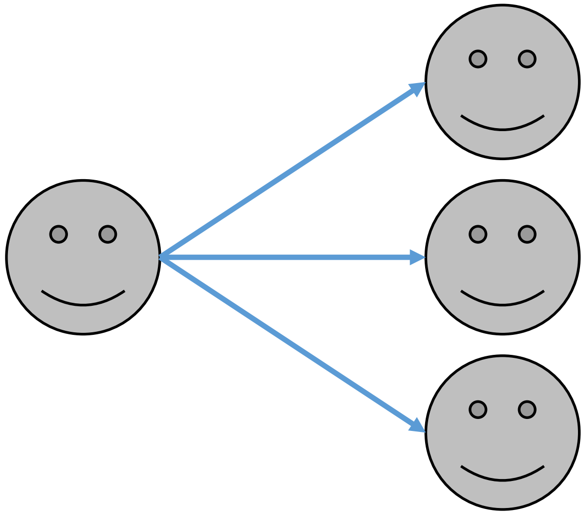
R_0 or R_e or Doubling-Time: Which to Follow?

CA-ANG/SG

25 March 2020

R = reproduction number

= average number cases arising from one case



R = 3

R = reproduction number

= average number cases arising from one case

This number changes as the epidemic progresses

As people recover, more of the population is immune to the virus.
The virus has a harder time spreading.
R decreases.

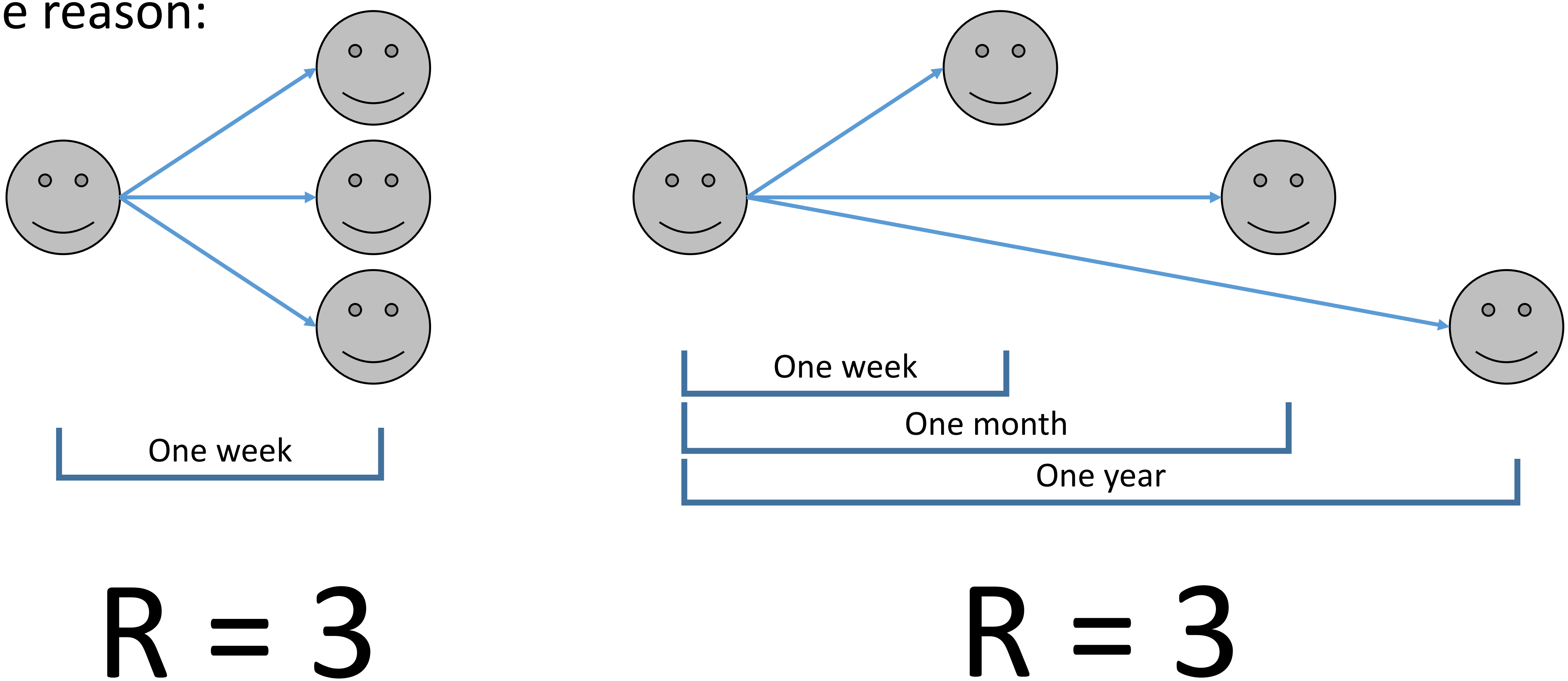
R_0 = initial reproduction number (at beginning of epidemic)

