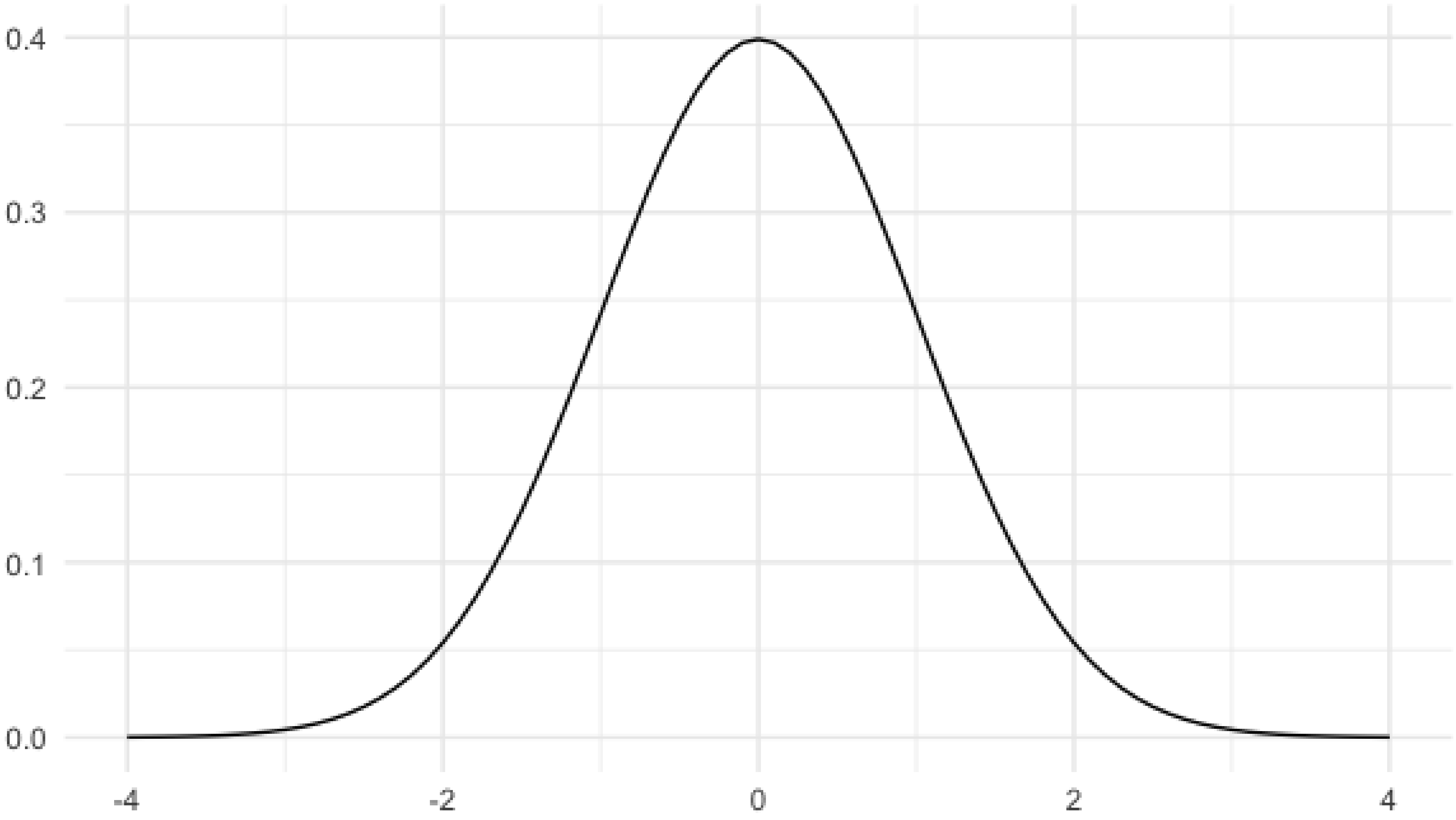
The normal distribution

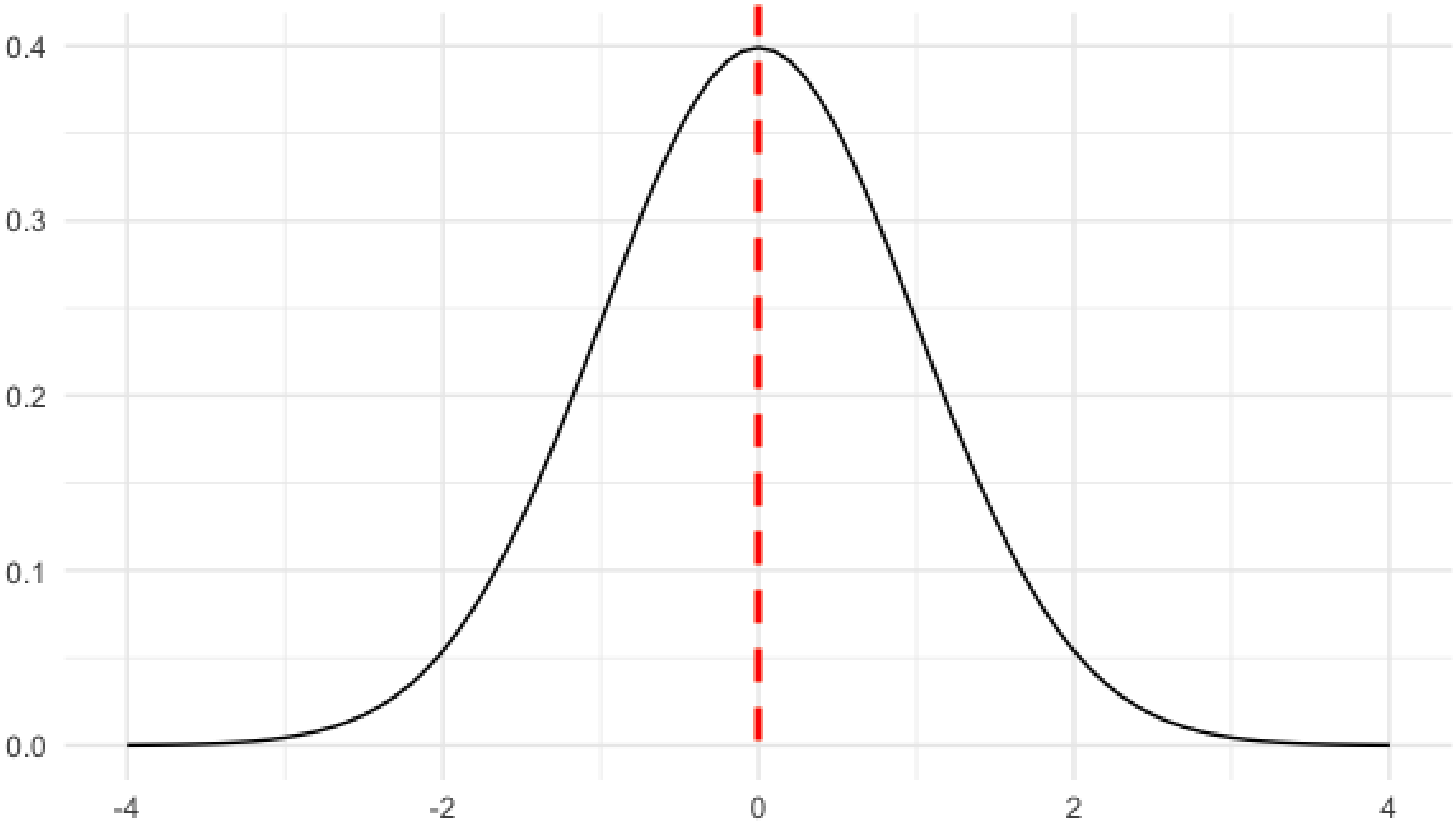
INTRODUCTION TO STATISTICS IN PYTHON

Maggie Matsui
Content Developer, DataCamp

What is the normal distribution? properties of normal distribution

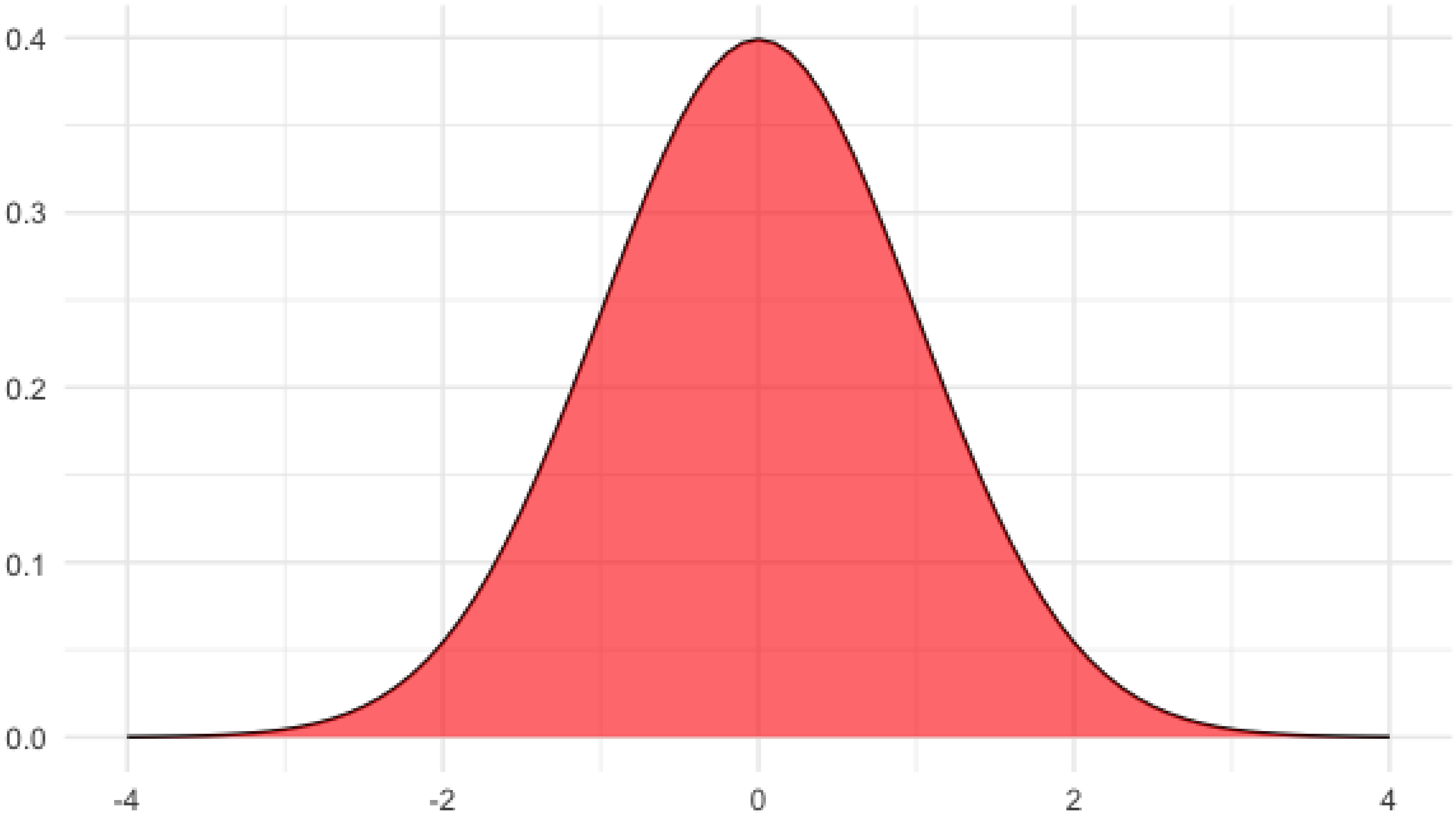


Symmetrical



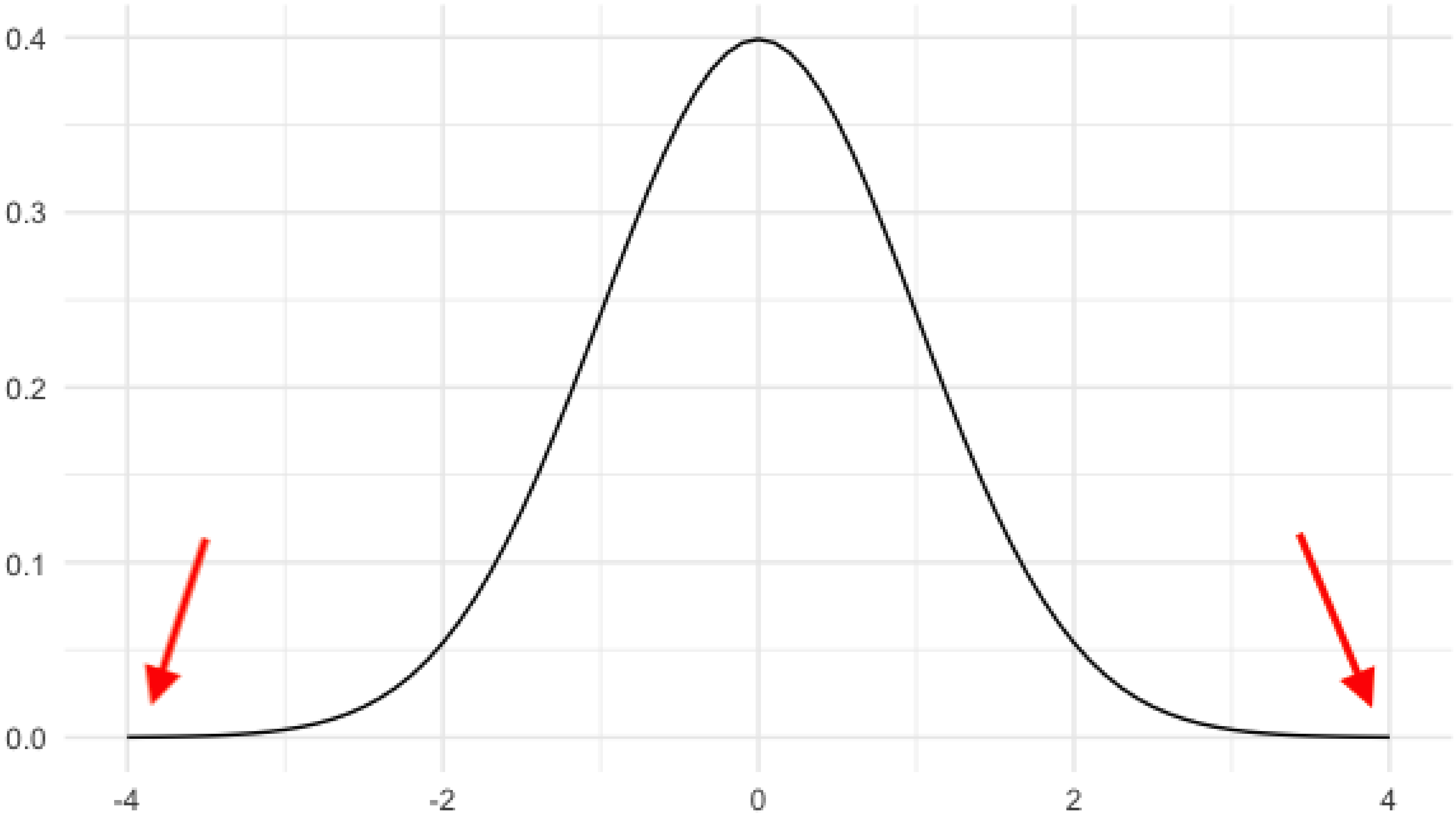
Area = 1

Prop - 0.2



Curve never hits 0

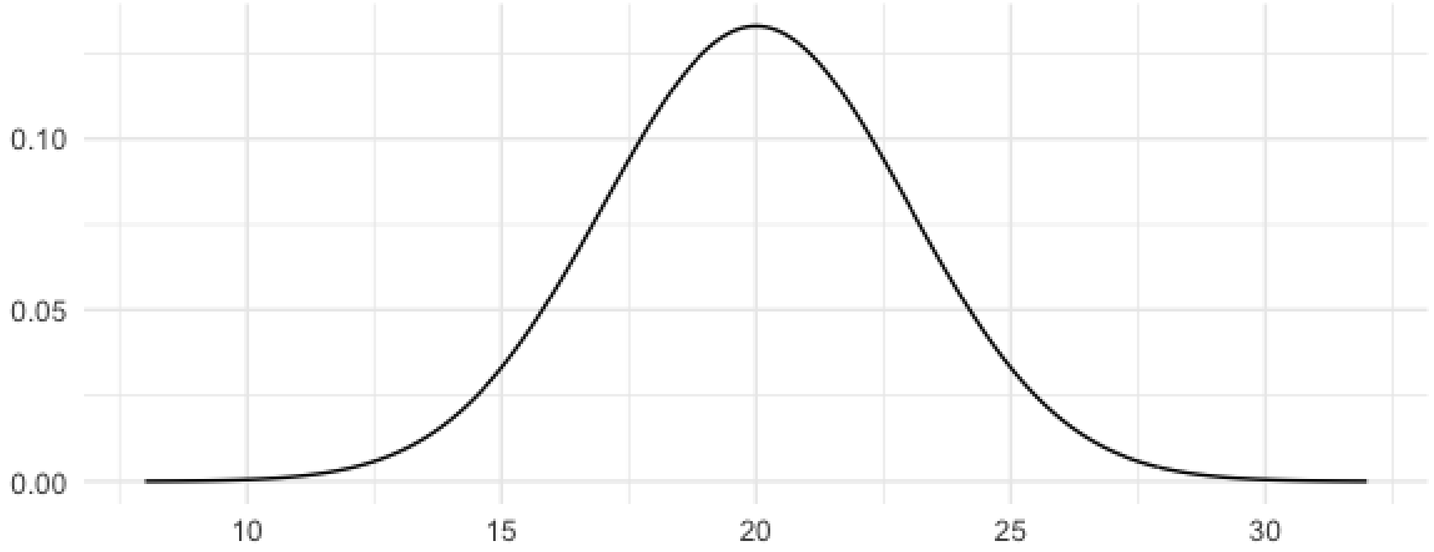
Prop - 03



Described by mean and standard deviation

Mean: 20

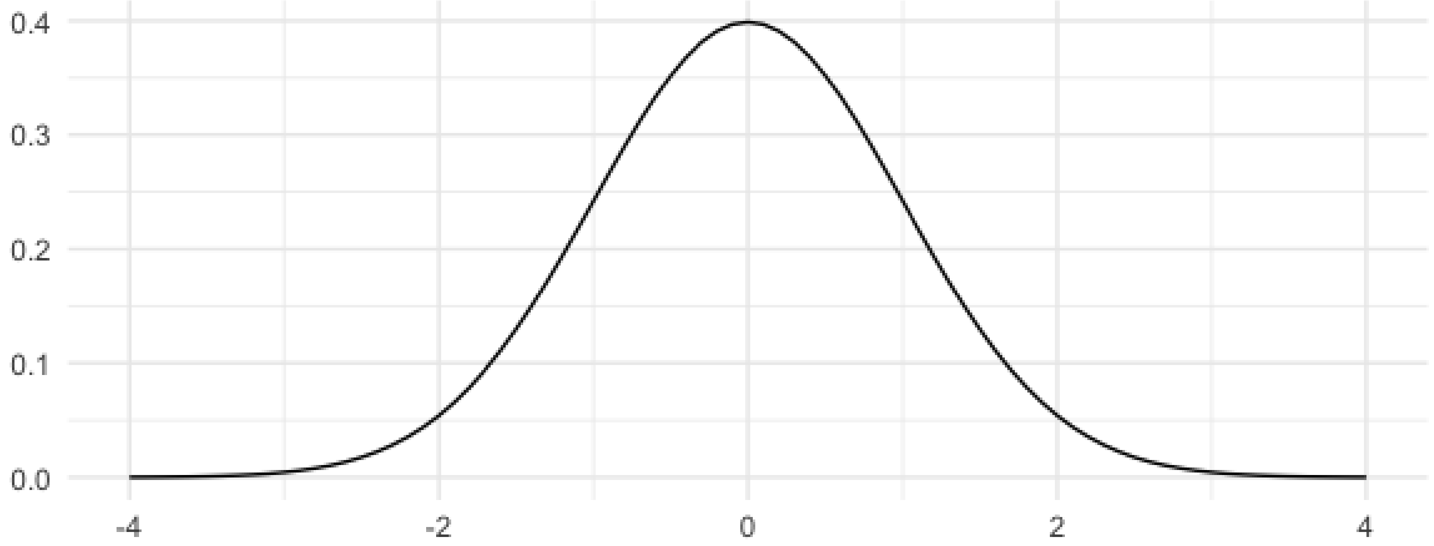
