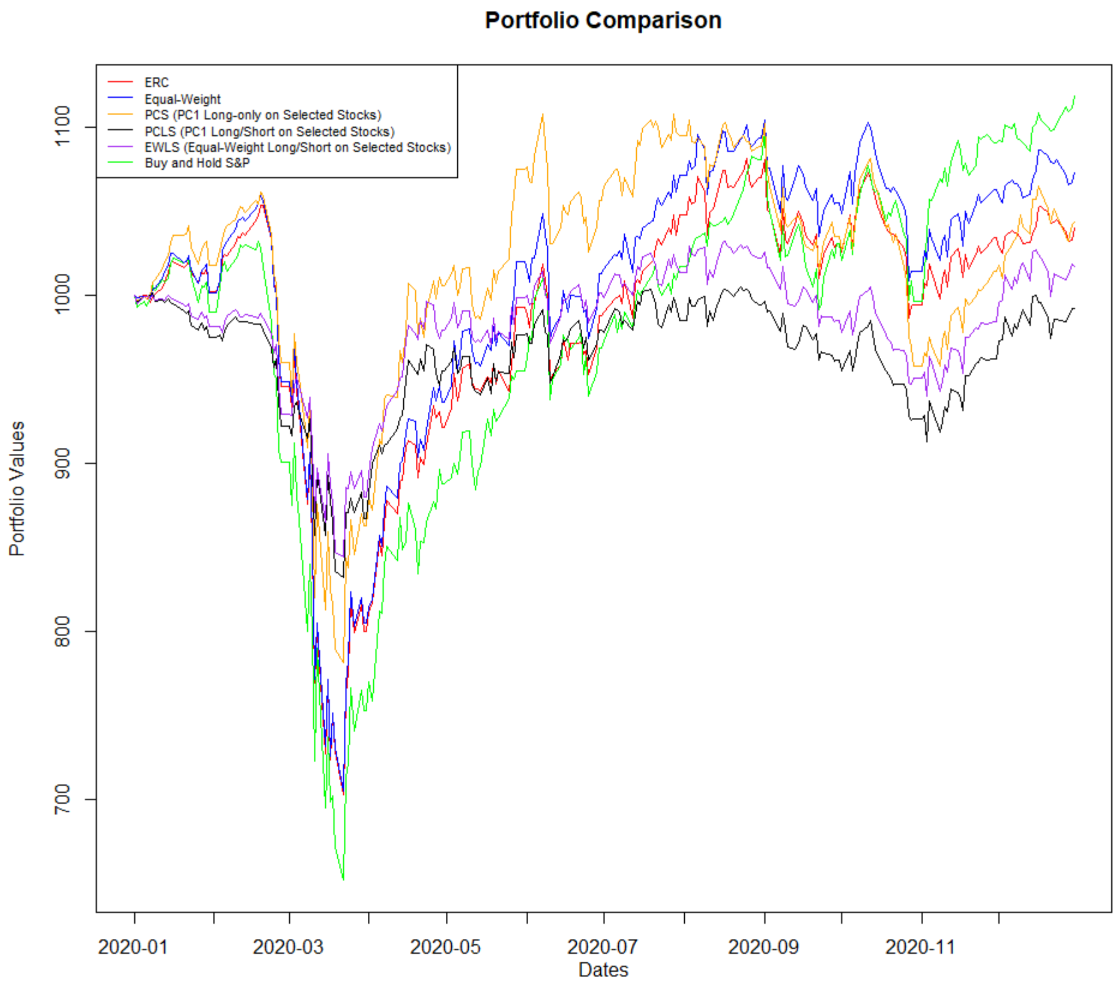
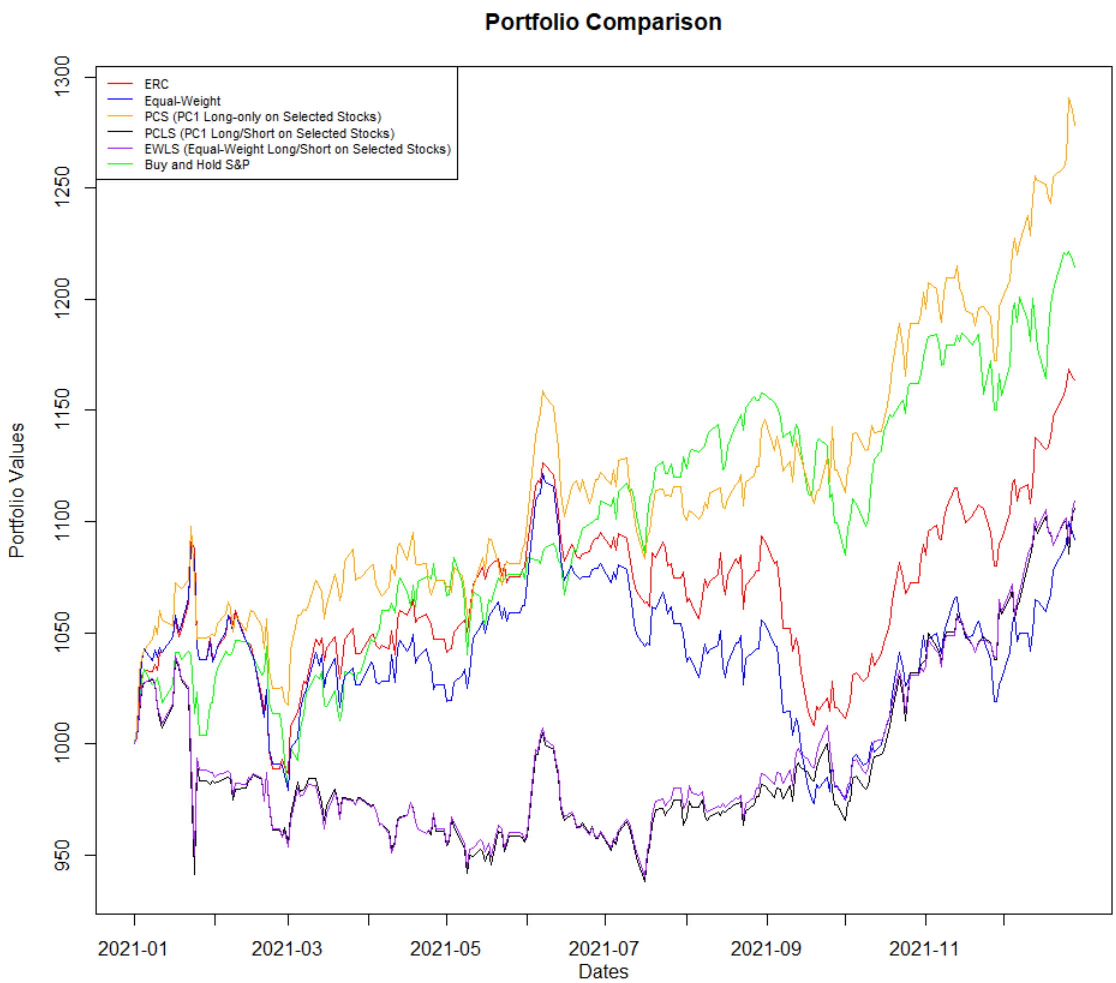
