Assessing accuracy of the LSR line

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Underlying assumption

- Modeled points in a plane as being from a line, but with noise in the y-coordinate direction. In other words, we assumed an underlying relationship

$$y = mx + b + \varepsilon$$

for some m and b, and a random variable ε[1] that has expected value 0. Alternatively, among the "entire population" there is an LSR line $mx + b$.

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  for some m and b, and a random variable ε[1] that has expected value 0. Alternatively, among the "entire population" there is an LSR line $mx + b$.
- Assumption: ε is independent of x.

When we have a data set $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, from the population, our procedure determines an LSR line $\hat{m}x + \hat{b}$. However, $\hat{m}$ and $\hat{b}$ are not the slope and intercept for the population curve m and b.

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```
x = np.random.uniform(0, 2, size=30)

def simulate_data(x, std):
    return -1.6*x + 0.8 + np.random.normal(0, std, size=len(x))
y = simulate_data(x, 0.5)
```

In groups, compute slope and intercept of the LSR line for a size 30 simulated data set; store $\hat{m}$ and $\hat{b}$ (in two lists). Iterate this 1000 times $\longrightarrow$ a list of 1000 slopes and intercepts.

What is the mean of the slopes and of the intercepts?

Sample statistic, relation to population statistic

This fundamental to statistics.

- Say that a sample of 2000 people are selected from around the country and their height is measured. Mean of these 2000 heights: sample mean.
- Sample mean differs from the true mean height of the entire population of the country. (Perhaps, not by much.)
  - Weak Law of Large Numbers: if s random samples of 2000 people taken, and each sample mean calculated, as $s \longrightarrow \infty$, mean of the sample means limits to population mean.
- Analogous thing happens with data from linear relationship with noise – think of parameters $\hat{m}$ and $\hat{b}$ as sample statistics (like sample mean).

Confidence intervals

How close do we suspect $\hat{m}$ and $\hat{b}$ to be to the "true" (population) slope and intercept?

Standard error (SE): Suppose that for our error term ε, we have $\mathrm{Var}(\varepsilon) = \sigma^2$. Sample size: n.

Using $\bar{x}$ for the average of $x_1, \ldots, x_n$,

$$SE(\hat{m})^2 = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2};$$

$$SE(\hat{b})^2 = \sigma^2 \left(\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \right).$$