Task 1:

Grand canonical ensemble

energy levels: $\epsilon, 2\epsilon, 3\epsilon, \ldots, M\epsilon$

N particles

Occupancy (n) is 0 or 1 for fermions

partition function for 1 particle, ground state energy

$$Z_\epsilon = 1 + e^{-\beta[\epsilon-\mu]}$$

$$Z_{2\epsilon} = 1 + e^{-\beta[2\epsilon-\mu]}$$

$$Z_{tot} = Z_\epsilon \cdot Z_{2\epsilon} \cdots Z_{M\epsilon}$$

$\cdots$

$$Z_{M\epsilon} = 1 + e^{-\beta[M\epsilon-\mu]}$$

$$Z_{tot} = \prod_{i=1}^{M}\left(1 + e^{-\beta[i\epsilon-\mu]}\right)$$

Task 2:

a) Microstates:

State	# particles w/ $E=0$	# particles w/ $E=\epsilon$
1	N	0
2	N-1	1
3	N-2	2
...	...	...
N	1	N-1
N+1	0	N

b) Canonical ensemble

$$Z = \sum_{k=0}^{N} \frac{N!}{k!(N-k)!} e^{-\beta k \epsilon} = (1+e^{-\beta\epsilon})^N$$

$$P(E_s) = \binom{N}{\frac{E}{\epsilon}} \frac{e^{-\beta E}}{Z}$$

c) $$\langle n_0 \rangle_c = \frac{1}{Z} \sum_{k=0}^{N} (N-k) \binom{N}{k} e^{-\beta k \epsilon} = \frac{N}{1+e^{-\beta\epsilon}}$$

$$\langle n_\epsilon \rangle_c = \frac{1}{Z} \sum_{k=0}^{N} k \binom{N}{k} e^{-\beta k \epsilon} = \frac{N}{1+e^{\beta\epsilon}}$$

d) $$Z = \sum_{k=0}^{N} e^{-\beta k \epsilon} = \frac{e^{\beta\epsilon} - e^{-N\beta\epsilon}}{e^{\beta\epsilon}-1}$$

$$P(\epsilon) = \frac{e^{-\beta\epsilon}}{Z} = \frac{e^{-\beta\epsilon}(e^{\beta\epsilon}-1)}{e^{\beta\epsilon} - e^{-N\beta\epsilon}}$$

e) $$\langle n_0 \rangle = N - \frac{1}{e^{\beta\epsilon}-1} + \frac{(N+1)e^{(N+1)\beta\epsilon}}{e^{(N+1)\beta}-1}$$

$$\langle n_\epsilon \rangle = \frac{1}{e^{\beta\epsilon}-1} - \frac{(N+1)e^{(N+1)\beta\epsilon}}{e^{(N+1)\beta\epsilon}-1}$$

f) $$\Omega_G = \sum_{N=0}^{\infty} e^{\beta\mu N} Z = \frac{1}{(e^{\beta(\mu-\epsilon)}-1)(e^{\beta\mu}-1)} \qquad \mu < 0$$