

# Contents

<b>Introduction</b>	<b>2</b>
<b>Numbers and Operations</b>	<b>3</b>
Place Value . . . . .	3
Reading and Writing Numbers . . . . .	4
Expanded Form . . . . .	6
Comparing and Ordering Numbers . . . . .	7

# Introduction

Welcome to Grade 3 Mathematics. This textbook is designed to enhance your mathematical knowledge with a comprehensive study of several key areas:

- **Numbers and Operations:**
  - **Place Value:** Understand the value of digits in larger numbers, which is fundamental for arithmetic operations and comparison.
  - **Reading and Writing Numbers:** Learn to express numbers in different forms, which enables clear communication of quantities.
  - **Expanded Form:** Break down numbers to show the value of each digit, aiding in comprehension and estimation.
  - **Comparing and Ordering Numbers:** Develop skills to assess which numbers are larger or smaller, an essential skill for practical decision-making.
  - **Addition and Subtraction:** Master these operations, starting with two-digit numbers and progressing to three-digit numbers. These are critical for tasks like budgeting and adjusting plans.
  - **Multiplication and Division:**
    - \* Learn multiplication as repeated addition and division as sharing or grouping.
    - \* Understand multiplication and division facts, which are important for efficient calculation and logical problem-solving.
    - \* Recognize the role of remainders and how they affect division results.
- **Fractions:**
  - Understand and compare fractions, and learn about equivalent fractions. This knowledge is used in tasks like splitting portions and understanding ratios.
  - Explore fractions on a number line to see their relation to whole numbers.
  - Practice adding and subtracting fractions with like denominators and working with mixed numbers, important in measurements and detailed calculations.
- **Measurement and Data:**
  - Measure length in inches, feet, centimeters, and meters, relevant for tasks from simple home projects to scientific experiments.
  - Understand weight and mass in ounces, pounds, grams, and kilograms.
  - Learn about volume and capacity with units like cups or liters, which are vital for cooking and chemistry.
  - Discover concepts of time, including reading clocks and calculating elapsed time, essential for scheduling and time management.
  - Study money, covering the identification of coins and bills, as well as making change, crucial for financial literacy.
  - Develop skills in reading and creating bar graphs and line plots to interpret and present data visually.
- **Geometry:**
  - Learn about plane shapes and solid shapes, which are foundational in understanding the spatial dimensions of objects around us.
  - Explore perimeter and area, key for design and layout tasks.
  - Study lines and angles, including concepts of types of angles and parallel vs perpendicular lines, which apply in fields like art and architecture.
- **Algebraic Thinking:**
  - Develop skills by identifying number patterns and understanding even and odd numbers, and practicing skip counting.
  - Tackle missing numbers, addends, and factors to strengthen problem-solving skills.
  - Work on word problems to apply math in real contexts, learning to identify keywords and approach multi-step problems.
- **Probability and Logic:**
  - Introduction to probability, focusing on concepts such as likely vs unlikely and cer-

- tain vs impossible events.
- Logic and reasoning with exercises like if-then statements and true or false evaluations which aid in structured thinking and prediction-making.

Throughout these topics, mathematics will be linked to practical applications, helping you see how these concepts are used in everyday life. The goal is to establish a robust mathematical foundation that will serve you in education and real-world contexts.

## Numbers and Operations

Numbers and operations are foundational elements in mathematics, shaping how we understand and engage with the world. Whether calculating the distance between stars or the ingredients needed in a recipe, numbers are omnipresent. Operations such as addition, subtraction, multiplication, and division are the tools that enable us to manipulate and interpret these numbers effectively.

Numbers are more than mere symbols; they have a fascinating history. The earliest recorded use of numbers dates back to ancient Sumerians around 4000 BCE, where they were used for trade and record-keeping. Over time, different cultures developed unique numerical systems, such as the Roman numerals and the Hindu-Arabic numeral system we use today.

“Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding.” — William Paul Thurston

Operations allow us to solve real-world problems efficiently. For instance, ancient Egyptians used basic arithmetic to construct the pyramids, demonstrating an early application of mathematical principles to engineering.

In this section, we will explore how numbers and operations function as the building blocks of mathematics. From tracking the growth of a garden over time to computing the trajectory of a space probe, they are essential tools in a child’s journey into mathematics. Understanding how they work enables us to not only solve problems but also to appreciate the underlying order and patterns in the natural and man-made world.

## Place Value

Place value is a fundamental concept in mathematics that helps us understand the value of digits in numbers based on their position. This concept is vital because it forms the basis of our number system, which is known as the base-10 or decimal system.

