Second Grade Mathematics

A Comprehensive Curriculum for Early Learners

Skylar Saveland

Corpora Inc

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Contents

Welcome to Second Grade Mathematics

Welcome to second grade mathematics. In this unit, we will introduce the main topics you will learn and explain why they are important. This unit gives you a roadmap to help you understand the big ideas in math and shows you how these ideas work in everyday life.

What You Will Learn:

- **Numbers and Operations**: Learn about numbers and how to add, subtract, multiply, and divide them. These skills help you solve problems like sharing snacks or counting objects.
- **Measurements**: Understand how to measure lengths, weights, and times. You will learn to use rulers, clocks, and more.
- **Shapes and Patterns**: Discover different shapes and learn how to recognize patterns. These skills help you with art, design, and solving puzzles.
- **Graphs and Data**: Learn how to organize information in graphs and charts. This makes it easier to see how things compare.
- **Time and Money**: Practice telling time and using money by solving simple problems about buying things and giving change.

Why This Matters:

Learning math is like learning a new language. It gives you the tools to solve problems and think clearly. Math helps you make decisions every day, whether you are sharing, measuring, or planning activities. As you learn these skills, you build a strong foundation for more advanced math in the future.

How It Applies in Real Life:

- When you measure ingredients for a recipe, you use math skills.
- Deciding how to share toys or treats involves addition and subtraction.
- Recognizing patterns can help you in art and music.
- Using clocks and calendars is a daily way to practice math when you manage your time.
- When you count coins or make purchases, you use money math.

> "Pure mathematics is, in its way, the poetry of logical ideas." — Albert Einstein

In each lesson, you will see clear, step-by-step examples that show how these ideas work in simple problems. This approach will help you build your skills one step at a time. As you proceed, remember that every math problem is an opportunity to learn and grow. Enjoy your journey through math!

How This Book is Structured

This book is designed to help you learn math in a clear and organized way. It is divided into units, and each unit is made up of lessons that focus on specific math ideas. The book is organized so that you can gradually learn new concepts by following simple steps.

Each unit represents a big math idea, and each lesson breaks that idea into smaller parts. When you open a lesson, the title tells you what you will learn. Then, the lesson explains the steps needed to understand and solve problems. This structured layout makes it easier to follow the lessons and helps you understand the material one step at a time.

The book uses clear examples that show exactly how to solve math problems. Every explanation is written in simple language so you can easily follow along. When you see a new concept, take your time to read the explanation and follow each example carefully.

To use this book well, start by reading each lesson completely. Focus on understanding each step. If something seems confusing, review that part again or ask for help from your teacher. Staying organized by keeping track of the lessons you have completed will also help you learn faster.

Regular review is very important. As you review, you will notice that ideas become clearer and you can solve problems more easily. Remember, learning math is a process, and each step builds on the one before it. By following this book carefully, you will improve your math skills and become more confident in solving problems.

An Introduction to Mathematical Thinking, Problem Solving, and Real-World Application

Mathematical thinking is the way we use numbers and ideas to understand the world. This lesson shows how to look at problems, make a plan, and use math to solve them. You will learn how to break problems into smaller parts and see how math makes everyday tasks simpler.

What is Mathematical Thinking?

Mathematical thinking means using clear steps to solve problems. It helps us:

- Understand numbers, shapes, and patterns.
- Think of solutions in many different ways.
- Organize our ideas so we can solve challenges step by step.

Every time you face a problem, you can use math to find an answer. This way of thinking is like using a map to guide you through a maze of numbers and ideas.

Solving Problems with Math

When solving problems, it is important to follow clear steps. Here is a method you can use:

1. **Understand the Problem**: Read the problem carefully. Make sure you know what you are trying to find. Sometimes, underlining or circling important words can help.
2. **Choose a Strategy**: Think about which math operation or method will help. You might add, subtract, count, or use another method you know well.
3. **Solve It**: Do the math one small step at a time. Write down each step so you can follow your work.
4. **Check Your Work**: Look at the answer and review your steps. Ask yourself if the answer makes sense.

Following these steps makes solving problems clear and organized.

Real-World Application

Math is used in many parts of our daily lives. Here are some ways you see math in the real world:

- **Cooking**: Measuring ingredients requires careful counting and measurements.
- **Shopping**: Adding up prices and figuring out change uses math skills.
- **Building and Designing**: Measuring lengths and spaces helps in making and designing objects.
- **Games and Sports**: Keeping score and calculating times involve math.
- **Everyday Planning**: Organizing schedules and planning trips use numbers and patterns.

> **Key Insight**: "The essence of mathematics is not to make simple things complicated, but to make complicated things simple." — Stan Gudder

Detailed Example: Solving a Simple Problem

Let's see how these steps work with a clear example.

Problem: Sam has 3 red apples and 4 green apples. How many apples does he have altogether?

Step 1: Understand the Problem

We need to count all the apples to know the total amount.

Step 2: Choose a Strategy

We can use addition because we are combining two groups of apples.

Step 3: Solve It

Add the number of red apples to the number of green apples:

$$3 + 4 = 7$$

Step 4: Check Your Work

Count the apples again in your mind to be sure there are 7 in total.

This example shows the step-by-step process to solve a problem and how math makes it easy to get the correct answer.

Mathematical thinking and problem solving are powerful tools. By learning and practicing these steps, you will be prepared to tackle many different types of problems, both in school and in everyday life.

Key Math Terms, Symbols, and Notational Conventions Used in This Textbook

In this lesson, you will learn about the math words, symbols, and ways of writing that you will see in this book. Knowing these helps you follow and solve problems step by step.

Key Math Terms

- **Number**: A symbol or word that shows how many things there are. For example, 1, 2, and 3.
- **Addition**: Putting numbers together to find a total.
- **Subtraction**: Taking one number away from another to find the difference.
- **Multiplication**: Adding equal groups quickly. This means you add the same number several times.
- **Division**: Splitting a number into equal parts or groups.
- **Equals**: A word or symbol that means both sides are the same.

Important Math Symbols

- **Plus Sign (+)**: Used for addition.

  Example:

  $$3 + 2 = 5$$

- **Minus Sign (−)**: Used for subtraction.

  Example:

  $$5 - 2 = 3$$

- **Multiplication Sign** $\times$: Used for multiplication.

  Example:

  $$4 \times 3 = 12$$

- **Division Sign (÷ or /)**: Used for division.

  Example:

  $$12 \div 4 = 3$$

  or

  $$12/4 = 3$$

- **Equals Sign (=)**: Shows that the numbers on each side have the same value.

  Example:

  $$3 + 2 = 5$$

Notational Conventions

- **Writing Numbers**: We use digits like 1, 2, 3, etc., to show amounts.
- **Using Symbols for Operations**: Write + for addition, − for subtraction, $\times$ or * for multiplication, and ÷ or / for division.
- **Order of Operations** (Simple for Grade 2):
  - When numbers are added or subtracted, work from left to right.
  - In many problems, multiplication or division is done by grouping numbers together before adding or subtracting.

More Things You Will See

- **Equal Parts**: When you split a number into groups that are the same, each group has equal parts.
- **Grouping**: Putting numbers together to add them or multiply them easily.
- **Word Problems**: You will see stories or questions that use these words and symbols to ask questions about everyday situations.

> Learning these math words, symbols, and ways of writing will help you understand every math lesson in this book.

Review these terms and examples often. They are the tools you need to start your math journey.

Using Tables, Graphs, and Charts to Organize and Interpret Data

This unit introduces ways to display information using tables, graphs, and charts. You will learn how to arrange data in neat formats and understand what the data tells you.

What: This unit covers organizing numbers and facts into tables, drawing simple graphs, and reading charts. You will see different methods to sort and present information.

Why: Organizing data helps us see patterns and make decisions. Tables and graphs make complex information easy to understand.

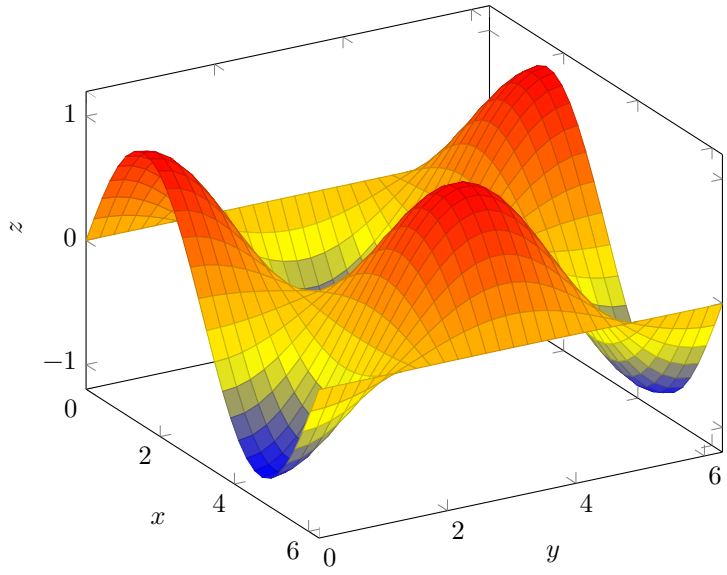
How:

- **Tables**: Arrange data in rows and columns for clear comparisons.
- **Graphs**: Use pictures and lines to show changes and differences.
- **Charts**: Visual tools that help us quickly compare numbers or categories.

