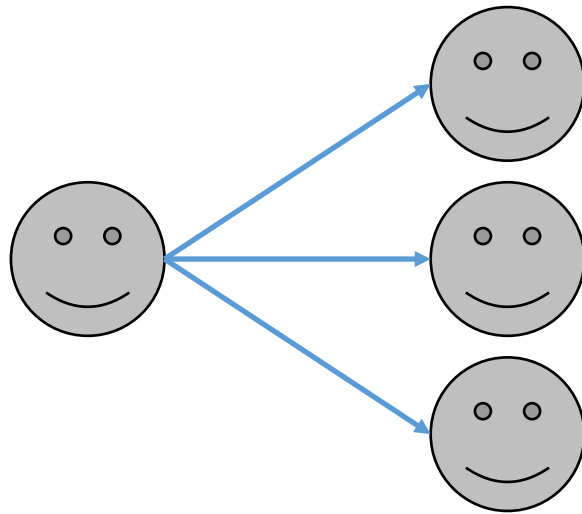


R_0 or R_e or Doubling-Time: Which to Follow?

CA-ANG/SG

25 March 2020

R = reproduction number
= average number cases arising from one case



$$R = 3$$

R = reproduction number

= average number cases arising from one case



This number changes as the epidemic progresses

As people recover, more of the population is immune to the virus.
The virus has a harder time spreading.
R decreases.

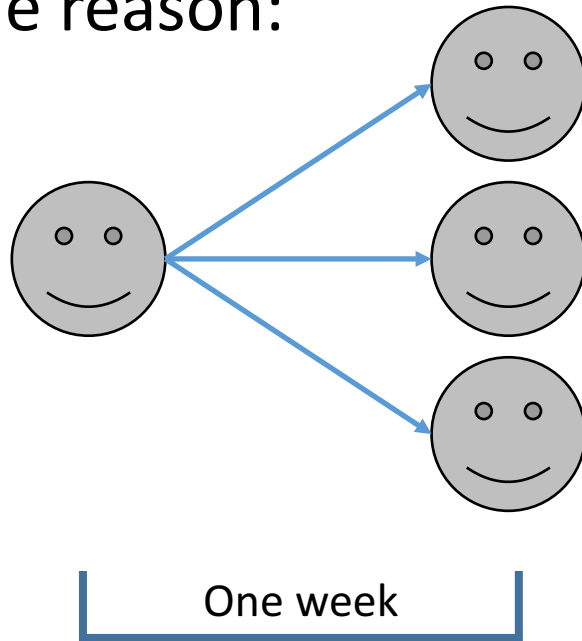
R_0 = initial reproduction number
(at beginning of epidemic)

R_e = effective reproduction number
(at later times in the epidemic)