### Understanding Place Value

In the base-10 system, each digit in a number has a value that depends on its position or ‘place.’ Starting from the right, the first place is the ‘ones,’ the second is the ‘tens,’ the third is the ‘hundreds,’ and so on. For example, in the number 345, the digit 5 is in the ‘ones’ place, 4 is in the ‘tens’ place, and 3 is in the ‘hundreds’ place. This means that the number represents  $3 \times 100 + 4 \times 10 + 5 \times 1$ .

### Historical Context

The concept of place value has evolved over thousands of years. Here’s a glimpse into its fascinating history:

- **Ancient Civilizations:** Early civilizations, like the Egyptians and the Romans, used different methods and symbols for counting, but those lacked a true place value system. Roman numerals, for example, represent different values through combinations of letters, but their system does not positionally determine numerical value.

- **Babylonians and Base-60:** The Babylonians were early pioneers of a place value system, using a base-60 (sexagesimal) system. This choice was partially due to 60's divisibility by many numbers, making calculations of fractions more manageable. Their numerals were written using a combination of two symbols in a vertical and horizontal arrangement.
- **Emergence of Zero:** A crucial development in place value was the introduction of zero. Around the 5th century AD, Indian mathematicians recognized zero as a number and a placeholder, allowing for more sophisticated calculations and the propagation of the fully positional decimal system. This innovation was essential for distinguishing quantities such as 40 from 400.
- **Spread to the Western World:** The decimal system, along with the concept of zero, eventually spread to the Arab world and later to Europe. It greatly enhanced mathematical calculations, paving the way for the scientific and commercial advancements of the Renaissance.

## Different Number Systems

While the base-10 system is widely used today, it is not the only possible system.

- **Binary System:** Suitable for computers, this system uses only two digits, 0 and 1, representing numbers through combinations of these digits.
- **Hexadecimal and Beyond:** Other systems like base-16 (hexadecimal) are used in computing, employing digits 0-9 and the letters A-F to represent values.

## Why Place Value Matters

Understanding place value is crucial because:

- It helps in comprehending the size and scale of numbers.
- It simplifies arithmetic operations like addition, subtraction, multiplication, and division.
- It is foundational for understanding more complex mathematical concepts, such as algebra and calculus.

## Real-World Applications

- **Currency and Finance:** Place value helps us understand money, as different positions of a number in currency units denote vastly different wealth amounts.
- **Measurements:** Place value assists in accurately reading and writing measurements in various units of distance, weight, or time.

The history and development of place value benefited many aspects of daily life and technology and continue to be an essential part of education and mathematics today. Understanding how numbers work, not just what they are, allows for greater numeric literacy and problem-solving skills.

## Reading and Writing Numbers

**Understanding Numbers up to 10,000** In this lesson, we will explore how to read, write, and represent whole numbers up to 10,000. This involves understanding the number in different forms, such as standard form, word form, and expanded form. We will also discuss place value, which is crucial in understanding how numbers work.

**Standard Form** **Standard form** is the way numbers are commonly written using digits. For example, the number three thousand four hundred seventy-eight is written as 3,478 in standard form. Each digit has a specific place value, determining its meaning based on its position.

- Example: 7,890
  - 7 is in the thousands place
  - 8 is in the hundreds place
  - 9 is in the tens place
  - 0 is in the ones place

**Word Form** **Word form** involves writing the number in words. Understanding how to convert a number to word form requires familiarity with terms for different place values.

- Example: 6,532 is written as “six thousand five hundred thirty-two.”

This form is often used in writing checks or formal documents.

**Expanded Form** In the **expanded form**, a number is broken down into the sum of values of each digit, based on its place value. This form helps in understanding how each digit contributes to the overall value of the number.

- Example: 4,206 is expanded as  $4,000 + 200 + 6$

**Place Value** **Place value** is the foundation of reading and writing numbers correctly. It refers to the value of a digit based on its position within a number.

- Example: In 5,647
  - 5 is in the thousands place, meaning it represents 5,000
  - 6 is in the hundreds place, meaning it represents 600
  - 4 is in the tens place, meaning it represents 40
  - 7 is in the ones place, meaning it represents 7

Place value helps us compare numbers to determine which is larger or smaller.

**Practice Problems** Convert the following numbers to all three forms (standard, word, and expanded):

1. 7,301
  - Standard: 7,301
  - Word: “seven thousand three hundred one”
  - Expanded:  $7,000 + 300 + 1$
2. 9,425
  - Standard: 9,425
  - Word: “nine thousand four hundred twenty-five”
  - Expanded:  $9,000 + 400 + 20 + 5$
3. 8,060
  - Standard: 8,060
  - Word: “eight thousand sixty”
  - Expanded:  $8,000 + 60$
4. 2,917
  - Standard: 2,917
  - Word: “two thousand nine hundred seventeen”
  - Expanded:  $2,000 + 900 + 10 + 7$

**Application** Understanding how to represent and interpret numbers up to 10,000 is essential in everyday tasks such as reading population data, budgeting finances, and measuring distances. Mastery of these skills enhances accuracy in both academic contexts and real-world applications, leading to better decision-making.

