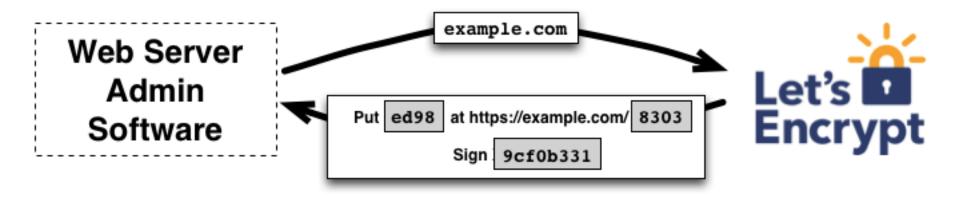
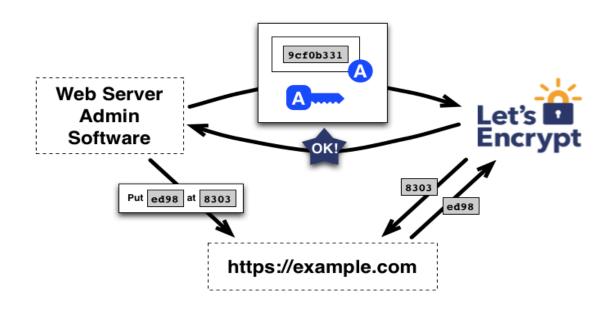
Today in Cryptography (5830)

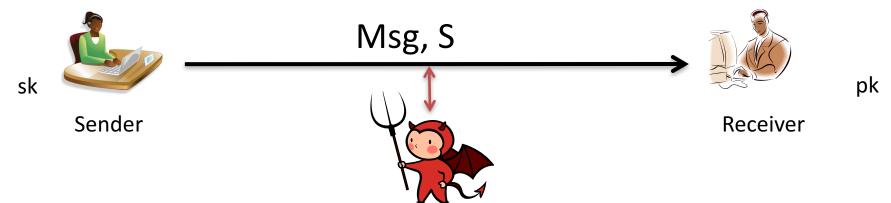
Digital signatures
Schnorr signatures, DSA
Encryption messaging

Free CAs





Digital signatures



Two algorithms:

- (1) Key generation outputs (pk,sk)
- (2) Sign (sk, Msg) outputs a signature S (may be randomized)
- (3) Verify(pk,Msg,S) outputs 0/1 (invalid / valid)

Correctness: Verify(pk,Msg,Sign(sk,Msg)) = 1 always

Security: No computationally efficient attacker can forge signatures for a new message even when attacker gets

$$(Msg_1, S_1), (Msg_2, S_2), ..., (Msg_q, S_q)$$

for messages of his choosing and reasonably large q.

Groups for Schnorr and DSA Signatures

Let p be a large prime number Let q be a prime such that q divides p-1 Example: p = 2q + 1 (so-called safe prime p)

Fix the group $G = \mathbf{Z}_p^* = \{1,2,3,..., p-1\}$ Let g be generator of sub-group of order q:

 $\{g^0, g^1, g^2, ..., g^{q-1}\}\$ subset of G

How to pick g? $g = h^{(p-1)/q} \mod p$ for some h and check $g \ne 1 \mod p$ If so, try repeat with another h. Usually start with h = 2

(Variant of) Schnorr signatures

```
p,q,g specified

sk = x chosen randomly from \mathbf{Z}_q pk = X = g^x
```

```
\frac{Sign(x, M)}{r < -\$ \mathbf{Z}_{q}}
R = g^{r} ; c = H(M \mid\mid R) ; z = r + cx \mod q
Return (R,z)
```

```
\frac{\text{Ver}(X, M, (R,z))}{c = H(M \mid \mid R)}
\text{If } g^z = RX^c \text{ then Return 1}
\text{Return 0}
```

Correctness? $g^z = g^{r+cx} = g^r g^{xc} = RX^c$

Security intuition



Assume an adversary that can output forgery (M,(R,z))

Then to be valid:

$$g^z = RX^c$$
 implies $z = r + cx$

for
$$c = H(M \mid\mid R)$$
.

Assume c is random (H is random oracle)

Imagine we can run adversary twice but force forgery to be on same R, different c.

