Introduction to Regression Models for Panel Data Analysis

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Professor Patricia A. McManus

What are Panel Data?

Panel data are a type of *longitudinal data*, or data collected at different points in time. Three main types of *longitudinal data*:

- Time series data. Many observations (large t) on as few as one unit (small N). Examples: stock price trends, aggregate national statistics.

- Pooled cross sections. Two or more *independent* samples of many units (large N) drawn from the same population at different time periods:
    - General Social Surveys
    - US Decennial Census extracts
    - Current Population Surveys*

- Panel data. Two or more observations (small t) on many units (large N).

    - Panel surveys of households and individuals (PSID, NLSY, ANES)
    - Data on organizations and firms at different time points
    - Aggregated regional data over time

- This workshop is a basic introduction to the analysis of *panel data*. In particular, I will cover the *linear error components model*.

Why Analyze Panel Data?

- We are interested in *describing* change over time
  - social change, e.g. changing attitudes, behaviors, social relationships
  - individual growth or development, e.g. life-course studies, child development, career trajectories, school achievement
  - occurrence (or non-occurrence) of events

- We want *superior estimates* trends in social phenomena
  - Panel models can be used to inform policy – e.g. health, obesity
  - Multiple observations on each unit can provide superior estimates as compared to cross-sectional models of association

- We want to estimate *causal models*
  - Policy evaluation
  - Estimation of treatment effects

What kind of data are required for panel analysis?

- Basic panel methods require at least two "waves" of measurement.

  Consider student GPAs and job hours during two semesters of college.

- One way to organize the panel data is to create a single record for each combination of unit and time period:

StudentID	*Semester*	*Female*	*HSGPA*	*GPA*	*JobHrs*
17	5	0	2.8	3.0	0
17	6	0	2.8	2.1	20
23	5	1	2.5	2.2	10
23	6	1	2.5	2.5	10

- Notice that the data include:
  - A *time-invariant* unique identifier for each unit (*StudentID*)
  - A *time-varying* outcome (*GPA*)
  - An indicator for time *(Semester*).

- Panel datasets can include other time-varying or time-invariant variables

- An alternative way to structure the data is to keep all the measures related to each student in a single record. This is sometimes called "wide" format.

StudentID	*Female*	*HSGPA*	*GPA5*	*JobHrs5*	*GPA6*	*JobHrs6*
17	0	2.8	3.0	0	2.1	20
23	1	2.5	2.2	10	2.5	10

  - Why are there two variables for *GPA* and *JobHrs* ?
  - Why is there only one variable for gender and high school GPA?
  - Where is the indicator for time?

Estimation Techniques for Panel Models

We can write a simple panel equation predicting GPA from hours worked:

$$GPA_{it} = \beta_0 + TERM_{it}\beta_T + HSGPA_{it}\beta_H + JOB_{it}\beta_J + v_{it}$$

- General Linear Model is the foundation of linear panel model estimation
  - Ordinary Least Squares (OLS)
  - Weighted least squares (WLS)
  - Generalized least squares (GLS)

Least-squares estimation of panel models typically entails three steps:

(a) Data transformation or first-stage estimation

(b) Estimation of the parameters using Ordinary Least Squares

(c) Estimation of the variance-covariance matrix of the estimates (VCE)

Parameter estimates are sometimes refined using iteratively reweighted least squares (IRLS), a maximum likelihood estimator.

Basic Questions for the Panel Analyst

What's the story you want to tell?

- Is this a descriptive analysis? Less worry, fewer controls are usually better.
- Is this an attempt at causal analysis using observational data? Careful specification *AND* theory is essential.

How does time matter?

- Some analyses, e.g. difference-in-difference analysis associates time with an event (before and after)
- Some analyses may be interested in growth trajectories.
- Panel analysis may be appropriate even if time is irrelevant. Panel models using cross-sectional data collected at fixed periods of time generally use dummy variables for each time period in a two-way specification with fixed-effects for time.

Are the data up to the demands of the analysis?

- Panel analysis is data-intensive. Are two waves enough?
- Can you perform the necessary specification tests?
- How will you address panel attrition?