Standard deviation: 3



Standard normal distribution

Mean: 0

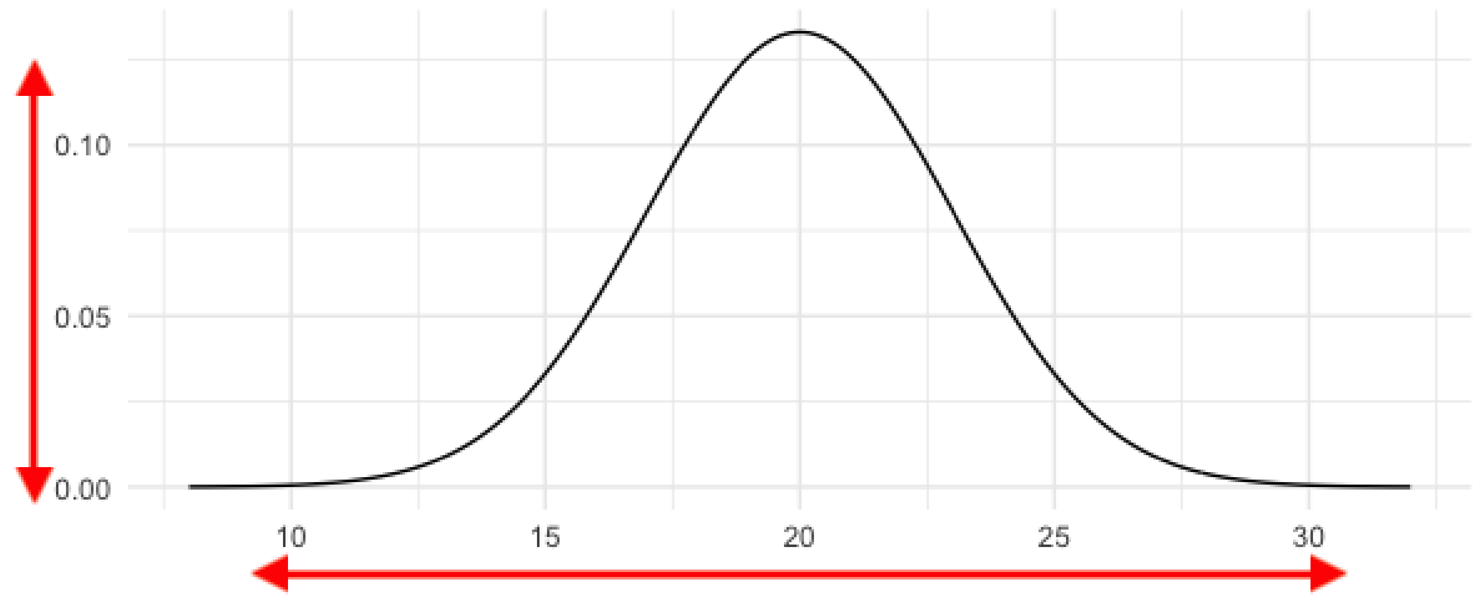
Standard deviation: 1



Described by mean and standard deviation

Mean: 20

Standard deviation: 3



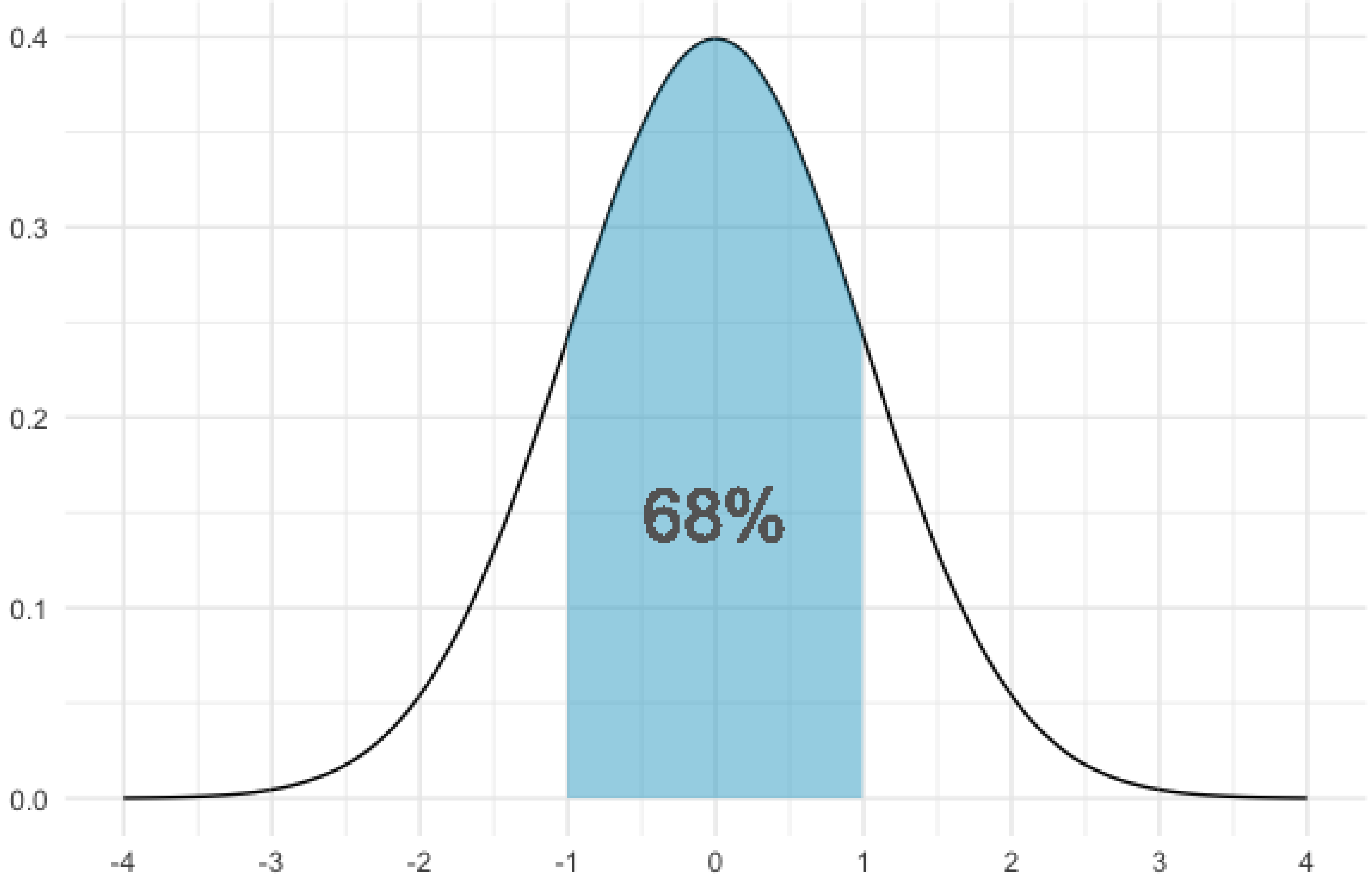
Standard normal distribution

Mean: 0

Standard deviation: 1

Areas under the normal distribution

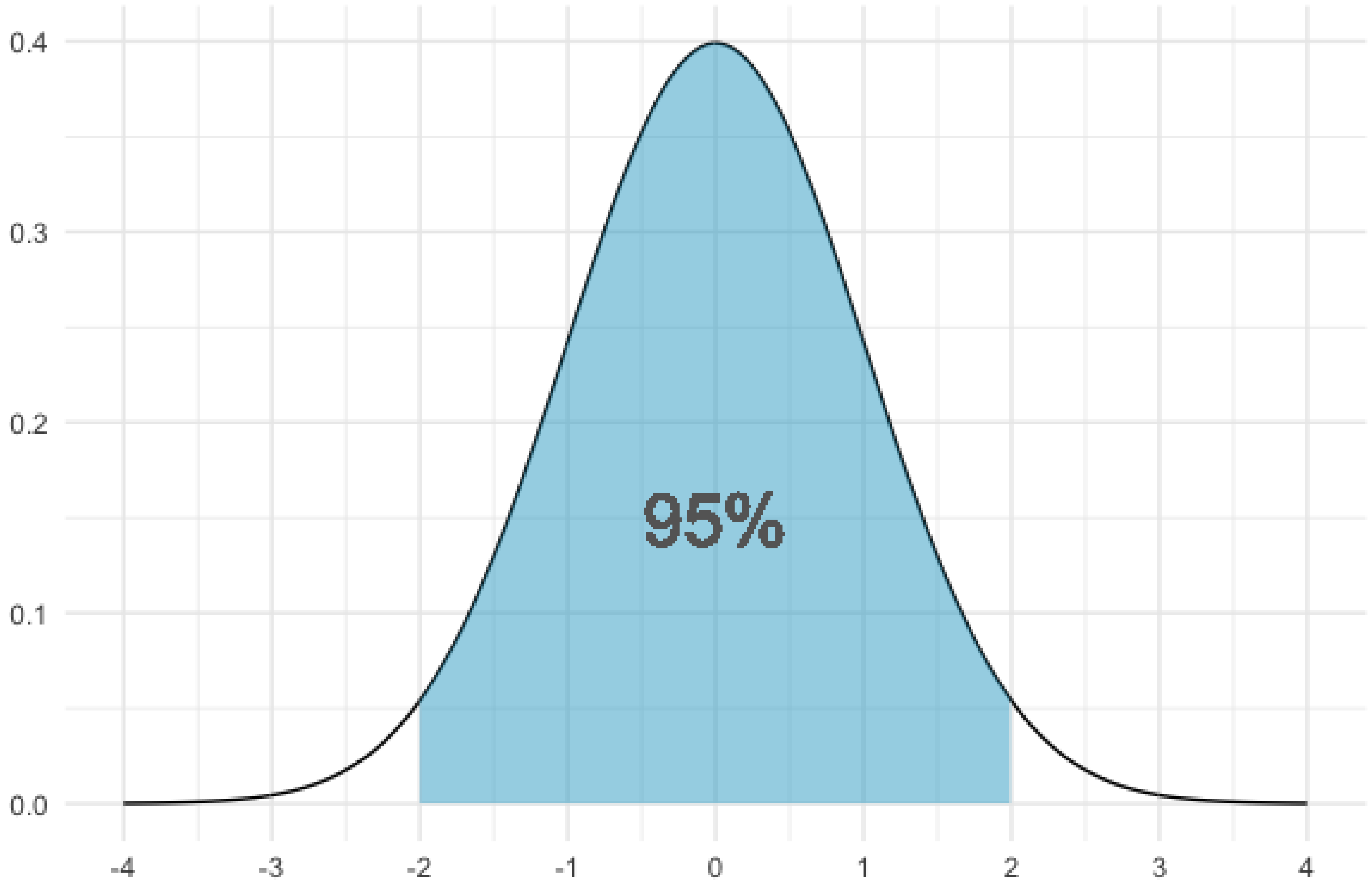
68% falls within 1 standard deviation



Areas under the normal distribution

Prop-4

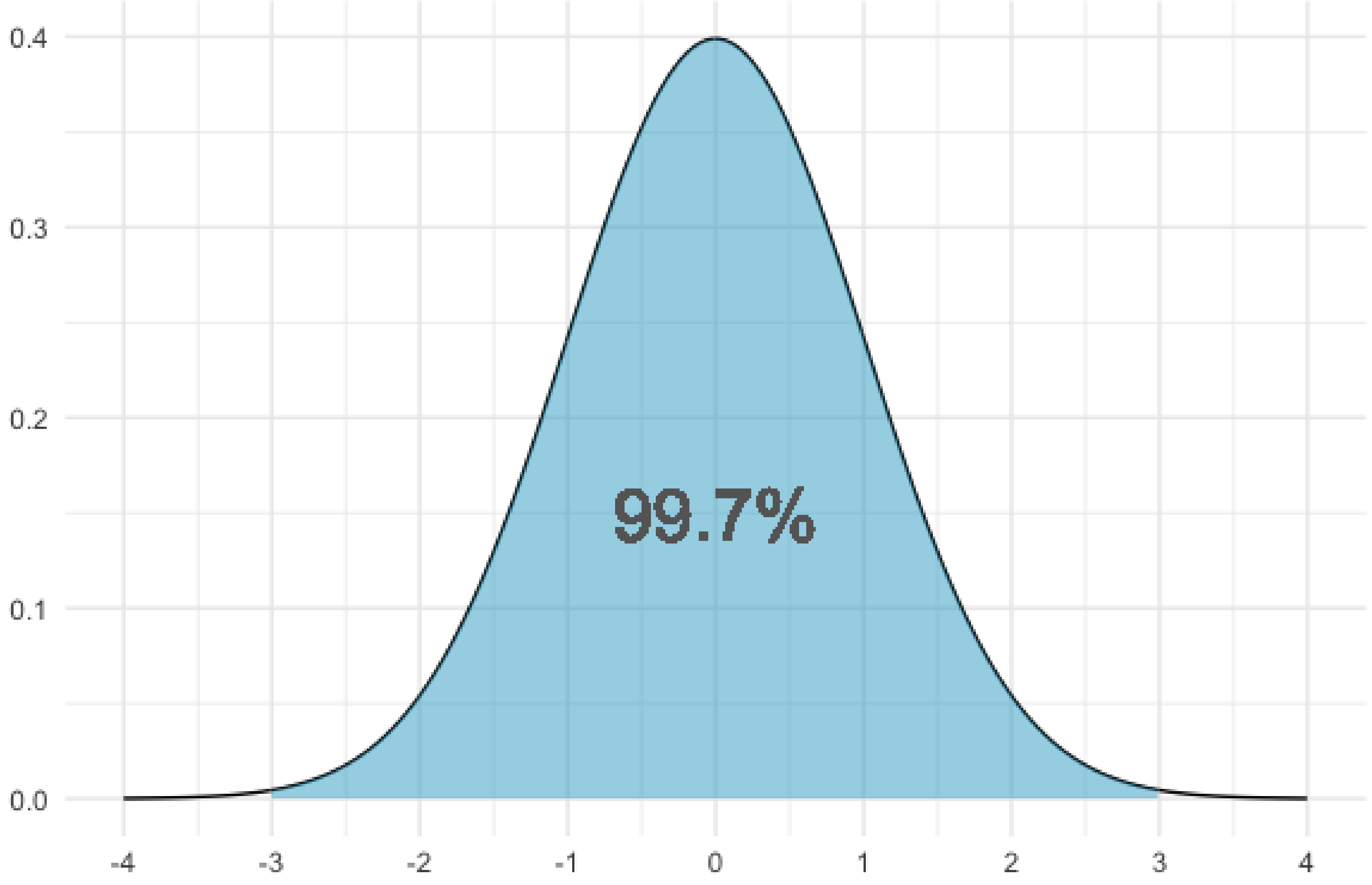
95% falls within 2 standard deviations