> “Without data, you’re just another person with an opinion.” — W. Edwards Deming

By learning these skills, you will be able to look at everyday information—like weather, class votes, or sports scores—and understand it clearly.

Surface Plot: $z = \sin(x)\cos(y)$



Understanding What Data Is and How It Informs Our Daily Decisions

What Is Data?

Data is information that we collect. It can be numbers, words, or facts. For example, a list of temperatures or scores in a game are pieces of data.

Why Is Data Important?

Data helps us see patterns and make choices. When you know the numbers, you can decide what to do next. For example, knowing the weather helps you choose what to wear.

How We Use Data

There are clear steps to use data:

1. Collect the data. This could be by counting, measuring, or asking questions.
2. Organize the data into a table or list.
3. Create a graph to show the data in pictures.
4. Look at the graph or table to decide what comes next.

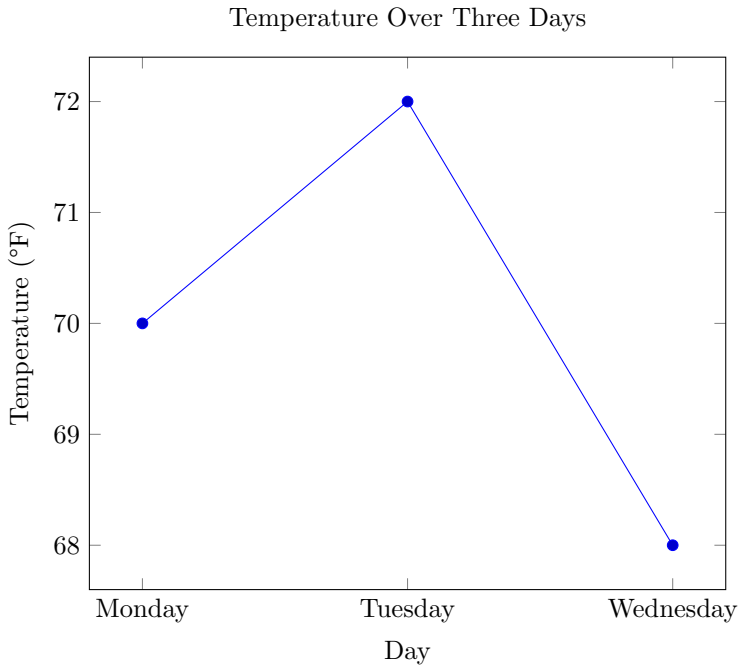
Organizing Data in a Table

A table arranges data neatly in rows and columns. For example:

Day	Temperature (°F)
Monday	70
Tuesday	72
Wednesday	68

Displaying Data as a Graph

A graph helps us see changes quickly. For example, a line graph of temperature might look like this:



How Data Guides Decisions

When we see organized information like a table or graph, we can make choices. For example:

- Data about sunny and rainy days helps us decide if we need an umbrella.
- Data about how many fruits we eat helps us know if we are eating healthy.

> "In God we trust; all others must bring data." — W. Edwards Deming

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Example: Choosing an Outfit

Imagine you have a list of temperatures in the morning:

Time	Temperature (°F)
Morning 1	60
Morning 2	67
Morning 3	63
Morning 4	70

You can use this simple rule:

If temperature < 65, wear a jacket.

If temperature ≥ 65, no jacket is needed.

This rule is a simple way to use data to decide how to dress.

Summary of Decisions

Below is a table summarizing the decisions based on temperature:

Time	Decision
Morning 1 (60)	Wear a jacket
Morning 2 (67)	No jacket needed
Morning 3 (63)	Wear a jacket
Morning 4 (70)	No jacket needed

Exploring Different Types of Data: Categorical vs Numerical

Data comes in different types. In this lesson, we learn about two main types: categorical data and numerical data.

What is Data?

Data is information. It can be written as words or numbers. Understanding data helps us learn more about the world.

Categorical Data

Categorical data contains names or labels. It does not use numbers for calculation. Examples include colors, types of pets, or names of fruits.

> Categorical data groups items by characteristics.

For example, consider a list of favorite fruits: Apple, Banana, and Orange. This list shows names of fruits, not numbers.

Numerical Data

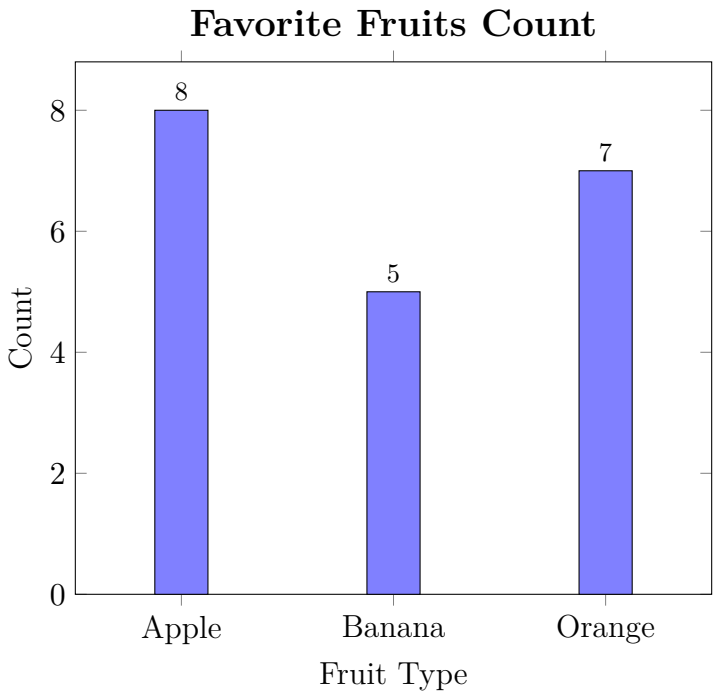
Numerical data uses numbers. We can do math with these numbers. Examples include age, height, or the number of items in a group.

> Numerical data tells us how many, how much, or how long.

For instance, the ages of students (7, 8, 9) are numerical data.

Visual Example of Categorical Data

Below is a bar graph that shows a count of favorite fruits. The fruits are the categories and the count is numerical.



Summary

- **Categorical Data**: Data that names groups or categories (e.g., fruit types).
- **Numerical Data**: Data that uses numbers to show amounts or measurements (e.g., age, count).

Use these ideas to look at everyday information. Identify if the data is categorical or numerical by checking if it uses words or numbers.

Methods for Collecting Data Through Surveys and Experiments in the Classroom

Data collection means gathering information. In this lesson, we learn two ways to collect data: surveys and experiments.

What is Data Collection?

Data collection is the process of gathering information. We do this to learn more and make decisions.

Surveys

Surveys ask people questions to get their answers. Here are simple steps to do a survey:

- Decide one or more questions.
- Ask your classmates or friends the questions.
- Write down their answers.
- Count how many times each answer appears.

For example, you may ask, “What is your favorite snack?” and then list the answers.

Experiments

Experiments test one idea to see what happens. Follow these steps for a simple experiment:

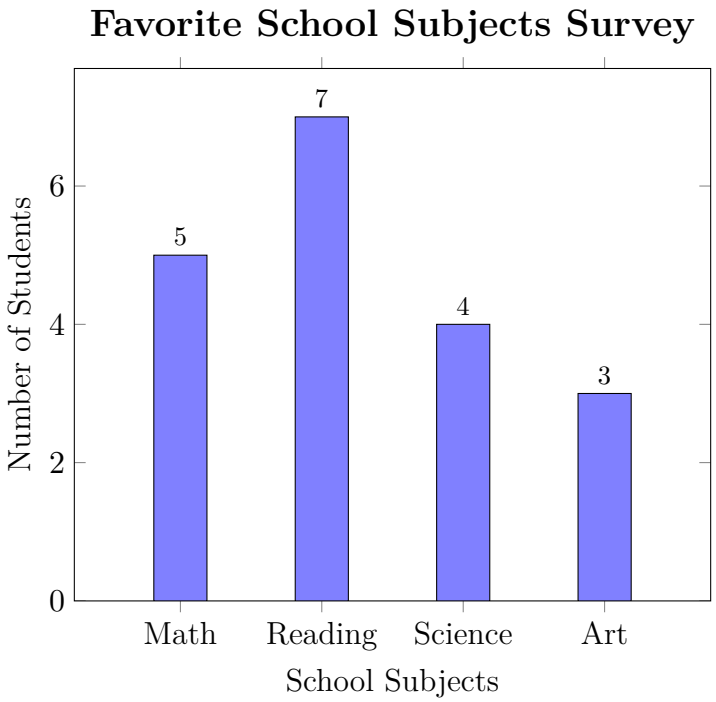
- Choose one thing to test (for example, how much water a plant needs).
- Do the test by changing one part at a time.
- Watch what happens and record the results.