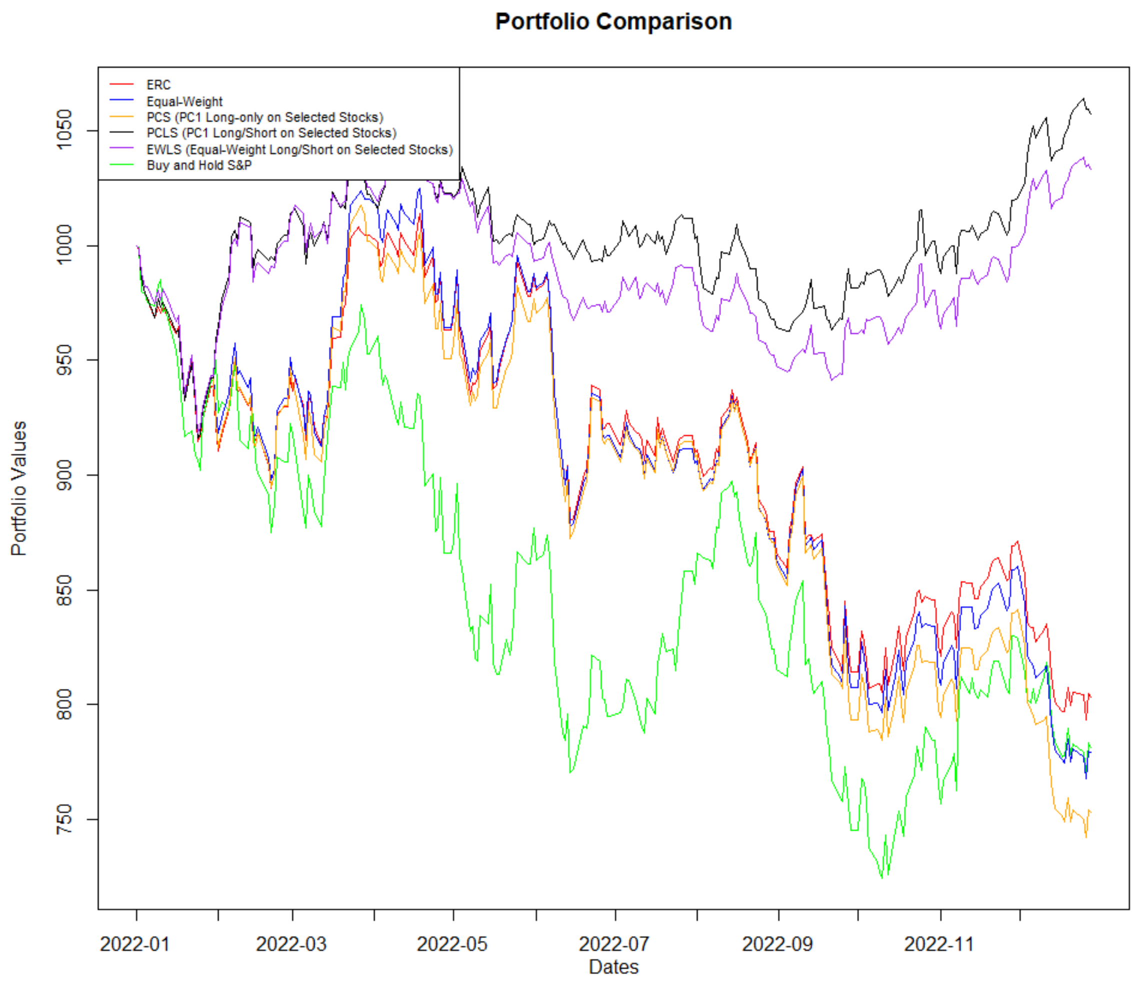
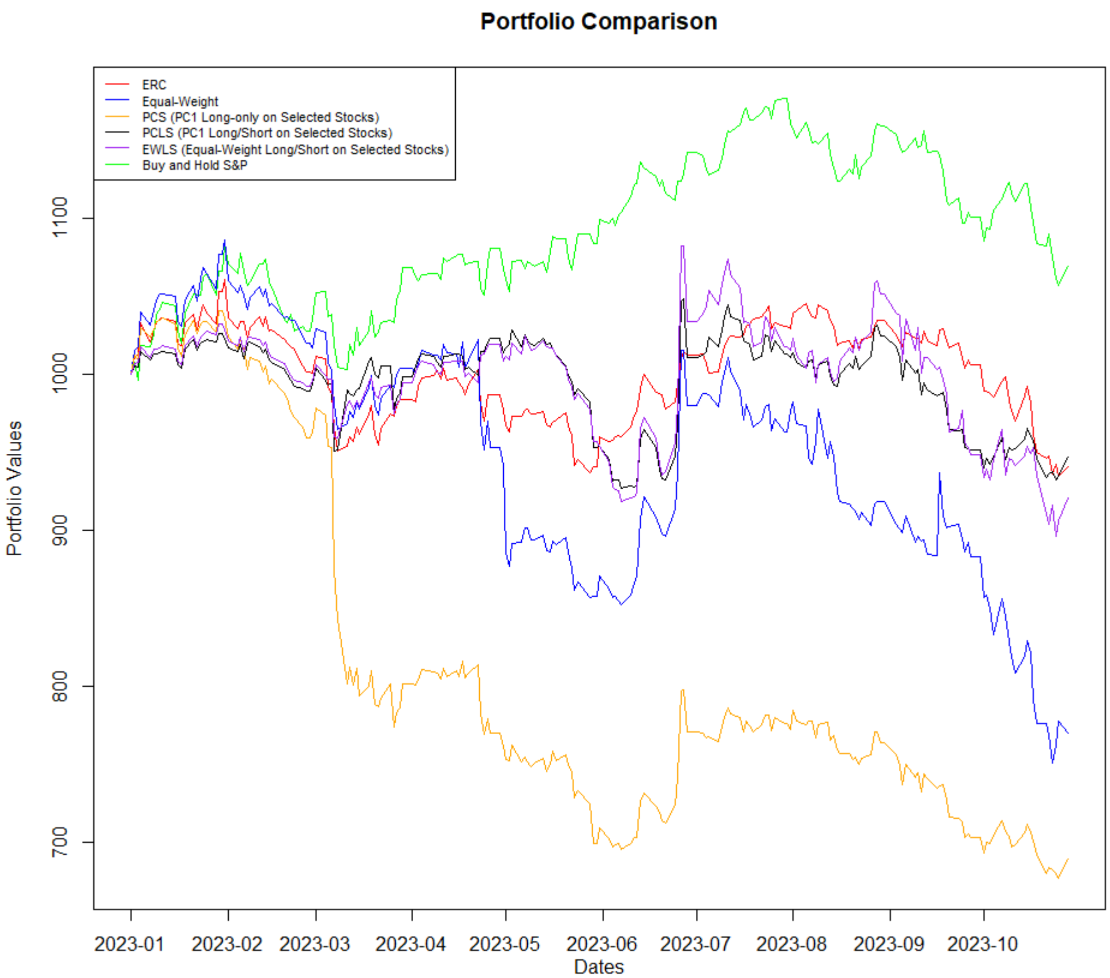
2019-09
2019-11
Dates