In second execution, getting (M',(R,z'))

Then success second time around gives:

$$g^{z'} = RX^{c'}$$
 implies $z' = r + c'x$

But now can compute z - z' / (c - c') = x the secret key

Fragility of signatures

Repeat randomness failure:

```
Sign two messages M \neq M' and reuse randomness
Sign(x,M) -> (R,z) = (R, r + cx \mod q)
Sign(x,M') -> (R,z') = (R, r + c'x \mod q)
```

Then:
$$x = (z - z') / (H(M||R) - H(M'||R))$$

If r is predictable/leaked, can recover secret from (R,z)

```
Can improve security by "hedging":
choose r = H(x || M || randomness)
```

Actual Schnorr signatures

```
p,q,g specified

sk = x chosen randomly from \mathbf{Z}_q pk = X = g^x
```

```
\frac{Sign(x, M)}{r < -\$ \mathbf{Z}_{q}}
R = g^{r} ; c = H(M \mid\mid R) ; z = r + cx \mod q
Return (c,z)
```

```
\frac{\text{Ver}(X, M, (c,z))}{R' = g^s X^{-c}}
c' = H(M \mid \mid R')
If c' = c then Return 1
Return 0
```

Correctness?
$$R' = g^s X^{-c} = g^{r + cx} g^{x/(H(M||R))} = g^r$$

DSA (digital signature algorithm)

p,q,g specified sk = x chosen randomly from \mathbf{Z}_a

$$pk = X = g^x$$

Sign(x, M) r <-\$ \mathbf{Z}_q ; R = (g^r mod p) mod q $z = r^{-1}$ (H(M) + x R) mod q Return (R,z) Ver(X, M, (R,z)) $w = z^{-1} \mod q$ $u1 = H(m) * w \mod q$ $u2 = R*w \mod q$ If $R = (g^{u1} X^{u2} \mod p) \mod q$ then Return 1 Else Return 0

Correctness?

$$g^{u1} X^{u2} = g^{H(M)} w g^{x R w} = g^{(H(M)+xR)} w$$

= $g^{(H(M)+xR)} (H(M)+xR)^{-1} r = g^{r}$

Fragility of DSA

Repeat randomness failure:

```
Sign two messages M \neq M' and reuse random

Sign(x,M) -> (R,z) = (R, r^{-1}(H(M) + x R) mod q)

Sign(x,M') -> (R,z') = (R, r^{-1}(H(M') + x R) mod q)
```

Then: Solve for r^{-1} , solve for x

If r is predictable/leaked, can recover secret from (R,z)

```
Again, can improve security by "hedging":
choose r = H(x || M || randomness)
```

Hackers Describe PS3 Security As Epic Fail, Gain Unrestricted Access

BY MIKE BENDEL

DECEMBER 29, 2010 @ 11:19 AM



http://psx-scene.com/forums/content/sony-s-ps3-security-epic-fail-videos-within-581/

Application-layer crypto

- So far focused on TLS as running example
 - Transport Layer Security
 - Provides network socket style stream interface
- What about if an application wants to encrypt discrete messages (as opposed to stream)?
 - Email
 - Text messages
 - Etc.

Email encryption



Sender

 pk_A , sk_A



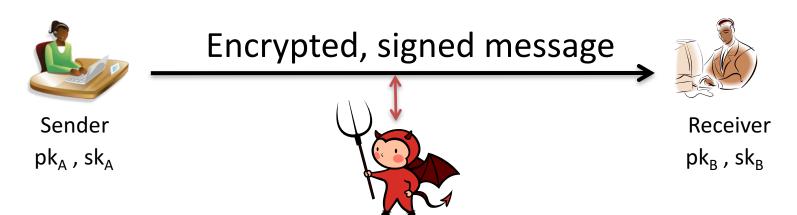


Receiver pk_B, sk_B



- Message may be large (body of email, PDF of attachments)
- Desire authenticity and confidentiality
- Public-keys delivered out-of-band
 - Websites, key parties, key directory servers

Email encryption



How should we design a solution?

ElGamal public-key encryption

g is generator for group of order p Kg outputs $pk = (g,X = g^x)$ and sk = (g,x)

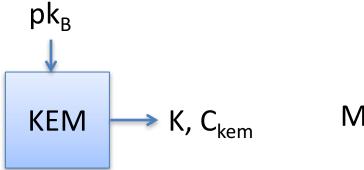
> Enc((g,X), M, R) $r < -\$ \mathbf{Z}_p$ $C1 = g^r$ $C2 = X^r * M$ Return C1, C2

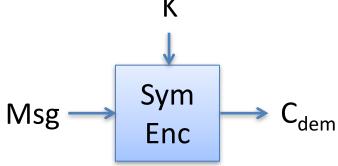
Dec((g,x), C1, C2): Return C2 * C1-x

This is only at most chosen-plaintext attack secure. CCA attacks?