Areas under the normal distribution

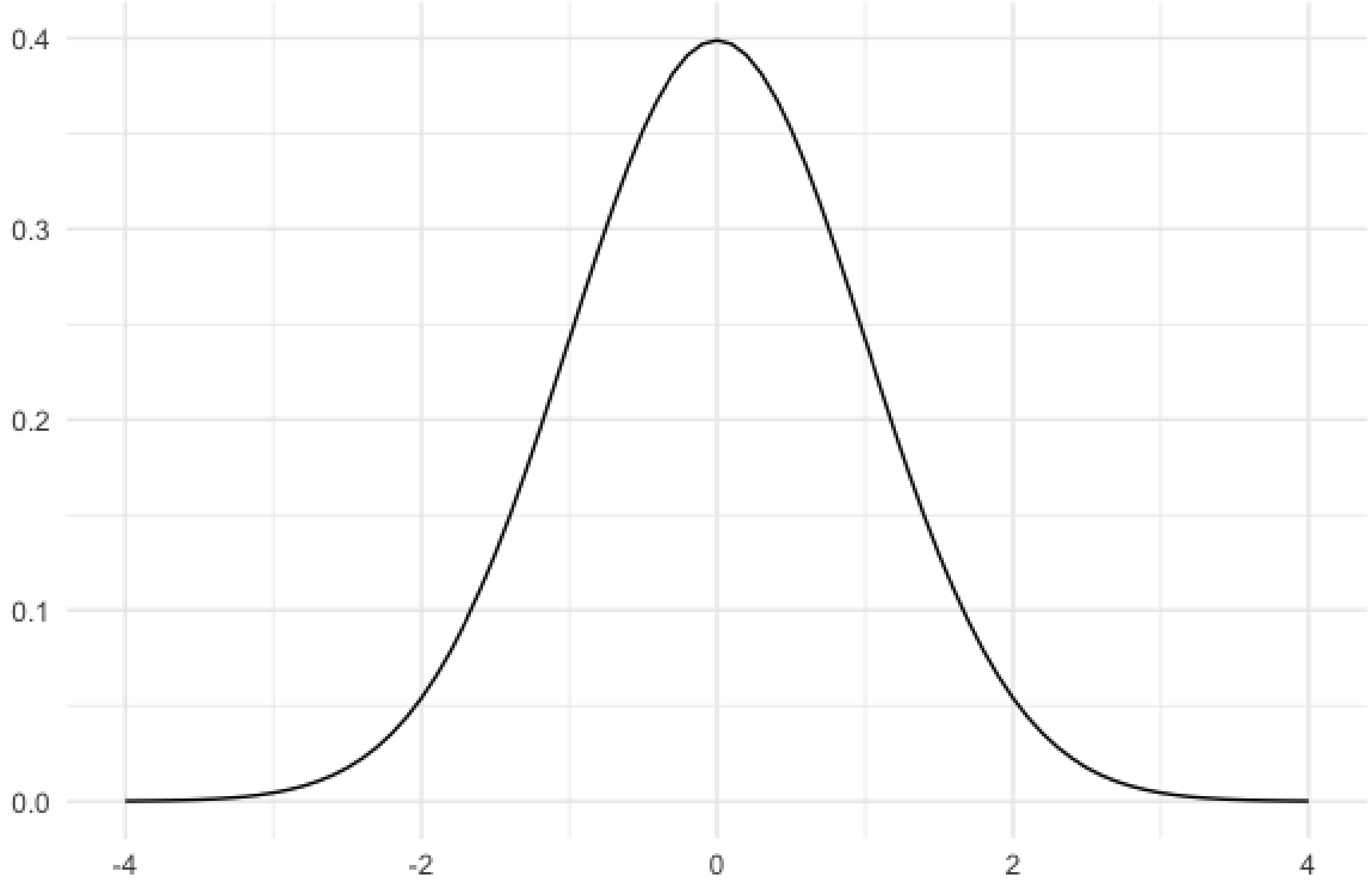
Prop-5

99.7% falls within 3 standard deviations

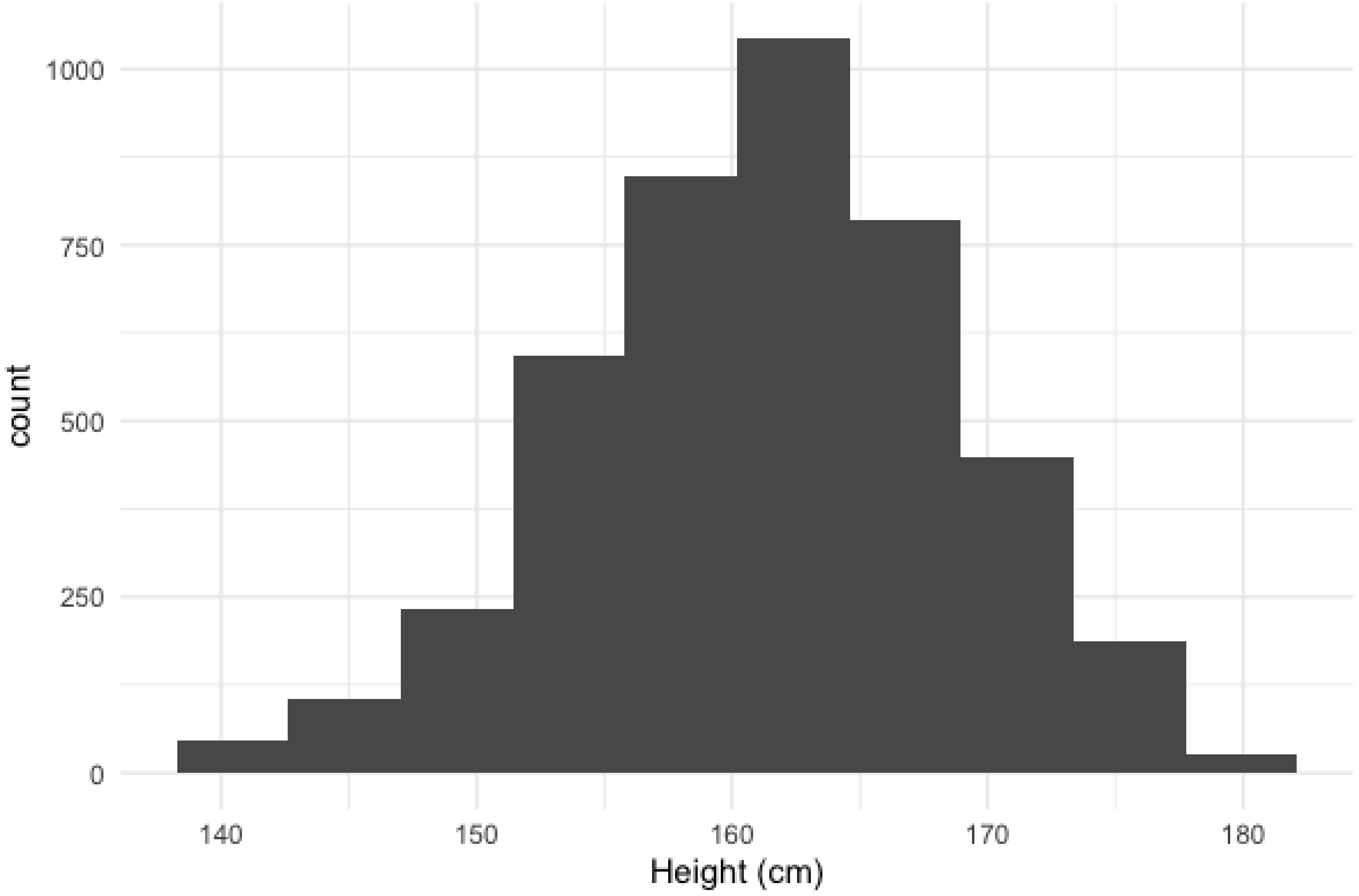


Lots of histograms look normal

Normal distribution

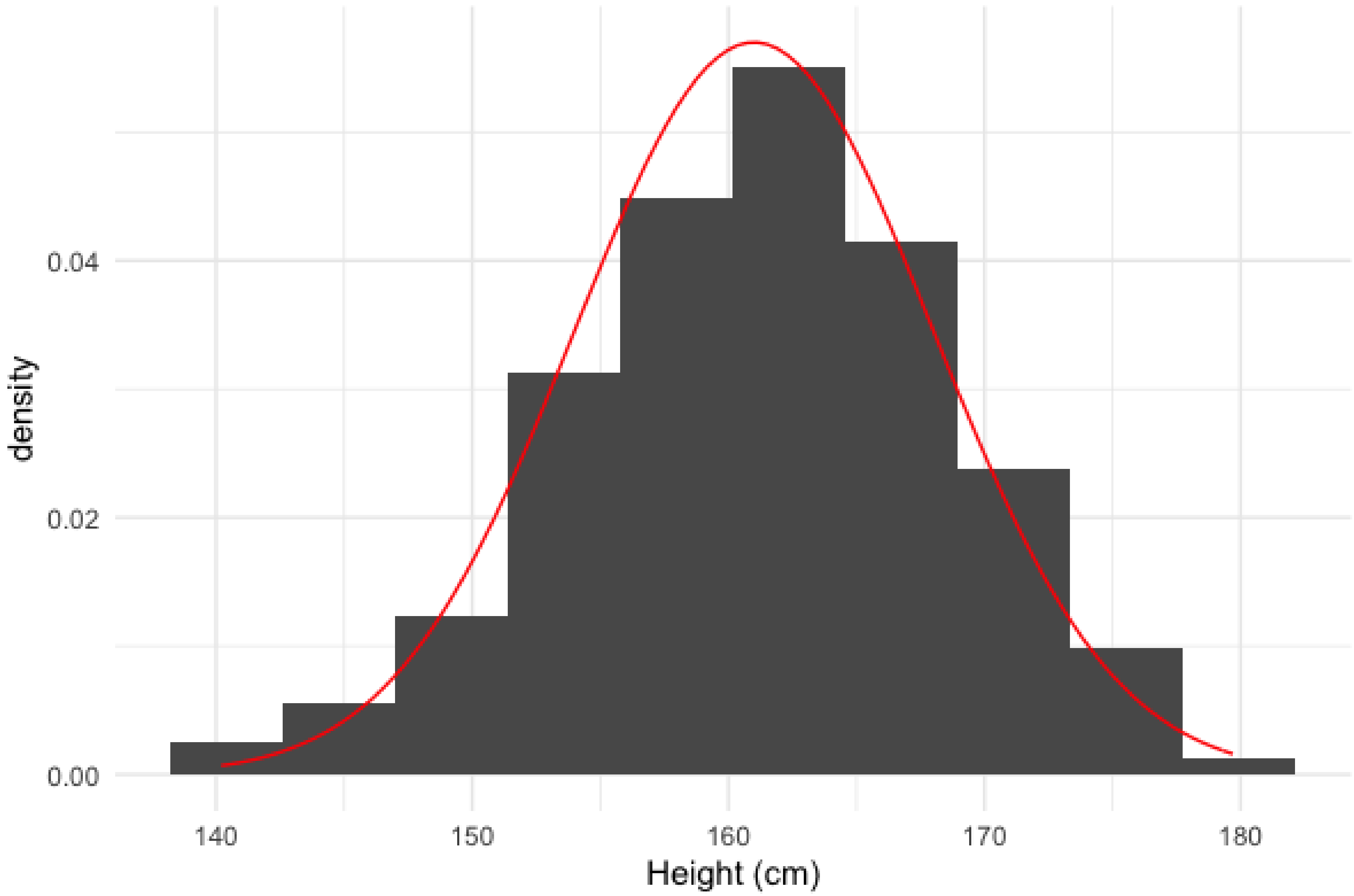


Women's heights from NHANES

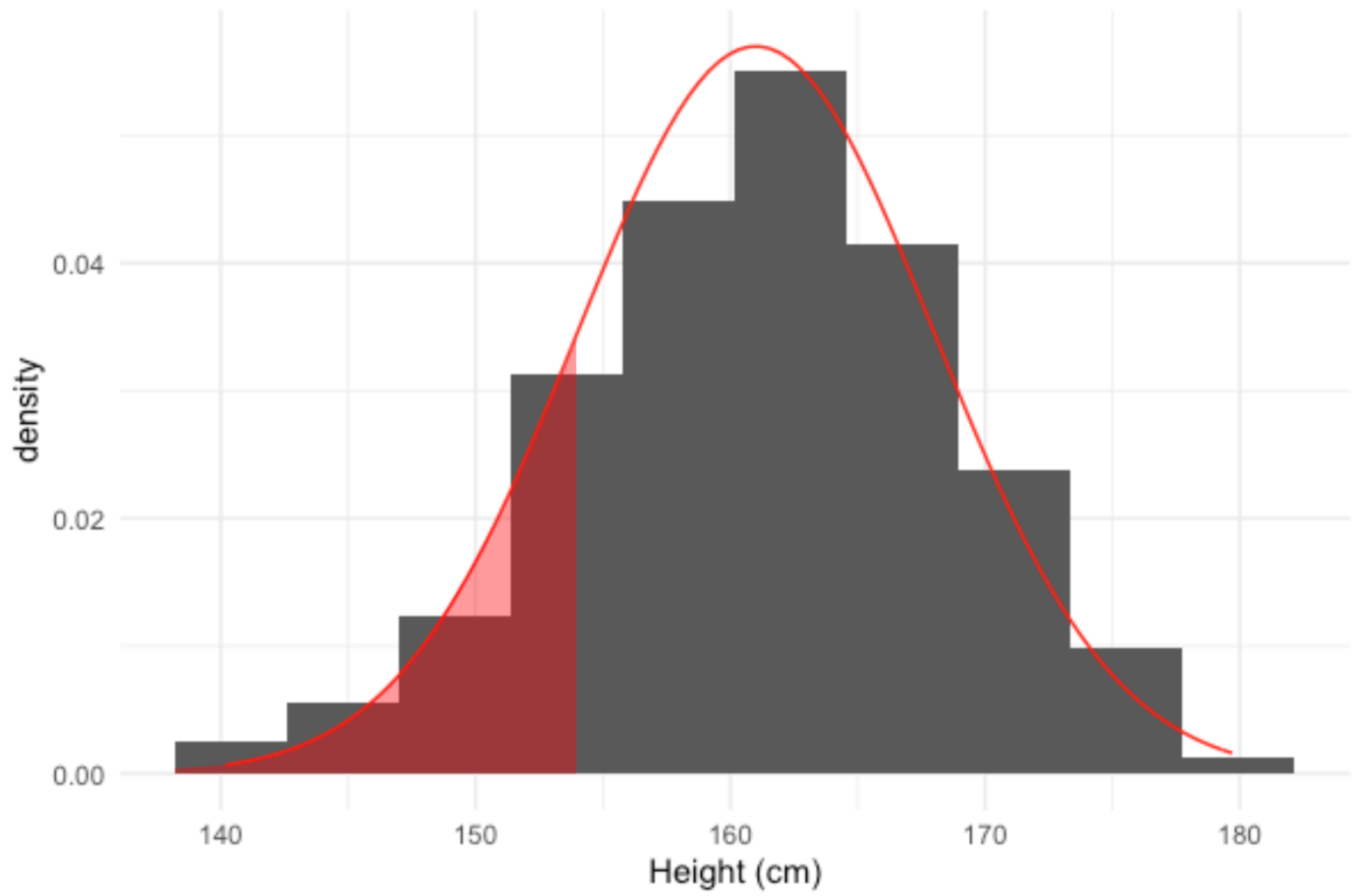


Mean: 161 cm ***Standard deviation: 7 cm***

Approximating data with the normal distribution



What percent of women are shorter than 154 cm?




```python
from scipy.stats import norm
norm.cdf(154, 161, 7)
```

```
0.158655
```