This method shows you cause and effect. For example, you can water one plant a little and another a lot. Then, you see which plant grows better.

Visual Example: Survey on Favorite School Subjects

Imagine you ask your classmates: "What is your favorite school subject?" You might get answers like Math, Reading, Science, or Art. Count the answers to see which subject is most liked.

Below is a bar graph that shows the survey results.



Collecting and Using Data

After you collect data from a survey or experiment, you count and record your results. This information helps you learn which answer is the most common or which experiment worked best.

Use surveys to ask about opinions and experiments to test ideas. Both methods help us understand the world around us.

Techniques for Organizing Information Using Charts and Tables

Organizing information helps us understand data better. In this lesson, we learn two methods: using tables and using charts.

Using Tables for Organizing Data

Tables arrange information in rows and columns. They help us compare items easily.

Steps to create a table:

1. List the items or categories you want to compare.
2. Create columns and rows. Columns hold one type of information, and rows hold different items.
3. Fill in the table with your data.

For example, suppose we want to show our class favorite fruits. A table can look like this:

Fruit	Count
Apple	8
Banana	5
Orange	7

This table clearly shows the favorite fruits and their counts.

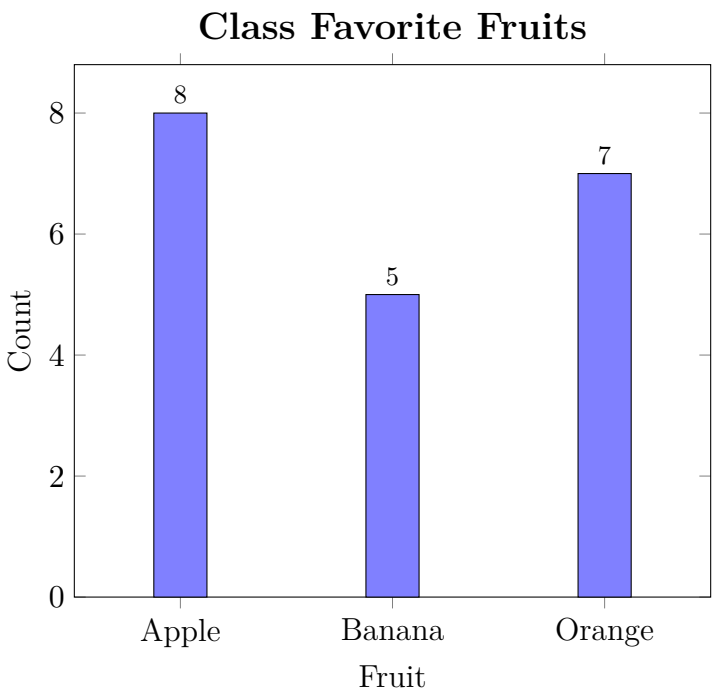
Using Charts for Organizing Data

Charts turn data into pictures. They can show changes, differences, or comparisons quickly.

A bar graph is one common chart. In a bar graph:

- The horizontal line (x-axis) shows the items or categories.
- The vertical line (y-axis) shows numerical values, such as counts or amounts.

Below is an example of a bar graph that shows favorite fruits count:



Organizing Information Step by Step

1. Decide what information you need to show.
2. Choose a method: table for clear comparisons or chart for visual data.
3. Organize your data carefully with clear labels.
4. Check your work to make sure the information is easy to read.

> A chart is a picture of data that makes numbers easier to see and understand.

Using tables and charts makes complex information simple. Follow these steps to work with your own data and see the patterns in everyday information.

How to Create Picture Graphs with Real World Examples and Interpret Them

A picture graph uses simple symbols to show data. Each symbol stands for one or more items. This lesson explains how to create and read a picture graph step by step.

What is a Picture Graph?

A picture graph is a chart that uses symbols instead of bars or lines. It makes data easy to understand by repeating a symbol for each count.

> A picture graph turns numbers into clear pictures.

Steps to Create a Picture Graph

1. Collect the data you need to show.
2. Choose a symbol for each item. Decide if one symbol equals one item or several items.
3. Write the names of the categories on the left side.
4. Place the symbols next to each category to match the count.

Real World Example: Favorite Pets

Suppose we ask classmates about their favorite pet. The data is:

- Dogs: 4
- Cats: 3
- Fish: 2

We will use a picture graph where:

• The letter D represents one dog, • The letter C represents one cat, and • The letter F represents one fish.

Follow these steps to make the graph:

1. Write the names of the pet categories on the left side.
2. Decide that one symbol equals one pet.
3. Place the symbols in a row next to each category according to the count.

Creating the Picture Graph

Below is an example of a picture graph for our favorite pets:

Dogs	D	D	D	D
Cats	C	C	C	
Fish	F	F		

Interpreting a Picture Graph

To read a picture graph:

- Look at the symbols in each row.
- Count the symbols next to each category.
- Compare the counts to understand the data.

In our example, counting the D symbols tells us there are 4 dogs. This makes the information clear and easy to understand.

By following these steps, you can create your own picture graphs to display data from everyday surveys or observations. The use of symbols helps turn numbers into clear pictures that are simple to see and compare.

Strategies for Reading and Interpreting Picture Graphs Accurately

Picture graphs use simple pictures or symbols to show data. Each picture represents a certain number as given in the key. This lesson explains how to read these graphs step by step.

Understanding Picture Graphs

In a picture graph:

- A picture or symbol stands for one or more items.
- A key tells you what each picture means.
- The graph has labels and a title to explain the data.

Steps to Read a Picture Graph

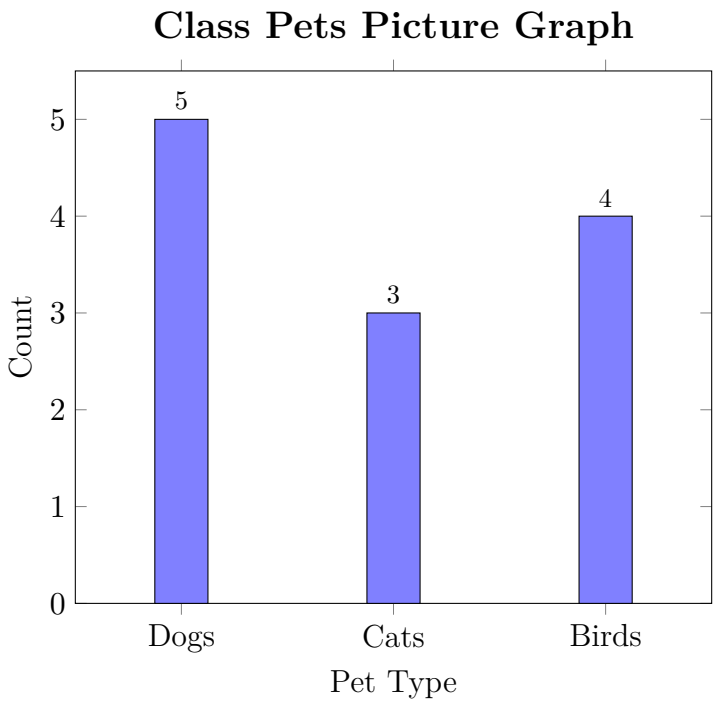
1. Read the title to know what information is shown.
2. Look at the key. Note what each picture represents. For example, one picture may equal 2 items.
3. Count the pictures in each category.
4. Multiply the count by the value given in the key if each picture represents more than one item.
5. Compare the totals to understand the differences between categories.

> Always check the key before counting the pictures. It tells you exactly what each symbol means.

Example of a Picture Graph

Suppose we have a picture graph showing the number of pets in a class. The key tells us that one picture (icon) equals 1 pet. The graph shows three categories: Dogs, Cats, and Birds.

Below is a bar graph that looks like a picture graph. In a real picture graph, the bars would be replaced by repeated icons. Here, each bar's height represents the count of pictures.



How to Use the Picture Graph

- Look at the title: It shows that the graph is about class pets.
- Check the key: One picture equals 1 pet.
- Count the pictures (or look at the bar heights):
  - Dogs: 5 pictures mean 5 dogs.
  - Cats: 3 pictures mean 3 cats.
  - Birds: 4 pictures mean 4 birds.
- Use this information to compare which pet is most or least common.

Following these steps, you can accurately read and interpret picture graphs. They help make data clear and simple by turning numbers into pictures.

How to Create and Read Bar Graphs to Compare Data Sets

Bar graphs are pictures made of rectangles. Each rectangle shows a number. This lesson explains how to make a bar graph and how to read one.

What is a Bar Graph?

A bar graph shows information by using bars. Each bar represents a category or a group. The longer the bar, the bigger the number it shows.

