The Random package, Defining Custom Functions

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Outline

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   Commands above will sample with replacement.
   - To sample without replacement, use argument `replace=False`.

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(b-a)*np.random.random(size=m) + a
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Larger the sample, the closer the distribution of that sample (i.e., normalized histogram) should be to the theoretical pdf. Can we display this in a plot?

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Below: sample of size 50, from `uniform(0, 5)`.

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xx = np.linspace(0,5)
sample = np.random.uniform(0, 5, size=50)
# will also plot horizontal line at height 1/(b-a)
plt.plot(xx, [1/5]*len(xx), color='black')
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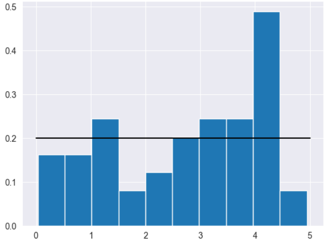

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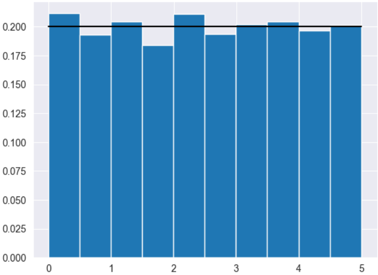

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Using the normal distribution, mean μ and std.deviation σ, the density function is

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

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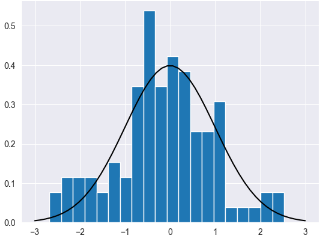

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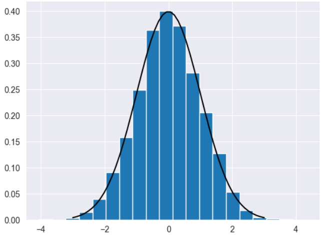
A larger sample size gives a distribution that is closer to the bell curve.



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  - This changes `the_array` in place.
- Not just for 1d arrays; can get a matrix (or any order tensor) with entries sampled from the distribution by adjusting the `size` argument.

```
# to get 30x2 matrix, entries from normal N(0,1)
np.random.normal(size=(30,2))
```

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How to define a custom function

The basic structure for a definition of a custom function has four parts: the function name, a list of arguments it takes, the “body” of the function (what happens when it is called), and what it returns.

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def my_function(v):
    reverse_v = v[::-1]
    multiplied = reverse_v * v
    zeros = np.zeros(len(v))
    return np.maximum(multiplied, zeros)
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```
def coloring(array_of_points):
    ref_vector = np.array([2,-2/3])
    dotvalues = array_of_points@ref_vector
    colors = np.array(['darkblue' if v < 0 else 'salmon' for v in dotvalues])
    return colors
```

Additional arguments, default values, keyword arguments

```
def my_function(v, *, translate=2):
    reverse_v = v[::-1]
    multiplied = reverse_v * v + translate
    zeros = np.zeros(len(v))
    return np.maximum(multiplied, zeros)
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A column of a dataframe has a method, .to_numpy(), that converts it to a NumPy array.

Regression line on data in plane

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Getting the "best fit" regression line[4] is straightforward in NumPy.

```
# returns slope and intercept of best fit line
np.polyfit(so2, pm10, 1)
```

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