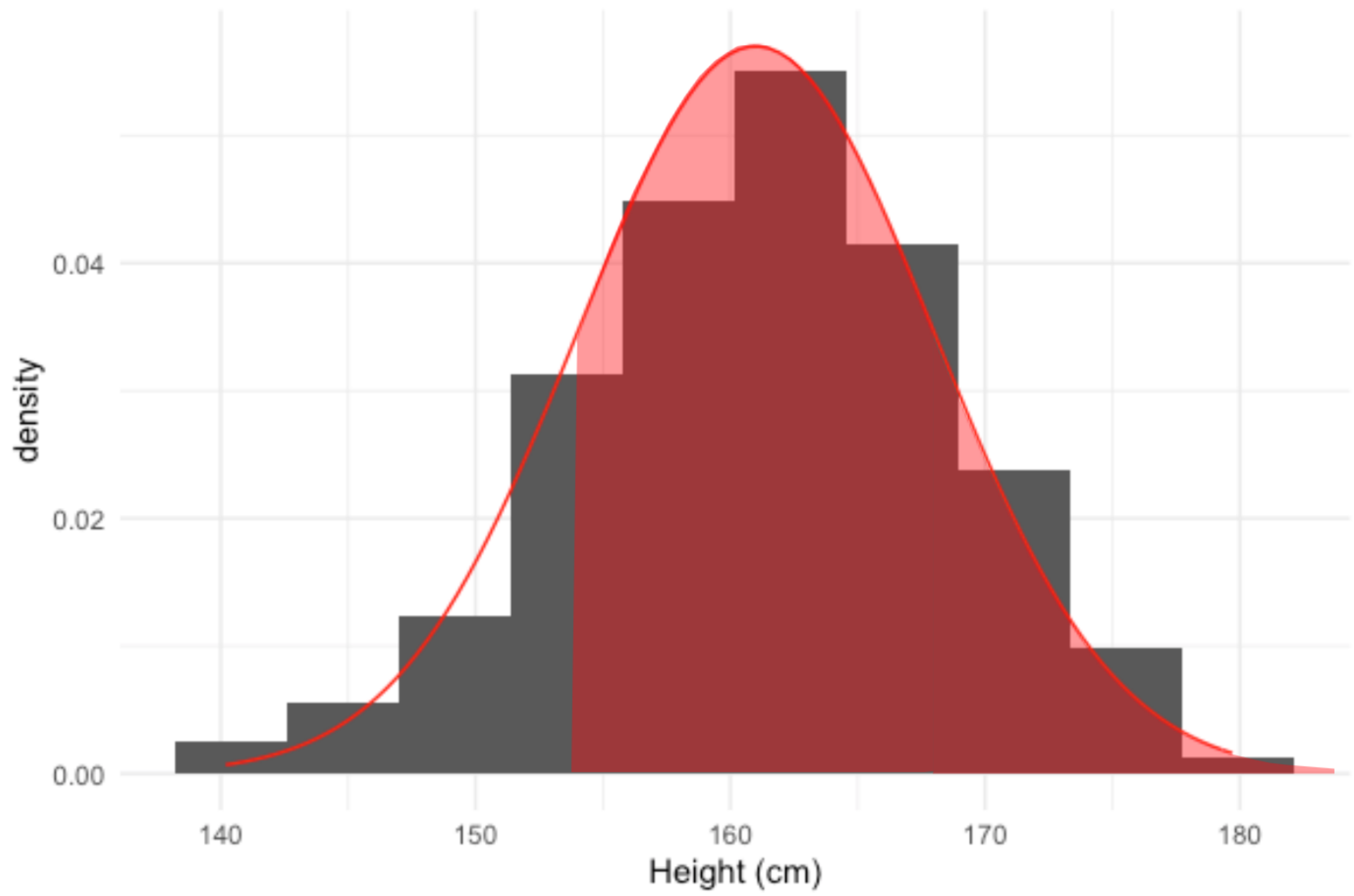
mean

std_dev

16% of women in the survey are shorter than 154 cm

it all about calculating the area

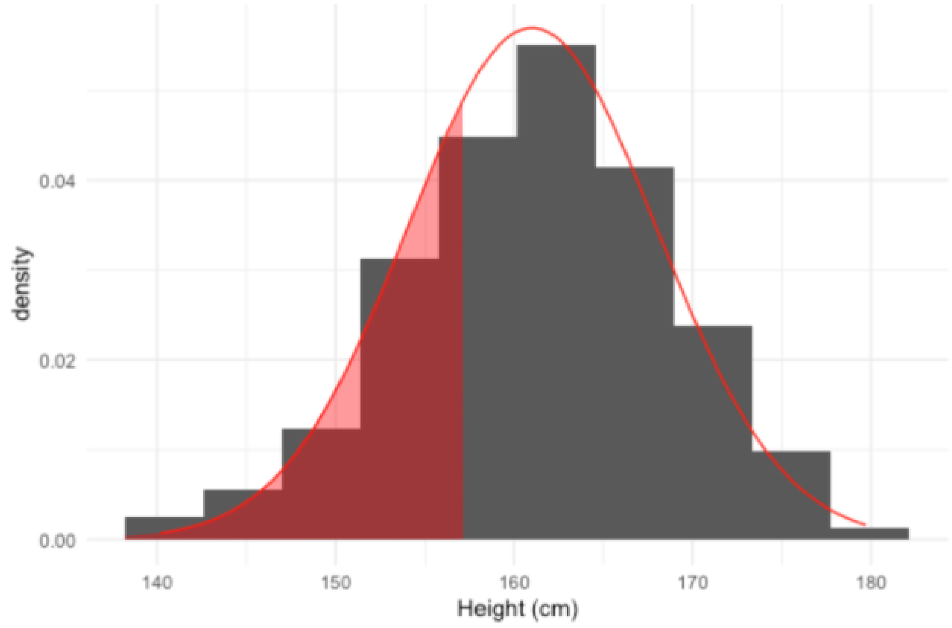
What percent of women are taller than 154 cm?




```python
from scipy.stats import norm
1 - norm.cdf(154, 161, 7)
```

```
0.841345
```

What percent of women are 154-157 cm?



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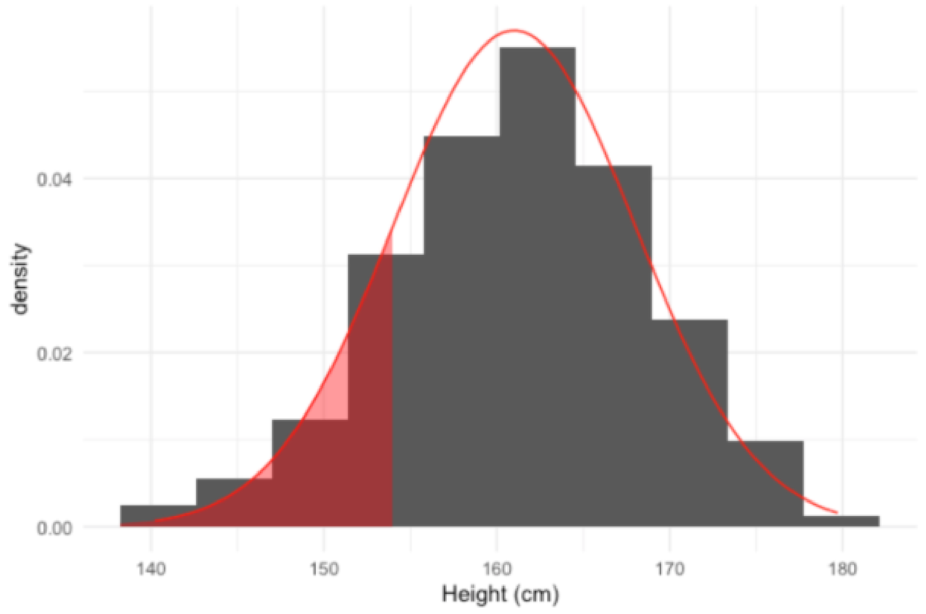
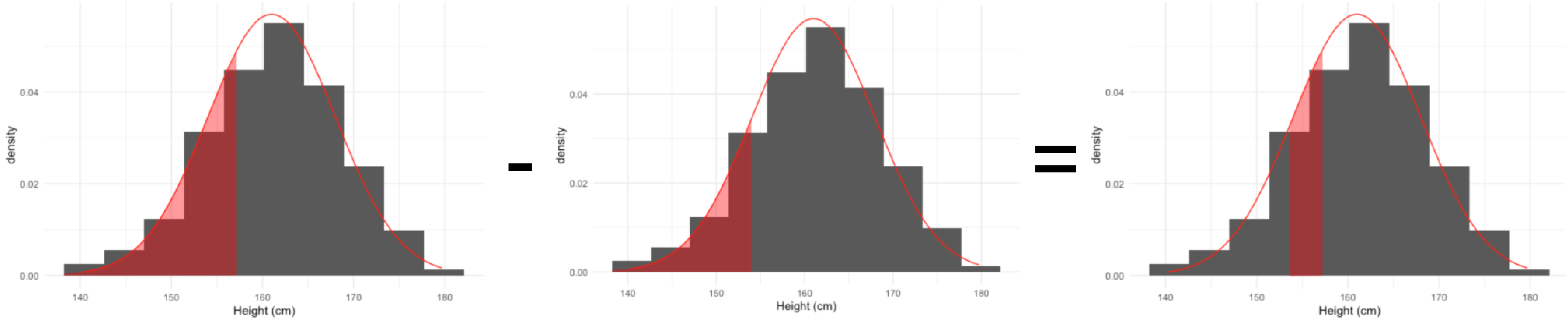



```
norm.cdf(157, 161, 7) - norm.cdf(154, 161, 7)
```

it all about calculating the area

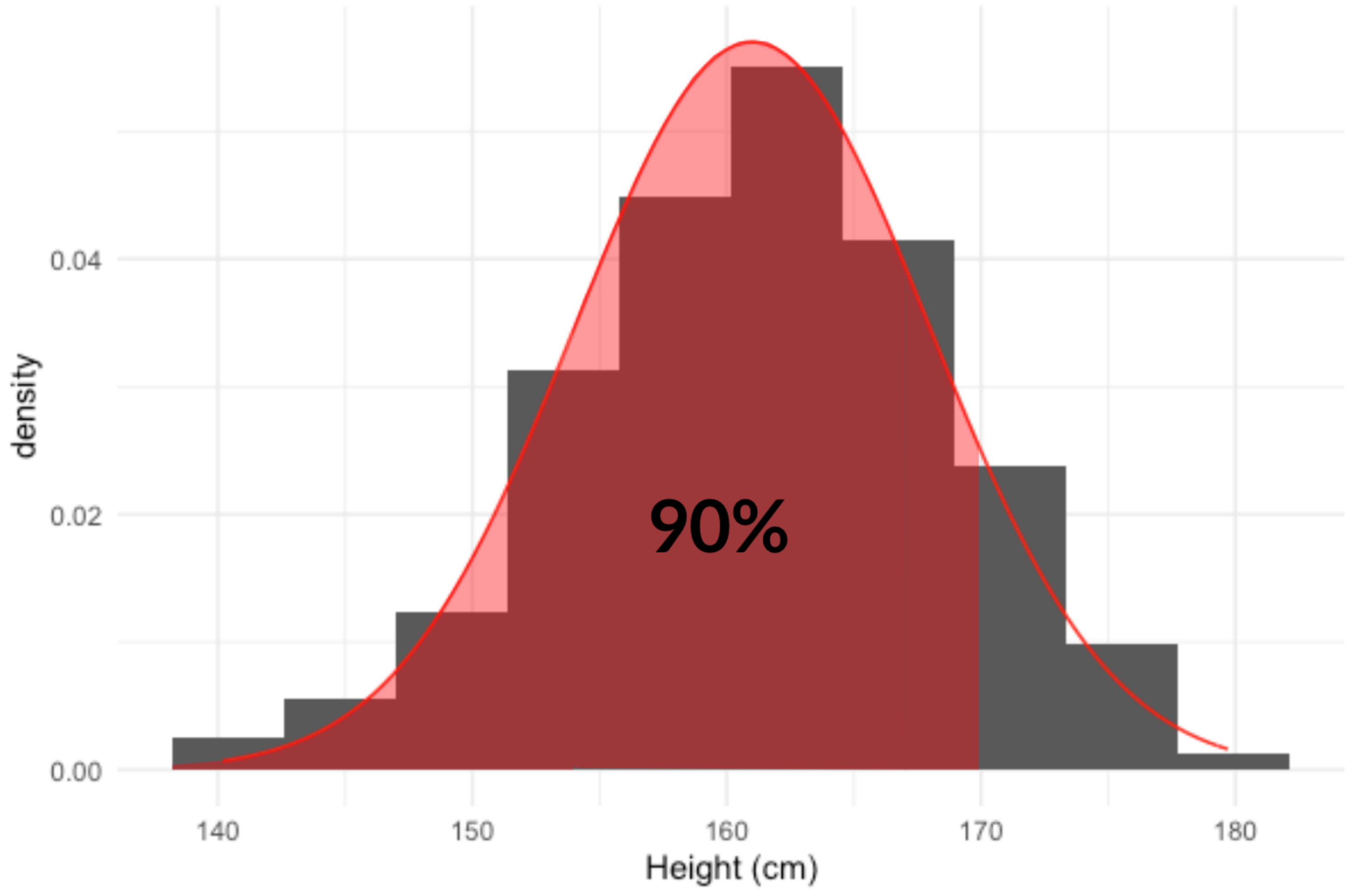
What percent of women are 154-157 cm?




```
norm.cdf(157, 161, 7) - norm.cdf(154, 161, 7)
```

```
0.1252
```

What height are 90% of women shorter than?



