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A Genetic Algorithm for the Multi-Depot Vehicle Routing Problem

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Abstract. In this paper, we focus on the optimization of the system of the spare parts distribution for authorized garages in the Czech Republic. A spare parts market belongs to one of the key elements of the car industry. However, it has to adapt to still higher requirements on accuracy, speed and minimum error rate of the deliveries with keeping the costs at its minimum at the same time. The distribution of products from depots to customers is a practical and challenging problem in logistics that opens a significant space for application of software products.

The design of optimal routes of vehicles from two depots can be formulated in combinatorial optimization as a multi-depot vehicle routing problem.

The goal of a multi-depot vehicle routing problem is to design routes that start and end in one of the depots and visit a subset of customers in a specific sequence. Every customer has to be visited on one of the routes and the total costs for the delivery should be minimal.

Vehicle routing problems belong to the class of NP-hard problems which means that there is no efficient algorithm for finding optimal solution available. To find a solution in an efficient way, we propose an approximate method based on a genetic algorithm.

Introduction

In logistics systems, physical transport of goods from manufacturer or distributor to the consumer is one of the most important functions. Consider a network of depots with known location and unlimited capacity, set of consumers with known location and demand and fleet of vehicles with known capacity of transported goods. Each vehicle leaves a depot, serves a subset of consumers and returns to depot. In the literature, this problem is called vehicle routing problem (VRP) and it was first described by Dantzig and Ramser in 1959 [1].

The VRP derives from travelling salesman problem (TSP), where the vehicle has ho limitation of capacity, so one vehicle can serve all consumers in single route, and arc routing problem. The VRP expands to the TSP with addition of depots, from which vehicles have to start and return to. So, the simplest type of VRP problem is that when we have only one depot and unlimited capacity (sometimes called single depot vehicle routing problem (SDVRP) [2]). In addition, many other variants of VRP are known.

One of the variants is capacitated VRP (CVRP), which adds the capacity to the travelling vehicles. Every consumer has a fixed volume of demand, so the problem is how to construct the routes in such a way that the length of it is minimal and the capacity limit of the vehicle will not be exceeded.

Another variant is VRP with time windows (VRPTW). In this variant, the consumers have given time window when they are available. Variation of this kind of problem also implements the soft time windows, where consumer still operates outside of the time windows but visiting them in that time involves some form of penalty.

Multi depot VRP (MDVRP) extends the basic VRP in such a way, that more than one depot is considered. Depending on the situation, vehicles are obligated to return to the starting depot (fixed) or another depot (non-fixed). Extended variant is when the locations of the depots are not fixed and the goal is to find the optimal locations for them (cost of the traveling will be the smallest). The fixed MDVRP with capacities is the focus of our research.

Review of multi-depot VRP

First works about meta-heuristic solutions for MDVRP were issued in 1996 by Renaud, Laporte et al. [3]. They used meta-heuristic method called Tabu-Search (TS) and they studied the MDVRP with constraints of vehicle capacities and maximum duration of routes. First paper about usage of genetic algorithms (GA) for MDVRP was published by Filipec, Skrlec and Krajcar (1997) [4]. Their objective was minimizing total travel distance. Thangiah and Salhi (2001) [5] introduced solution of this problem by method called Genetic Clustering (GenClust). It means that they used genetic algorithms to define clusters of clients and then the routes are found by TSP.

The most studied type of problem has been the capacitated MDVRP. First papers for this variant were published by Bae, Hwang, Cho, and Goan [6] in 2007 and Vidal, Crainic, Gendreau, Lahrichi, and Rei [7] in 2010. Both work focused on optimizing total route distance or cost.

Another big group of works has been published for the problem variant with heterogeneous fleet (HFMDVRP). We can highlight the work of Jeon, Leep, and Shim [8] from 2007. They used a hybrid genetic algorithm that minimizes the total distance traveled.

Some authors also propose some hybrid meta-heuristic algorithms, where the GA was used. A hybridized genetic algorithm – ant colony optimization procedure was presented by Liu and Yu [9] in 2013. This algorithm minimizes the maximum travel distance of a vehicle in a system with heterogeneous fleet of vehicles. Another work from Liu [10] was published also in 2013 and it solving the classical MDVRP with use of genetic algorithm with bee colony optimization and simulated annealing.

Multi-Depot Vehicle Routing Problem

Vehicle Routing Problem (VRP) can be described as a problem seeking to service a number of customers with a fleet of vehicles. Every route starts in a depot; afterwards a subset of customers is visited in a specific sequence followed by the return to the depot. Every customer has to be assigned to exactly one route and the total amount of shipments of customers assigned to a given vehicle cannot exceed the capacity of the vehicle. The routes should be chosen so that the total costs for delivery are the minimum.

Vehicle routing problem can be seen as a generalization of the traveling salesman problem. The difference from the traveling salesman problem is a number of further constraints and extensions that can often be found in real life problems.

Multi-Depot Vehicle Routing Problem (MDVRP) extends the problem to seeking optimal routes from multiple depots. As in VRP, every route has to start and end in the same depot. Therefore the fleets are independent for every depot.

