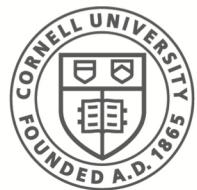


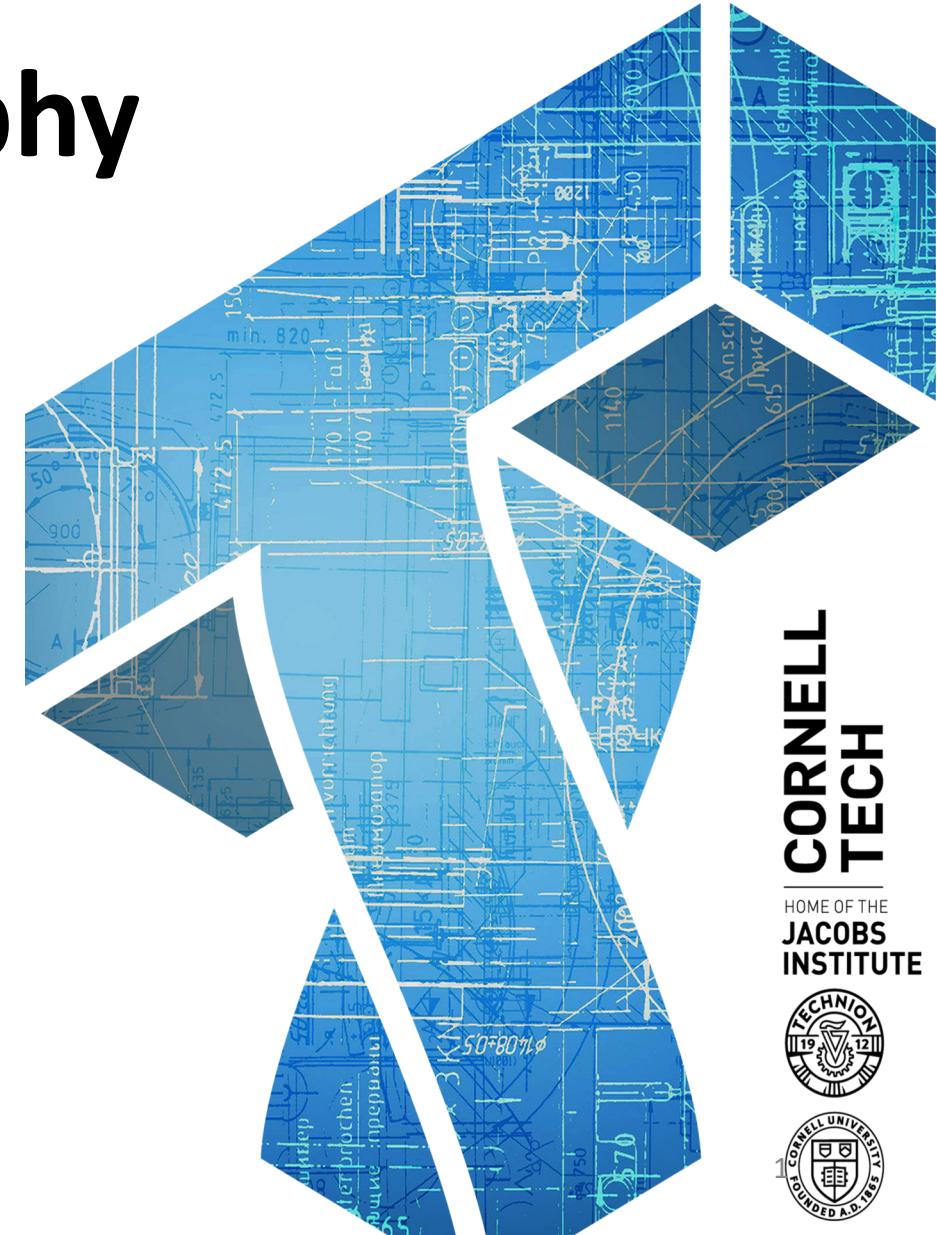
# CS 6831:

# Designing Secure Cryptography

Tom Ristenpart



Cornell CIS  
**Computer Science**



**CORNELL  
TECH**

HOME OF THE  
**JACOBS  
INSTITUTE**



# Cryptography: “Hidden writing”

- Study and practice of building security protocols that resist adversarial behavior
- Blend of mathematics, engineering, computer science

# Crypto is foundational

Failures highlight dependency on good crypto:

- WWII
- Government sabotage
- Playstation 3 crack
- Wireless keys for cars (Tesla)
- Password cracking
- WEP attacks
- Many TLS vulnerabilities
- Stuxnet malware attack  
on Iranian nuclear program
- ...