R_e = effective reproduction number (at later times in the epidemic)

- Generally $R_0 > R_e$
- Generally R_e falls as epidemic progresses
- Epidemic ends when $R_e < 1$: spread is no longer exponential

R_0 & R_e: Hard to Measure, Sometimes Misleading

- One reason:



Yes, R_0 and two more quantities define epidemic

- And it should be obvious that:

$$I_{total} = \int_{t=0}^{t=\infty} \left[\frac{R_0}{(1+d)^t}\right]^t dt$$

$$I_{total} = \frac{\exp\left(\frac{\ln(R_o)^2}{4\ln(1+d)}\right)\sqrt{\frac{\pi}{\ln(1+\mathrm{d})}}}{2} \bullet \left[erf(x-\mu)\sqrt{\ln(1+d)} - erf(-\mu)\sqrt{\ln}(1+d)\right]$$

Where

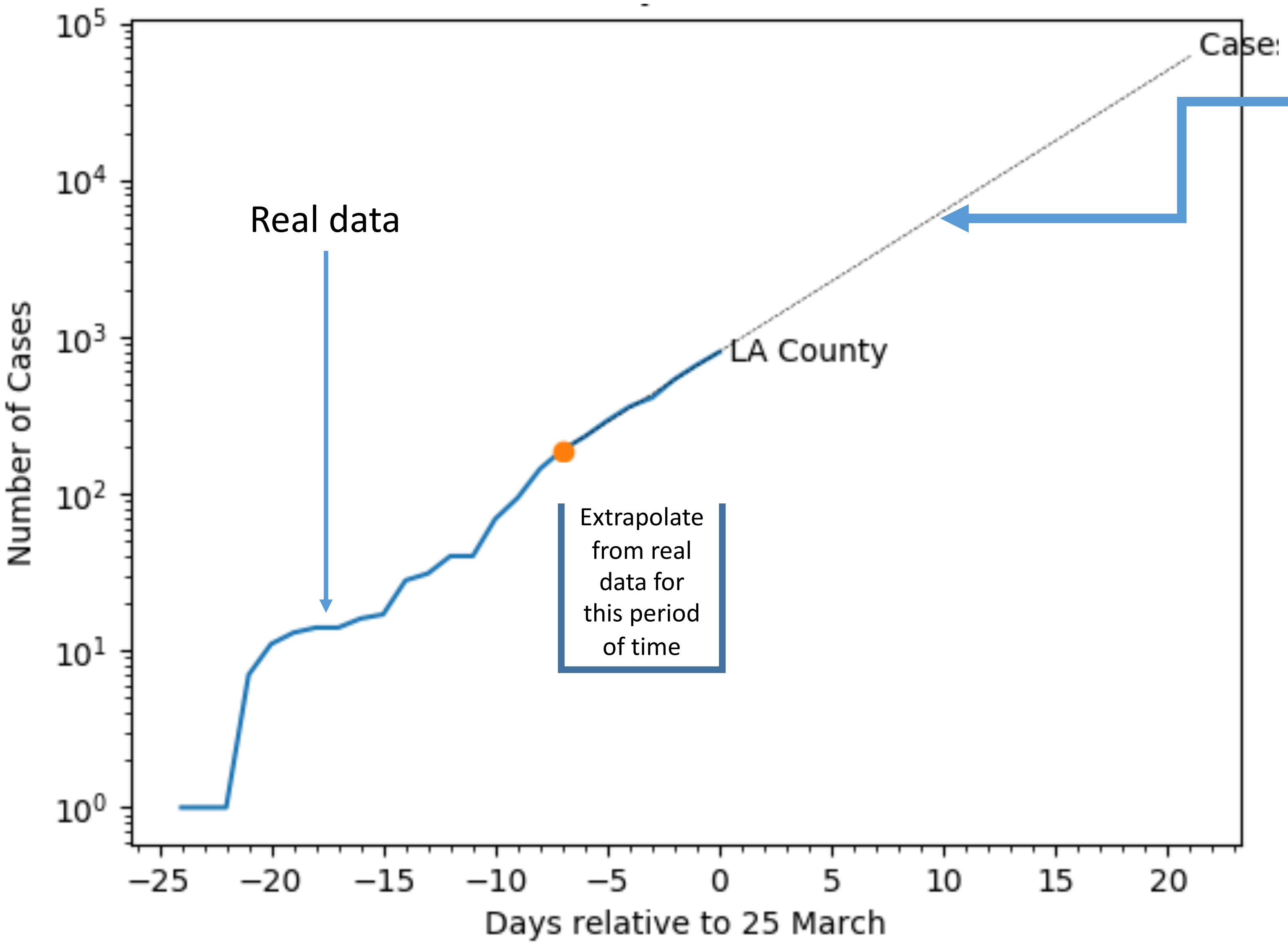
$$\mu = \frac{\ln(R_0)}{2\ln(1+d)}$$

“Doubling Time” is a simpler way

- Graph the number of total cases over the course of the epidemic
- Extrapolate from the last few days of the curve
- From this extrapolation, calculate how long it will take for the number of cases to double
  - This is the “doubling time”