Review of the Classical Linear Regression Model

$$y_i = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + ... + x_{ki}\beta_k + u_i \ , \ i=1,2,3,...N$$

Where we assume that the linear model is correct and:

- Covariates are Exogenous: $E\ u_i \mid x_{1i}, x_{2i}, .., x_{ki} = 0$
- Uncorrelated errors: $Cov\ u_i, u_j = 0$
- Homoskedastic errors: $Var\ u_i = Var\ y_i \mid x_{1i}, x_{2i}, ..., x_{ki} = \sigma^2$

If assumptions do not hold, OLS estimates are BIASED and/or INEFFICIENT

- Biased - Expected value of parameter estimate is different from true.
  - Consistency. If an estimator is unbiased, or if the bias shrinks as the sample size increases, we say it is *CONSISTENT*
- Inefficient - (Informally) Estimator is less accurate as sample size increases than an alternative estimator.
  - Estimators that take full advantage of information more efficient

OLS Bias Due to Endogeneity

- Omitted Variable Bias
  - Intervening variables, selectivity
- Measurement Error in the Covariates
- Simultaneity Bias
  - Feedback loops
  - Omitted variables

Conventional regression-based strategies to address endogeneity bias

- Instrumental Variables estimation
- Structural Equations Models
- Propensity score estimation
- Fixed effects panel models

OLS Inefficiency due to Correlated Errors

Many data structures are susceptible to error correlation:

- Hierarchical data sample multiple individuals from each unit, e.g. household members, employees in firms, multiple pupils from each school.
- Multistage probability samples often incorporate cluster-based sampling designs with errors that may be correlated within clusters.
- Repeated observations data often show within-unit error correlation.
- Time series data often have errors that are serially correlated, that is, correlated over time.
- Panel data have errors that can be correlated within unit (e.g. individuals), within period.

Conventional regression-based strategies to address correlated errors

- Cluster-consistent covariance matrix estimator to adjust standard errors.
- Generalized Least Squares instead of OLS to exploit correlation structure.

Linear Panel Data Model (LPM)

Suppose the data are on each cross-section unit over *T* time periods:

$$
\begin{aligned}
y_{i,t1} &= \mathbf{x}'_{i,t1}\boldsymbol{\beta}_{t1} + u_{i,t1} \\
y_{i,t2} &= \mathbf{x}'_{i,t2}\boldsymbol{\beta}_{t2} + u_{i,t1} \\
&\vdots \\
y_{i,T} &= \mathbf{x}'_{i,T}\boldsymbol{\beta}_{T} + u_{i,T}
\end{aligned}
\quad , \qquad t=1,2,\ldots,T
$$

We can express this concisely using $\mathbf{y}_i$ to represent the vector of individual outcomes for person *i* across all time periods:

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{u}_i, \text{ where } \mathbf{y}'_i = y_{i,t1}, y_{i,t2}, \ldots, y_{iT}$$

For comparison, begin with two conventional OLS linear regression models, one for each period. Note that the variables `female highgpa (HS GPA)` is time-invariant.

OLS Results for each term:

	Term 5 GPA			Term 6 GPA		
	Estimate	SE	*t*-stat	Estimate	SE	*t*-stat
Intercept	3.02	0.17	17.8	3.02	0.17	18.3
jobhrs	-0.182	0.05	-4.0	-0.174	0.05	-3.6
female	0.108	0.04	2.5	0.145	0.05	3.2
highgpa	-0.004	0.04	-0.1	0.003	0.04	0.1

Pooled OLS Results for both terms:

	Term 5&6 GPA			Term 5&6 GPA (Clustered SE)		
	Estimate	SE	*t*-stat	Estimate	SE	*t*-stat
Intercept	2.97	0.17	25.1	2.97	0.17	17.2
jobhrs	-0.178	0.05	-5.4	-0.178	0.05	-5.8
female	0.125	0.04	4.1	0.125	0.04	3.0
highgpa	-0.0001	0.03	-0.01	0.0001	0.03	-0.0004
term6	0.095	0.016	6.1	0.095	0.016	6.1

Linear Unobserved Effects Panel Data Model

- Motivation: Unobserved heterogeneity

Suppose we have a model with an unobserved, time-constant variable *c*:

$$y = \beta_0 + x_1\beta_1 + x_2\beta_2 + ... + x_k\beta_k + c + u$$

Where u is uncorrelated with all explanatory variables in **x**.

Because c is unobserved it is absorbed into the error term, so we can write the model as follows:

$$y = \beta_0 + x_1\beta_1 + x_2\beta_2 + ... + x_k\beta_k + v$$
$$v = c + u$$

The error term v consists of two components, an "idiosyncratic" component *u* and an "unobserved heterogeneity" component c.