# This class

- It is really, really, really hard to get crypto right
- Need principled approaches to help us  
*design secure cryptography*
- Approaches themselves are fascinating

# A bit of history

## Classical cryptography

1940's Shannon "Communication Theory of Secrecy Systems"

1970's digital asymmetric cryptography & symmetric cryptography  
(Diffie-Hellman, RSA, DES)

1980's computational complexity security analyses (Micali-Goldwasser)  
symbolic analyses approaches (Dolev-Yao)

1990's concrete security analyses (Bellare-Rogaway)



# Gaining confidence in security

Design A

↓ break

Provably secure

Formal definition of  
security

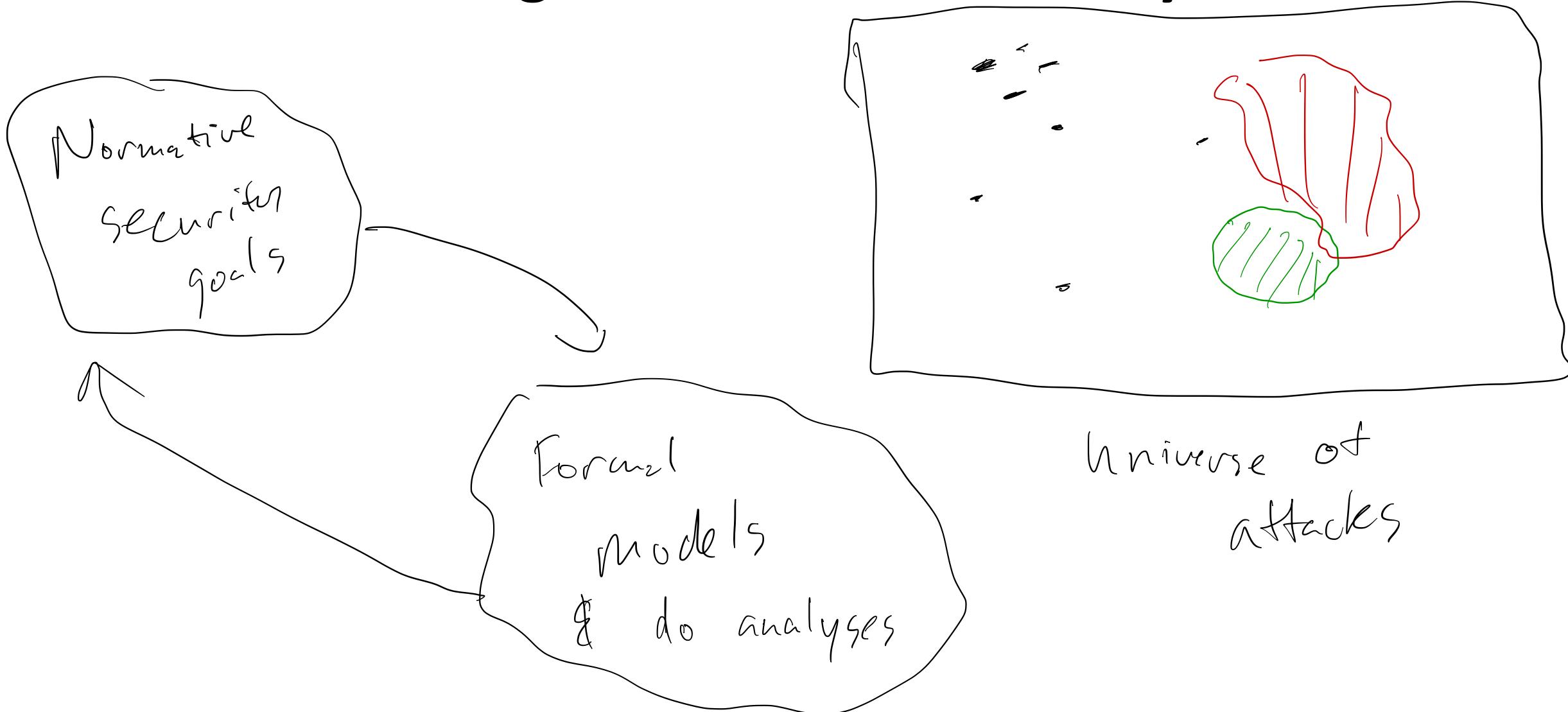
Design B

↓ break

Proof that Design A

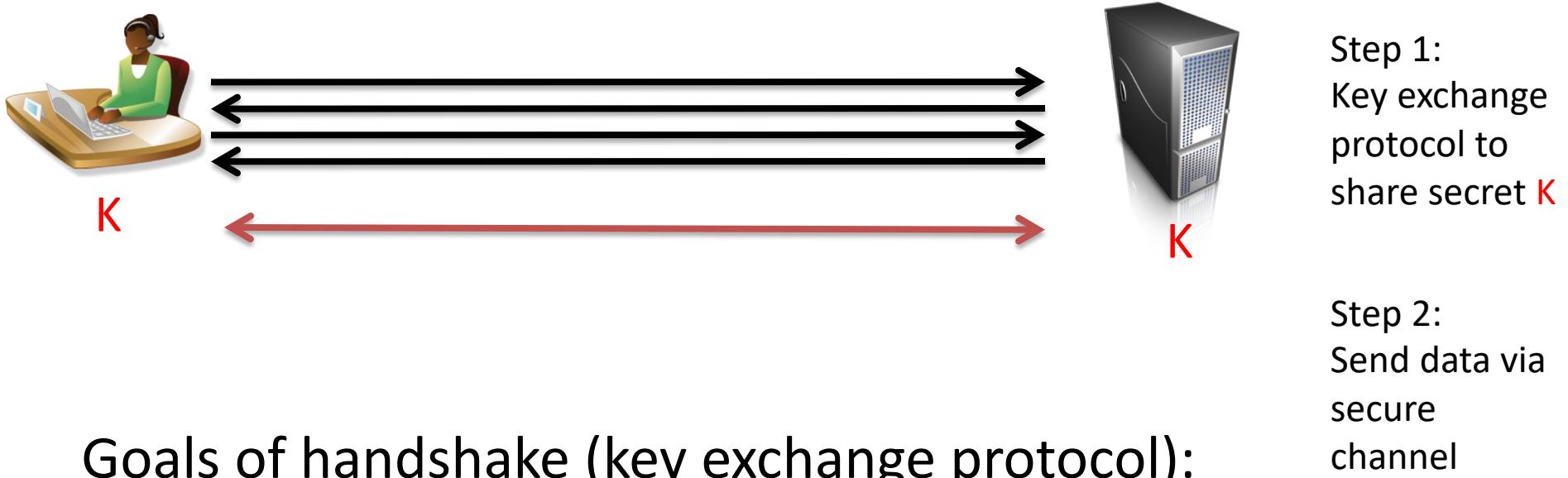
meets it

# Gaining confidence in security



# How TLS works (high level view)

<https://amazon.com>

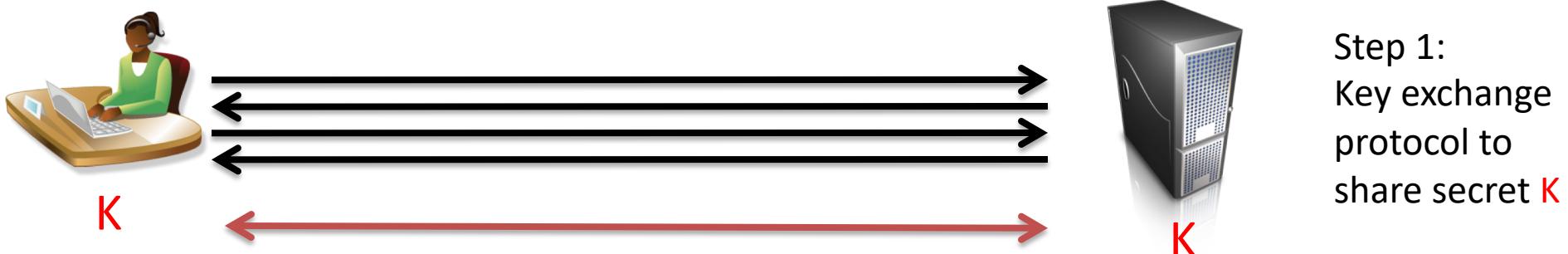


## Goals of handshake (key exchange protocol):

- Negotiate version
- Negotiate parameters (crypto to use)
- Authenticate server (Is server actually Amazon.com?)
  - Digital signatures and certificates
- Establish shared secret
  - Asymmetric encryption primitives

