

1.a

- [Top5Tweets.java](#)
- [Top5Followers.java](#)

1.b. #Followee and #Tweets have the highest correlation with a value of 0.1936.

- ([AggregateUserData.java](#) for code to generate user.txt)

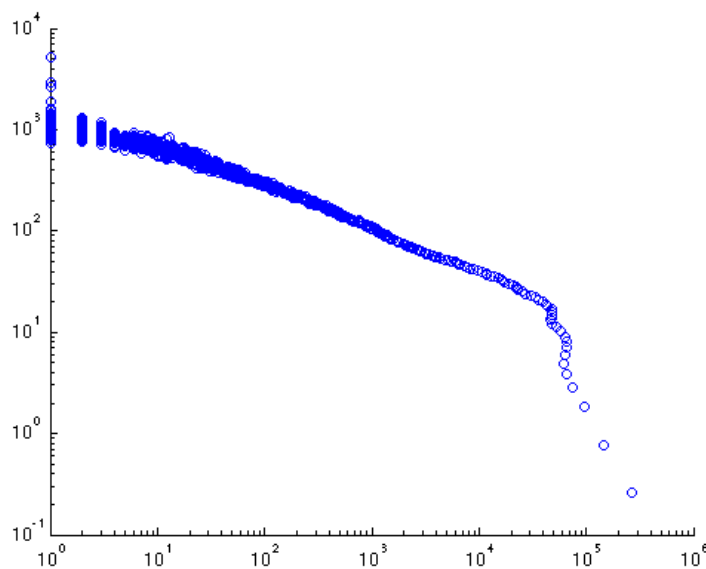
```
>> A = load('user.txt');  
>> [r,p] = corrcoef( A(:,2:4) )  
r =
```

1.0000	0.0086	0.0217
0.0086	1.0000	0.1936
0.0217	0.1936	1.0000

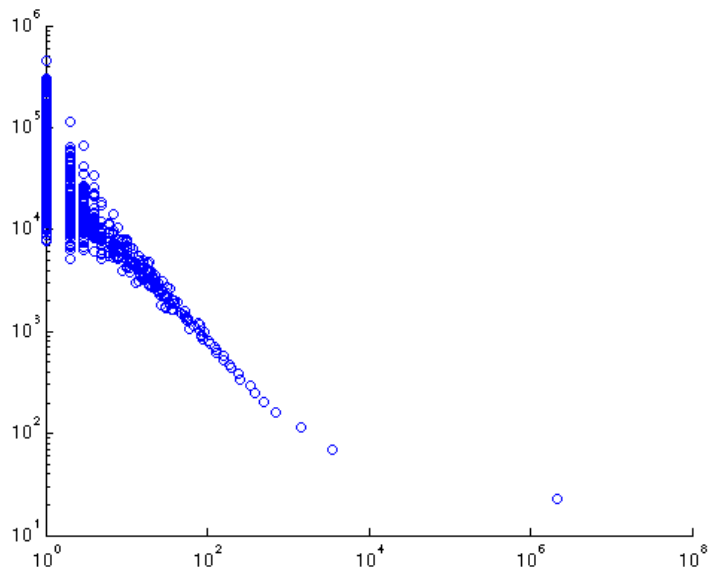
- Script to generate scatter plots:

```
A = load('user.txt');  
% Column 2 for followers, 3 for followees, 4 for tweets  
[f,v] = hist( A(:,2), 10000 );  
scatter(f,v);  
set( gca, 'XScale', 'log' );  
set( gca, 'YScale', 'log' );
```

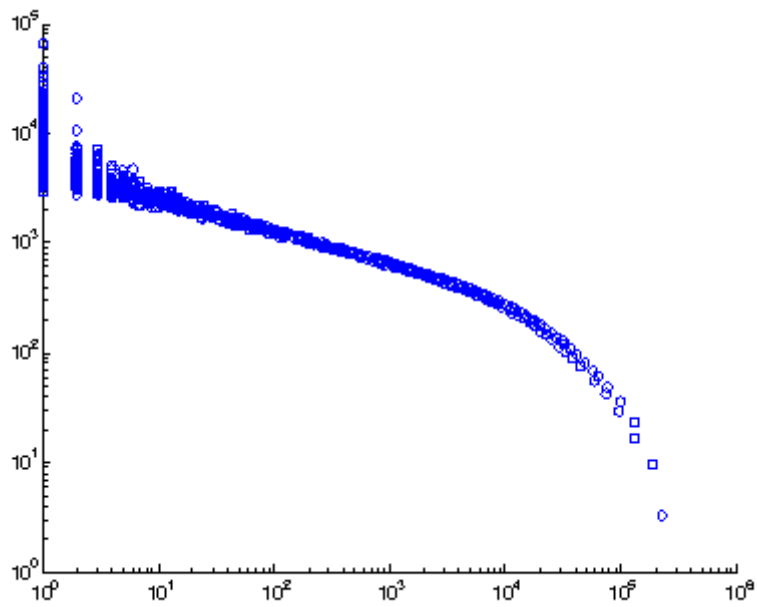
Followees - Follows a power-law distribution