How to Create a Bar Graph

1. Identify the groups you want to compare. For example, types of fruit: Apple, Banana, and Orange.

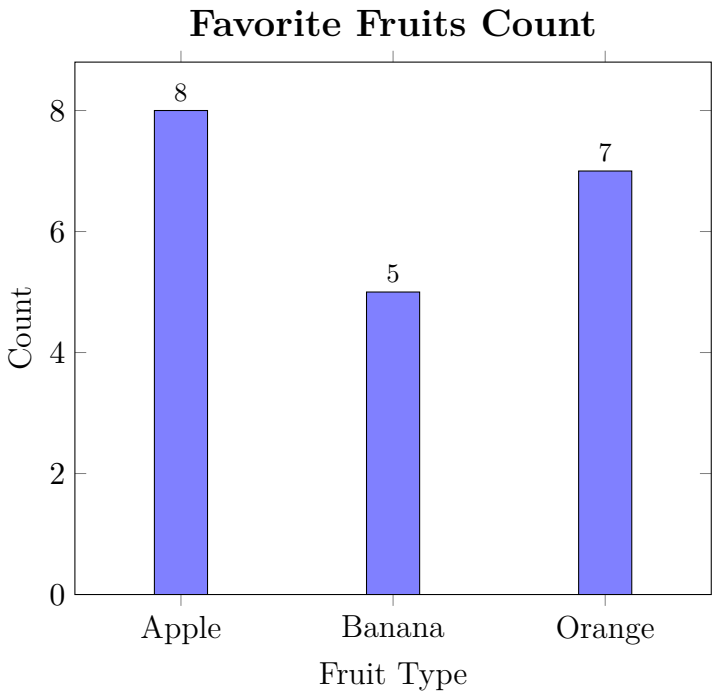
2. Write the names of the groups on the horizontal line (x-axis).
3. Decide what number each bar will show. Write numbers on the vertical line (y-axis).
4. Draw a bar for each group. The height of each bar shows its number.

How to Read a Bar Graph

1. Look at the names under each bar. They tell you what the bar is about.
2. Look at the height of the bar. Match the top of the bar with the numbers on the y-axis.
3. Compare the heights. A taller bar shows a larger number. A shorter bar shows a smaller number.

Example: Favorite Fruit Count

Below is a bar graph showing how many students like each fruit.



Step-by-Step Example

Imagine you want to compare the number of books read by three students: Sam, Alex, and Lee.

1. List the names: Sam, Alex, Lee on the x-axis.
2. Use the y-axis to show the number of books. For example, the numbers go from 0 to 10.
3. If Sam read 4 books, Alex read 6 books, and Lee read 3 books, you draw a bar for each student with that height.
4. Now you can see that Alex read the most books because his bar is the highest.

> A bar graph is a simple way to see differences between groups.

This is how you create and read bar graphs to compare data sets.

Methods for Creating and Interpreting Line Plots to Display Data

A line plot is a graph that shows a trend or change over time by connecting data points with a line. This type of plot is useful for tracking how things change, such as the price of eggs over a week.

Creating a Line Plot

1. Collect your data. For example, record the price of eggs each day for a week.
2. Draw horizontal and vertical axes. The horizontal axis (x-axis) represents time (days) and the vertical axis (y-axis) represents the price.
3. Mark the data points on the graph. Each point represents the price on a certain day.
4. Connect the points with a line to show the trend.

Example: Egg Prices Over a Week

Suppose you recorded the price of eggs over 7 days as follows:

- Day 1: $2.00
- Day 2: $2.20
- Day 3: $2.15
- Day 4: $2.30
- Day 5: $2.50
- Day 6: $2.45
- Day 7: $2.60

To create the line plot:

- Plot each day on the x-axis and the corresponding price on the y-axis.
- Connect the points with a line to show the rising and falling trend in the price.

Below is an example that shows the line plot for the egg prices:

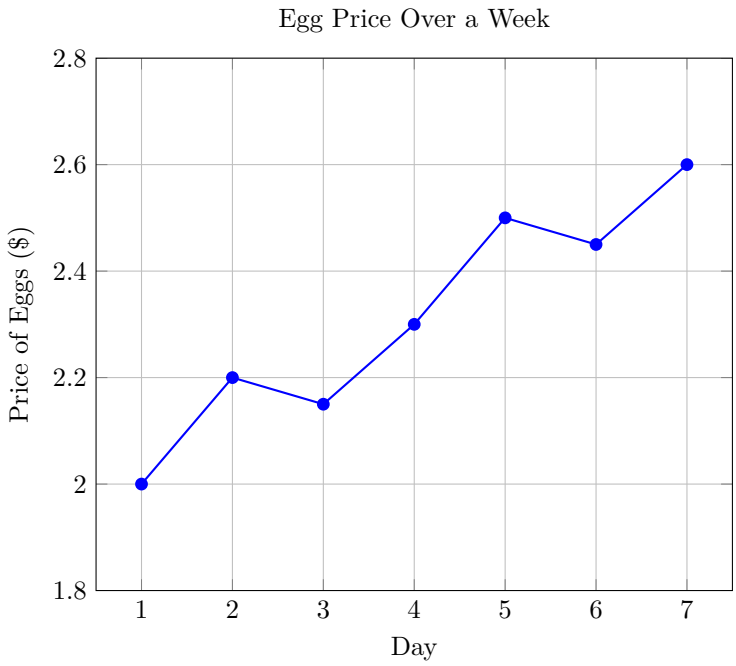


Figure: Line plot showing the change in egg prices over one week.

Interpreting a Line Plot

1. Look at the connected line to see if the price is going up or down over time.
2. Identify peaks and valleys. The highest point shows the highest price and the lowest shows the lowest price.
3. Use the trend to understand how the price changes. For example, you can tell if the price slowly increases or if there are any dips mid-week.

> A line plot helps us see trends over time by connecting data points in order.

By following these steps, you can create and understand line plots that show changes over time. Use line plots to track trends in data, such as changes in prices, temperatures, or any other measurements over a period.

Comparing and Contrasting Information from Multiple Graph Types

Graphs help us see information in different forms. In this lesson, we learn to compare two graph types: bar graphs and line plots. Each graph works best for different kinds of data.

What Are Graphs?

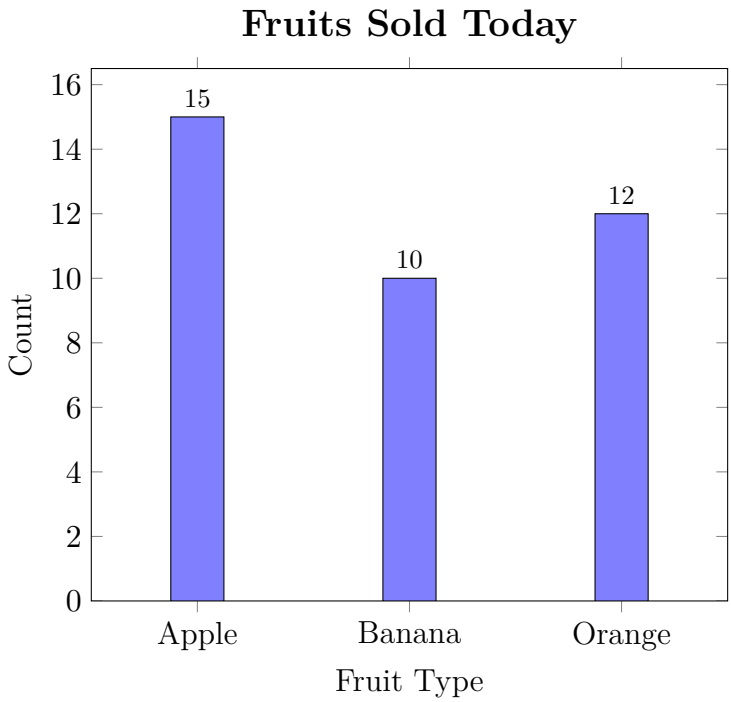
A graph is a picture that shows numbers and facts in a clear way. Some common graph types are:

- **Bar Graphs**: These show data with separate bars. They are good for comparing different groups at one moment in time.
- **Line Plots**: These connect points with lines. They are useful for showing changes over time.

Each graph organizes information in its own clear way.

When to Use a Bar Graph

A bar graph is best when you have different groups or categories. For example, imagine you want to show how many apples, bananas, and oranges were sold at a fruit stand today.