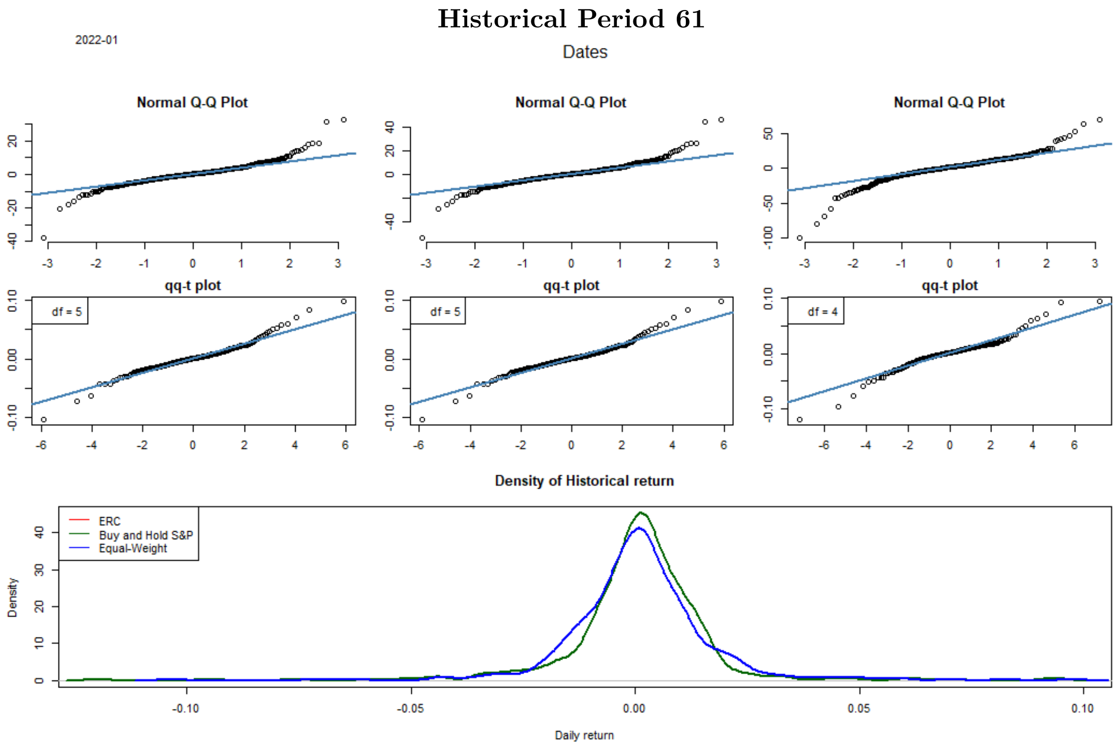

Portfolio Comparison
ERC
Equal-Weight
PCS (PC1 Long-only on Selected Stocks)
PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P
Portfolio Values
700
800
900
1000
1100
2020-01
2020-03
2020-05
2020-07
2020-09
2020-11
Dates