# How TLS works (high level view)

<https://amazon.com>

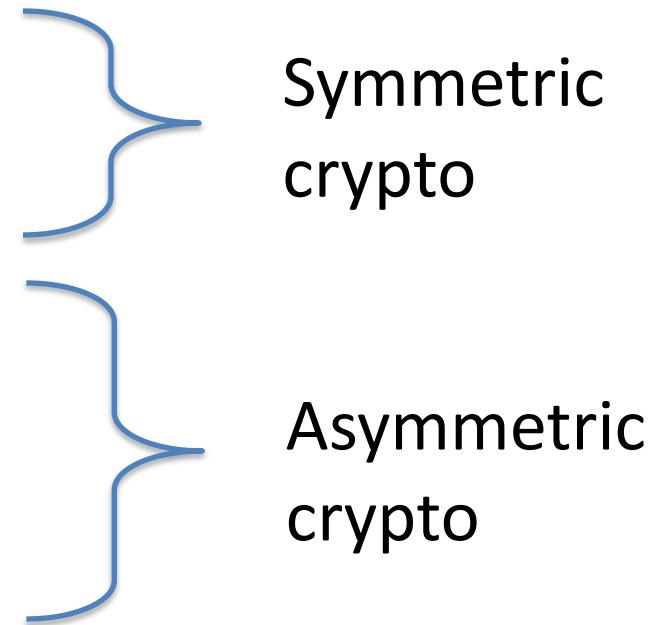


## Goals of secure channel (record layer protocol):

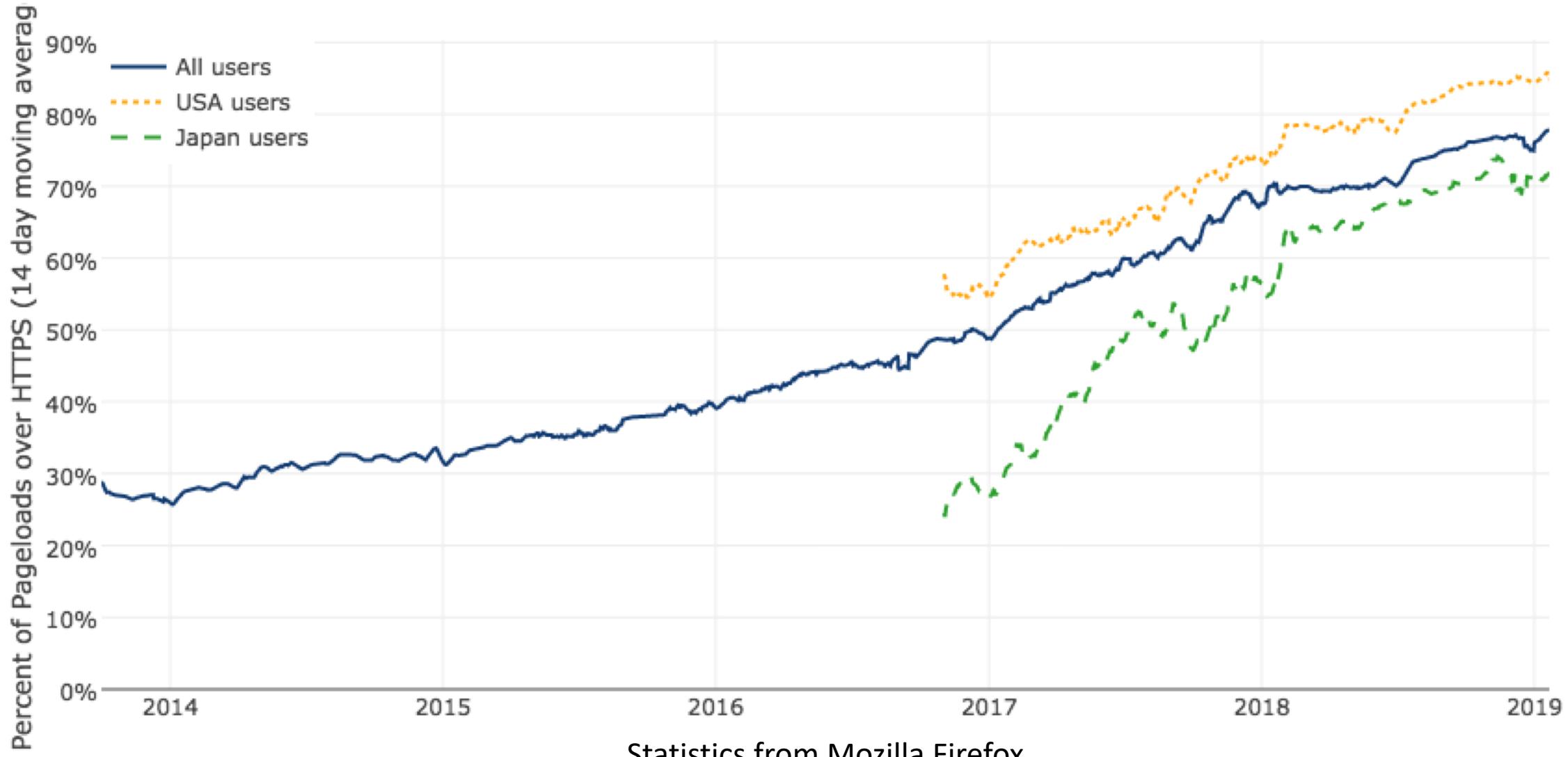
- Confidentiality
  - Only sender/recipient can learn information about plaintext
- Authenticity / integrity
  - Only sender/recipient can generate valid ciphertext

# Crypto primitives used in TLS

- Symmetric encryption
- Cryptographic hashing
- Public-key encryption
- Key-exchange
- Digital signatures