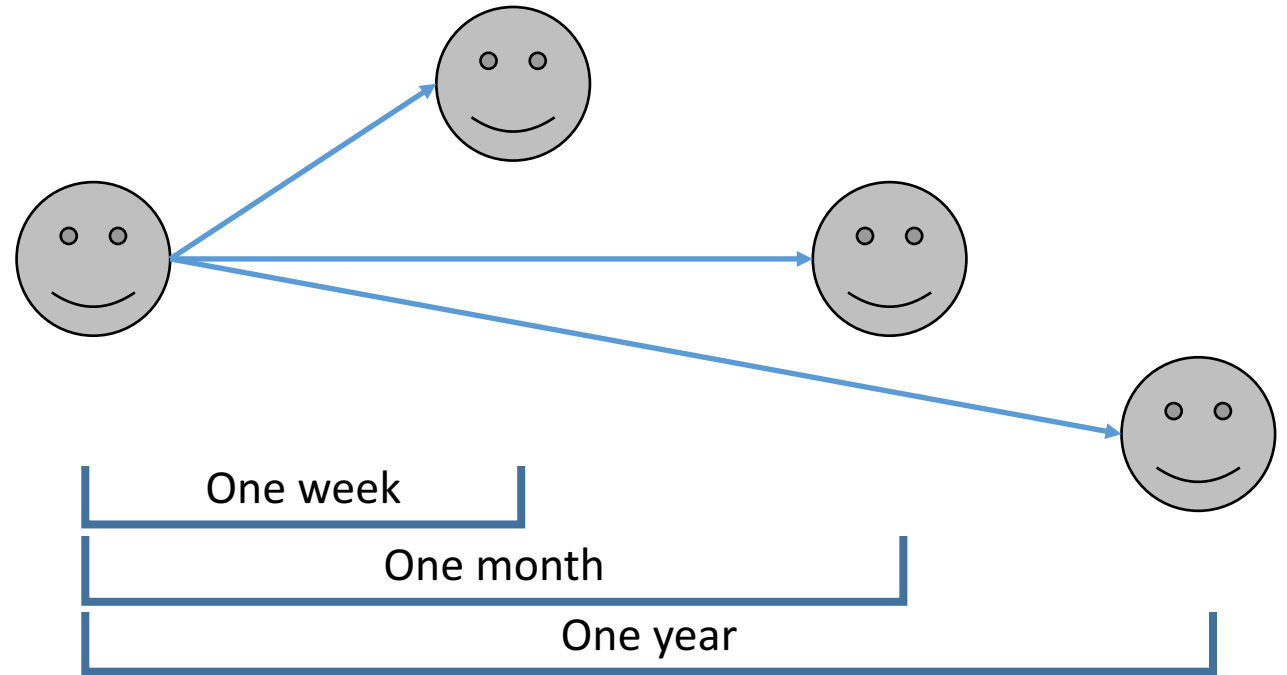
- Generally $R_0 > R_e$
- Generally R_e falls as epidemic progresses
- Epidemic ends when $R_e < 1$: spread is no longer exponential

R_0 & R_e : Hard to Measure, Sometimes Misleading

- One reason:



$$R = 3$$



$$R = 3$$

Yes, R_0 and two more quantities define epidemic

- And it should be obvious that:
- $$I_{total} = \int_{t=0}^{t=\infty} \left[\frac{R_0}{(1+d)^t} \right]^t dt$$

$$I_{total} = \frac{\exp\left(\frac{\ln(R_0)^2}{4\ln(1+d)}\right) \sqrt{\frac{\pi}{\ln(1+d)}}}{2} \bullet$$
$$\left[\operatorname{erf}(x - \mu) \sqrt{\ln(1+d)} - \operatorname{erf}(-\mu) \sqrt{\ln(1+d)} \right]$$