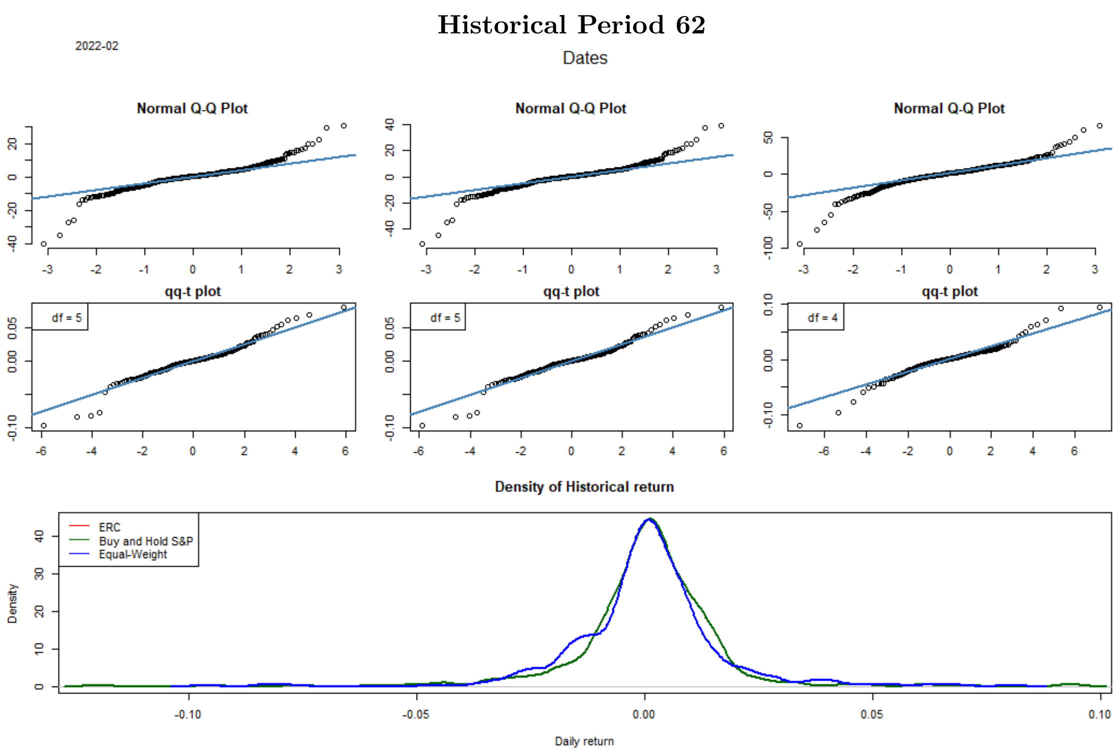

Portfolio Comparison
ERC
Equal-Weight
PCS (PC1 Long-only on Selected Stocks)
PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P
Portfolio Values
950
1000
1050
1100
1150
1200
1250
1300
2021-01
2021-03
2021-05
2021-07
2021-09
2021-11
Dates

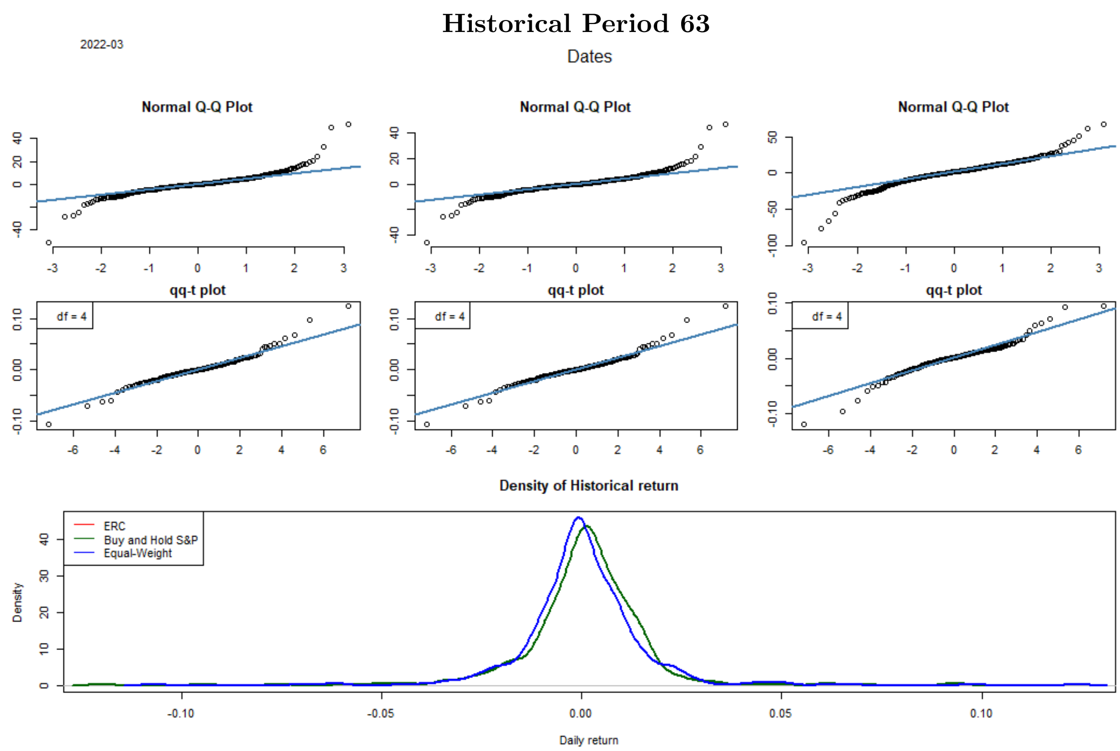

Portfolio Comparison
ERC
Equal-Weight
PCS (PC1 Long-only on Selected Stocks)
PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P
Portfolio Values
1050
1000
950
900
850
800
750
2022-01
2022-03
2022-05
2022-07
2022-09
2022-11
Dates