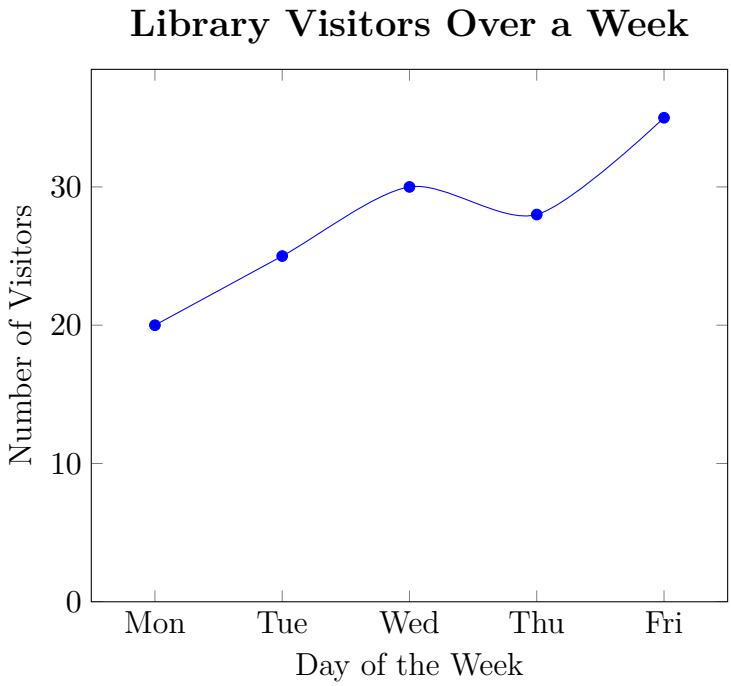


In this graph, you can easily see which fruit sold the most.

When to Use a Line Plot

A line plot is useful for showing a trend or change over time. For example, if you want to see how many students visited the school library each day during the week.

Let's look at a line plot that shows library visits from Monday to Friday.



In this graph, you can see how the number of visitors changes each day. It shows a clear trend over time.

Comparing the Two Graphs

- **Bar Graph**: Best for comparing different categories at one moment in time. It shows clear differences between groups.
- **Line Plot**: Best for tracking changes over time. It shows trends and patterns clearly.

When you compare graphs, look at these points:

1. Read the labels and title to know what is shown.
2. Notice how the graph displays data: bars for separate groups vs. points connected by lines for trends.
3. Choose the graph that fits your data best.

> A clear graph makes information easier to understand.

Steps to Compare Graphs

1. Look at each graph carefully.
2. Check the labels on the axes.
3. Notice how the data is shown.
4. Decide which graph makes the information easiest to read.

This lesson shows that choosing the right graph helps tell the story of your data more clearly. Use bar graphs for clear category comparisons and line plots to see trends over time.

Introducing the Concepts of Likely Versus Unlikely Outcomes with Examples

This lesson explains what it means for an outcome to be likely or unlikely. We will learn which events happen often and which happen only a little.

What Does 'Likely' Mean?

An outcome is likely if it happens most of the time. It means it happens often.

Example: Imagine a spinner with 5 equal parts. If 4 parts are red and 1 part is blue, the spinner usually lands on red.

What Does 'Unlikely' Mean?

An outcome is unlikely if it happens only a little. It means it does not happen very often.

Example: In a bag with 10 marbles, if 8 are green and 2 are purple, drawing a purple marble happens less often than drawing a green one.

Step-by-Step Example with Marbles

1. You have a bag with 10 marbles.
2. Count the marbles: 8 are green and 2 are purple.
3. When you pick one marble, you will most likely get a green marble because there are more green marbles.
4. A purple marble is less likely because there are only 2 purple marbles.

Step-by-Step Example with a Spinner

1. Imagine a spinner divided into 5 equal parts.
2. 4 parts are colored red and 1 part is blue.
3. When you spin the spinner, it will often point to red because most parts are red.
4. It is less common for the spinner to stop on blue since only one part is blue.

> Outcomes that happen often are likely. Outcomes that happen just a little are unlikely.

Certain Events and Impossible Events

This lesson explains two types of events: certain events and impossible events. We will learn the difference between events that always happen and events that can never happen.

Understanding Certain Events

A certain event is one that always happens. This means there is no chance for it not to happen.

For example:

- When you wake up in the morning, the sun will rise. This is a certain event because the sun always rises.
- If you have 2 apples and you do not eat any, you will still have 2 apples. This is certain.

> Certain events are like rules that always work.

Understanding Impossible Events

An impossible event is one that can never happen. There is no chance for it to happen.

For example:

- Rolling a regular 6-sided die and getting a 7 is impossible because the die has only the numbers 1 to 6.
- A fish cannot fly in the sky. This is impossible under normal conditions.

> Impossible events are things that do not follow the rules of our world.

Step-by-Step Comparison

1. Identify the event you are thinking of.
2. Decide if the event always happens. If yes, it is a certain event.
3. Decide if the event can never happen. If yes, it is an impossible event.
4. Use a simple example to check your idea.

For instance, consider the event: “A square has four sides.” This event is certain because every square always has four sides.

Now consider: “A square has five sides.” This event is impossible because it does not follow the rules of geometry.

These ideas help us understand how to look at events in everyday life and in math. By knowing if an event is certain or impossible, we can make better decisions and check our work in problems.

Building Fluency with Addition and Subtraction Through Various Strategies

This unit introduces methods to strengthen your addition and subtraction skills. You will learn strategies that help you solve problems easily and confidently.

What: This unit covers different ways to add and subtract numbers. It highlights methods like counting on, using number lines, and using mental math shortcuts.

Why: Developing fluency in addition and subtraction makes everyday math more understandable. It helps when you shop, share, and solve real problems.

How:

- **Counting On and Back**: Use your fingers or a number line to count forward or backward.
- **Breaking Numbers Apart**: Split numbers into smaller parts to add or subtract them step by step.
- **Mental Math Strategies**: Learn shortcuts to quickly solve problems in your head.

> "Arithmetic is the seed from which all mathematics grows. With addition and subtraction, our ancestors built the foundations of civilization and unlocked the secrets of the universe." — Anonymous

Building fluency in addition and subtraction through these strategies helps you understand how numbers work together. As you explore this unit, remember that each method is a tool you can choose when solving math problems.

Building a Solid Foundation of Addition and Subtraction Facts

In this lesson, we will learn how addition and subtraction facts work. We will see how to combine numbers and take them apart step by step.

Understanding Addition

Addition means combining two groups of items to find out how many there are in total. For example, if you have 2 apples and then get 3 more apples, you have:

$$2 + 3 = 5$$

> Addition helps us see the total when we combine things.

Here is a step-by-step guide for addition:

1. Start with the first number.
2. Count up by the second number.
3. The number you reach is the total.

For example:

- Start with 2.
- Count up 3 steps: 3, 4, 5.
- You reach 5, so $2 + 3 = 5$.

Understanding Subtraction

Subtraction means taking away a part from a whole to find out what remains. If you have 5 apples and give away 3, you have:

$$5 - 3 = 2$$

> Subtraction helps us know how many are left when something is removed.

Follow these steps for subtraction:

1. Start with the total number.
2. Count back by the number you take away.
3. The number you land on is what remains.

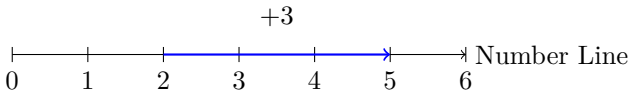
For example:

- Start with 5.
- Count back 3 steps: 4, 3, 2.
- You reach 2, so $5 - 3 = 2$.

Using a Number Line

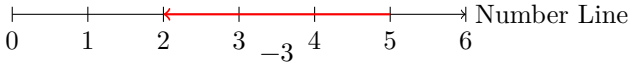
A number line is a visual tool to help understand addition and subtraction. Draw a line with numbers in order. When adding, move to the right. When subtracting, move to the left.

Below is an example number line for $2 + 3 = 5$:



The arrow shows that we move from 2 to 5 by adding 3.

Now, see a number line for $5 - 3 = 2$:



Relating Addition and Subtraction

Addition and subtraction are like two sides of the same coin. If $a + b = c$, then $c - b = a$. For example, since $2 + 3 = 5$, it also means that $5 - 3 = 2$.

> Remember: Finding the missing number is easier when you know how addition and subtraction work together.

Step-by-Step Example

Let us solve an example together:

$$4 + 2 = ?$$

Step 1: Start with 4.

Step 2: Count up 2 steps: 5, 6.

Step 3: The answer is 6, so

$$4 + 2 = 6$$

Now, if we reverse the operation:

$$6 - 2 = ?$$

Step 1: Start with 6.

Step 2: Count back 2 steps: 5, 4.

Step 3: The answer is 4, so

$$6 - 2 = 4$$

This shows how addition and subtraction are connected.

Practicing with Real-Life Examples

Imagine you have 4 pencils. You find 2 more pencils on the desk. When you count all your pencils, you have 6:

$$4 + 2 = 6$$

If you then give 2 pencils to a friend, you are left with:

$$6 - 2 = 4$$

Understanding these steps will help you build a solid foundation for both addition and subtraction facts.

How to Use Doubles and Near-Doubles to Boost Addition Skills

Doubles are pairs of the same number added together. They are easy to remember. For example,

$$4 + 4 = 8$$

When you know the double, you can use it to add numbers that are close to it. These are called near-doubles.