Recognizing and working with numbers is a crucial skill developed over time. Explore datasets or interactive applications that highlight these concepts to enrich your learning experience.

## Expanded Form

**Introduction to Expanded Form** Expanded form is a way of breaking down a number to show the value of each digit. It helps us to understand the role of each place value in forming the entire number. For example, the number 345 can be expressed in expanded form as:

$$300 + 40 + 5$$

This format makes it clear that the 3 represents three hundred, the 4 represents forty, and the 5 represents five.

**How to Write Numbers in Expanded Form** To write a number in expanded form:

1. **Identify the place value** of each digit in the number.
2. **Multiply each digit** by its place value.
3. **Create a sum** of these values.

### Example:

Consider the number 2,581. To express it in expanded form, follow these steps:

- The digit 2 is in the thousand's place:

$$2 \times 1000 = 2000$$

- The digit 5 is in the hundred's place:

$$5 \times 100 = 500$$

- The digit 8 is in the ten's place:

$$8 \times 10 = 80$$

- The digit 1 is in the one's place:

$$1 \times 1 = 1$$

Combine these values to write the expanded form:

$$2000 + 500 + 80 + 1$$

## Practice Problems

1. Write the number 3,746 in expanded form.
2. Convert the expanded form

$$400 + 30 + 2$$

into standard form.

3. Express the number 5,082 in expanded form.
4. Write the expanded form for the number 6,109.
5. Given the expanded form

$$7,000 + 500 + 60 + 9$$

, write it in standard form.

**Real-World Application** Understanding expanded form is crucial for learning how to manipulate numbers in math problems effectively. In real life, this concept is particularly useful in financial contexts, such as:

- **Accounting and Finance:** Breaking down amounts helps in understanding the distribution of figures in budgets and invoices.
- **Measurements:** In scientific contexts, scientific notation—which is a form of expanded form—is used to express large numbers concisely.

**Historical Insight** The concept of place value and expanded notation has roots in ancient civilizations such as the Babylonians and Egyptians. They used different systems to signify numbers which laid the groundwork for our modern numeral system. The Hindu-Arabic numeral system, which we use today, was further developed around the 6th century AD, incorporating a zero which allowed for a positional number system, a crucial aspect of our modern numeric expansions.

## Comparing and Ordering Numbers

**Understanding the Basics** When we compare numbers, we determine which number is greater, lesser, or if they are equal. This is an essential skill as it helps us make decisions based on numerical values. For example, knowing that 20 is less than 50 can help you understand that 20 candies are fewer than 50 candies.

**Symbols Used in Comparison** There are three main symbols used to compare numbers:

- Greater than (  $>$  )
- Less than (  $<$  )
- Equal to (  $=$  )

Here are examples of how they are used:

- $5 < 9$   
(5 is less than 9)
- $12 > 3$   
(12 is greater than 3)
- $7 = 7$   
(7 is equal to 7)

**Real-World Example** Imagine you have two snack boxes. One box contains 15 chocolates and the other contains 18. To compare these, you calculate:

$$\begin{array}{r} 15 \\ < \\ 18 \end{array}$$

This tells you the first box has fewer chocolates than the second.

**Ordering Numbers** Ordering numbers means arranging them from the smallest to the largest (ascending order) or from the largest to the smallest (descending order).

- **Ascending Order:** 3, 7, 12, 25
- **Descending Order:** 25, 12, 7, 3

### Steps to Compare and Order Numbers

#### 1. Comparing Two Numbers:

- Look at the highest place value (like hundreds, tens, ones) first.
- Compare digits starting from the leftmost place. If they are the same, move to the next place.
- Use the correct symbol (  $<$  or  $>$  ) to indicate which is greater or lesser.

#### 2. Ordering Several Numbers:

- Write down the numbers.
- Compare each pair using place value.
- Arrange them in the required order, either ascending or descending.

### Practice Problems

1. Compare these pairs of numbers using  $<$ ,  $>$ , or  $=$ .
  - $\frac{1}{2}$   $\frac{1}{4}$
  - 8  $\frac{1}{2}$
  - 10  $\frac{1}{2}$
  - 45  $\frac{1}{2}$



•

67

—

76

2. Arrange these numbers in ascending order:

14, 3, 9, 27

3. Order these numbers in descending order:

43, 18, 25, 32

**Historical Context** The symbols for greater than and less than (

>

and

<

) were introduced by Thomas Harriot, an English mathematician, in the 16th century. These symbols have made mathematical communication clearer and are now used worldwide.

**Why It Matters** Comparing and ordering numbers is crucial in everyday activities such as shopping, where you compare prices, or in scheduling, where you arrange events by time. Understanding these concepts will help you make informed and efficient choices.