Where

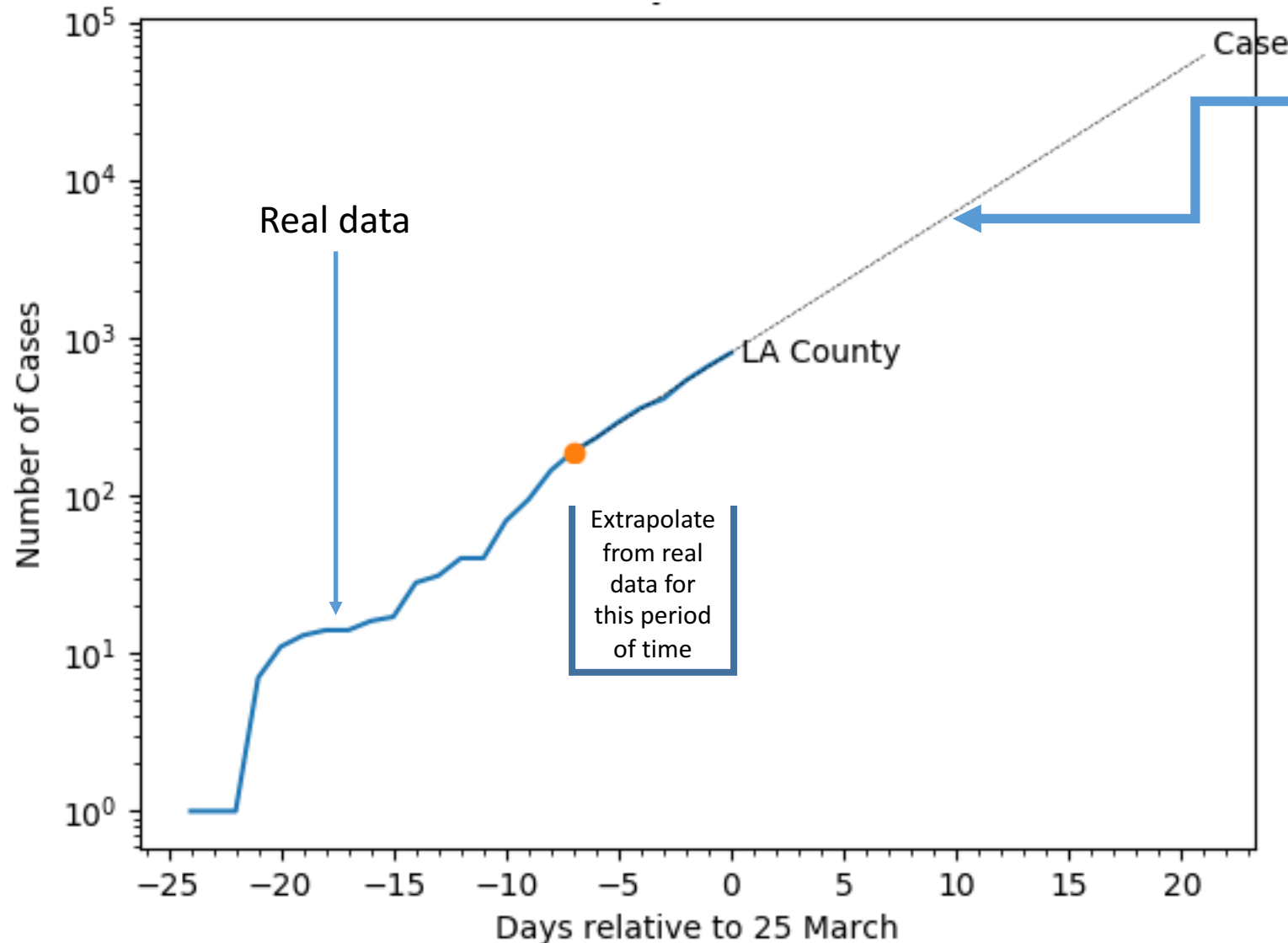
$$\mu = \frac{\ln(R_0)}{2\ln(1+d)}$$

“Doubling Time” is a simpler way

- Graph the number of total cases over the course of the epidemic
- Extrapolate from the last few days of the curve
- From this extrapolation, calculate how long it will take for the number of cases to double
 - This is the “doubling time”