Using Doubles

A double is when the two numbers are the same:

- Example:

$$3 + 3 = 6$$

Knowing this fact can help you add quickly because you remember that doubling 3 gives 6.

Using Near-Doubles

A near-double is when one number is one more or one less than its double. You can use the double and then adjust by 1.

- Example: To add

$$3 + 4$$

, first think of the double of 3:

$$3 + 3 = 6$$

Since 4 is one more than 3, add an extra 1:

$$6 + 1 = 7$$

Another example:

- Example: For

$$5 + 4$$

, start with the double of 4:

$$4 + 4 = 8$$

Since 5 is one more than 4, add 1 to 8:

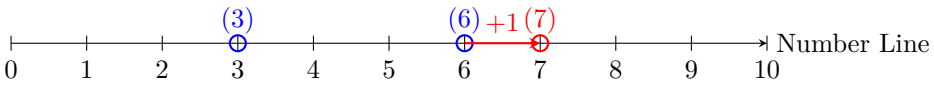
$$8 + 1 = 9$$

Step-by-Step Method

1. Identify if the problem is a double or near-double.
2. If it is a double, simply add the two same numbers.
3. If it is a near-double, find the double, then adjust by adding or subtracting 1.

Visual Example

Below is a revised diagram to help you see the process for near-doubles:



This diagram shows how you start with the double (3 + 3 = 6) and then add 1 to move to the near-double result.

Practice Together

Let's solve one together:

- Problem:

$$6 + 7$$

  Think of the double of 6:

$$6 + 6 = 12$$

  Since 7 is one more than 6, add 1:

$$12 + 1 = 13$$

Using doubles and near-doubles makes addition faster and easier. Remember to look for patterns and adjust when needed.

Mastering the Make-a-Ten Strategy for Improved Addition

The make-a-ten strategy helps us add numbers by first making a sum of 10. This makes the problem easier.

What Is the Make-a-Ten Strategy?

This strategy means we look for a pair of numbers that add up to 10. Once we make a 10, we add the rest of the number.

For example, with $8 + 6$, we see that 8 needs 2 to make 10. We can break 6 into $2 + 4$. Then, we add

$$8 + 6 = (8 + 2) + 4 = 10 + 4 = 14$$

Step-by-Step Guide

1. **Identify the Number to Complete 10:** Look at the first number. Ask, "How many more do I need to reach 10?"
2. **Break the Second Number:** Split the second number into two parts. One part fills the gap to 10; the other is extra.
3. **Add in Two Steps:** First, add to make 10. Then add the extra part.

Detailed Example

Let's add $7 + 5$ using the make-a-ten strategy.

1. Start with 7. It needs 3 to reach 10.
2. Break 5 into 3 and 2.
3. Add $7 + 3$ to get 10.
4. Then, add the remaining 2.

$$7 + 5 = (7 + 3) + 2 = 10 + 2 = 12$$

Another Example

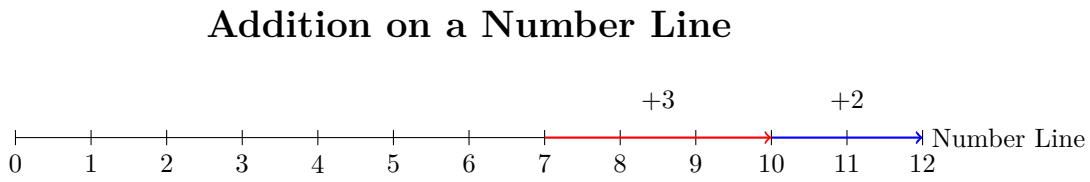
Now, try adding $9 + 4$.

1. Start with 9. It needs 1 to reach 10.
2. Break 4 into 1 and 3.
3. Add $9 + 1$ to get 10.
4. Then, add the remaining 3.

$$9 + 4 = (9 + 1) + 3 = 10 + 3 = 13$$

Visual Representation

Below is a number line that shows how the make-a-ten strategy works. First, move from the first number to 10, then add the remaining amount.



Practice in Your Mind

When you see an addition problem, try to see if one number can be paired with part of the other to form 10. This technique makes adding faster and easier.

Developing Counting On and Counting Back Techniques for Mental Math

Counting on and counting back are strategies that help you solve math problems quickly in your head. In this lesson, we will learn how to add by counting on and subtract by counting back.

Counting On

Counting on means starting at one number and counting forward to add another number. This method is useful when adding two numbers mentally.

For example, to calculate:

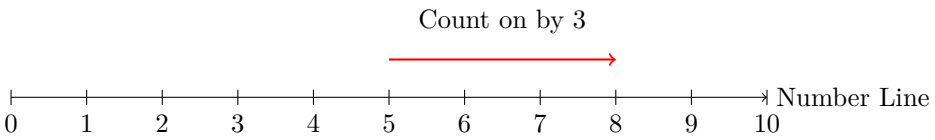
$$5 + 3$$

Follow these steps:

1. Start at 5 on the number line.
2. Count forward three steps: 6, 7, 8.
3. You have reached 8, so $5 + 3 = 8$.

> Counting on helps us add numbers by moving forward one number at a time.

Below is a visual number line showing this idea:



Counting Back

Counting back means starting at a number and moving backwards to subtract a smaller number. This method is useful when subtracting mentally.

For example, to calculate:

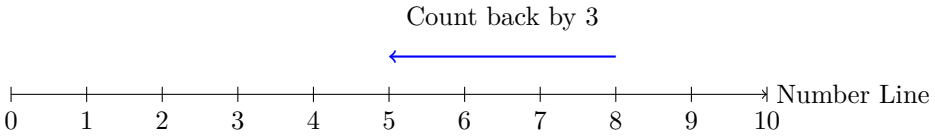
$$8 - 3$$

Follow these steps:

1. Start at 8 on the number line.
2. Count backward three steps: 7, 6, 5.
3. You have reached 5, so $8 - 3 = 5$.

> Counting back helps us subtract numbers by moving one number at a time in reverse order.

Below is a visual number line showing this idea:



Step-by-Step Approach

1. Identify the starting number.
2. If you are adding, count up one number at a time.
3. If you are subtracting, count backward one number at a time.
4. Stop after you have counted the required amount.

> These methods can make solving addition and subtraction problems easier and faster.

Real Life Applications

Counting on and counting back can be used when:

- Adding small groups of objects, like counting the total number of apples in two baskets.
- Subtracting items, like figuring out how many toys are left after giving some away.

By practicing these strategies, you can improve your mental math skills and become quicker at solving everyday problems.

Understanding Subtraction through the Unknown Addend Approach

Subtraction can be made easier by finding the missing number that makes an addition true.

When we see a subtraction problem like:

$$a - b = c$$

This means:

- The first number, a, is the starting number.
- The second number, b, is the number you take away.
- The answer, c, is what is left.

We can also think of this as an addition problem:

$$b + c = a$$

Here, a, b, and c are symbols. They stand for numbers. In our words:

- The starting number minus the number you take away equals the answer.
- Or, the number you take away plus the answer makes the starting number.

How to Solve Using the Unknown Addend Approach

1. Look at the first number. This is your starting number.
2. Look at the second number. This is the number you take away.
3. Ask: What number do I add to the second number to make the first number?

Example 1: Solving 13 - 5 = ?

We start with the problem:

$$13 - 5 = ?$$

This tells us:

- 13 is the starting number.
- 5 is the number you take away.

We ask: What do we add to 5 to get 13?

Rewrite it as:

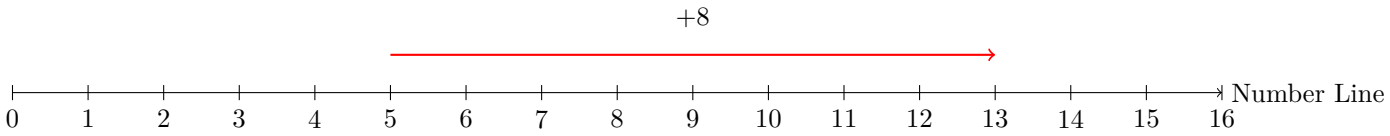
$$5 + ? = 13$$

The missing number is:

$$? = 13 - 5 = 8$$

So, 13 - 5 = 8.

Below is a number line that shows this process. The arrow starts at 5 and ends at 13, showing the addition of 8:



Example 2: Solving 18 - ? = 14

Now, look at this problem:

$$18 - ? = 14$$

Here:

- 18 is the starting number.
- 14 is the number left.

We ask: What number must be added to 14 to get 18?

Write it like this:

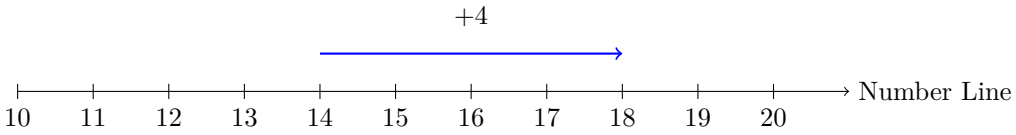
$$14 + ? = 18$$

The missing number is:

$$? = 18 - 14 = 4$$

So, 18 - 4 = 14.