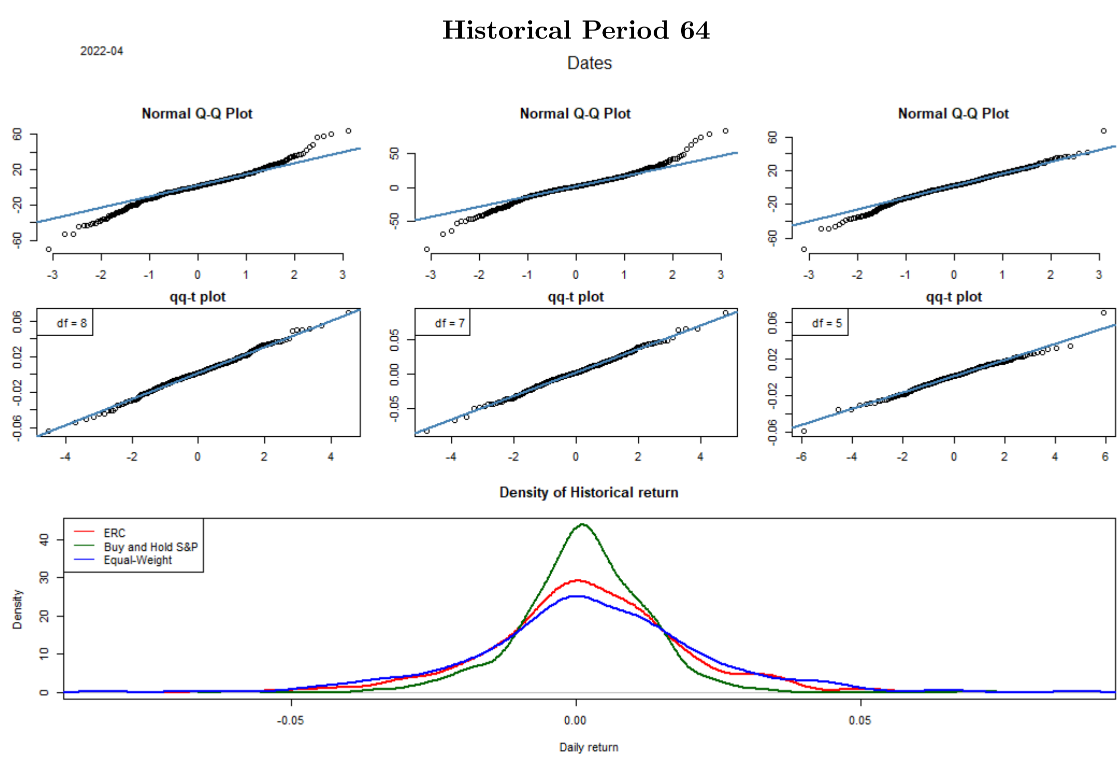

Portfolio Comparison
ERC
Equal-Weight
PCS (PC1 Long-only on Selected Stocks)
PCLS (PC1 Long/Short on Selected Stocks)
EWLS (Equal-Weight Long/Short on Selected Stocks)
Buy and Hold S&P
Portfolio Values
700
800
900
1000
1100
2023-01
2023-02
2023-03
2023-04
2023-05
2023-06
2023-07
2023-08
2023-09
2023-10
Dates

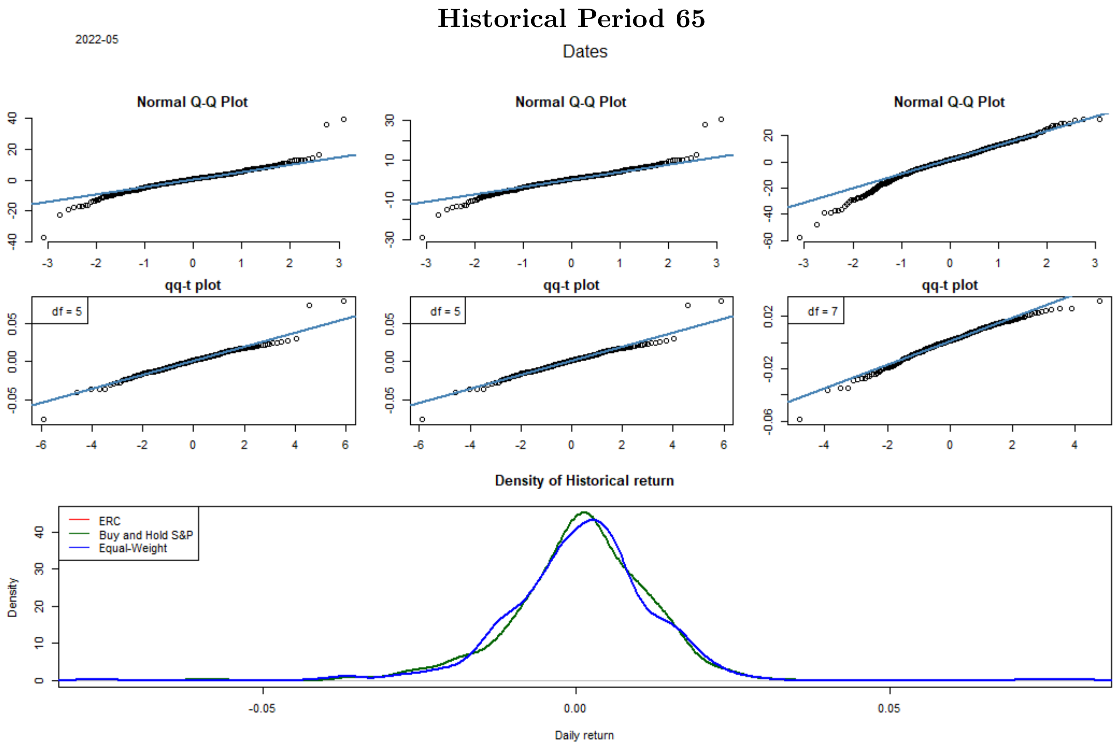
11.5 Return Distribution in 2022 (Historical Period 61-72)