Only encrypts messages of size up to about log p bits

Hybrid encryption (KEM/DEM)





KEM = key encapsulation mechanism Randomized public-key primitive DEM = data encapsulation mechanism
One-time secure authenticated encryption

HybEnc(pk, M)

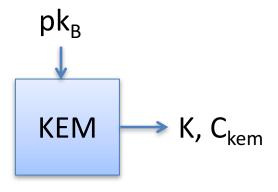
K, C_{kem} <-\$ KEM(pk)

C_{dem} <- Enc(K,M)

Return C_{kem}, C_{dem}

HybDec(sk, C_{kem}, C_{dem_})
K <- KEM⁻¹(sk, C_{kem})
M <- Dec(K, C_{dem})
Return M

KEM from PKE



KEM = key encapsulation mechanism Public-key primitive

KEM(pk)

Choose randomness R C_{kem} <- PKE-Enc(pk,R) Return H(R), C_{kem}

ElGamal KEM

Kg outputs $pk = (g,X = g^x)$ and sk = (g,x)g is generator for group of order prime p

EG-KEM((g,X), R)

 $r = R \mod p$

 $C_{kem} = g^r$

 $K = X^r$

Return H(K), C_{kem}

 $\underline{Dec((g,x), C_{kem})}$:

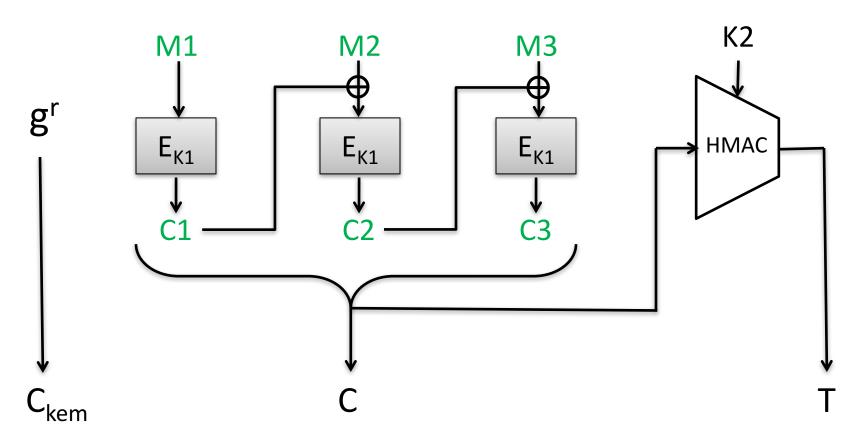
Return $H(C_{kem}^{x})$

Secure if computational Diffie-Hellman assumption holds in group

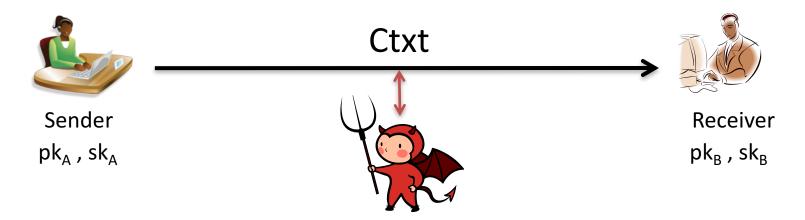
Example hybrid encryption

Enc(X,M):

$$K1 \mid \mid K2 = SHA256(g^{xr})$$



Email encryption



- To digitally sign, let M = Msg | | Sign(sk_A, Msg)
- Ctxt = Encrypt(pk_B, M)

PGP history

 Phil Zimmerman released "Pretty Good Privacy" in 1991 on a USENET post marked as "US only"

- 1993: Criminal investigation by US government for munitions export without a license.
 - Printed PGP source code into a book. First amendment gambit

OpenPGP overview

- Standard for PGP is RFC 4880
- Key encapsulation mechanism:
 - RSA PKCS#1 v1.5 encryption
 - ElGamal over finite field or elliptic curve
- Digital signatures:
 - RSA PKCS#1 v1.5 signatures
 - DSA
- Symmetric encryption:
 - Password-based key derivations using iterated hashing
 - CFB mode using block cipher (variant of CBC mode)

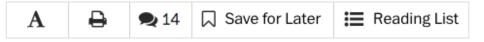
OpenPGP overview

- Security problems:
 - Padding oracle attacks against CFB & PKCS#1 v1.5
 - Attacks against home-brewed integrity checks (modification detection check, MDC)
 - Subject lines always in the clear
- Usability problems:
 - Users must manage their own keys
 - Copying private keys to each device
 - Checking validity of other recipient's public key