This number line shows the process. The arrow begins at 14 and ends at 18:



Recap

We learned that subtraction can be seen as finding the missing addend. In the equation:

$$a - b = c$$

- a is the starting number.
- b is the number you take away.
- c is the answer (what is left).

We can also write it as:

$$b + c = a$$

This idea helps us think of subtraction as the reverse of addition.

Remember:

- Look at the starting number.
- See how much you take away.
- Find the missing number that makes the addition true.

Techniques for Adding Two-Digit Numbers Without Regrouping

When we add two-digit numbers without regrouping, the ones (the right digits) add to less than 10. This makes the addition simple.

Understanding Two-Digit Addition

Two-digit numbers have a tens digit and a ones digit. For example, in the number 23, the digit 2 is in the tens place and 3 is in the ones place.

> In these problems, we only add numbers where the ones digits add to less than 10. This means we do not need to carry a number to the tens place.

Step-by-Step Process

1. Write the numbers in a column so that the tens and ones digits are aligned.
2. Add the ones digits. Since there is no regrouping, the sum is less than 10.
3. Add the tens digits. Multiply the tens digit by 10 to find its value.
4. Combine the tens sum and the ones sum to get the final answer.

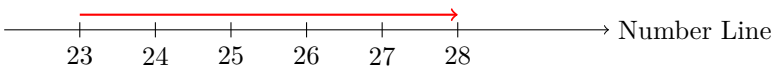
We can express the steps as follows:

$$\text{For } 23 + 15: \quad \text{Ones: } 3 + 5 = 8 \quad \text{Tens: } 20 + 10 = 30 \quad \text{Total: } 30 + 8 = 38$$

Visualizing with a Number Line

A number line helps us see the addition process. Start at the first number and count forward by the ones digit.

For example, to add the ones digits in $23 + 15$:



This shows that adding 5 to 23 gives 28. Then, when we add the tens (which is $20 + 10$), we combine them with the ones sum to get 38.

Worked Example

Let's work through another example: $42 + 31$.

1. Break down each number into tens and ones:
   - 42 is 40 (tens) and 2 (ones).
   - 31 is 30 (tens) and 1 (ones).
2. Add the ones digits:

$$2 + 1 = 3$$

3. Add the tens digits:

$$40 + 30 = 70$$

4. Combine the sums:

$$70 + 3 = 73$$

These steps show how we add two-digit numbers when no regrouping is needed.

Practice in Your Mind

Remember, when the ones digits add up to less than 10, you can add each column separately. This method keeps the addition simple and clear.

Keep practicing these steps to build strong addition skills!

Strategies for Adding Two-Digit Numbers with Regrouping and Carrying Over

In this lesson, we learn how to add two-digit numbers when the sum of the ones column is more than 9. This is called regrouping or carrying over. We will use a step-by-step method with clear examples.

Adding Two Numbers with Regrouping

We start by writing the numbers one under the other so that the ones and tens are in the correct columns.

For example, consider adding 47 and 38:

$$\begin{array}{r} 47 \\ +38 \\ \hline 85 \end{array}$$

Step 1: Add the ones column. Here, $7 + 8 = 15$. Write 5 in the ones place and carry 1 to the tens column.

Step 2: Add the tens column. Now add $4 + 3$ and the carried 1: $4 + 3 + 1 = 8$. Write 8 in the tens place. The answer is 85.

Using a detailed array, we can show the carried number above the tens column:

$$\begin{array}{r} 1 \\ 47 \\ +38 \\ \hline 85 \end{array}$$

Another Example

Let us add 36 and 47 using the same method.

Step 1: Write the numbers:

$$\begin{array}{r} 36 \\ +47 \\ \hline 83 \end{array}$$

Step 2: Add the ones column: $6 + 7 = 13$. Write 3 in the ones place and carry 1 to the tens column.

Step 3: Add the tens column: $3 + 4 = 7$, then add the carried 1 for a total of 8. Write 8 in the tens place.

A complete written method with the carried number looks like this:

$$\begin{array}{r} {\scriptstyle 1} \\ 36 \\ +47 \\ \hline 83 \end{array}$$

Summary of the Steps

1. Write the two-digit numbers aligned by their ones and tens places.
2. Add the ones column. If the sum is 10 or more, write the ones digit and carry the tens digit to the next column.
3. Add the tens column along with any carried number to get the final answer.

Practice these steps carefully to become confident in adding two-digit numbers with regrouping.

Methods for Subtracting Two-Digit Numbers Without Regrouping

Subtracting two-digit numbers without regrouping means that in each column the top digit is greater than or equal to the bottom digit. We work with the tens and ones places separately.

Understanding Place Value

Each two-digit number has a tens place and a ones place. In subtraction without regrouping, the digit in the tens and ones places of the top number is at least as big as the corresponding digit in the bottom number.

Example 1: Subtracting 65 - 32

Step 1: Write the numbers in column form.

$$\begin{array}{r} 65 \\ -32 \\ \hline \end{array}$$

Step 2: Subtract the ones place.

$$5 - 2 = 3$$

Step 3: Subtract the tens place.

$$6 - 3 = 3$$

The answer is 33.

Example 2: Subtracting 84 - 42

Step 1: Write the numbers in column form.

$$\begin{array}{r} 84 \\ -42 \\ \hline \end{array}$$

Step 2: Subtract the ones place.

$$4 - 2 = 2$$

Step 3: Subtract the tens place.

$$8 - 4 = 4$$

The answer is 42.

Visual Method Using a Number Line

A number line can help you see subtraction in action. For instance, with $65 - 32$, you can start at 65 and count backward by 32. Break the subtraction into parts:

- Count back 30 from 65 to land on 35.
- Then count back 2 more to reach 33.

Steps to Subtract Without Regrouping

In subtraction without regrouping:

- Write the numbers one under the other.
- Make sure the top digit in each column is not smaller than the bottom digit.
- Subtract the ones place.
- Subtract the tens place.

Follow these steps to solve such problems quickly and accurately.

How to Perform Subtraction with Regrouping and Borrowing

When subtracting two-digit numbers, sometimes the top number's ones digit is smaller than the bottom number's ones digit. In this case, we borrow 1 ten from the tens place to help with the subtraction.

Example: 47 — 38

Step 1: Write the numbers in columns.

$$\begin{array}{r} 4\ 7 \\ -\,3\ 8 \\ \hline \end{array}$$

Step 2: Look at the ones place. Since 7 is less than 8, there is not enough to subtract. We borrow 1 ten from the tens place.

- The tens digit 4 decreases by 1 to become 3.
- The ones digit 7 gains 10, becoming 17.

Now the problem looks like this:

$$\begin{array}{r} 3\ 17 \\ -\,3\ \ 8 \\ \hline \end{array}$$

Step 3: Subtract the ones place.

$$17 - 8 = 9$$

Step 4: Subtract the tens place.

$$3 - 3 = 0$$

The final answer is 9.

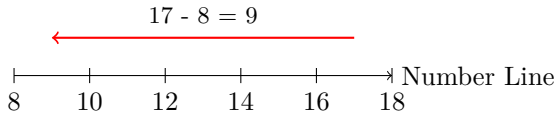
Visualizing the Borrowing

Borrowing means taking 1 ten from the tens place and adding it to the ones place, making it easier to subtract numbers that originally seem too small.

> Borrowing helps break numbers into groups of tens and ones so the subtraction can be done step by step.

Number Line Example

The following number line shows how we subtract 8 from 17 in the ones place:



Using this method, you can borrow and subtract numbers step by step. Practice with different problems to become more confident with subtraction with regrouping and borrowing.

Mental Math Strategies to Improve Speed and Accuracy

Mental math means solving problems in your head quickly and correctly. In this lesson, you will learn different ways to think about numbers easily. We will use step-by-step examples to show you how to add and subtract faster.

Strategy 1: Counting On for Addition

When you add two numbers, start from the larger number and count on. This is called counting on.

Example: Find $7 + 4$.

Step 1: Start at 7 on the number line.

Step 2: Count four numbers forward from 7.

7 (start), 8, 9, 10, and 11. So, $7 + 4 = 11$.

> Counting on means you add by moving step by step from the larger number.

Strategy 2: Counting Back for Subtraction

When you subtract, you can count back. This helps you find the answer faster.

Example: Find $12 - 3$.

Step 1: Start at 12 on the number line.

Step 2: Count back three steps from 12.

12 (start), 11, 10, and 9. So, $12 - 3 = 9$.

> Counting back means you subtract by moving step by step to a smaller number.

Strategy 3: Breaking Numbers Into Parts

Sometimes it helps to break numbers into smaller parts. You can add or subtract the parts and then join them together.

Example for Addition: Find $8 + 5$.

Step 1: Break 5 into 2 and 3.