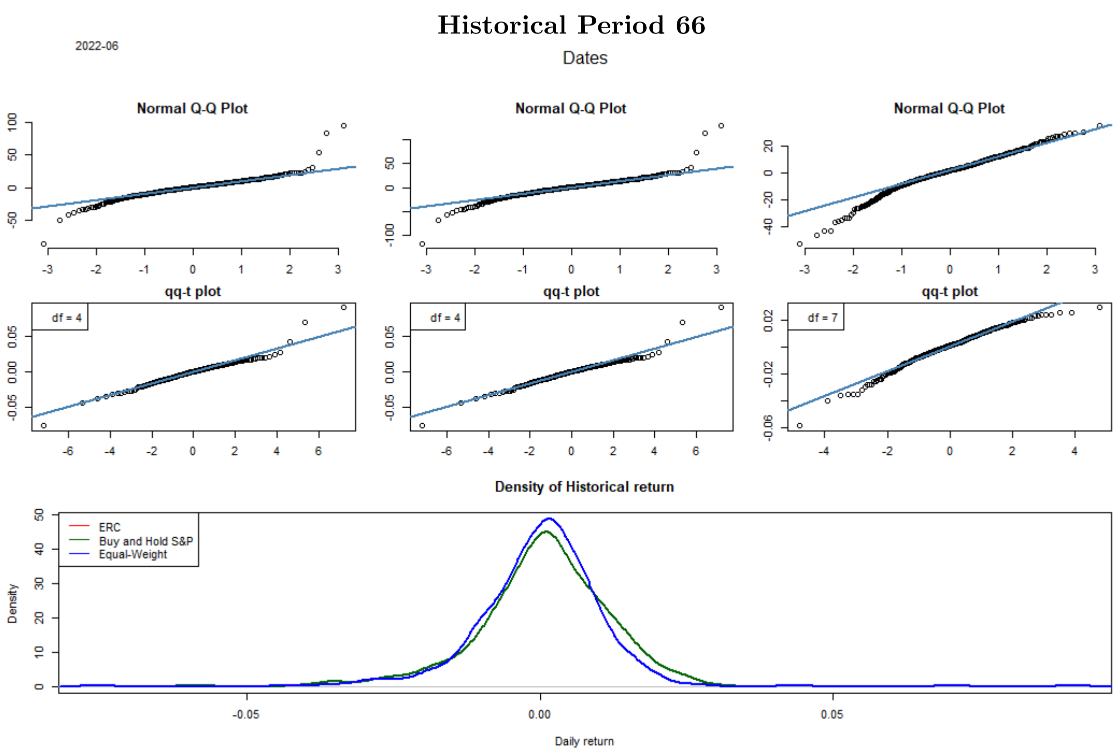


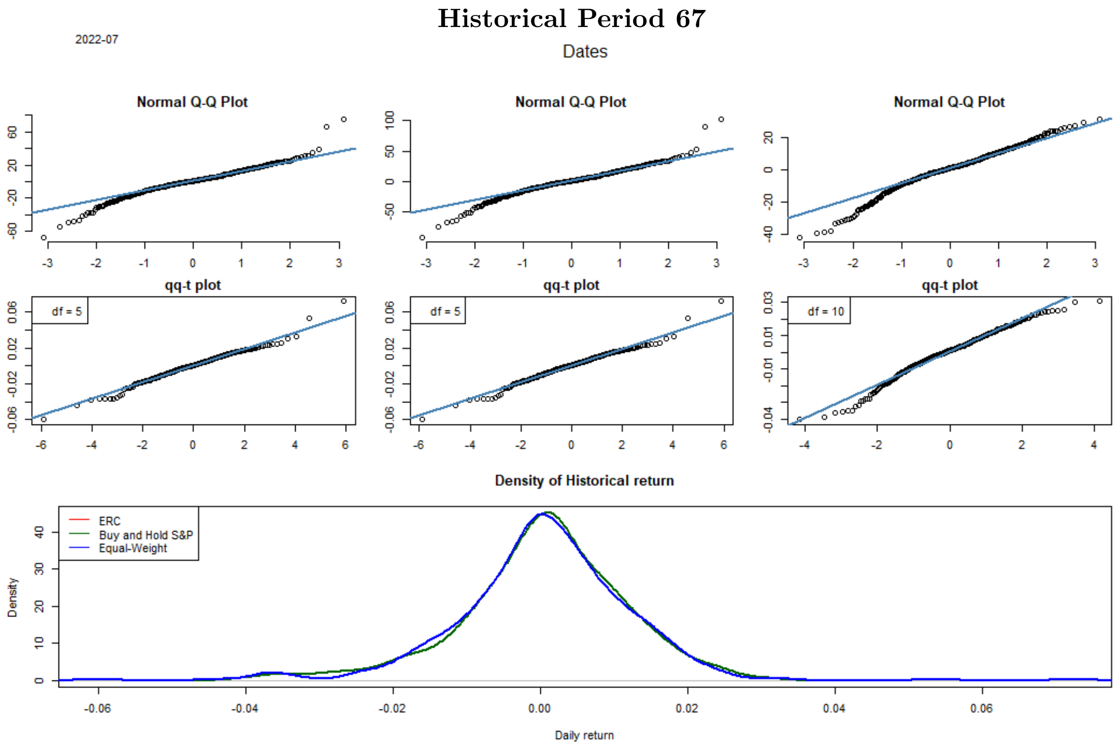


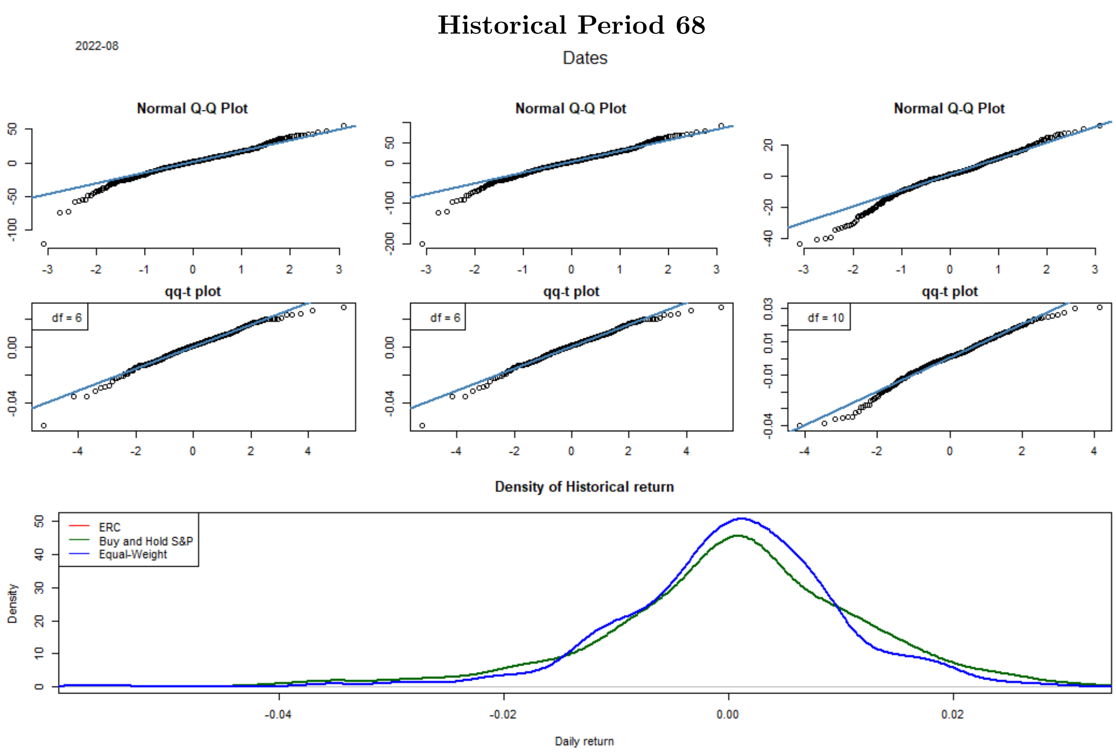

Historical Period 63
2022-03
Dates
Normal Q-Q Plot
Normal Q-Q Plot
Normal Q-Q Plot
qq-t plot
qq-t plot
qq-t plot
df = 4
df = 4
df = 4
Density of Historical return
ERC
Buy and Hold S&P