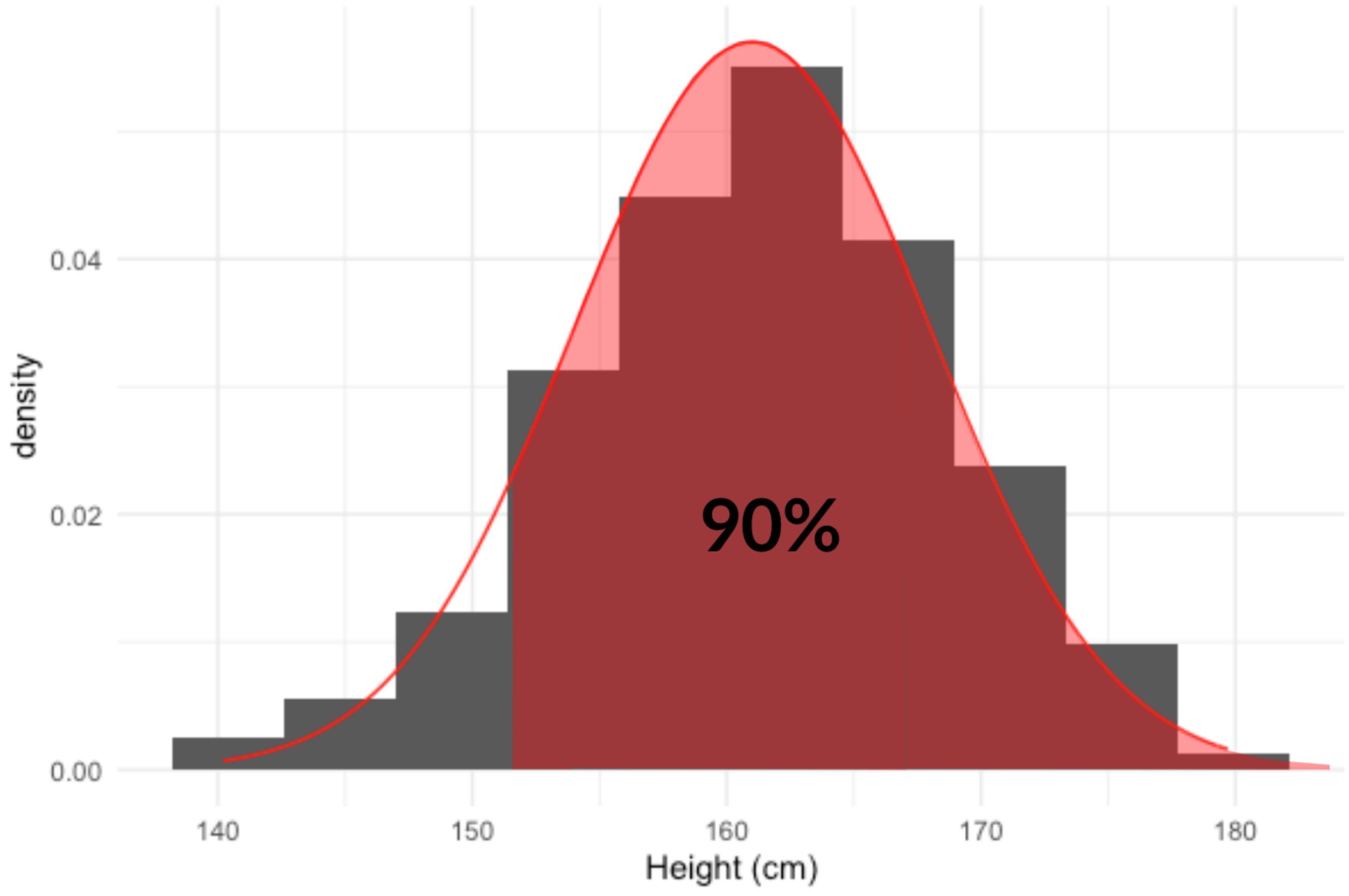

```
norm.ppf(0.9, 161, 7)
```

```
169.97086
```

mean

std_dev

What height are 90% of women taller than?




```
norm.ppf((1-0.9), 161, 7)
```

```
152.029
```

mean std dev

Generating random numbers

```
# Generate 10 random heights
norm.rvs(161, 7, size=10)
```

random values if more likely to be from congested area

```
array([155.5758223 , 155.13133235, 160.06377097, 168.33345778,
       165.92273375, 163.32677057, 165.13280753, 146.36133538,
       149.07845021, 160.5790856 ])
```

norm.rvs(mean , std dev ,size =n)

161 -> Mean : The average value (center) of the distribution is 161.

7 -> Standard deviation : The spread of the distribution is 7.

size=10 -> Generate 10 random samples from this distribution.

Let's practice!

INTRODUCTION TO STATISTICS IN PYTHON

The central limit theorem

INTRODUCTION TO STATISTICS IN PYTHON

Maggie Matsui
Content Developer, DataCamp

Rolling the dice 5 times

```python
die = pd.Series([1, 2, 3, 4, 5, 6])
# Roll 5 times
samp_5 = die.sample(5, replace=True)
print(samp_5)
```

```
array([3, 1, 4, 1, 1])
```

```python
np.mean(samp_5)
```

```
2.0
```

Rolling the dice 5 times

```
# Roll 5 times and take mean
samp_5 = die.sample(5, replace=True)
np.mean(samp_5)
```

```
4.4
```

```
samp_5 = die.sample(5, replace=True)
np.mean(samp_5)
```

```
3.8
```

Rolling the dice 5 times 10 times

Repeat 10 times:

- Roll 5 times
- Take the mean

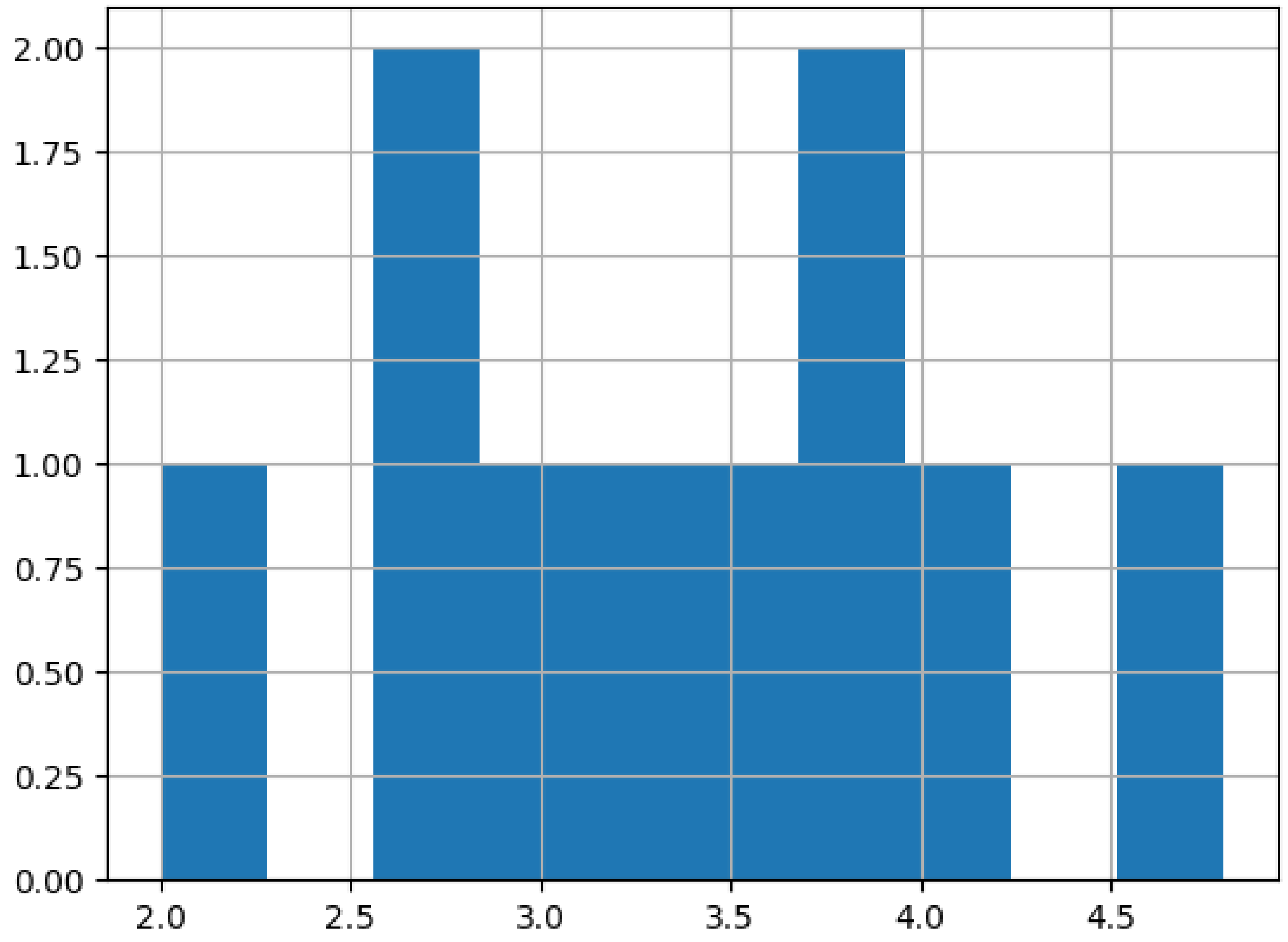
```
sample_means = []
for i in range(10):
  samp_5 = die.sample(5, replace=True)
  sample_means.append(np.mean(samp_5))
print(sample_means)
```

```
[3.8, 4.0, 3.8, 3.6, 3.2, 4.8, 2.6,
3.0, 2.6, 2.0]
```

what is clt : roll a dice 5 times
and take mean , repeat this
10 times , 100 times , 1000 times
the more you repeat...the more the
distribution gets closer to normal
distribution

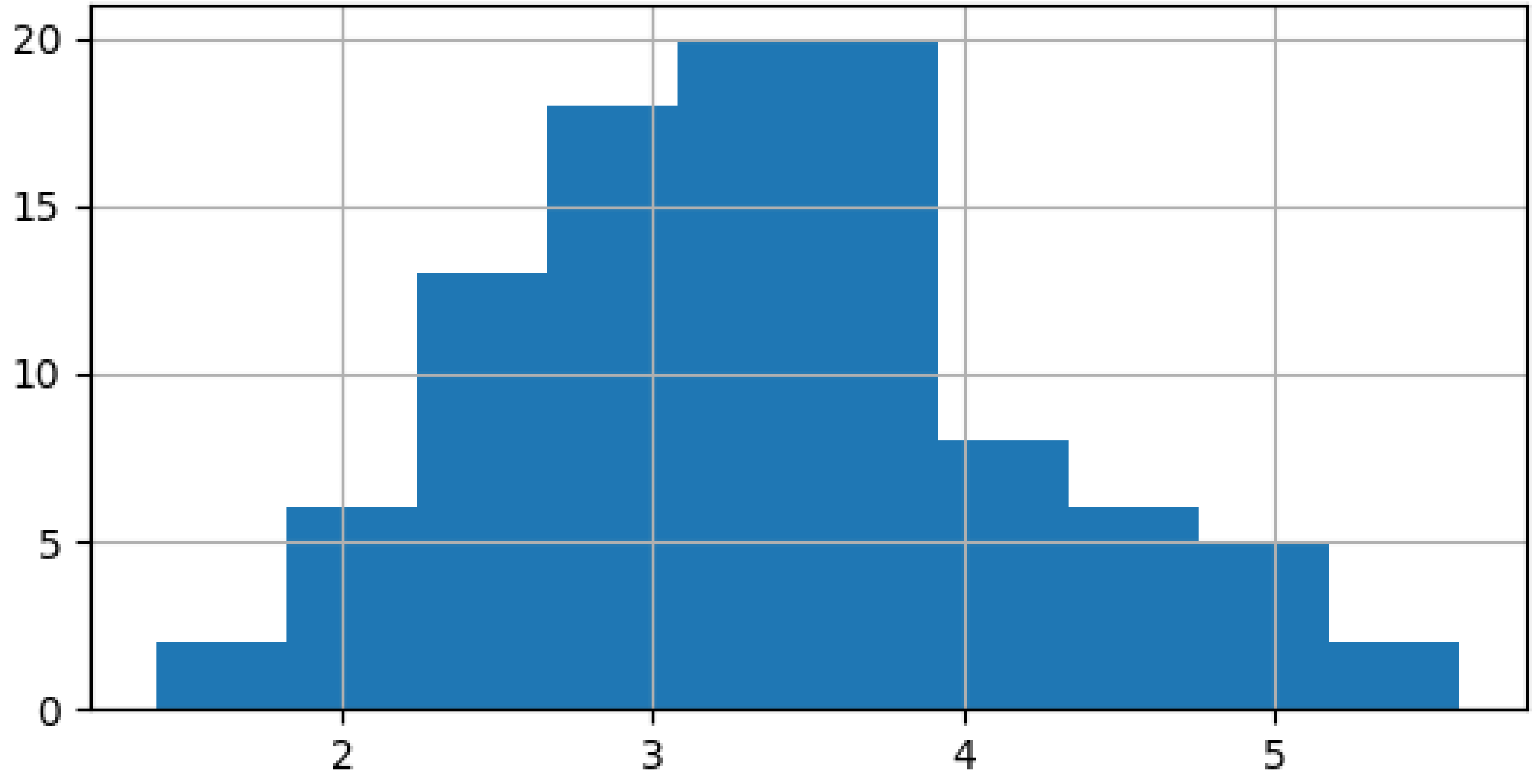
Sampling distributions

Sampling distribution of the sample mean



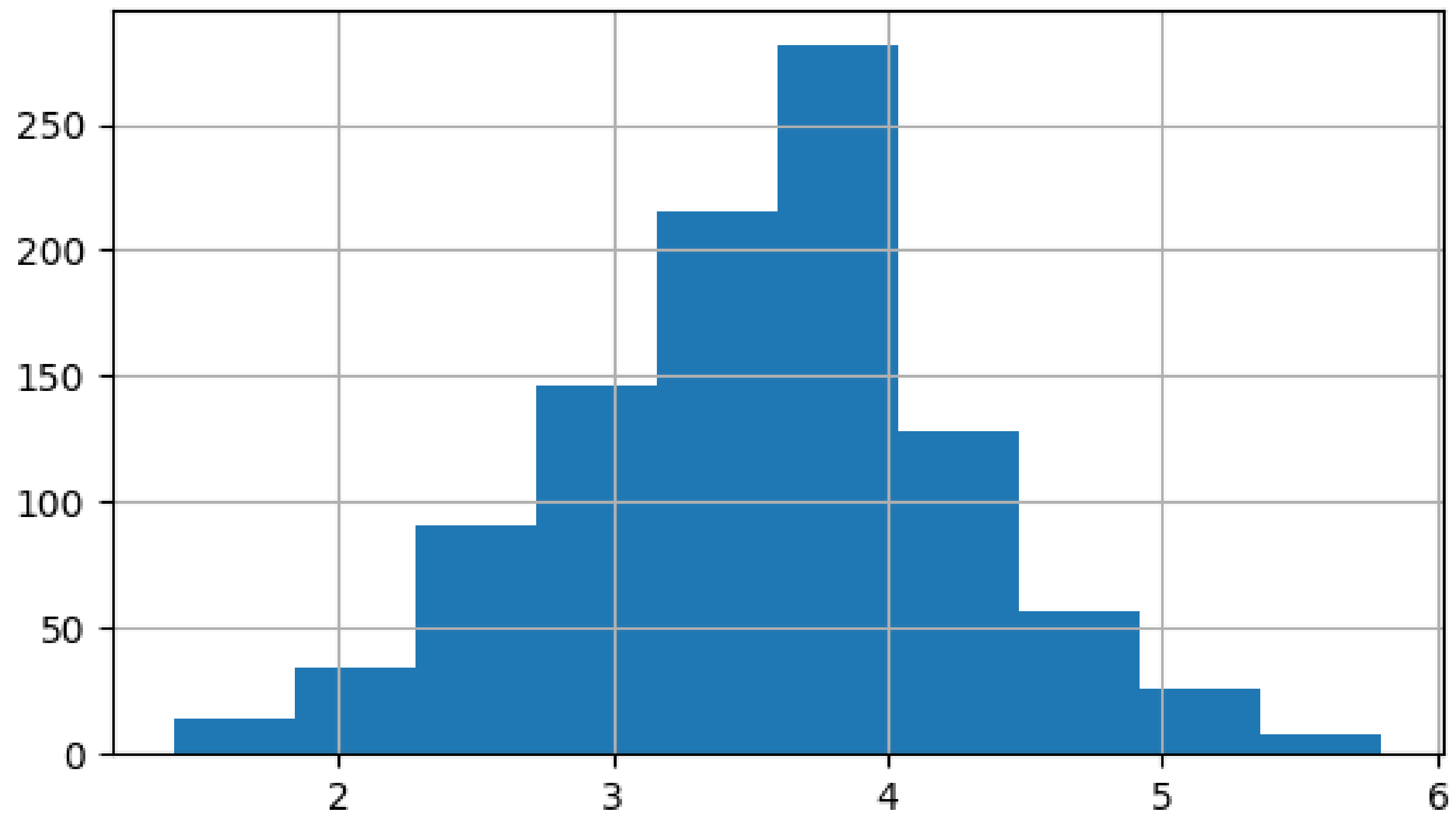
100 sample means

```
sample_means = []
for i in range(100):
    sample_means.append(np.mean(die.sample(5, replace=True)))
```