The Switch

Yahoo's plan to get Mail users to encrypt their email: Make it simple



Messaging encryption



Sender pk_A, sk_A





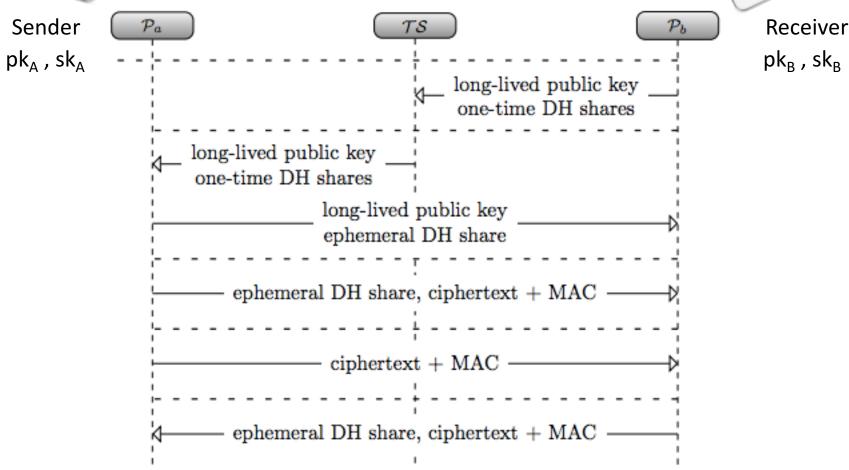
Receiver pk_B , sk_B

- End-to-end encrypted messaging is a big topic
- TextSecure is protocol adopted by WhatsApp (~1 billion users)

TextSecure



Encrypted/Signed SMS or IM



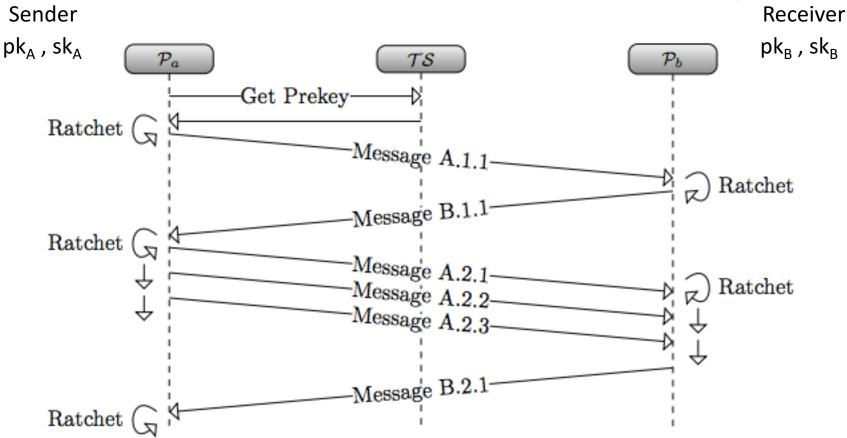
https://eprint.iacr.org/2014/904.pdf

TextSecure



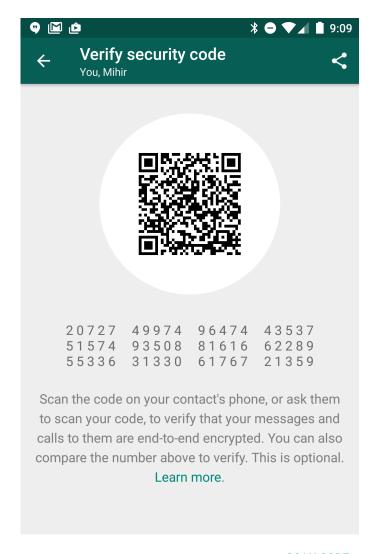
Encrypted/Signed SMS or IM





https://eprint.iacr.org/2014/904.pdf

Verifying public keys



SCAN CODE





Summary

- Schnorr and DSA allow discrete-log based digital signatures, but are fragile without hedging
- Hybrid encryption uses combination of asymmetric and symmetric cryptography
 - Key encapsulation mechanisms (KEM) based on secure PKE, (elliptic curve) Diffie-Hellman
 - Use an authenticated encryption scheme for data encapsulation mechanism (DEM)
- PGP is historical example (and still somewhat widely used)
- End-to-end messaging for IM, chat hotter topic, now widely deployed