OLS Estimation of the Error Components Model

- If the unobserved heterogeneity c_i is correlated with one or more of the explanatory variables, OLS parameter estimates are biased and inconsistent.

- If the unobserved heterogeneity c is uncorrelated with the explanatory variables in $\mathbf{x}_i$, OLS is unbiased even in a single cross-section.

- If we have more than one observation on any unit, the errors will be correlated and OLS estimates will be inefficient

$$
\begin{aligned}
y_{i,1} &= \beta_0 + x_{1_{i1}}\beta_1 + x_{2_{i1}}\beta_2 + \ldots + x_{k_{i1}}\beta_k + v_{i,1} \\
y_{i,2} &= \beta_0 + x_{1_{i2}}\beta_1 + x_{2_{i2}}\beta_2 + \ldots + x_{k_{i2}}\beta_k + v_{i,2} \\
v_{i,1} &= c_i + u_{i,1} \\
v_{i,2} &= c_i + u_{i,2} \\
cov(v_{i,1}, v_{i,2}) &\neq 0
\end{aligned}
$$

- Unobserved Heterogeneity in Panel Data

Suppose the data are on each cross-section unit over *T* time periods.

This is an **unobserved effects model** (UEM), also called the **error components model.** We can write the model for each time period:

$$\begin{aligned} y_{i1} &= \mathbf{x}_{i1}\boldsymbol{\beta} + c_i + u_{i1} \\ y_{i2} &= \mathbf{x}_{i2}\boldsymbol{\beta} + c_i + u_{i2} \\ &\vdots \\ y_{iT} &= \mathbf{x}_{iT}\boldsymbol{\beta} + c_i + u_{iT} \end{aligned} ,$$

Where there are *T* observations on outcome *y* for person *i*,

$\mathbf{x}_{it}$ is a vector of explanatory variables measured at time *t*,

c_i is unobserved in all periods but constant over time

u_{it} is a time-varying idiosyncratic error

Define $v_{it} = c_i + u_{it}$ as the composite error.

Consistent estimation of the Error Components Model with Pooled OLS

If we assume no contemporaneous correlation of the errors and the explanatory variables, pooled OLS estimation is *consistent:*

$E(\mathbf{x}_{it}^{'} u_{it}) = \mathbf{0}$ and $E(\mathbf{x}_{it}^{'} c_i) = \mathbf{0}$, t=1,2,…,T