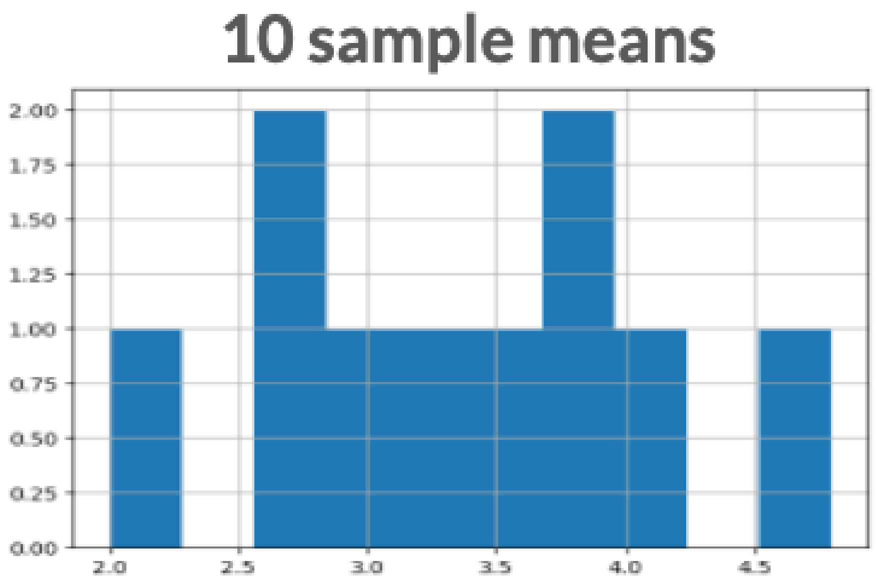
1000 sample means

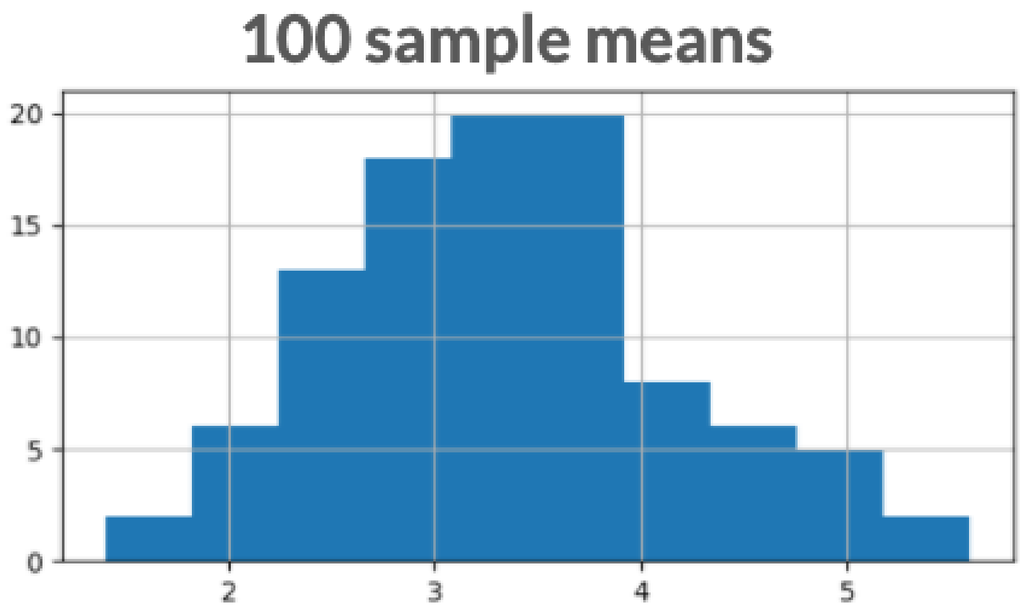
```
sample_means = []
for i in range(1000):
    sample_means.append(np.mean(die.sample(5, replace=True)))
```

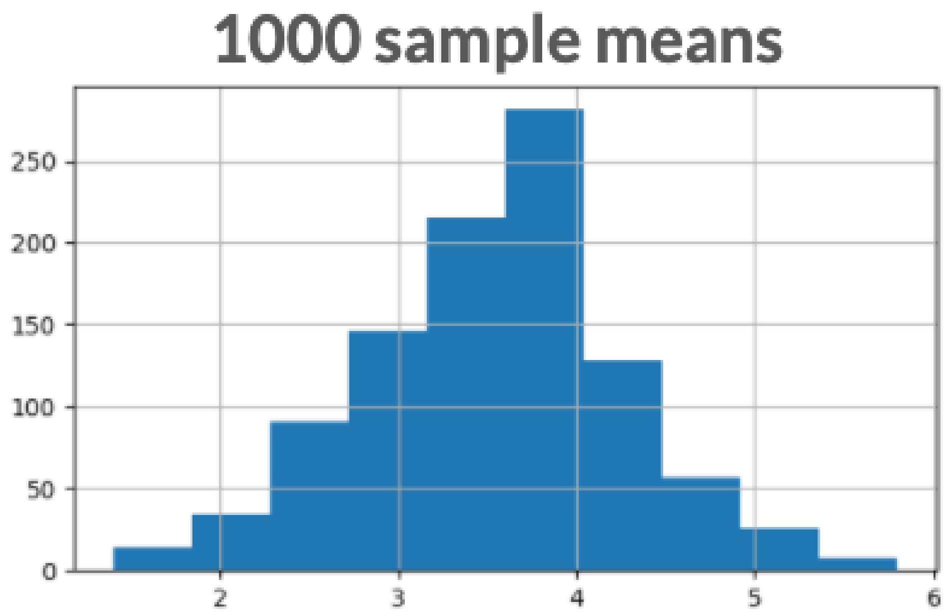


Central limit theorem

The sampling distribution of a statistic becomes closer to the normal distribution as the number of trials increases.



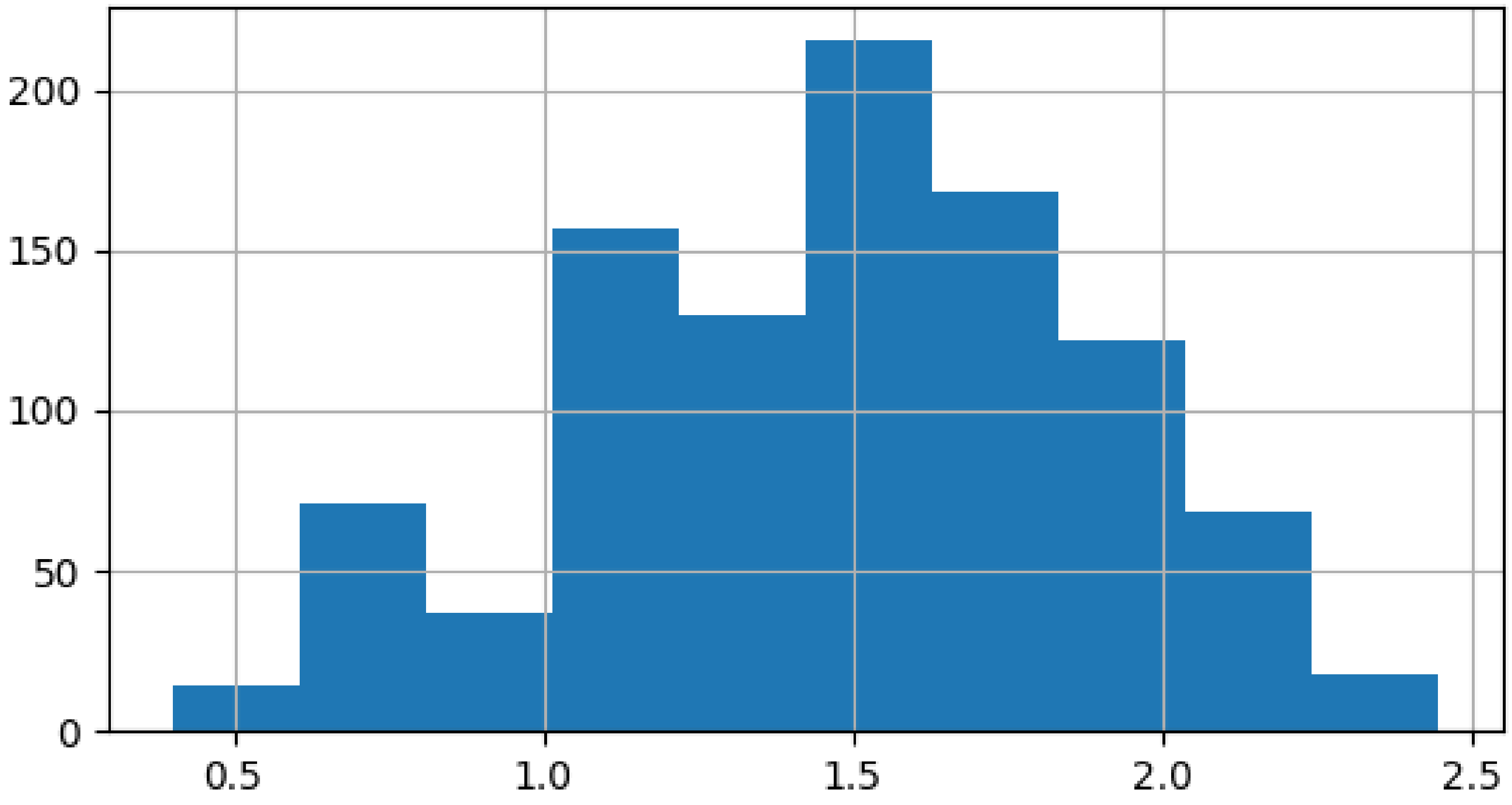




* *Samples should be random and independent*

Standard deviation and the CLT

```
sample_sds = []
for i in range(1000):
  sample_sds.append(np.std(die.sample(5, replace=True)))
```



Proportions and the CLT

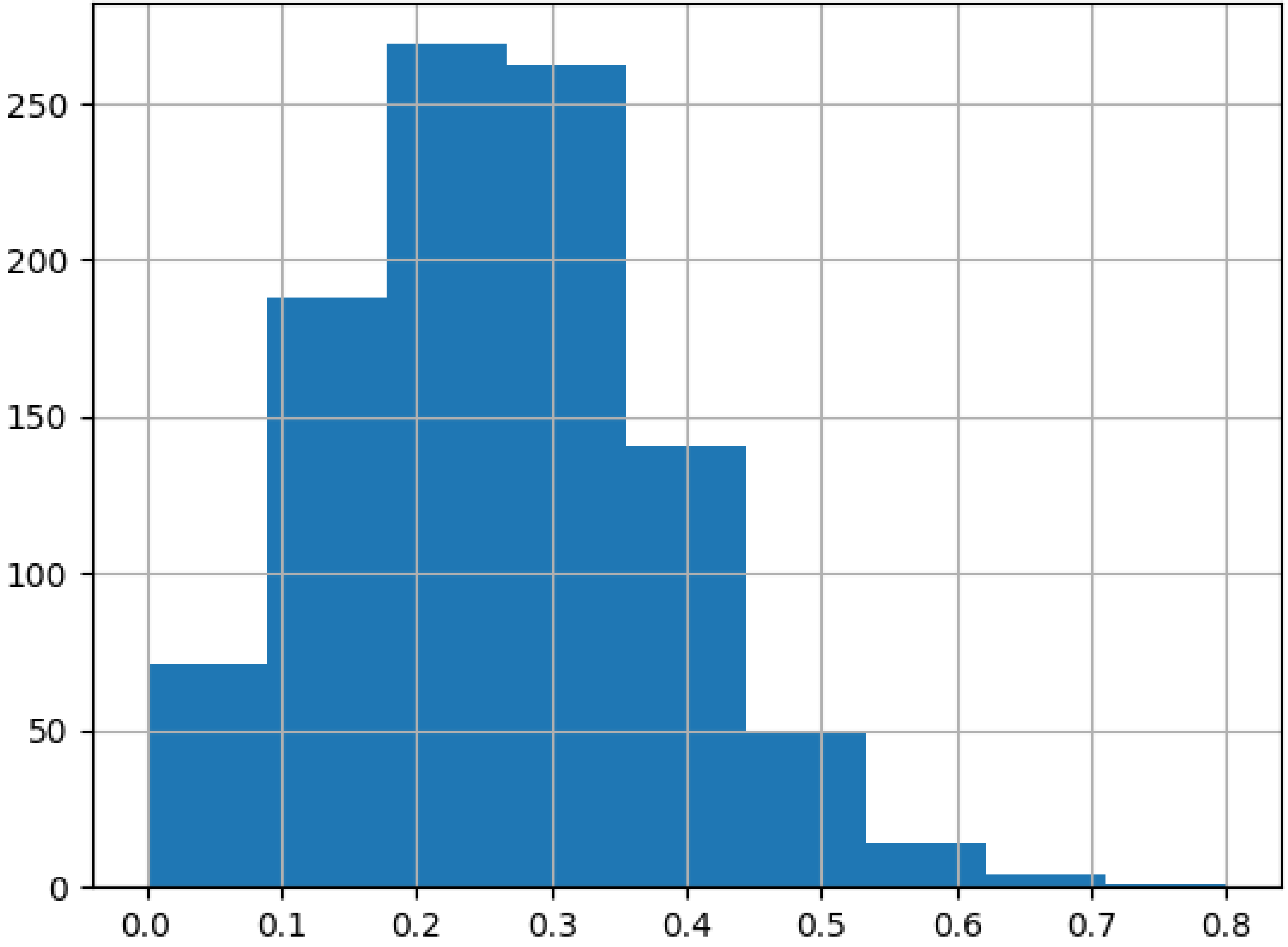
```
sales_team = pd.Series(["Amir", "Brian", "Claire", "Damian"])
sales_team.sample(10, replace=True)
```

```
array(['Claire', 'Damian', 'Brian', 'Damian', 'Damian', 'Amir', 'Amir', 'Amir',
       'Amir', 'Damian'], dtype=object)
```

```
sales_team.sample(10, replace=True)
```

```
array(['Brian', 'Amir', 'Brian', 'Claire', 'Brian', 'Damian', 'Claire', 'Brian',
       'Claire', 'Claire'], dtype=object)
```

Sampling distribution of proportion



Mean of sampling distribution

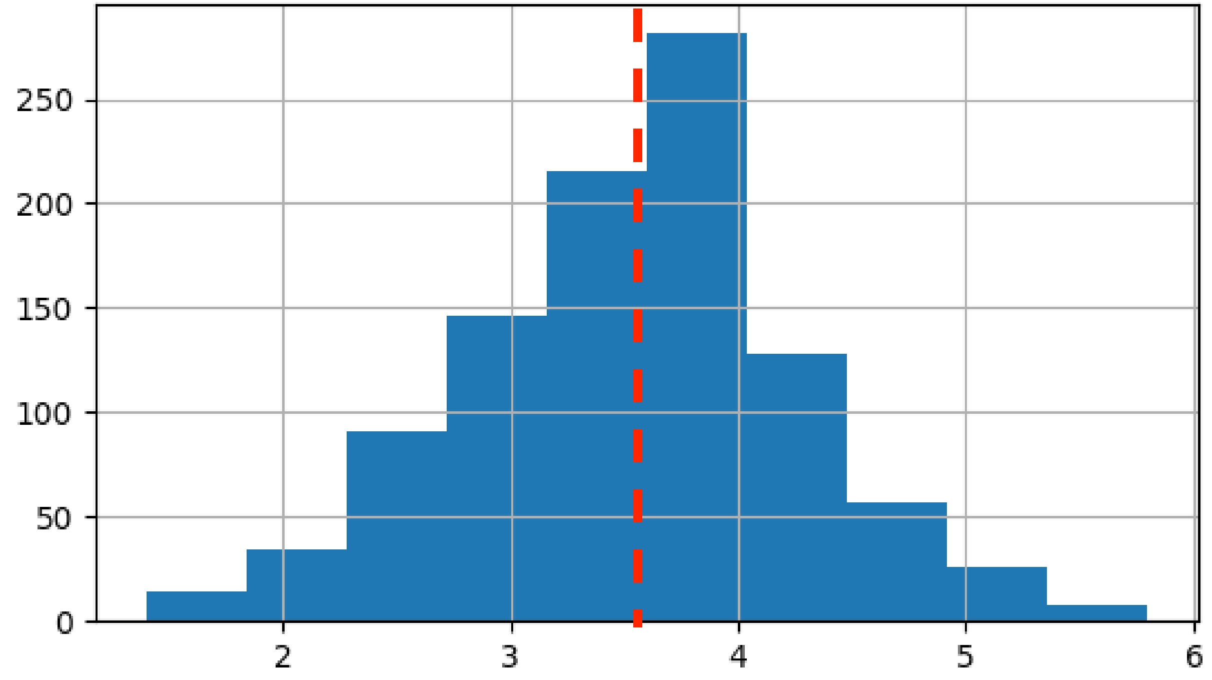
```
# Estimate expected value of die
np.mean(sample_means)
```

```
3.48
```

```
# Estimate proportion of "Claire"s
np.mean(sample_props)
```

```
0.26
```

we use this , when we want to estimate of the whole population just collecting data from small groups



- Estimate characteristics of unknown underlying distribution
- More easily estimate characteristics of large populations

Let's practice!