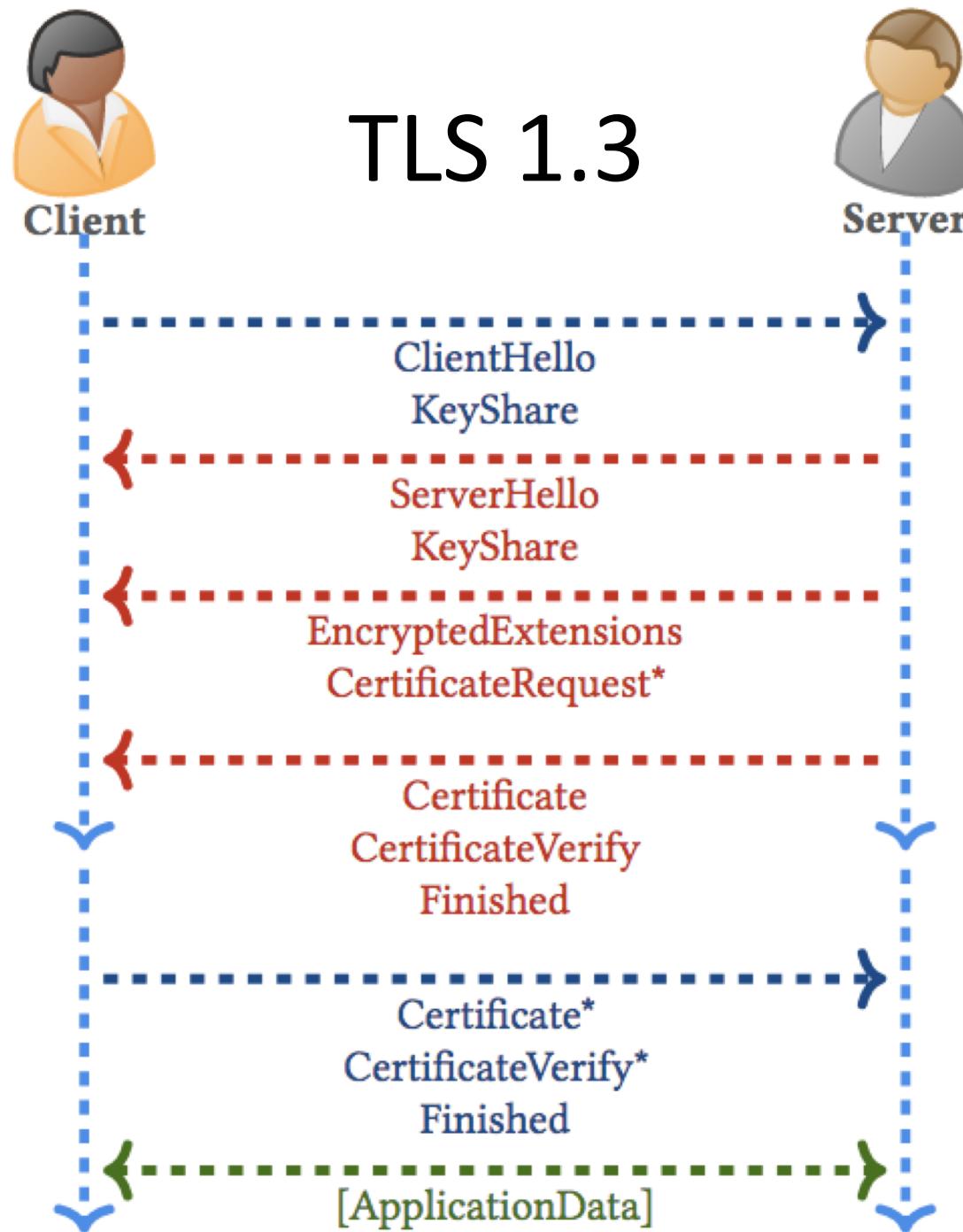
# % of Firefox pageloads using HTTPS



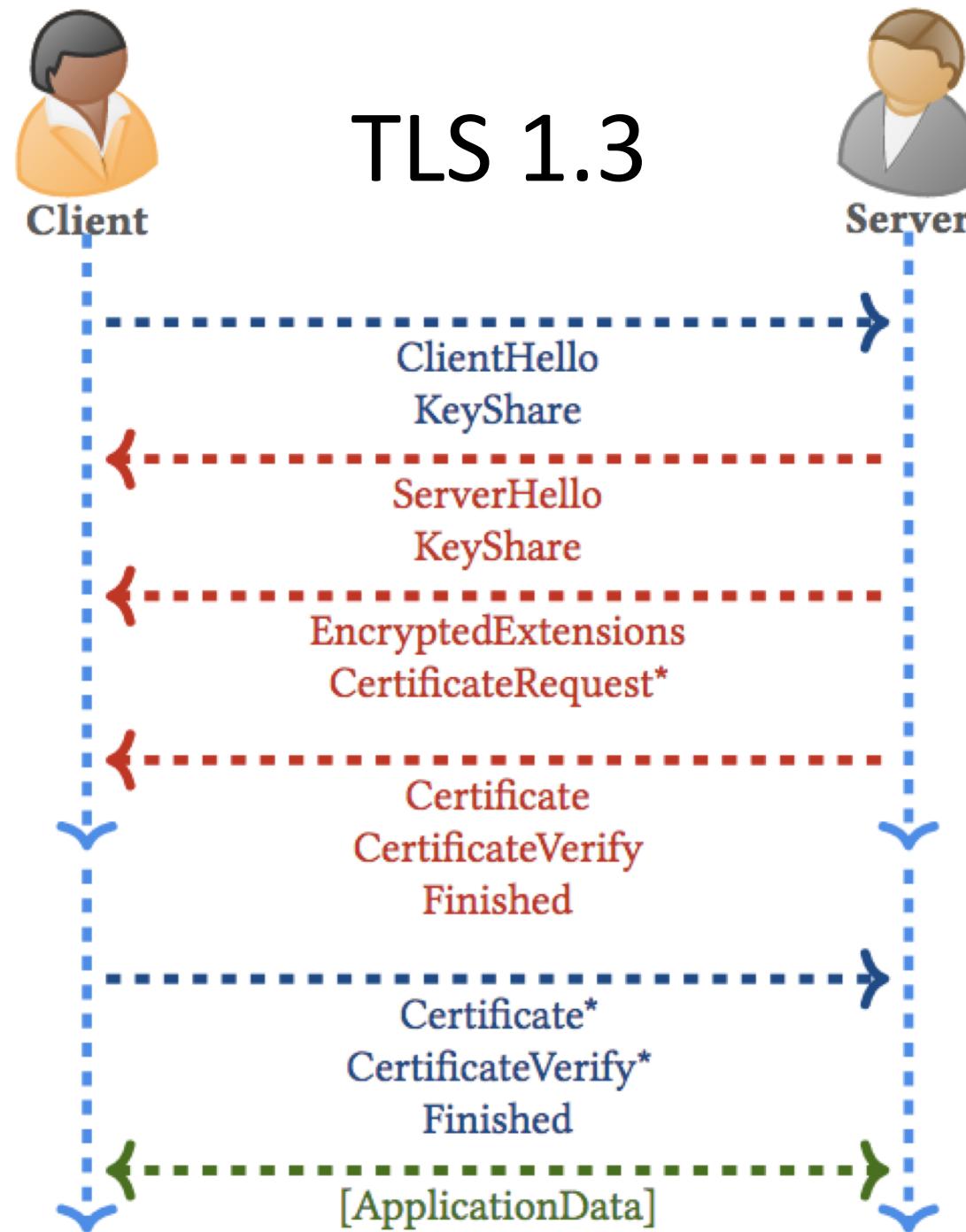
Statistics from Mozilla Firefox  
<https://letsencrypt.org/stats/>