Equal-Weight
Density
Daily return

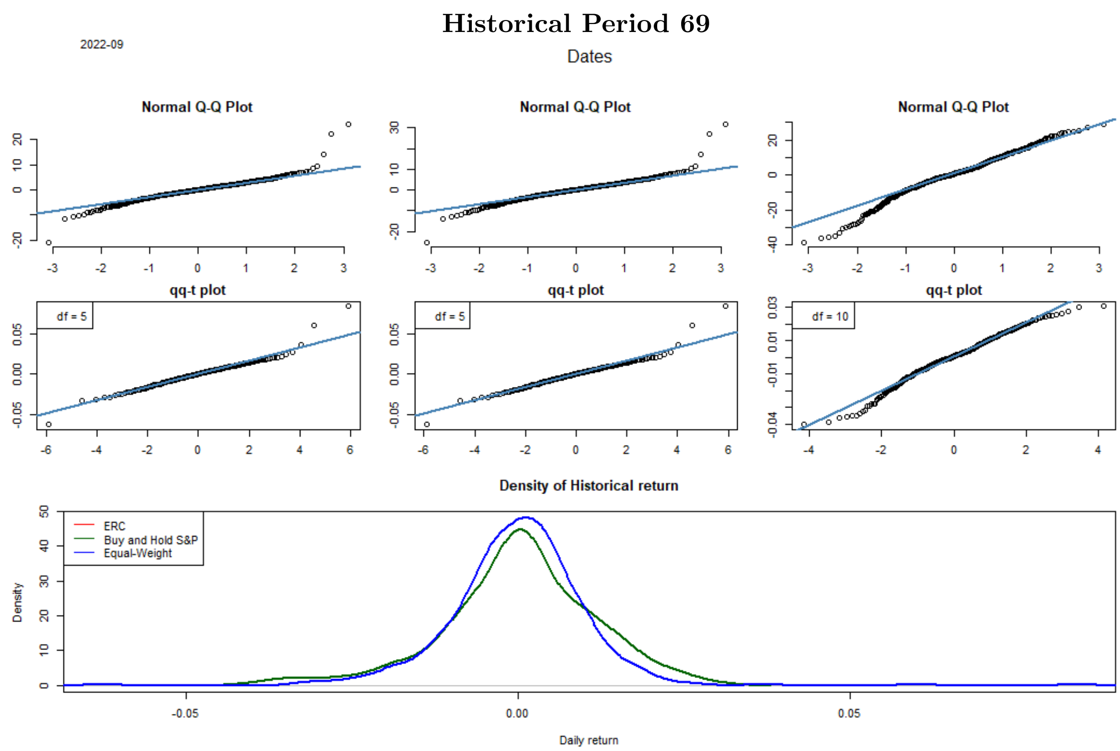

Historical Period 64
2022-04
Dates
Normal Q-Q Plot
Normal Q-Q Plot
Normal Q-Q Plot
qq-t plot
qq-t plot
qq-t plot
df = 8
df = 7
df = 5
Density of Historical return
ERC
Buy and Hold S&P
Equal-Weight
Density
Daily return

