CECS 451
Assignment 7
Total: 60 Points

General Instruction

- Submit your work in the Assignment folder via Canvas (Not email)
- Simple calculation is required, otherwise you will get half of the points
- Submit separate files: a PDF file for (1) and (2), hmm.ipynb for (3)

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1. (6 points) In Figure 1, suppose we observe an unending sequence of days on which the umbrella appears. As the days go by, the probability of rain on the current day increases toward a fixed point, we expect that $\vec{P}(R_t|u_{1:t}) = \vec{P}(R_{t-1}|u_{1:t-1}) = \langle \rho, 1-\rho \rangle$. Find ρ with a scale of 4, i.e., 0.1234

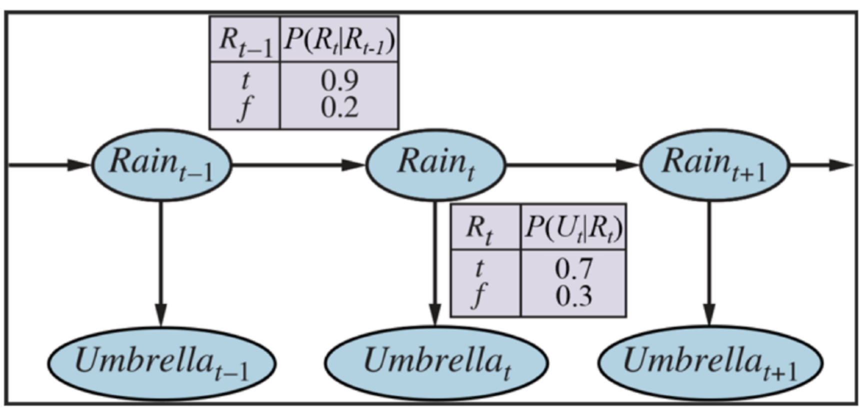

Figure 1: Bayesian network structure and conditional distributions describing the umbrella world
The transition model is $\vec{P}(R|R_{t-1})$ and the sensor model is $\vec{P}(U|R_t)$

2. A professor wants to know if students are getting enough sleep. Each day, the professor observes whether they have red eyes. The professor has the following domain theory:
   - The prior probability of getting enough sleep, with no observations, is 0.7
   - The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not
   - The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not

   a. (6 points) Formulate this information as a hidden Markov model. Give a Bayesian network and conditional distributions.

b. (8 points) Consider the following evidences, and compute $\vec{P}(ES_2|e_{1:2})$ with a scale of 4

   i. $e_1 = red\ eyes$

   ii. $e_2 = not\ red\ eyes$

3. (40 points) Implement a program to perform filtering in the hidden Markov model (HMM).

   a. Assume that the hidden state variable and the evidence variable are binary variables.

   b. The program should compute $\vec{P}(X_t|\vec{e}_{1:t})$ when $\vec{e}_{1:t}$ is given.

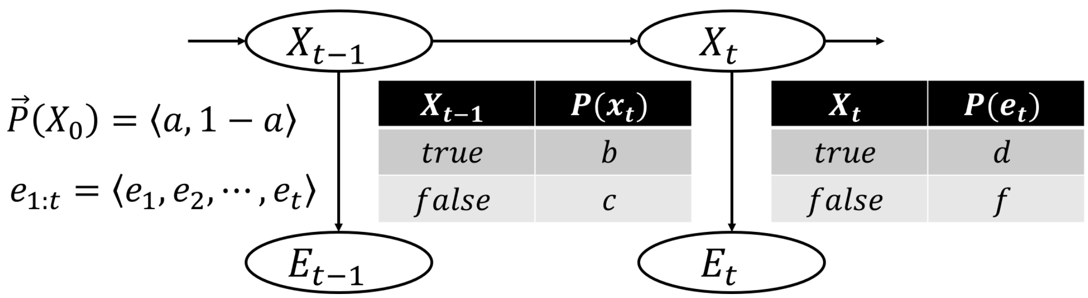

Figure 1: HMM of binary variables.

   c. The program receives user input as a string containing independent variables $a, b, c, d, f, e_1, e_2, \ldots, e_t$ in Figure 1 in that order. For example,

```
Enter the values: 0.5,0.7,0.3,0.9,0.2,t,t
```

   means $a = 0.5,\ b = 0.7,\ c = 0.3,\ d = 0.9,\ f = 0.2,\ e_1 = t,\ e_2 = t$

   d. Show the probability $\vec{P}(X_t|\vec{e}_{1:t})$ for these inputs:

      i. `0.5,0.7,0.3,0.9,0.2,t,t`

      ii. `0.5,0.7,0.3,0.9,0.2,t,t,f`

   Please follow the output format. (Fix precisions using `"{:.4f}".format`)

```
P(X_t|e_1:t) = <#.####,#.####>
```

   a. Submit **`hmm.ipynb`** (including the outputs)