Step 2: Add 2 to 8 first: $8 + 2 = 10$.

Step 3: Then add the remaining 3: $10 + 3 = 13$.

So, $8 + 5 = 13$.

Example for Subtraction: Find $15 - 7$.

Step 1: Break 7 into 5 and 2.

Step 2: Subtract 5 from 15 first: $15 - 5 = 10$.

Step 3: Then subtract 2 from 10: $10 - 2 = 8$.

So, $15 - 7 = 8$.

> Breaking numbers apart makes it easier to see the smaller steps of a problem.

Strategy 4: Compensation Strategy

Sometimes it's easier to adjust the numbers to round figures and then compensate. For example, to compute $9 + 7$: - Think of 9 as 10, so $10 + 7 = 17$. - Since you added one too many, subtract 1: $17 - 1 = 16$.

This "round and adjust" method can speed up your calculations!

Strategy 5: Friendly Numbers and Doubling

If you recognize doubles, you can use them to your advantage. For example: - You know that $6 + 6 = 12$. - So, if you need to compute $6 + 7$, it's just one more than 12, which is $12 + 1 = 13$.

This works well for numbers that are close to doubles.

Strategy 6: Inverse Operations for Checking

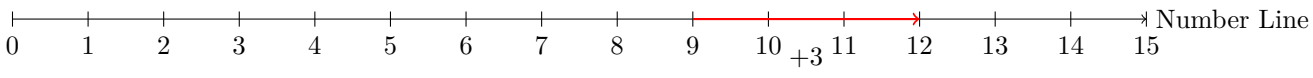
After solving a problem, use the opposite operation to check your answer. For instance: - If you calculate $15 - 7 = 8$, check by adding: $8 + 7$ should equal 15. - This helps ensure you didn't make a mistake.

These strategies, along with counting on, counting back, and breaking numbers apart, give you multiple tools to solve addition and subtraction problems quickly and accurately!

Using a Number Line to Visualize Problems

A number line is a great tool to see the steps in your head. Look at the number line and move along it.

For example, let's use a number line for $9 + 3$.



Start at 9 and move three steps to the right to reach 12. So, $9 + 3 = 12$.

Tips for Improving Speed and Accuracy

- Practice each strategy until you feel comfortable.
- Use a number line to see your steps clearly.
- Try different strategies to see which one works best for you.
- Always check your answer by thinking through each step again.

Each strategy helps you see the problem in a simple way. With these methods, you can solve addition and subtraction problems faster and more accurately!

Solving Real-World Word Problems Using Addition and Subtraction Techniques

Word problems use stories to help us practice math. In these problems, you read about a situation, decide what math to use, and then solve it step by step.

Step 1: Read the Problem Carefully

Read the problem slowly. Look for key words that tell you if you need to add or subtract.

> Key words for addition: together, in all, more, total.
>
> Key words for subtraction: left, remain, fewer, lost.

Step 2: Identify the Important Numbers

Write down the numbers from the story. Decide what each number means in the problem.

For example:

Lucy has 5 apples. She buys 3 more apples. What is the total number of apples?

The numbers are 5 and 3.

Step 3: Decide Which Operation to Use

Use addition when the problem tells you to combine numbers. Use subtraction when you remove some from a group.

In Lucy's problem, she is adding her apples, so you add the numbers.

Step 4: Solve Using Addition

Write the addition problem:

$$\begin{array}{r} 5 \\ +3 \\ \hline \end{array}$$

Now, add them:

$$\begin{array}{r} 5 \\ +3 \\ \hline 8 \end{array}$$

Lucy now has 8 apples.

Example 2: Using Subtraction

Michael had 10 marbles. He lost 4 marbles. How many marbles does he have left?

Here, you need to subtract because some items are taken away.

Write the subtraction problem:

$$\begin{array}{r} 10 \\ -\,4 \\ \hline \end{array}$$

Subtract step by step:

$$\begin{array}{r} 10 \\ -\,4 \\ \hline 6 \end{array}$$

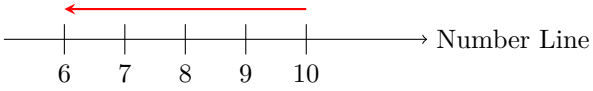
Michael has 6 marbles left.

Visualizing the Problem with a Number Line

Using a number line can help you see how numbers change when you add or subtract.

For Lucy's problem (5 + 3):

For Michael's problem (10 - 4):



Step 5: Check Your Work

Look over your numbers and steps. Answer the question in your own words:

- Lucy's problem: 5 apples plus 3 apples equal 8 apples.
- Michael's problem: 10 marbles minus 4 marbles equal 6 marbles.

By breaking the problem into small parts, you can use addition and subtraction to solve real-world problems.

Recognizing Repeating Patterns in Addition and Subtraction Problems

Repeating patterns show us a rule that happens over and over. In these problems, the rule is to add or subtract the same number each time.

What Is a Repeating Pattern?

A repeating pattern is a process that repeats in the same way. In addition and subtraction, it means adding or subtracting the same value every time.

A pattern is simply a rule that tells us what comes next.

Recognizing a Repeating Pattern in Addition

Consider this example:

$$4 + 2 = 6$$

Now, add 2 again to the result:

$$6 + 2 = 8$$

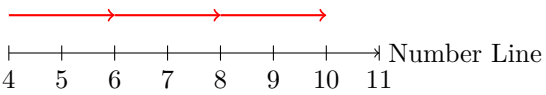
And one more time:

$$8 + 2 = 10$$

Each step adds 2. The pattern is: start with 4 and add 2 many times. This repeating action helps us know the next number.

Visualizing the Addition Pattern

A number line can show the same idea. Look at the number line below for the addition example:



Each red arrow shows that we add 2 to move forward.

Recognizing a Repeating Pattern in Subtraction

Now, look at a subtraction example. Suppose we start with 15 and subtract 3 each time.

$$15 - 3 = 12$$

Subtract 3 again:

$$12 - 3 = 9$$

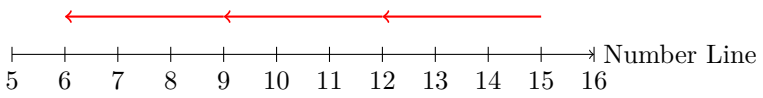
And one more time:

$$9 - 3 = 6$$

The rule is clear: start at 15 and subtract 3 every step. This is a repeating subtraction pattern.

Visualizing the Subtraction Pattern

We can also use a number line to see subtraction. Imagine a number line where we move to the left by 3 each time.



Each red arrow shows a jump backward by 3.

Key Points

- A repeating pattern means doing the same addition or subtraction each time.
- In our addition example, we add 2 repeatedly.
- In our subtraction example, we subtract 3 repeatedly.
- Understanding the rule helps us predict what comes next in the pattern.

Recognizing these patterns makes solving math problems easier because you learn the rule and can use it to find missing numbers.

An Introduction to Balancing Equations and Finding Missing Numbers

An equation is like a scale. Both sides must be equal. In an equation, we add numbers and sometimes a missing number makes one side lighter. We find the missing number to balance the equation.

What Is a Balanced Equation?

A balanced equation looks like this:

$$3 + \ ? \ = 7$$

This means when you add the number that is missing (the question mark) to 3, you get 7. To find the missing number, we subtract 3 from 7.

$$? = 7 - 3$$

So, the missing number is 4. We can check it:

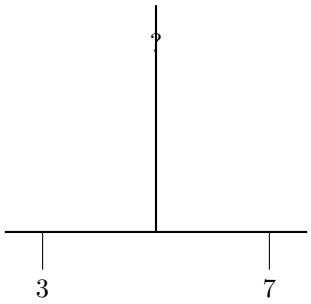
$$3 + 4 = 7$$

Step-by-Step Explanation

1. Look at the equation. One side is missing a number.
2. Find the known number on that side (here, it is 3).
3. Subtract the known number from the total on the other side (7 - 3).
4. The result is the missing number (4).

Visualizing the Equation

Imagine a balance scale. One side has the numbers you know, and the other side shows the total value. The missing number is what you need to add to the first side so the scale stays balanced.



Another Example

Consider this equation:

$$? + 5 = 10$$

To find the missing number, follow these steps:

1. Identify the known number on the left side (5).
2. Subtract 5 from 10.

$$? = 10 - 5$$

So, the missing number is 5. We can check:

$$5 + 5 = 10$$

Practice in Your Head

Think about the equation:

$$2 + ? = 5$$

Ask yourself: What number added to 2 gives 5? You can solve it by subtracting 2 from 5:

$$? = 5 - 2$$

So, the missing number is 3. Check it by adding:

$$2 + 3 = 5$$

Key Ideas

> A balanced equation means both sides have the same value.

We use subtraction to find the missing number when one side of the equation is incomplete.

This concept helps us understand that every action in math has an opposite action. In these problems, subtraction is the opposite action of addition.

By practicing these steps, you learn to think about numbers as part of a balanced game, where both sides must be equal.