# A brief history of TLS

1994-1996	SSL 1.0, 2.0, 3.0	1996 Bleichanbacher RSA attack 2002 Vaudeny CBC padding oracle 2002 Rogaway IV-reuse theory vuln
1999	TLS 1.0	2011 BEAST attack
2006	TLS 1.1	2012 CRIME/BREACH compression attack
2008	TLS 1.2	2013 RC4 attacks
2018	TLS 1.3	2013 Lucky13 CBC padding oracle 2014 FREAK downgrade attack 2015 Logjam downgrade attack ...



From [Cremers et al. 2017]  
<https://tls13tamarin.github.io/TLS13Tamarin/docs/tls13tamarin-draft21.pdf>



From [Cremers et al. 2017]  
<https://tls13tamarin.github.io/TLS13Tamarin/docs/tls13tamarin-draft21.pdf>

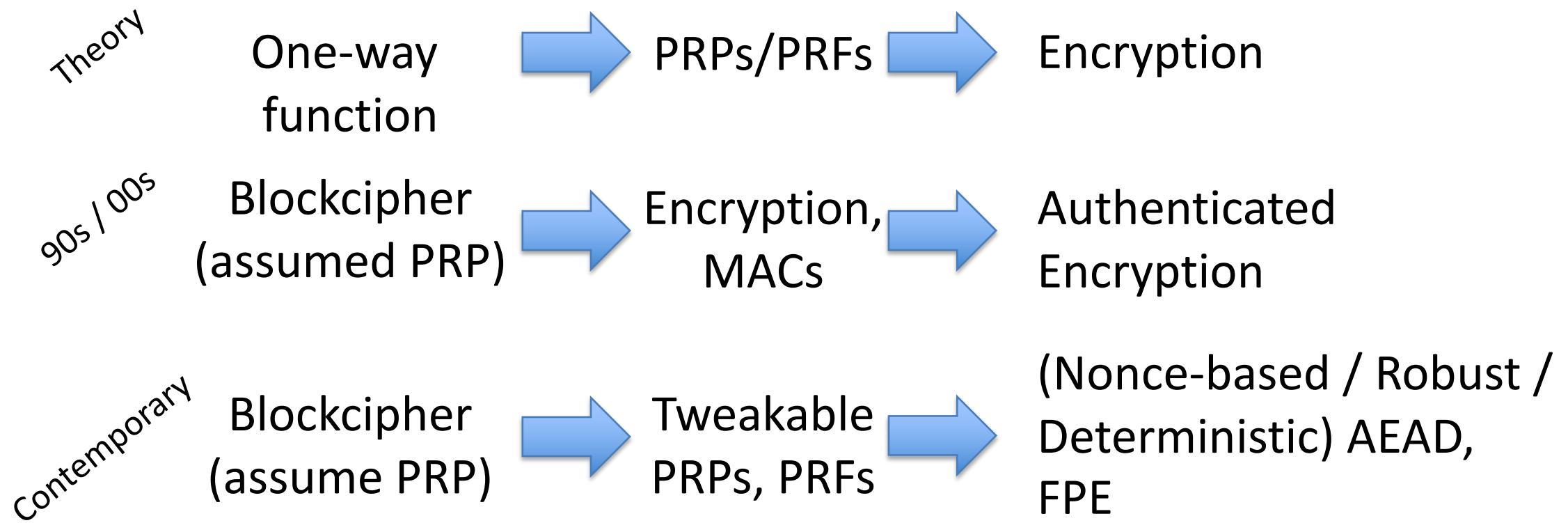
# Process for TLS 1.3

- Unprecedented collaboration
- Almost 100 contributors to RFC 8446
- Measurement
- Symbolic analyses (Dolev-Yao)
- Reductionist security analyses
- Cryptanalytic work