Part-03

The Poisson distribution

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Maggie Matsui
Content Developer, DataCamp

Poisson processes

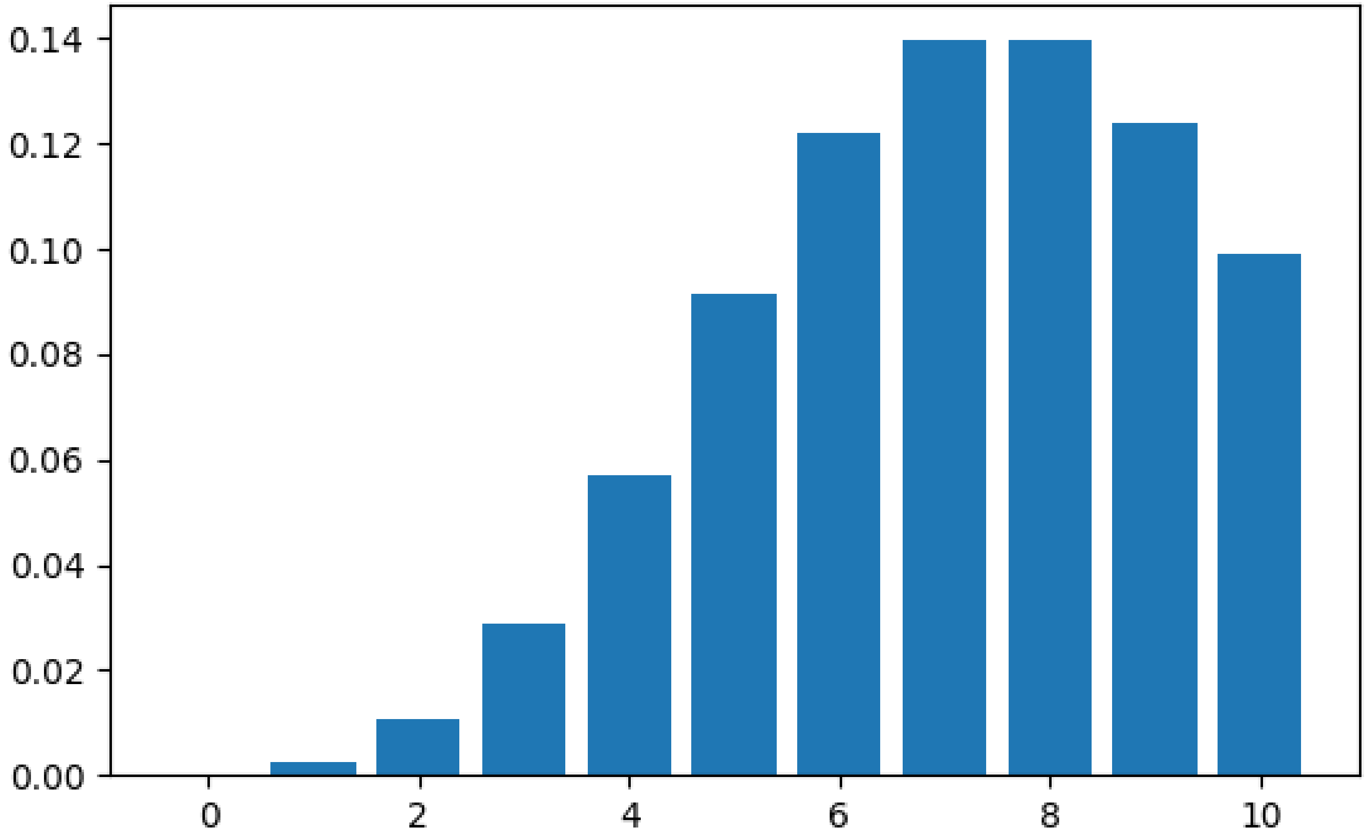
- Events appear to happen at a certain rate, but completely at random
- Examples
  - Number of animals adopted from an animal shelter per week
  - Number of people arriving at a restaurant per hour
  - Number of earthquakes in California per year
- Time unit is irrelevant, as long as you use the same unit when talking about the same situation

Poisson distribution

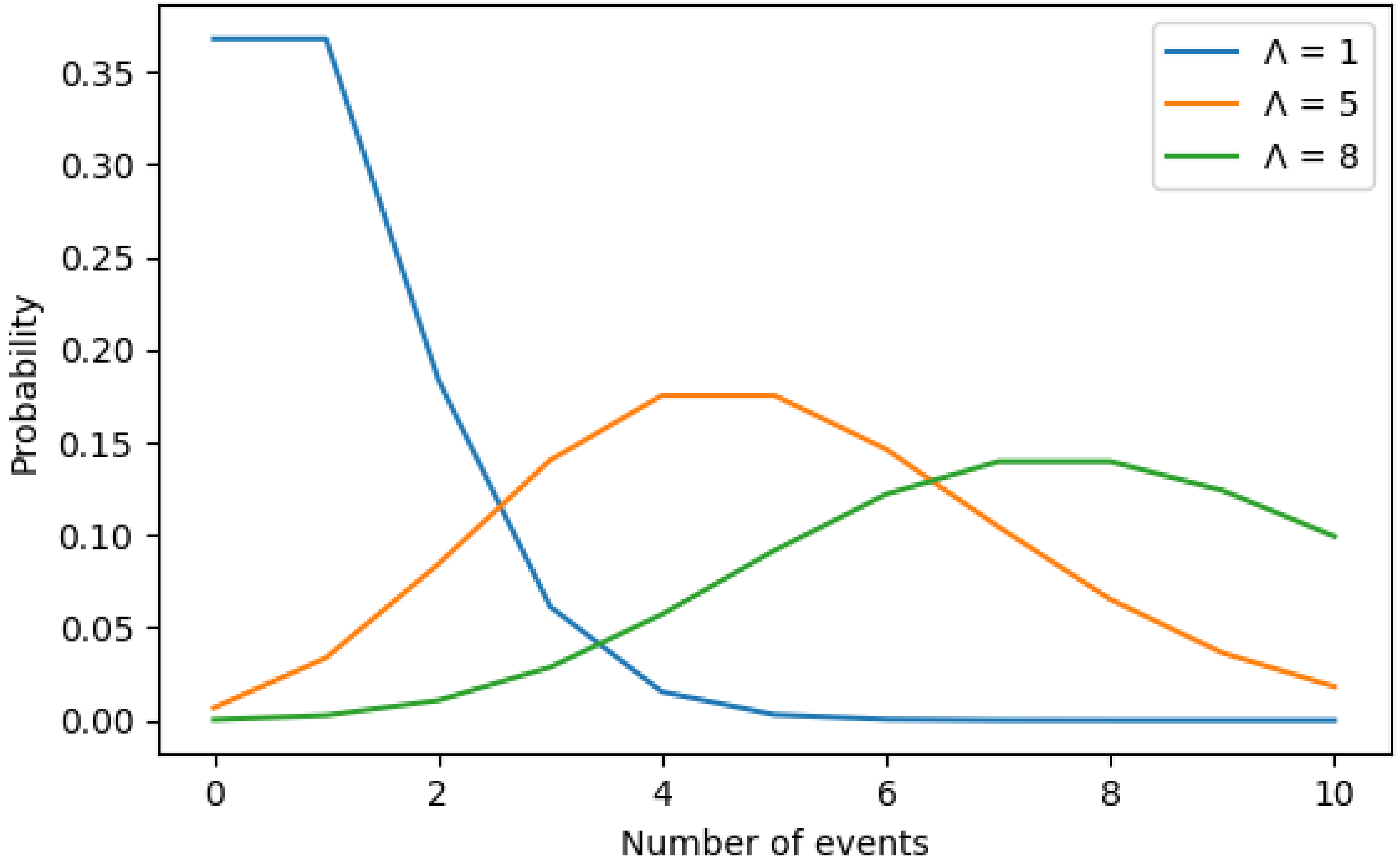
- Probability of some # of events occurring over a fixed period of time
- Examples
  - Probability of $\geq$ 5 animals adopted from an animal shelter per week
  - Probability of 12 people arriving at a restaurant per hour
  - Probability of $<$ 20 earthquakes in California per year

Lambda (λ)

- λ = average number of events per time interval
  - Average number of adoptions per week = 8



Lambda is the distribution's peak



Probability of a single value

% of

If the average number of adoptions per week is 8, what is $P(\#\text{ adoptions in a week} = 5)$?

```
from scipy.stats import poisson
poisson.pmf(5, 8)
```

λ

```
0.09160366
```

Probability of less than or equal to

→ % of

If the average number of adoptions per week is 8, what is $P(\#\text{ adoptions in a week} \leq 5)$?

```
from scipy.stats import poisson
poisson.cdf(5, 8)
```

```
0.1912361
```

Probability of greater than

$\rightarrow$ % of

If the average number of adoptions per week is 8, what is $P(\#\text{ adoptions in a week} > 5)$?

```
1 - poisson.cdf(5, 8)
```

```
0.8087639
```

If the average number of adoptions per week is **10**, what is $P(\#\text{ adoptions in a week} > 5)$?

$\rightarrow$ % of

```
1 - poisson.cdf(5, 10)
```

```
0.932914
```

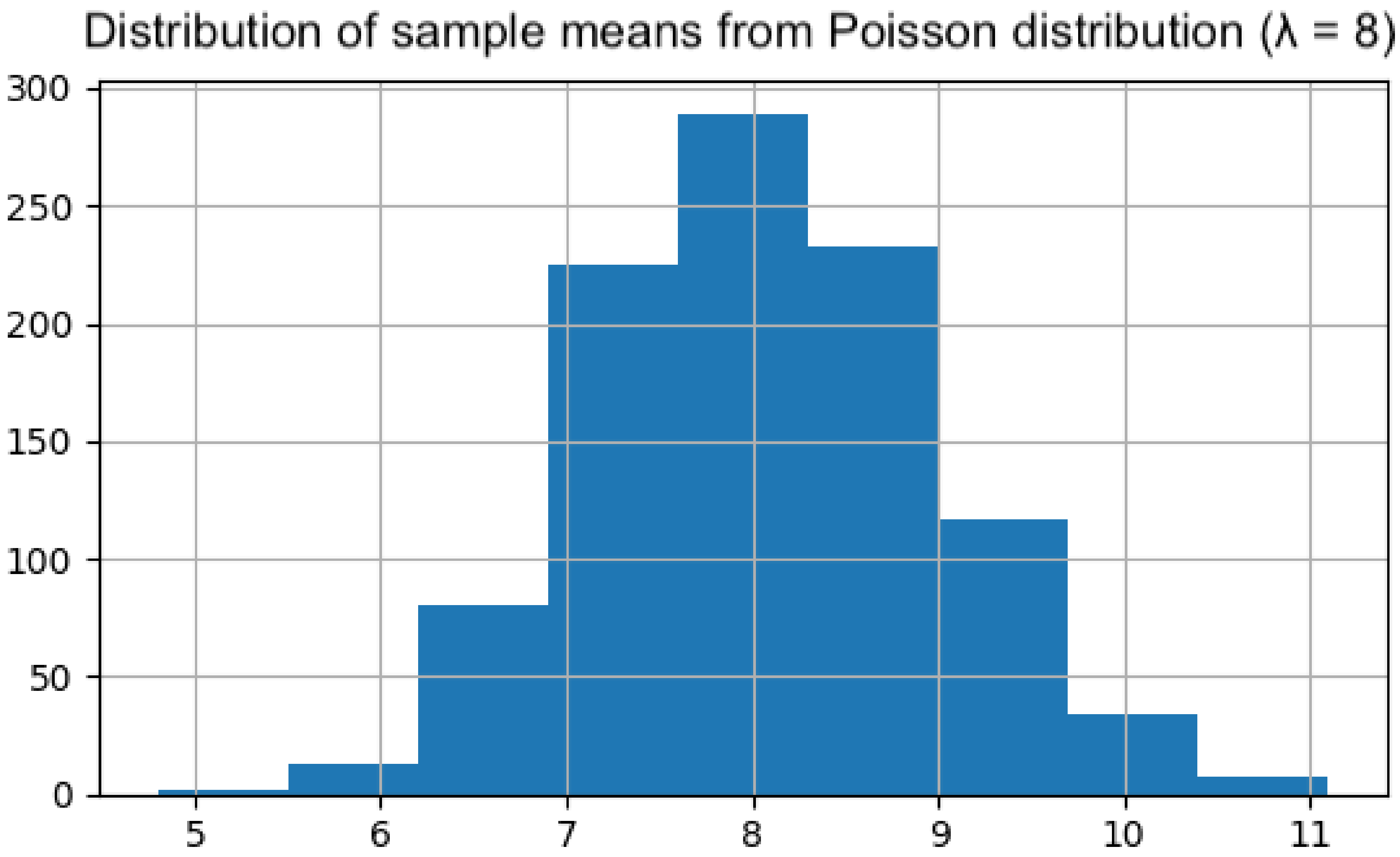
Sampling from a Poisson distribution

```
from scipy.stats import poisson
poisson.rvs(8, size=10)
```

```
array([ 9,  9,  8,  7, 11,  3, 10,  6,  8, 14])
```

$\lambda = 8$

The CLT still applies!



Let's practice!

INTRODUCTION TO STATISTICS IN PYTHON

Part-04

More probability distributions

INTRODUCTION TO STATISTICS IN PYTHON

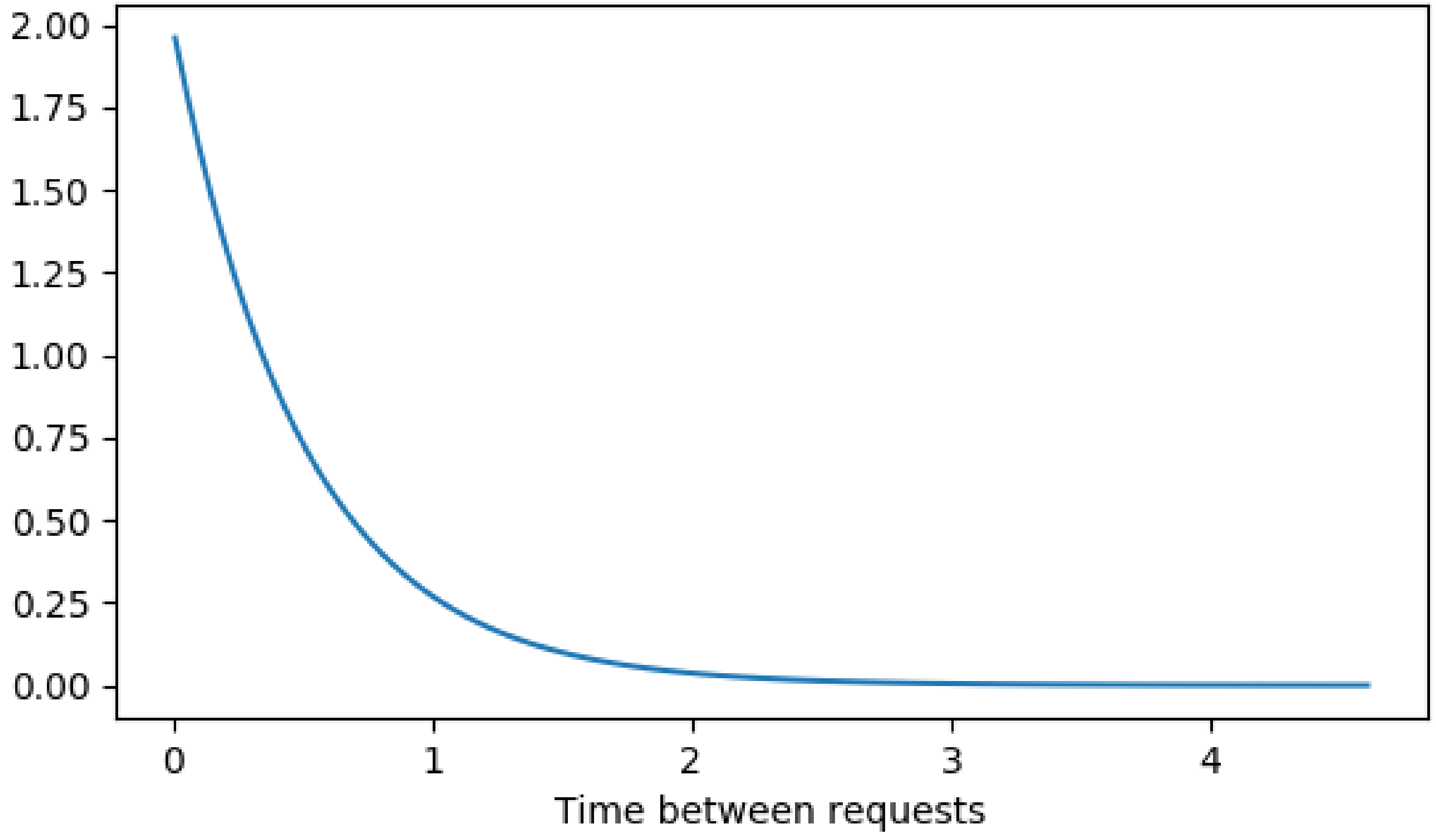
Maggie Matsui
Content Developer, DataCamp

