

Cost Functions for Half-space Models

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Outline

The Perceptron Cost and Softmax

The Margin Perceptron and Other Cost Functions

Accuracy and Counting Costs

The Logistic Regression Cost

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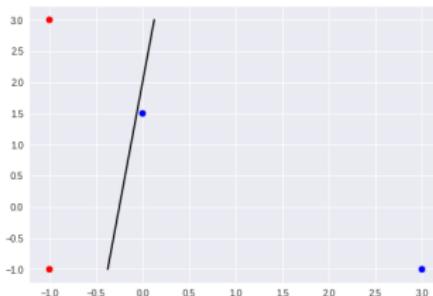
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Returning to the “Simple Example” for Perceptron algorithm

Example in \mathbb{R}^2 , with $P = 4$ points.

$$\mathbf{x}: \begin{bmatrix} -1 & 3 \\ -1 & -1 \\ 3 & -1 \\ 0 & 1.5 \end{bmatrix} \quad \mathbf{y}: \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

Final $\tilde{\mathbf{w}} = \begin{bmatrix} 1 \\ 4 \\ -0.5 \end{bmatrix}$; hyperplane in \mathbb{R}^2 (a line) shown below.



What if this data were part of a sample, which has noise? What about the point near the hyperplane?

Getting a Buffer – a “margin”

Say that $S = \{\mathbf{x}_i, y_i\}_{i=1}^P$ (with $y_i = \pm 1$) is linearly separable. Instead of simply trying to find a hyperplane that successfully separates the data, try to find one that has some distance from points on each side.

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Can express this as wanting no points between two parallel hyperplanes – between the set of \mathbf{x} where $\tilde{\mathbf{x}}^\top \tilde{\mathbf{w}} = 1$ and where $\tilde{\mathbf{x}}^\top \tilde{\mathbf{w}} = -1$.

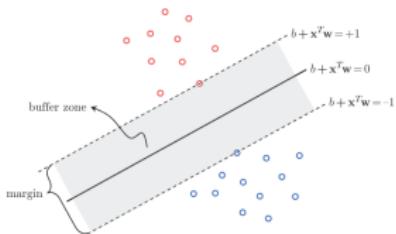


Figure: Taken from Figure 4.4 in textbook, p. 79. (Note: they used blue for the -1 label.)

This is equivalent to wanting, for all $i = 1, \dots, P$, $\tilde{\mathbf{x}}_i^\top \tilde{\mathbf{w}} \geq 1$ if $y_i = 1$ and $\tilde{\mathbf{x}}_i^\top \tilde{\mathbf{w}} \leq 1$ if $y_i = -1$. That is, we want $y_i(\tilde{\mathbf{x}}_i^\top \tilde{\mathbf{w}}) \geq 1$. Recall, S being linearly separable implies that there must be a $\tilde{\mathbf{w}}$ that achieves this.

Cost function for margin halfspace model

As we are looking for $\tilde{\mathbf{w}}$ so that $y_i(\tilde{\mathbf{x}}_i^\top \tilde{\mathbf{w}}) \geq 1$ for all $i = 1, \dots, P$, it makes sense to have (\mathbf{x}_i, y_i) contribute 0 to our cost function if this inequality works for that i . So, we might set

$$g_3(\tilde{\mathbf{w}}) = \sum_{i=1}^P \max \left(0, 1 - y_i(\tilde{\mathbf{x}}_i^\top \tilde{\mathbf{w}}) \right).$$

(Note that, when not using a margin and using the max cost function, rather than softmax, there was a trivial minimizer of $\tilde{\mathbf{w}} = \vec{0}$, which we do not want to use.)

Softmax approximation for margin halfspace: As before, if we want a smooth function we can replace the max in the cost function with softmax:

$$(Alternate) \quad g_3(\tilde{\mathbf{w}}) = \sum_{i=1}^P \text{softmax} \left(0, 1 - y_i(\tilde{\mathbf{x}}_i^\top \tilde{\mathbf{w}}) \right).$$

Squared max for margin halfspace: Another option for a

$$g_4(\tilde{\mathbf{w}}) = \sum_{i=1}^P \left(\max \left(0, 1 - y_i(\tilde{\mathbf{x}}_i^\top \tilde{\mathbf{w}}) \right) \right)^2.$$

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