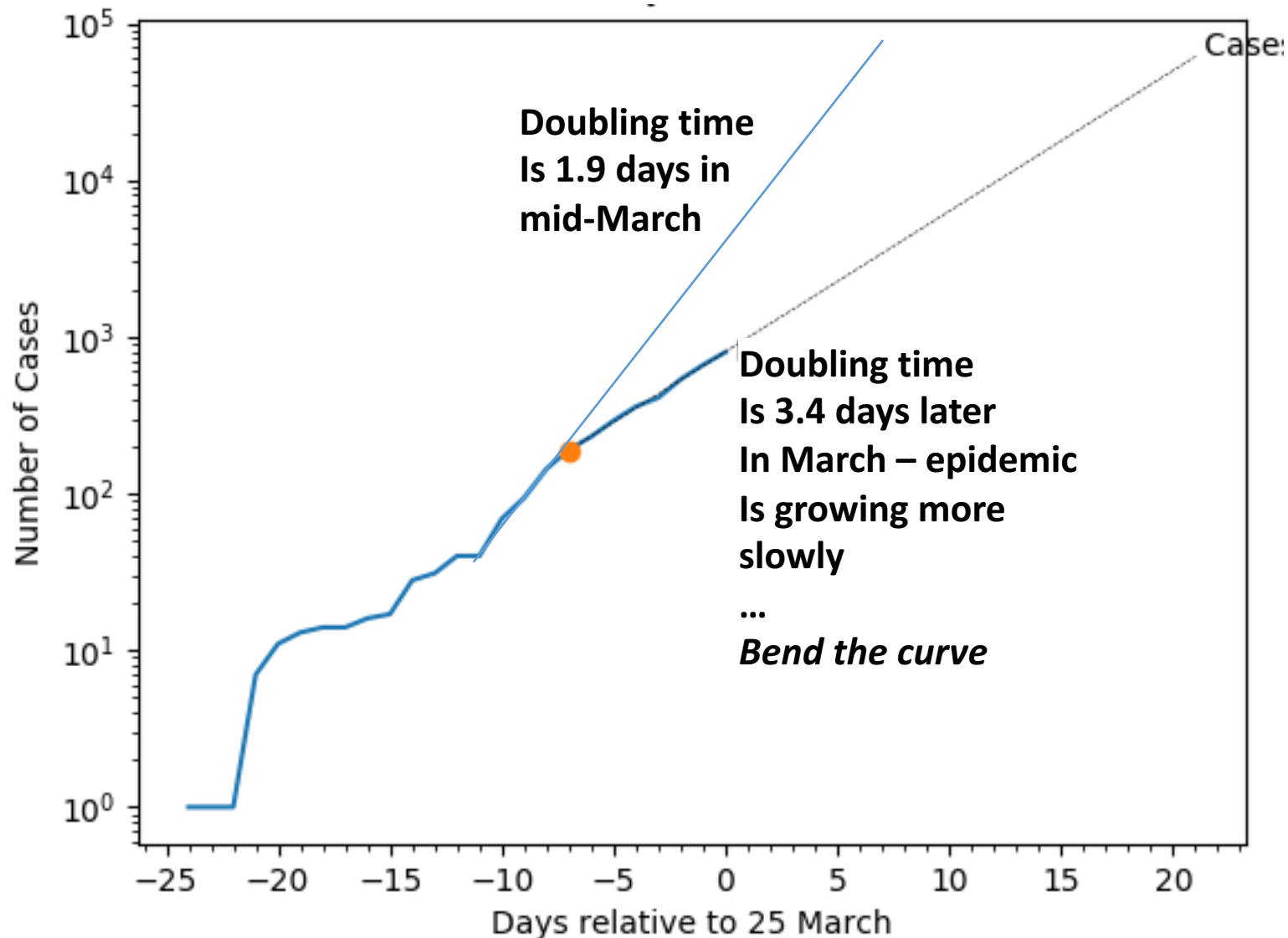
Graphical approach

(note logarithmic y-axis)



- Extrapolation
- Slope of extrapolated line determines the doubling time
- One simple division to compute it
- Here, number of cases doubles every 3.4 days

Doubling Time changes



- Extrapolation
- Slope of extrapolated line determines the doubling time
- Divide $\log 2$ by the slope to get doubling time

Governor Cuomo uses doubling times



PLAN OF ACTION

Evidence suggests density control plan working:

- **Sunday:** hospitalizations doubling every 2 days
- **Monday:** hospitalizations doubling every 3.4 days
- **Tuesday:** hospitalizations doubling every 4.7 days

Summary

- R_0 is good for describing early stages of epidemic
- R_e is hard to know
- Doubling time
 - Is easy to calculate
 - Has an easily understood meaning
 - Is well-suited to making decisions

References

- The monster equation
 - <https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0083622>