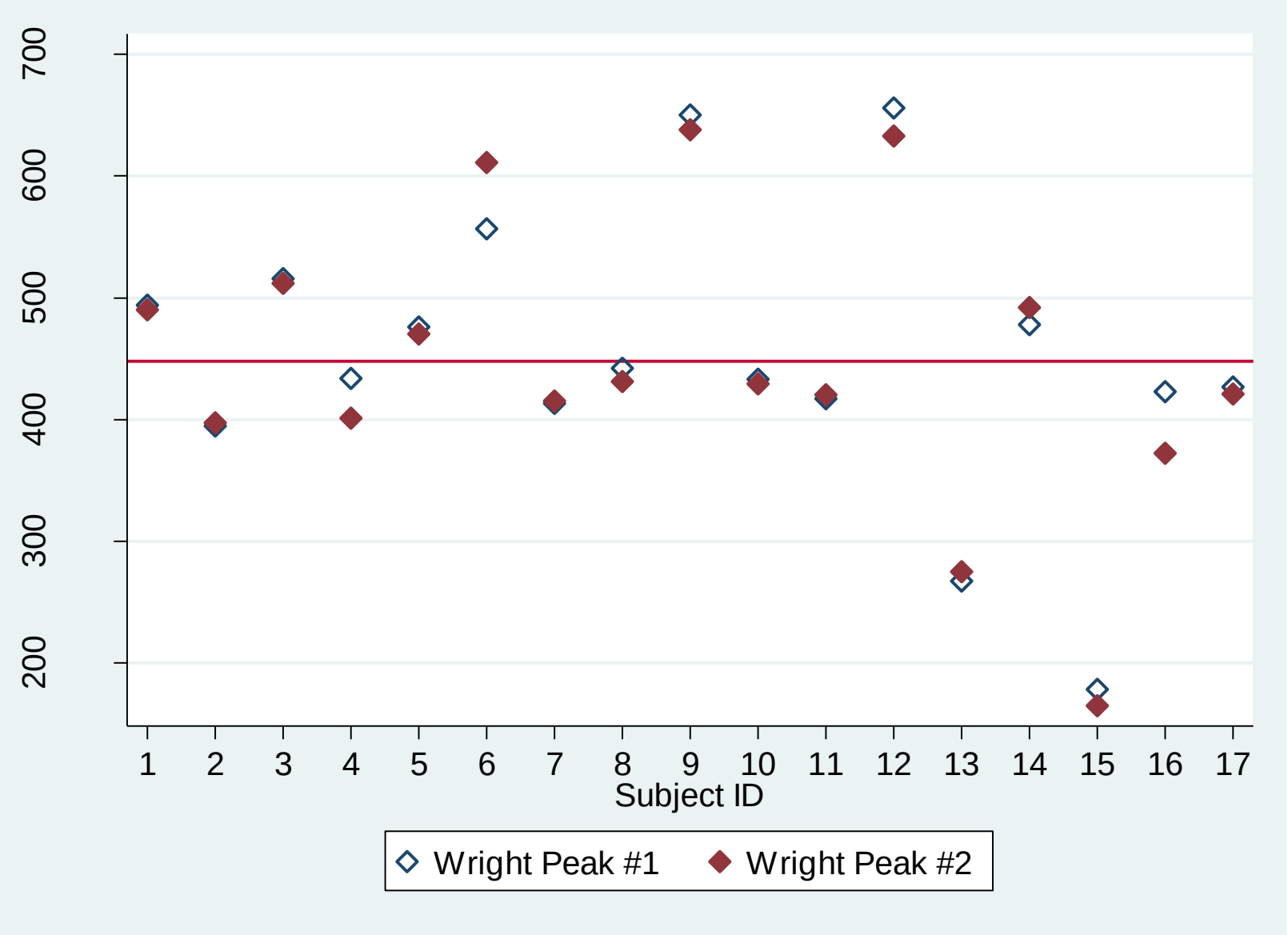
Efficient estimation of the Error Components Model with Pooled OLS

Even if estimation is consistent, pooled OLS may not be efficient.

- One strategy is to combine pooled OLS with cluster-consistent standard errors.
- Panel GLS methods may be preferred.

In the next sections, we consider the dominant approaches to estimation of the error components panel model: **fixed effects** and **random effects.**

Illustration of Within-unit correlation. Peak-flow Measurements



Just a few panel data examples:

Propper and Van Reenen (2010)

Effect of regulation of nursing pay on hospital quality

Data: 209 NHS Hospitals in the UK 1997-2005

Western, Bruce (2002)

Effect of Incarceration on wages and income inequality

Data: NLSY

Cherlin, Chase-Lansdale and McRae (1998)

Effect of parental divorce on mental health over life-course

Data: British Cohort Study

Jacobs and Carmichael (2002)

Determinants of Death Penalty in US states

Data: US Census 1970, 1980, 1990 + other sources

Baum and Lake (2003)

Effect of Democracy on Human Capital and Economic Growth

Data: Aggregate data on 128 countries over 30 years

Fixed Effects Methods for Panel Data

Suppose the unobserved effect c_i is correlated with the covariates.

Example: Motherhood wage penalty

- We observe that mothers earn less than other women, cet par.

  $\hat{\beta}_{KIDS_{OLS}} = -0.08$ in a log wage model suggests that each additional child reduces mothers' hourly wages by about 8%

  But if women who are less oriented towards work are also more likely to have more children, omitting "work orientations" from the model will bias the coefficient on children.

- Fixed-effects methods transform the model to remove c_i

  $\hat{\beta}_{KIDS_{FE}} = -0.03$ FE estimates a persistent but much smaller penalty.

- Caution: Fixed effects has some disadvantages
  - ⇨ FE is not a panacea for all sources of endogeneity bias.
    - *time-varying* unobserved effects
    - *time-varying* measurement error
    - *simultaneity* or feedback loops
  - ⇨ All time-constant effects are removed.
    - No estimation of effects of race, gender, birth order, etc.
    - Poor estimates if little variation (e.g. education in adulthood)
  - ⇨ FE trades consistency for efficiency.
    - FE uses *only* within-unit change, ignores between-unit variation.
    - Parameter estimates may be imprecise, standard errors large.
- Despite limitations, FE is an indispensable tool in the panel analyst's toolbox.