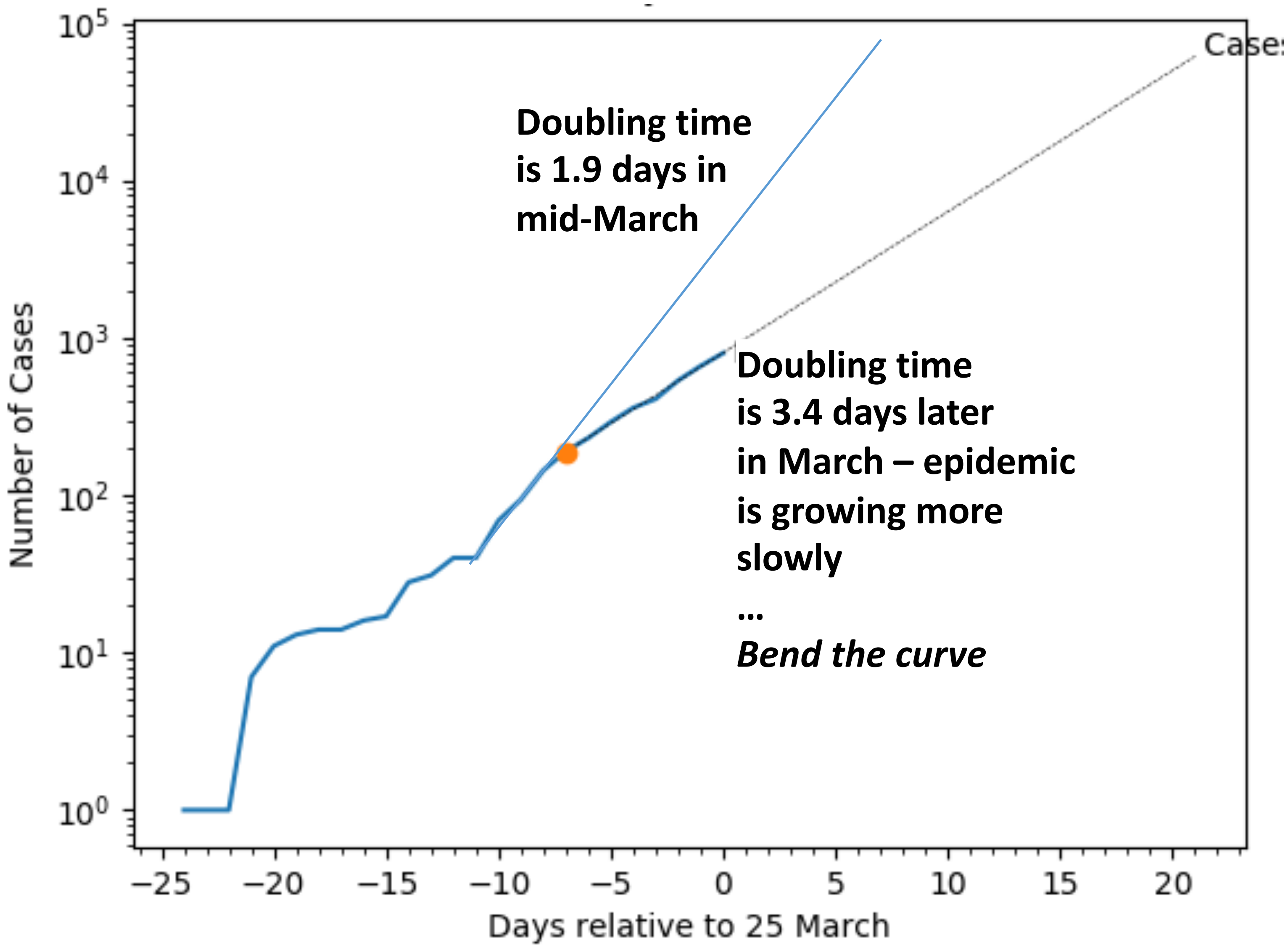
Graphical approach

(note logarithmic y-axis)

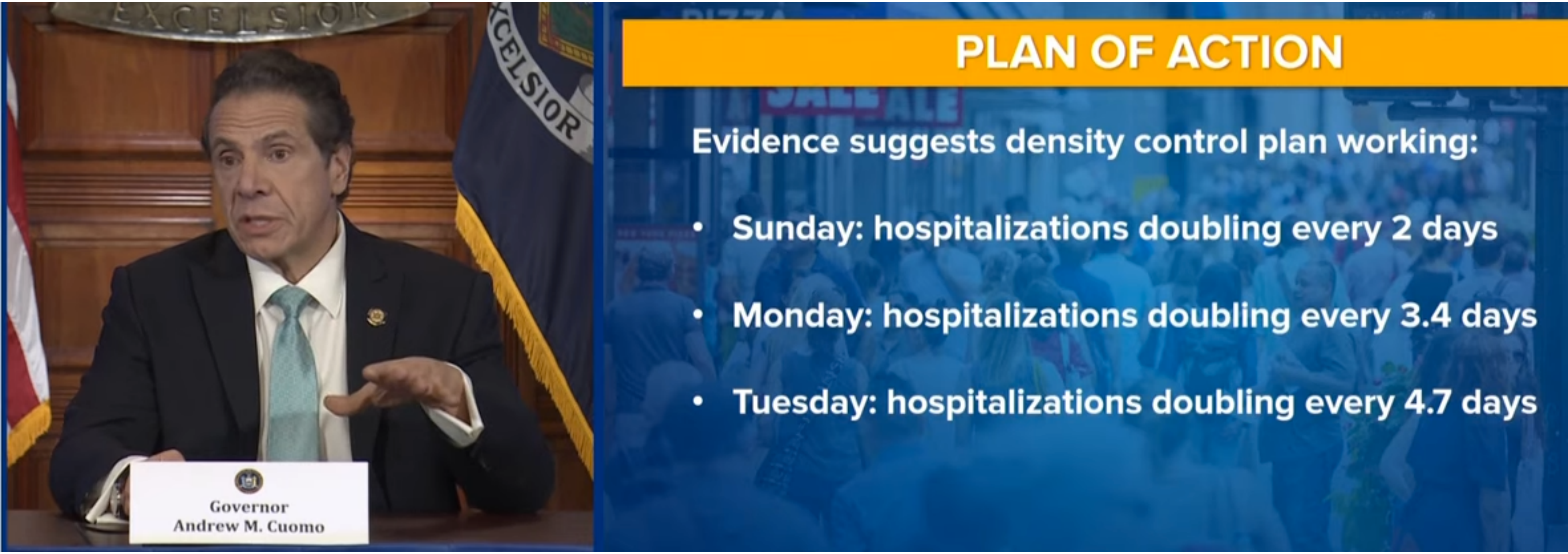


- Extrapolation
- Slope of extrapolated line determines the doubling time
- One simple division to compute it
- Here, number of cases doubles every 3.4 days

Doubling Time changes during epidemic



Governor Cuomo uses doubling times



https://www.cnbc.com/2020/03/25/watch-live-ny-gov-cuomo-holds-a-press-conference-on-the-coronavirus.html

Summary

- R_0 is good for describing early stages of epidemic
- R_e is hard to know

- Doubling time
  - Is easy to calculate
  - Has an easily understood meaning
  - Is well-suited to making decisions

References

- The monster equation
  - https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0083622