Followers - Does not follow a power law distribution



Tweets - Follows a power law distribution



2.

a. Orthogonal Projection:

i. $Y - Y_x$ perpendicular to Ω

ii. Y_x projected into Ω

i. Let $A = [X_1 | X_2]$

$$(Y - Y_x)A = 0$$

$$A^T Y = A^T Y_x$$

$$A^T Y = A^T Y_x$$

ii. Y_x projected into the subspace Ω

$$Y_x = y_1 X_1 + y_2 X_2 = A [y_1; y_2], \text{ Let } [y_1; y_2] = Z$$

$$Y_x = AZ$$

$$A^T Y = A^T AZ$$

$$(A^T A)^{-1} A^T Y = Z$$

$$A(A^T A)^{-1} A^T Y = AZ$$

$$A(A^T A)^{-1} A^T Y = Y_x$$

b. $X_1 \times X_2 - \omega = 0 \Rightarrow \omega = X_1 \times X_2$

and

$$\|\omega\|_2 = 1$$

c.

i. Using $A(A^T A)^{-1} A^T Y = Y_x$:

$$Yx =$$

$$0.4381$$

$$0.4190$$

$$0.3238$$

ii. $\omega_0 = X_1 \times X_2 = [0.02, 0.01, -0.04]$

$$\|\omega\| = 1 : \omega = \frac{\omega_0}{\|\omega_0\|} = [0.4364, 0.218, -0.8729]$$

3. Covariance

a. No. Nothing in the question indicates that the covariance for the age vs. number of years in the community is normalized in such a way that it can be accurately compared with the covariance for height vs. number of years in the community, so a value of 5 compared with 0.5 does not necessarily mean that height is a better predictor of community membership duration.

- b. using formula $\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$

Let $a = (x - \bar{x})$ (conforming to the redefinition of x)

Let $\bar{a} = \frac{\sum a}{n}$, $E[x] = \bar{x}$ (for sufficiently large n) $\Rightarrow \bar{a} = \frac{0}{n} = 0$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} = \frac{1}{n} \sum (a - \bar{a})(y - \bar{y}) - \bar{a}\bar{y}$$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} = \frac{1}{n} \sum (x - \bar{x} - 0)(y - \bar{y}) - 0\bar{y}$$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} < \frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) \text{ assuming } \bar{x} \text{ and } \bar{y} \text{ may never take on negative values}$$

- c. Let $a = cx$ (conforming to the redefinition of x where c is some constant)

Let $\bar{a} = c\bar{x}$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} = \frac{1}{n} \sum (a - \bar{a})(y - \bar{y}) - \bar{a}\bar{y}$$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} = \frac{1}{n} \sum (cx - c\bar{x})(y - \bar{y}) - c\bar{x}\bar{y}$$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} = \frac{1}{n} \sum c(x - \bar{x})(y - \bar{y}) - c\bar{x}\bar{y}$$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} = \frac{c}{n} \sum (x - \bar{x})(y - \bar{y}) - c\bar{x}\bar{y}$$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} < c \left(\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} \right)$$

- d. Let $a = \frac{x - \bar{x}}{\sigma_x}$ and $b = \frac{y - \bar{y}}{\sigma_y} \Rightarrow 0 \leq a, b \leq 1$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} = \frac{1}{n} \sum (a - \bar{a})(b - \bar{b}) - \bar{a}\bar{b}$$

Let $e = a - \bar{a}$ and $f = b - \bar{b}$ with $-1 \leq e, f \leq 1$

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} = \frac{1}{n} \sum ef - \bar{a}\bar{b}$$

Since $-1 \leq \frac{1}{n} \sum ef \leq 1$ and $0 \leq \bar{a}\bar{b} \leq 1$,

$$\frac{1}{n} \sum (x - \bar{x})(y - \bar{y}) - \bar{x}\bar{y} > \frac{1}{n} \sum (a - \bar{a})(b - \bar{b}) - \bar{a}\bar{b}$$

4. Attribute Classification:

- Number of years since 1 BC - Discrete, Quantitative, Interval
- GPA received by a student - Discrete, Quantitative, Ordinal
- Mood of blogger - Discrete, Qualitative, Nominal
- Sound intensity in dB - Continuous, Quantitative, Ratio