Fixed Effects Transformation - the "Within" Estimator

Suppose we have the UEM model:

$$y_{it} = \mathbf{x}_{it}'\beta + c_i + u_{it}, \qquad t=1,2,\ldots,T$$

For each unit, average this equation over all time periods *t*:

$$\bar{y}_i = \bar{\mathbf{x}}_i'\beta + \bar{c}_i + \bar{u}_i$$

Subtract the within-unit average from each observation on that unit:

$$y_{it} - \bar{y}_i = \left(\mathbf{x}_{it}' - \bar{\mathbf{x}}_i'\right)\beta + \left(c_i - \bar{c}_i\right) + \left(u_{it} - \bar{u}_i\right), \qquad t=1,2,\ldots,T$$

This is the **fixed effects transformation**. We can write it as:

$$\ddot{y}_{it} = \ddot{\mathbf{x}}_{it}'\beta + \ddot{u}_{it},$$

where $c_i - \bar{c}_i = 0$ and $\ddot{y}_{it} = y_{it} - \bar{y}_i$, $\ddot{\mathbf{x}}_{it} = \mathbf{x}_{it} - \bar{\mathbf{x}}_i$, $\ddot{u}_{it} = u_{it} - \bar{u}_i$

and $\ddot{\mathbf{x}}_{it}$ does not contain an intercept term.

The **fixed-effects estimator**, also called the **within estimator**, applies pooled OLS to the transformed equation:

$$\hat{\beta}_{FE} = \left(\sum_{i=1}^{N} \ddot{\mathbf{X}}_i' \ddot{\mathbf{X}}_i \right)^{-1} \left(\sum_{i=1}^{N} \ddot{\mathbf{X}}_i' \ddot{\mathbf{y}}_i \right) = \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \ddot{\mathbf{x}}_{it}' \ddot{\mathbf{x}}_{it} \right)^{-1} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \ddot{\mathbf{x}}_{it}' \ddot{y}_{it} \right)$$

Recall the student GPA Data:

StudentID	*Semester*	*Female*	*HSGPA*	*GPA*	*JobHrs*
17	5	0	2.8	3.0	0
17	6	0	2.8	2.1	20
23	5	1	2.5	2.2	10
23	6	1	2.5	2.5	10

After applying the fixed-effects transform, the demeaned (mean-centered) data:

StudentID	*Semester*	*CFemale*	*CHSGPA*	*CGPA*	*CJobHrs*
17	-.5	0	0	.45	-10
17	.5	0	0	-.45	10
23	-.5	0	0	-.15	0
23	.5	0	0	.15	0

Fixed Effects Dummy Variables Regression

Up to now, we've treated the unobservables c_i as random variables:

$$y_{it} = \mathbf{x}_{it}'\boldsymbol{\beta} + c_i + u_{it}$$

An alternative approach is to treat c_i as a fixed parameter for each unit. In this case, we can use dummy variables regression to estimate c_i.

Step one: Create a dummy variable for each of sample unit *i*

Step two: Substitute the vector of *N-1* dummies for c_i:

$$y_{it} = \gamma_1 + \mathbf{x}_{it}'\boldsymbol{\beta} + d2\gamma_2 + d3\gamma_3 + \ldots + dN\gamma_N + u_{it},$$

(where the intercept γ_1 estimates the effect when $d1$=1)

Step three: Estimate the equation using pooled OLS.

- The fixed effects dummy variables (FEDV) estimator produces precisely the same coefficient vector and standard errors as the FE estimator.

*** Practical asides

One way or two? Sometimes you will see "one-way" or "two-way" FE.

One-way fixed effects error components model - only the *unit* effects are conditioned out.

Two-way fixed effects error components model – both the *unit* effects and *period* effects are conditioned out.

In the conventional FE model with large *N* and small *T*, it is a simple matter to create dummy variables for each period, and most panel models will include controls for period effects.

Statistical Software

Most statistical packages offer several alternatives for estimating the FEM.