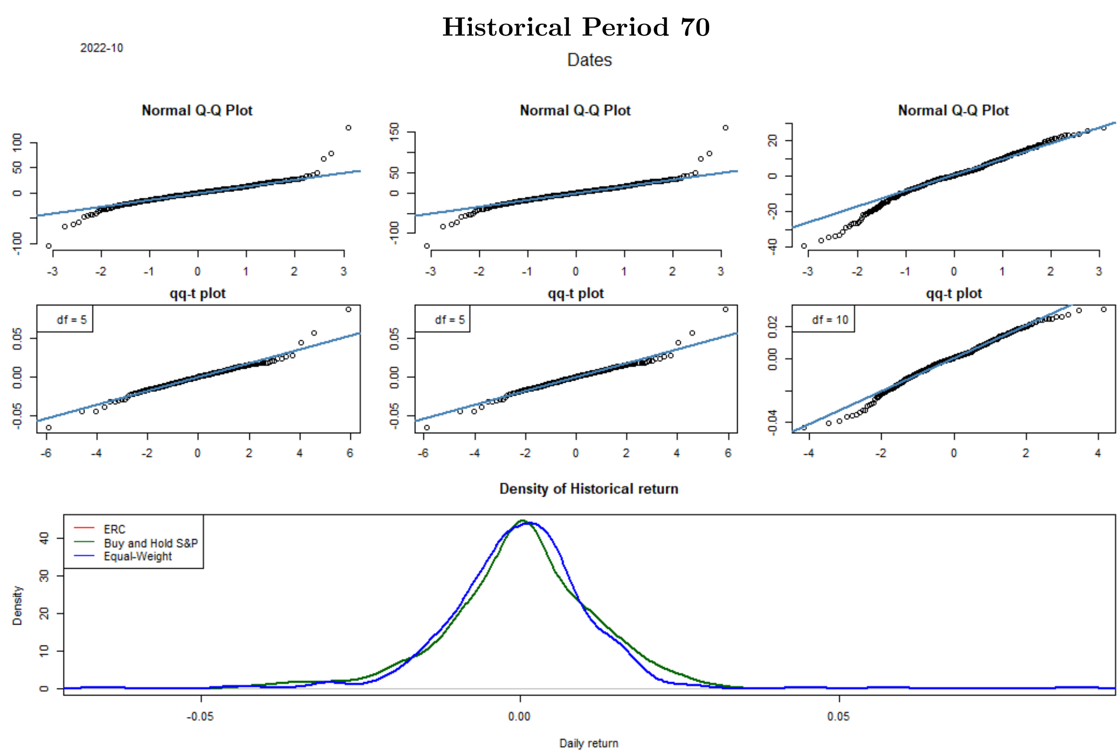

Historical Period 65
2022-05
Dates
Normal Q-Q Plot
Normal Q-Q Plot
Normal Q-Q Plot
qq-t plot
qq-t plot
qq-t plot
df = 5
df = 5
df = 7
Density of Historical return
ERC
Buy and Hold S&P
Equal-Weight
Density
Daily return


Historical Period 66
2022-06
Dates
Normal Q-Q Plot
Normal Q-Q Plot
Normal Q-Q Plot
qq-t plot
qq-t plot
qq-t plot
df = 4
df = 4
df = 7
Density of Historical return
ERC
Buy and Hold S&P
Equal-Weight
Density
Daily return

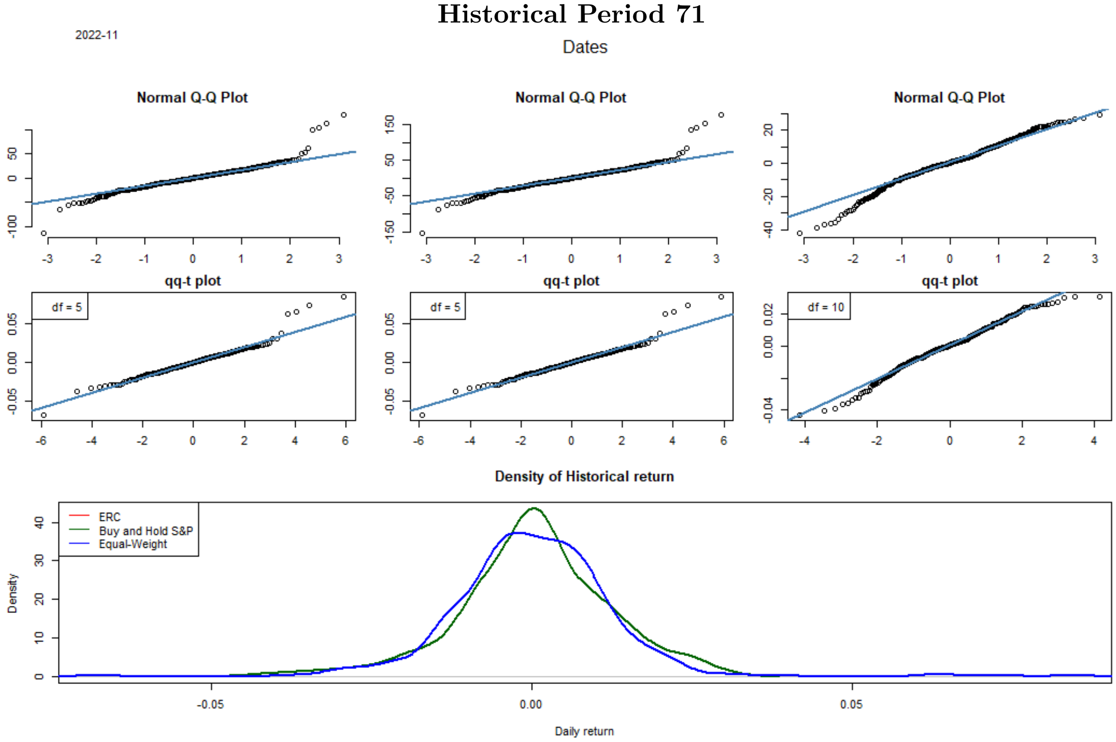

Historical Period 67
2022-07
Dates
Normal Q-Q Plot
Normal Q-Q Plot
Normal Q-Q Plot
qq-t plot
qq-t plot
qq-t plot
df = 5
df = 5
df = 10
Density of Historical return
ERC
Buy and Hold S&P
Equal-Weight
Density
Daily return


Historical Period 68
2022-08
Dates
Normal Q-Q Plot
Normal Q-Q Plot
Normal Q-Q Plot
qq-t plot
qq-t plot
qq-t plot
df = 6
df = 6
df = 10
Density of Historical return
ERC
Buy and Hold S&P
Equal-Weight
Density
Daily return


Historical Period 69
2022-09
Dates
Normal Q-Q Plot
Normal Q-Q Plot
Normal Q-Q Plot
qq-t plot
qq-t plot
qq-t plot
df = 5
df = 5
df = 10
Density of Historical return
ERC
Buy and Hold S&P
Equal-Weight
Density
Daily return


Historical Period 70
2022-10
Dates
Normal Q-Q Plot
Normal Q-Q Plot
Normal Q-Q Plot
qq-t plot
qq-t plot
qq-t plot
df = 5
df = 5
df = 10
Density of Historical return
ERC
Buy and Hold S&P
Equal-Weight
Density
Daily return

Historical Period 71



Historical Period 72