Exponential distribution

- Probability of time between Poisson events
- Examples
  - Probability of > 1 day between adoptions
  - Probability of < 10 minutes between restaurant arrivals
  - Probability of 6-8 months between earthquakes
- Also uses lambda (rate)
- Continuous (time)

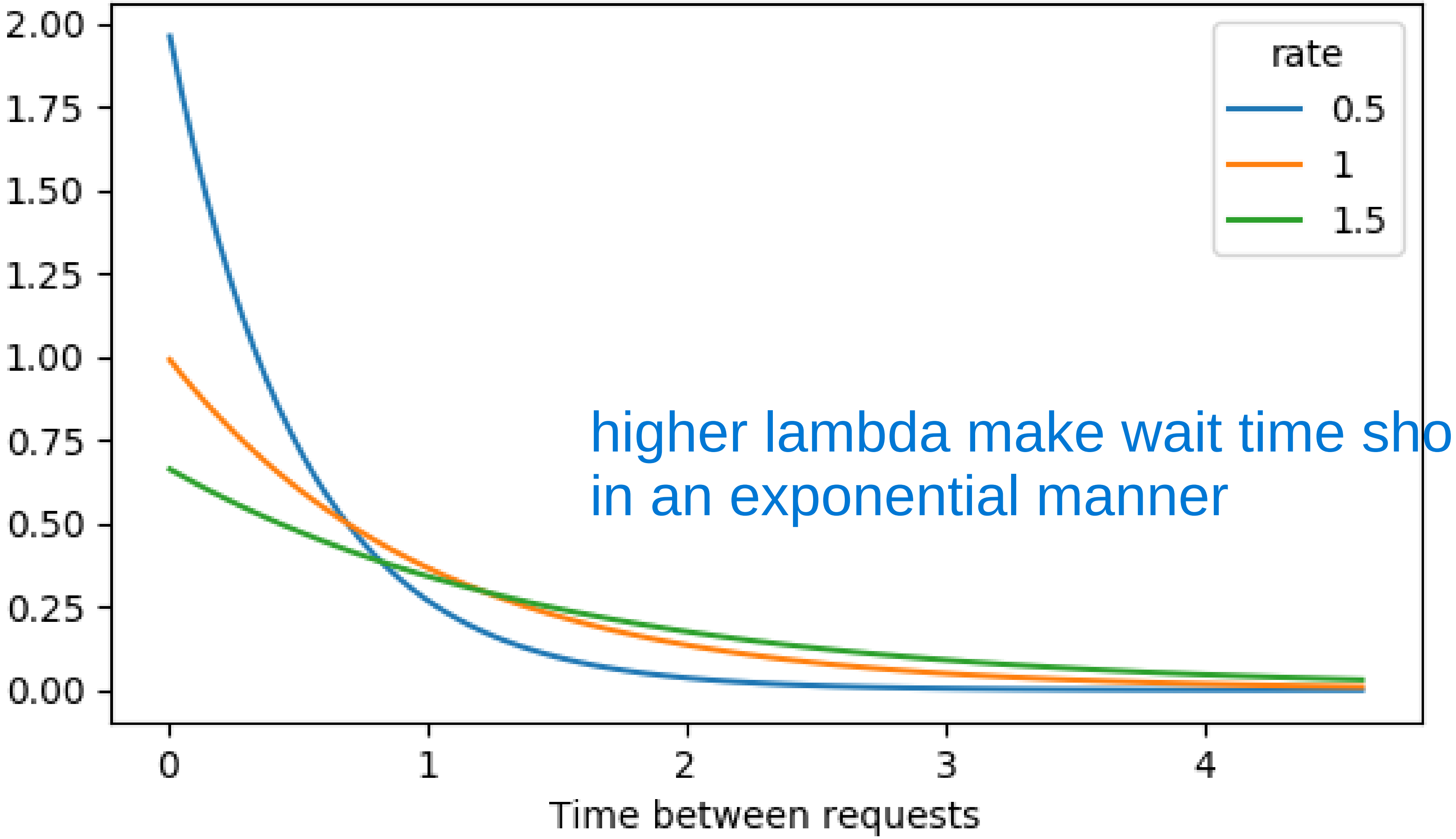
Customer service requests

- On average, one customer service ticket is created every 2 minutes
  - λ = 0.5 customer service tickets created each minute



Y-Axis: Probability Density (not probability)

Lambda in exponential distribution



higher lambda make wait time shorter
in an exponential manner

Y-Axis: Probability Density (not probability)

Expected value of exponential distribution

In terms of rate (Poisson):

- $\lambda = 0.5$ requests per minute

In terms of time between events (exponential):

- $1/\lambda$ = 1 request per 2 minutes
- $1/0.5 = 2$

How long until a new request is created?

Scale = 1/λ

```
from scipy.stats import expon
```

- `scale` $= 1/\lambda = 1/0.5 = 2$

$P(\text{wait} < 1 \text{ min}) =$

```
expon.cdf(1, scale=2)
```

```
0.3934693402873666
```

$P(\text{wait} > 4 \text{ min}) =$

```
1- expon.cdf(4, scale=2)
```

```
0.1353352832366127
```

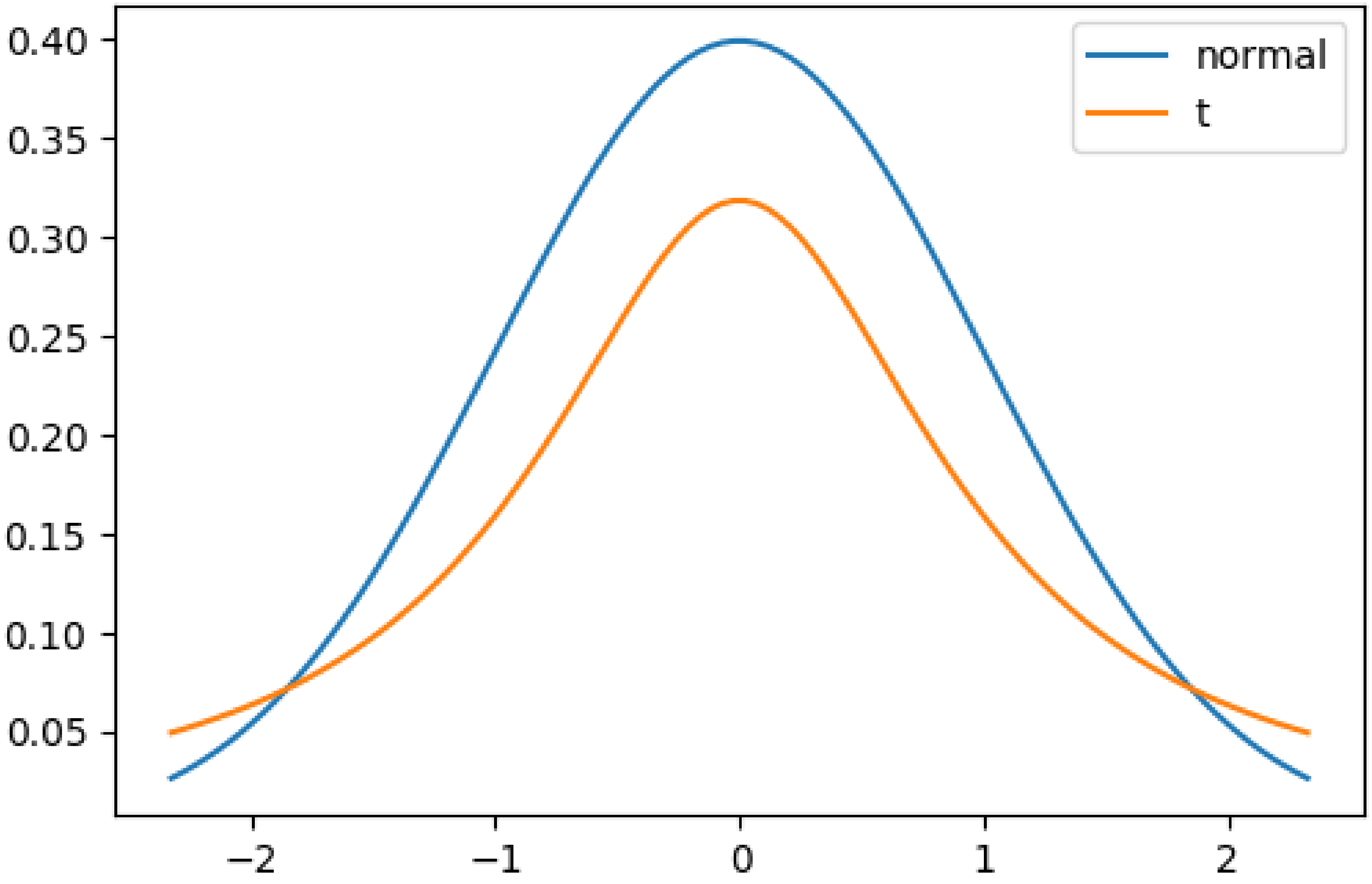
$P(1 \text{ min} < \text{wait} < 4 \text{ min}) =$

```
expon.cdf(4, scale=2) - expon.cdf(1, scale=2)
```

```
0.4711953764760207
```

(Student's) t-distribution

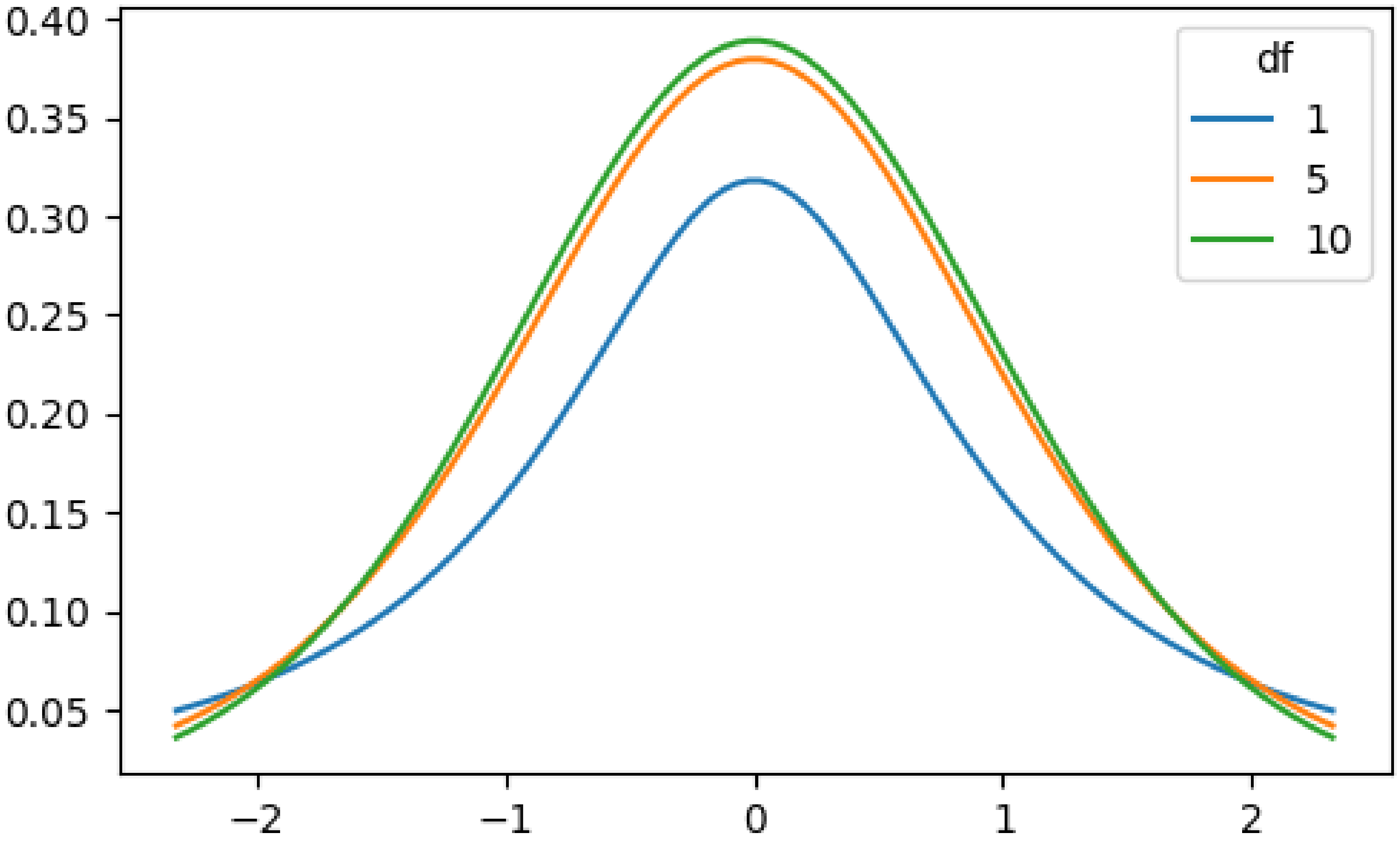
- Similar shape as the normal distribution



Degrees of freedom

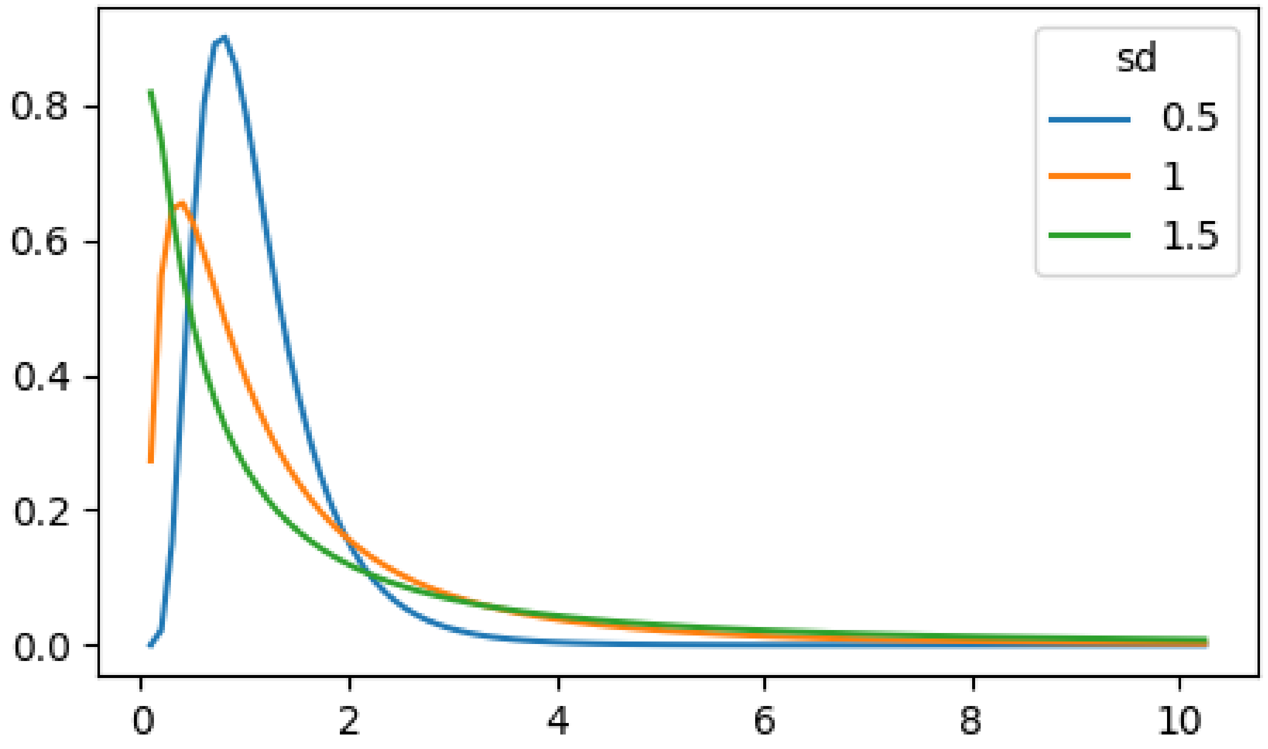
related to T distribution

- Has parameter degrees of freedom (df) which affects the thickness of the tails
  - Lower df = thicker tails, higher standard deviation
  - Higher df = closer to normal distribution



Log-normal distribution

- Variable whose logarithm is normally distributed
- Examples:
  - Length of chess games
  - Adult blood pressure
  - Number of hospitalizations in the 2003 SARS outbreak



Let's practice!

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