STATA	xtreg	areg	reg (with factor variables)
SAS	proc panel	proc glm (with "absorb" statement)	

First Differencing Methods

- The first difference (FD) model transforms the UEM model to remove the unobserved effects c_i
- FD is sometimes called a first-difference fixed effects model

Suppose we have the unobserved effects model (UEM):

$$y_{it} = \mathbf{x}_{it}'\beta + c_i + u_{it},$$

For each observation, subtract the previous within-unit observation:

$$y_{it} - y_{i,t-1} = \left(\mathbf{x}_{it}' - \mathbf{x}_{i,t-1}'\right)\beta + \left(c_i - c_i\right) + \left(u_{it} - u_{i,t-1}\right)$$

This is the **first-difference transformation**. We can write it as:

$$\Delta y_{it} = \Delta \mathbf{x}_{it}'\beta + \Delta u_{it},$$

where $\Delta c_i = 0$ and $\Delta \mathbf{x}_{it}$ does not contain an intercept term.

Consider the two-period data student GPA Data:

StudentID	*Semester*	*Female*	*HSGPA*	*GPA*	*JobHrs*
17	5	0	2.8	3.0	0
17	6	0	2.8	2.1	20
23	5	1	2.5	2.2	10
23	6	1	2.5	2.5	10

After applying the first-difference transform, the differenced data:

StudentID	*DSemester*	*dFemale*	*dHSGPA*	*dGPA*	*dJobHrs*
17	.	.	.	.	.
17	1	0	0	-.9	20
23	.	.	.	.	.
23	1	0	0	.3	0

Fixed Effects and First Differences in the Two-Period Case

FE (Within) Transform

StudentID	*Semester*	*CFemale*	*CHSGPA*	*CGPA*	*CJobHrs*
17	-.5	0	0	.45	-10
17	.5	0	0	-.45	10
23	-.5	0	0	-.15	0
23	.5	0	0	.15	0

FD (Differenced) Transform:

StudentID	*DSemester*	*dFemale*	*dHSGPA*	*dGPA*	*dJobHrs*
17	1	0	0	-.9	20
23	1	0	0	.3	0

Compare the transformed (FE) and differenced (FD) data. Each FD variable is equal to the difference between the second-period FE demeaned variable and the first-period demeaned variable.

This symmetry will always be present in the two-period panel model.

As a result, the parameter estimates for the two-period panel model can be obtained using FD or FE, with identical results. Not so if $T>2$!

FE and FD Results for two terms:

	Term 5&6 GPA (FE, *N*=400)		Term 5&6 GPA (FD, *N*=200)	
	Estimate	SE	Estimate	SE
jobhrs	-0.0640159	0.0223835	-0.0640159	0.0223835
term6	0.1133996	0.0125627	0.1133996	0.0125627

FE and FD Results for six terms:

	Terms 1-6 GPA (FE, *N*=1200)		Term 1-6 GPA (FD, *N*=1000)	
	Estimate	SE	Estimate	SE
jobhrs	-0.1285521	0.0188415	-0.087316	0.0174187
term	0.1037983	0.0040011	0.1066726	0.0091661

Difference-in-Difference Model for Panel Data using FD

Suppose we have a treatment that affects some but not all units in the population. The "difference-in-differences" estimator is the difference between the change over time in the treatment group and the change over time in the control:

$$DID = \bar{y}_{B,2} - \bar{y}_{B,1} - \bar{y}_{A,2} - \bar{y}_{A,1}$$

If we have panel data from a time period prior to treatment and a second observation drawn after the treatment event, we can study treatment effects using 2-period panel data *FD* and DID:

$$y_{i1} = \beta_0 + \beta_{PD} PERIOD_{i1} + \beta_{TREAT} treatment_{i1} + v_{i1}$$
$$y_{i2} = \beta_0 + \beta_{PD} PERIOD_{i1} + \beta_{TREAT} treatment_{i2} + v_{i2}$$

, and

Where $treatment_{i1}$ indicates treatment, zero for all at time *t*=1

$PERIOD$ is a dummy for the time period.

The first difference model for difference-in-difference:

$$\Delta y_i = \beta_{PD} + \beta_{TREAT}\Delta treatment_i + \Delta v_i$$

Where the intercept is replaced by a period effect ($\Delta PERIOD = 1$ for all units and the change in treatment is either 0 or 1.