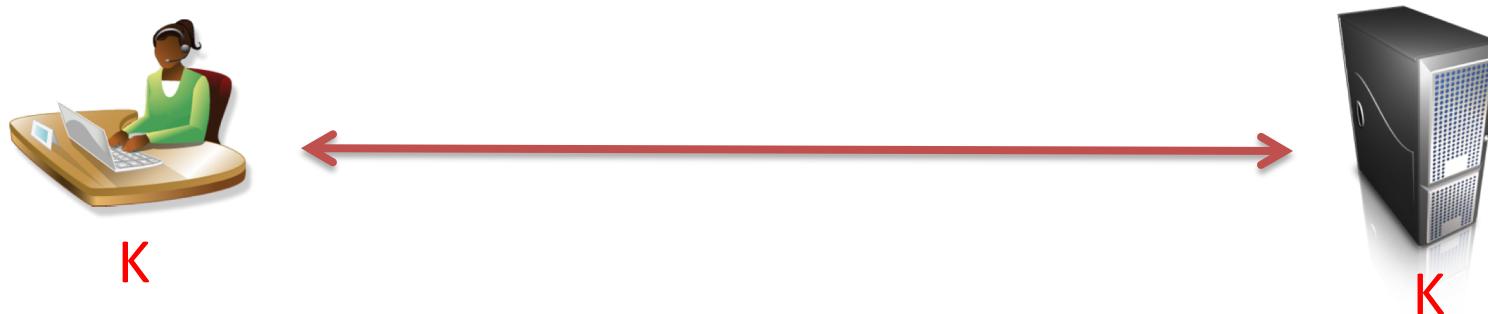
# Rough game plan

- Symmetric encryption
- Asymmetric encryption
- More esoteric stuff, time allowing

# High level views of building symmetric encryption



# Symmetric encryption



- Ciphers, basic definitions and computational viewpoint
- Block ciphers & tweakable block ciphers
  - PRPs / PRFs ; Feistel constructions ; Fast tweakable block ciphers
- Modes of operation, e.g., OCB “core”
- Active security & modern AEAD viewpoints

# Ciphers

$\Sigma = (E, D)$  is a pair of algs.

$E$  is enciphering  $E: K \times M \rightarrow C$

$D$  is deciphering  $D: K \times C \rightarrow M$

Associated to  $\Sigma$ :

$K$  is key space

$M$  is message space

$C$  is ctxt space

Correctness  $\forall m \in M, \forall k \in K$

$$D(k, E(k, m)) = m$$

$$E_k(m) = E(k, m)$$

$$D_k(c) = D(k, c)$$

# Ciphers

Simple OTP cipher.  $K = M = C = \{0, 1\}^n$

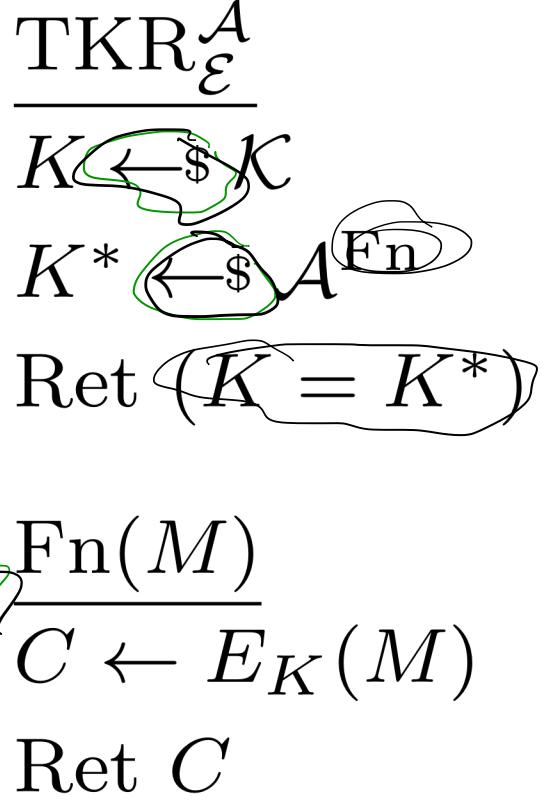
$$E(K, m) = m \oplus K$$

$$D(K, c) = c \oplus K$$

- (1) keys should remain secret
- (2) message confidentiality maintained

Identity cipher

# Target key recovery security



Chosen  
plaintext  
attack

$$\frac{\overbrace{A^{Fn}}{C \leftarrow Fn(0^n)} \leftarrow \text{Ret } C}{\overbrace{D_K(C) = C}{\Pr_{A \in \mathbb{Z}^n}[\text{Adv}_{\mathcal{E}, ID}^{\text{tkr}}(A) \leq \frac{1}{2^n}]}}$$

$$\text{Adv}_{\mathcal{E}}^{\text{tkr}}(A) = \Pr [ \text{TKR}_{\mathcal{E}}^A \Rightarrow \text{true} ]$$

$$\Pr[K = 1^n] = \frac{1}{2^n}$$





# Key recovery security

$$\text{KR}_{\mathcal{E}}^{\mathcal{A}}$$
$$K \xleftarrow{\$} \mathcal{K}$$
$$K^* \xleftarrow{\$} \mathcal{A}^{\text{Fn}}$$

