Task 2

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + w_0^2 x = Fe^{iw_f t} \quad , \quad -w^2\tilde{x} + i\gamma w\tilde{x} + w_0^2\tilde{x} = F\delta(w-w_f)$$

γ is damping coefficient
w_0 is the natural frequency
$Fe^{iw_f t}$ is driving force @ freq. w_f

Steady-state solution:

$$x(t) = Xe^{iw_f t}$$

$$(-w_f^2 + i\gamma w_f + w_0^2)X\cancel{e^{iw_f t}} = F\cancel{e^{iw_f t}}$$

$$X = \frac{F}{w_0^2 + w_f^2 + i\gamma w_f}$$

Velocity of oscillator:

$$V(t) = \frac{dx}{dt} = iw_f X e^{iw_f t} = iw_f \frac{F}{w_0^2 + w_f^2 + i\gamma w_f} e^{iw_f t}$$

Instantaneous power from driving force:

$$P(t) = Re\left[Fe^{iw_f t} v^*(t)\right]$$

$$V^*(t) = -iw_f \frac{F}{w_0^2 - w_f^2 - i\gamma w_f} e^{-iw_f t}$$

$$P(t) = Re\left[\frac{-iw_f F^2}{(w_0^2 - w_f)^2 - i\gamma w_f}\right]$$

$$* \quad \frac{1}{a-ib} = \frac{a+ib}{a^2+b^2}$$

$$P(t) = \frac{w_f F^2 \gamma}{(w_0^2 + w_f^2) + \gamma^2 w_f^2}$$

energy absorption per cycle is power integrated over full period $\longrightarrow$

$$E = \int_0^T P(t)\,dt = \int_0^{\frac{2\pi}{w_f}} w_f F^2 \frac{\gamma}{(w_0^2 - w_f^2)^2 + \gamma^2 w_f^2}\,dt \quad \left(\frac{2\pi}{w_f}\right)$$

$$\boxed{E = \frac{F\pi\gamma w_f}{(w_0^2 - w_f^2)^2 + \gamma^2 w_f^2}}$$