Designate *A* as the control group (i.e. $\Delta treatment_{i \in A} = 0$)

Designate *B* as the treatment group (i.e. $\Delta treatment_{i \in B} = 1$)

$$\Delta y_i = \beta_{PD} + \beta_{TREAT}\Delta treatment_i + \Delta v_i$$

Difference in differences estimator:

$$\begin{aligned} DID = & \ \overline{y}_{B,2} - \overline{y}_{B,1} \ - \ \overline{y}_{A,2} - \overline{y}_{A,1} \\ = & \ \hat{\beta}_{PD} + \hat{\beta}_{TREAT}\Delta treatment_{i \in B}) - (\hat{\beta}_{PD} + \hat{\beta}_{TREAT}\Delta treatment_{i \in A} \\ = & \ \hat{\beta}_{TREAT} \end{aligned}$$

e.g. Card & Krueger (2000) Minimum Wage increases & Employment

Choosing an Estimator: Fixed Effects vs. First Differences (FE vs FD)

- If T=2, (two period model) the *FE* and *FD* are identical

- If T>2 *FE* is more efficient than if there is no serial correlation of the idiosyncratic errors.

- If T>2 *FD* is more efficient if there is serial correlation.

- If the unobserved error is not correlated with the covariates, neither the *FE* nor the *FD* model is efficient.

Why not Just Use a Lagged Dependent Variable?

TABLE 4. ESTIMATES OF THE EFFECTS OF THE TRANSITION TO DIVORCE AND WIDOWHOOD ON CES-D IN THE LDV AND CS MODELS ($N = 3{,}904$)

	Model 1		Model 2	
Independent Variables	*B*	*SE B*	*B*	*SE B*
LDV model				
CES-D(t_1)	0.443**	0.015	0.427**	0.015
MD	0.298**	0.106	0.288**	0.106
MW	0.657**	0.129	0.630**	0.134
Female			0.375**	0.052
Age(t_1)			−0.004*	0.002
Constant	1.848	0.055	1.851	0.103
R^2	0.193**		0.205**	
CS model				
MD	0.054	0.112	−0.044	0.123
MW	0.629**	0.147	0.535**	0.150
Age			0.016**	0.005
Constant	3.248	0.016	2.525	0.239
R^2	0.005**		0.007**	

Note: CES-D = Center for Epidemiological Studies Depression; CS = change score; LDV = lagged dependent variable; MD = married to divorced; MW = married to widowed.

*$p < .05$. **$p < .01$.

Source: David Johnson. *Journal of Marriage and Family*, Vol. 67, No. 4 (Nov., 2005), pp. 1061-1075

Random Effects Methods

If we can assume that the unobserved heterogeneity will not bias the estimates:

- Fixed effects methods are *inefficient*. They throw away information.
- Pooled OLS is *inefficient* because it does not exploit the autocorrelation in the composite error term.
- Random effects methods use feasible GLS estimation (RE FGLS) to exploit within-cluster correlation
- Random effects estimation is more *efficient* than FE or OLS
- The "random effects assumption" of no bias due to c_i is more stringent

$$E(c_i \mid \mathbf{x}_{i1},\ldots,\mathbf{x}_{iT}) = E(c_i) = 0$$

A Conventional FGLS Random Effects Estimator

Assume the errors are correlated within each unit

Assume the errors are uncorrelated across units

Assume the variance in the composite errors is equal to the sum of the variances in the unobserved effect c_i and the idiosyncratic error u_i:

$$\sigma_v^2 = \sigma_u^2 + \sigma_c^2$$

RE strategy: If $\sigma_v^2 = \sigma_u^2 + \sigma_c^2$, find estimators such that $\hat{\sigma}_v^2 = \hat{\sigma}_u^2 + \hat{\sigma}_c^2$

Practical Feature of Random Effects Estimation

- Recall that the fixed effects "within" estimator essentially transforms the data by centering each variable on the unit-specific mean.

  OLS is then performed on the "fully demeaned" transformed data.

- The random effects estimator essentially transforms the data by "partially demeaning" each variable. Instead of subtracting the entire unit-specific mean, only part of the mean is subtracted.

  The demeaning factor λ is between 0 and 1, with the specific value based on the variance components estimation.