**win**  $\leftarrow$  true

For  $M \in \mathcal{X}$ :

If  $E_{K^*}(M) \neq E_K(M)$  then

**win**  $\leftarrow$  false

Ret **win**

$$\text{Fn}(M)$$
$$\mathcal{X} \leftarrow \mathcal{X} \cup \{M\}$$
$$C \leftarrow E_K(M)$$

Ret  $C$

$$\mathbf{Adv}_{\mathcal{E}}^{\text{kr}}(\mathcal{A}) = \Pr [ \text{KR}_E^{\mathcal{A}} \Rightarrow \text{true} ]$$





# Comparing security definitions

We can formally compare security definitions

$$\text{DEF1} \not\Rightarrow \text{DEF2}$$

We can show a *counter-example*:

- Scheme such that show that no (reasonable) DEF1-adversary gets good advantage
- We give DEF2-adversary that gets good DEF2 advantage

$$\text{DEF1} \Rightarrow \text{DEF2}$$

We can show a *reduction*:

- Convert DEF2-adversary A into DEF1-adversary B s.t. B's DEF1 advantage upper bounds A's DEF2 advantage



$\text{TKR}_{\mathcal{E}}^{\mathcal{A}}$  $K \leftarrow \$ \mathcal{K}$  $K^* \leftarrow \$ \mathcal{A}^{\text{Fn}}$ 

Ret  $(K = K^*)$

 $\text{Fn}(M)$  $C \leftarrow E_K(M)$ 

Ret  $C$

 $\text{KR}_{\mathcal{E}}^{\mathcal{A}}$  $K \leftarrow \$ \mathcal{K}$  $K^* \leftarrow \$ \mathcal{A}^{\text{Fn}}$ 

**win**  $\leftarrow$  true

For  $M \in \mathcal{X}$ :

If  $E_{K^*}(M) \neq E_K(M)$  then  
 $\text{win} \leftarrow \text{false}$

Ret **win**

 $\text{Fn}(M)$  $\mathcal{X} \leftarrow \mathcal{X} \cup \{M\}$  $C \leftarrow E_K(M)$ 

Ret  $C$

**Theorem 1** Let  $\mathcal{E}$  be a cipher. For any  $\text{TKR}_{\mathcal{E}}$ -adversary  $\mathcal{A}$ , we give a  $\text{KR}_{\mathcal{E}}$ -adversary  $\mathcal{B}$  such that  $\mathbf{Adv}_{\mathcal{E}}^{\text{tkr}}(\mathcal{A}) = \mathbf{Adv}_{\mathcal{E}}^{\text{kr}}(\mathcal{B})$ .

# **Key recovery doesn't imply message privacy**

- Say we have cipher that meets KR definition. Does this seem to imply message privacy?

# Towards message privacy definitions

$$\text{otIND}_{\mathcal{E}}^{\mathcal{A}}$$
$$K \xleftarrow{\$} \mathcal{K}$$
$$b \xleftarrow{\$} \{0, 1\}$$
$$b' \xleftarrow{\$} \mathcal{A}^{\text{Fn}}$$

Ret  $(b = b')$

$$\text{Fn}(M_0, M_1)$$
$$C \leftarrow E_K(M_b)$$

Ret  $C$

$$\mathbf{Adv}_{\mathcal{E}}^{\text{ot-ind}}(\mathcal{A}) = 2 \cdot \Pr \left[ \text{otIND}_E^{\mathcal{A}} \Rightarrow \text{true} \right] - 1$$

# OTP otIND security

**Theorem 1** Let  $\mathcal{E}$  be the OTP cipher. Then for any single-query otIND $_{\mathcal{E}}$ -adversary  $\mathcal{A}$  it holds that  $\text{Adv}_E^{\text{ot-ind}}(\mathcal{A}) = 0$ .



# Shannon's theorem

**Theorem 1** *Let  $\mathcal{E}$  be a cipher defined over  $(\mathcal{K}, \mathcal{M}, \mathcal{C})$  such that for any otIND<sub>E</sub>-adversary  $\mathcal{A}$  it holds that  $\mathbf{Adv}_E^{\text{ot-ind}}(\mathcal{A}) = 0$ . Then  $|\mathcal{K}| \geq |\mathcal{M}|$ .*

# Computational security

- Big paradigm shift is to focus on computationally bound adversaries
- Which adversaries that we've seen so far are computationally efficient?
- How do we measure computational costs?
  - Simple solution: assume abstract unit costs of (most) operations
  - Course-grained but useful

# Summary

- Even for simple primitives, security models have many subtleties
  - Multiple versions of KR
  - Easy to end up with mathematically fine, but security-wise uninteresting definitions
- Can formally compare definitions
- Can informally explore definitions
  - Build scheme that formally meets goal, but normatively insecure

# Logistics

- <https://github.com/cornelltech/CS6831-Spring2019>
- Grading:
  - Participation (20%)
  - Scribe notes (40%)
  - Project (40%)
- Class will involve some paper-reading, and you will get out of it what you put into it in terms of content
- I will set scribing schedule and milestones for project shortly, announce end of the week
  - Please let me know if you (not) enrolling ASAP