- Random effects routines are standard in statistical software packages:

  SAS: PROC GLM or PROC PANEL

  STATA: xtreg

RE Results compared to pooled OLS Results for two terms:

	RE Term 5&6 GPA			OLS Term 5&6 GPA		
	Estimate	SE	*z*-stat	Estimate	SE	*t*-stat
Intercept	2.81	0.16	18.0	2.97	0.17	17.2
jobhrs	-0.108	0.02	-4.8	-0.178	0.05	-5.8
female	0.126	0.04	3.0	0.125	0.04	3.0
highgpa	-0.001	0.03	-0.04	0.0001	0.03	-0.0004
term6	0.096	0.015	5.6	0.095	0.016	6.1

RE Results for six terms:

	Terms 1-6 GPA (FE, *N*=400)		
	Estimate	SE	
Intercept	2.41	0.10	23.5
jobhrs	-0.129	0.02	-7.0
female	0.086	0.03	2.8
highgpa	-0.030	0.02	1.2
term	0.088	0.006	13.6

Random Effects or Fixed Effects - How to decide?
Hausman test for the Exogeneity of the Unobserved Error Component

If the unobserved effects are exogenous, the FE and RE are asymptotically equivalent. This suggests the null hypothesis for the Hausman test:

$$H_0 : \hat{\beta}_{RE} = \hat{\beta}_{FE} \ ,$$

where $\hat{\beta}_{RE}$ and $\hat{\beta}_{FE}$ are coefficient vectors for the time-varying explanatory variables, excluding the time variables.

If the null hypothesis is rejected, we conclude that RE is inconsistent, and the FE model is preferred.

If the null hypothesis cannot be rejected, random effects is preferred because it is a more efficient estimator.

Hausman Test in Stata:

```
. xtreg gpa job sex highgpa,fe
. estimates store fe
. xtreg gpa job sex highgpa,re
. estimates store re

. hausman fe re
                ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |       fe           re         Difference          S.E.
-------------+----------------------------------------------------------------
         job |   -.0748115    -.1232374         .048426        .0088051
------------------------------------------------------------------------------
       b = consistent under Ho and Ha; obtained from xtreg
       B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =       30.25
                Prob>chi2 =      0.0000
```

- We reject the null and conclude the fixed effects estimator is appropriate.

Interpretation of Results from the Error Components Model

Since the UEM model is derived as a *levels* model, coefficients can be interpreted much the same as interpretations of a conventional OLS model, but there are nuances:

For example, suppose we estimate the relationship between marriage and men's wages, $\hat{\beta}_{MARRIED} \simeq 0.05$ in every model.

- **Pooled OLS** cross-section coefficients contain information about average differences between units.

  $E[y_{it} \mid \mathbf{x}_{it}] = \mathbf{x}_{it}\boldsymbol{\beta} + c_i$

  This is a *population-averaged effect*. On average, married men earn 5% more than men who are not married.

  This says nothing about the *causal* effect of marriage on men's earnings.

- **RE/FE/FD** estimate average effects *within* units.

  If the unobserved effects are exogenous these are asymptotically equivalent to the population averaged effect.

  $$E[y_{it} \mid \mathbf{x}_{it}, c_i] = \mathbf{x}_{it}\boldsymbol{\beta}$$

  This is sometimes called an *average treatment effect*. On average, entering marriage increases men's earnings by 5%.

- **RE** coefficients represent average change *within* units, estimated from all units whether they experience change or not.

- **FE and FD** coefficients represent average changes *within* units, only for units that did experience change

  This is akin to a *treatment effect among the treated.* On average, men who married increased their earnings by 5%.

Best Practices

Theorize the model

- What exactly does this unobserved heterogeneity represent?
- Why would you expect it to be correlated / uncorrelated with the regressors?

Specification Testing for Panel Analysis - Interval/Continuous Outcomes

- Before ruling out pooled OLS, test for appropriateness of panel methods vs. pooled ordinary least square.
- Optional: Obtain intraclass correlation coefficient (ICC) as indicator of the extent of within-unit clustering. This is a descriptive statistic, not a test.
- Specification tests for strict exogeneity
- Test for serial correlation in the idiosyncratic errors
- Hausman test for random effects vs. fixed effects

Extensions

FE Models with Time-Invariant Predictors

- Interactions between time and covariate

Panel Models for Categorical Outcomes

- Fixed effects logit and random effects logit for binary outcomes
- Fixed and random effects Poisson models can be used for count outcomes.
- Population averaged models can be estimated using General Estimation Equations (GEE).

Dynamic panel models i.e. lagged dependent variable as a covariate:

$$GPA_{it} = \beta_0 + GPA_{i,t-1}\beta_{GPA} + TERM_{it}\beta_T + HSGPA_{it}\beta_H + JOB_{it}\beta_J + v_{it}$$

- GLM models for instrumental variables (IV) estimation
- Generalized Method of Moments (GMM) is used for some dynamic panel models because it allows a flexible specification of the instruments