Mathematical formulation. MDVRP can be described as follows:

m depots d_m uses k independent vehicles with identical capacity K. Vehicles satisfy requirements r_i of n customers, $i = 1, \dots, n$. The solution of Multi-Depot Vehicle Routing Problem is a division of a set of n customers into k routes $R_1, \dots, R_k$ that minimizes an objective function of the total transportation costs and the capacity is not exceeded on any route $\sum_{v_p \in R_j} r_p \leq K$:

$$\min\ F_{MDVRP} = \sum_{j=1}^{k} C(R_j). \tag{1}$$

Costs of a single route $R_j = d_m, v_1, \dots, d_m$ are calculated in the following way:

$$C(R_j) = \sum_{i=0}^{l} c_{i,i+1} \tag{2}$$

where c_{ij} denotes transportation costs among customers i and j for $i, j = 1, \dots, n$ or customer and depot.

c_{ii} is valid for $\forall i = 1, \dots, n$ and moreover for a symmetric problem $\forall i, j : c_{ij} = c_{ji}$.

Description of the problem

The optimized logistic problem deals with the system of distribution of spare parts to authorized garages of two car manufacturers in the Czech Republic [11, 12].

Spare parts market is one of the key elements of the car industry and recently even gaining in its importance. As the customers demand fast and flexible repair services, also the whole distribution system of spare parts has to adapt to still higher requirements on accuracy, speed and minimum error rate of the deliveries with keeping the costs at its minimum at the same time.

Current distribution system

Spare parts of two types (modes) are ordered every working day by garages in the central depot. The following modes are distinguished:

Urgent - ordered spare parts must be prepared and sent to the Czech Republic on the same day. At night the trucks arrive at the distribution depots in Brno and Prague (garages are divided by region) and by 8:00AM of the following day the spare parts must be delivered to the garages.

Stock - applies to the spare parts used for more common repairs and therefore for the predictable demand. In this mode, spare parts have to be delivered to the garages within 48 hours from the order time.

The current system uses the same routes for everyday deliveries that supply the garages from Monday till Friday. Every day all garages in the system are visited, therefore there is no need to adapt the routes on daily basis. Ten routes are designed from Prague depot with the total of 64 visited garages and five routes from Brno depot with 33 garages.

The limiting conditions of the routes are mainly given by the short time for delivery that is required. On the other hand, capacity of vehicles, which is the basic constraint in VRP, has only little importance in our problem. On the basis of current design, the total distance traveled on one route and the number of garages visited on one route is chosen as the problem constraints.

Description of the algorithm

Genetic algorithm is a metaheuristic method that is inspired by nature processes. Genetic algorithms belong to a larger group of evolutionary algorithms that use techniques from natural evolution such as inheritance, mutation, selection, and crossover.

Using a genetic algorithm for solving a given problem consists mainly from the setting of the objective function (called fitness function), the design of the representation and the design of genetic operators (i.e. mutation, selection, and crossover.), that are all specific to the optimized problem.

Our present approach is based on our previous papers [13, 14] that proposed the solution for the same problem by a genetic algorithm, but the customers were first clustered and assigned to the depots and afterwards each depot was considered separately. The following genetic algorithm, on the other hand, solves the whole system interdependently and seeks the routes for all depots at the same time. Presented genetic algorithm is not limited to above described problem and can be used for a general MDVRP.

Representation. Each solution of the problem (called genome or individual) is represented by natural numbers. Every individual of the population consists of a set of routes and every route is constituted from a depot and a customer list with the customers listed in the same order, in which they are visited on the route. Both the depot and the customers on the route are selected from corresponding sets. Each customer can be assigned to only one route and each route has its own, independently chosen depot. An example of the representation of one individual with ten visited customers and two depots is given in Figure 1.

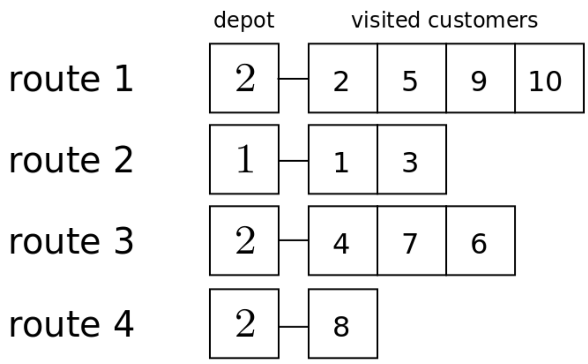

Fig. 1 Example of the representation

At the beginning of the algorithm, the population has to be filled by individuals, which are in this case created randomly but with ensuring that all given constraints are met (length of one route, number of points on one route). Also in the next steps of the algorithm, only feasible solutions are created. If some route of an individual exceeds the limiting constraints, then the route is divided into several new ones.

Fitness function. The choice of the fitness function corresponds to the aim of given problem which is to minimize the total traveled distance. The fitness function value is thus calculated as a sum of distances traveled on individual routes.

Selection. As a selection method, the tournament selection is used. For each parent, 3 individuals are chosen randomly and out of them the best one regarding the fitness value is selected. This individual then enters the crossover process as a parent.

Crossover. The crossover has to be designed with regard to feasibility of solutions. To avoid creating duplications, the following crossover operator is used. From the first parental solution, a random string is chosen and it is inserted in a random position in the second parent. All points in the inserted string are then deleted from the second parent so that no point occurs multiple times in the resulting individual. Second parental solution enters as an offspring the next generation. The result of this crossover is creating one offspring from two parental solutions. The crossover principle is demonstrated on example in Figure 2.

Second type of crossover that is used is described in [15]. The position in the second parent, to which the sequence is inserted, is not chosen randomly but as a point nearest to the first point of the inserted string. Both types of crossover are alternated in the algorithm with probability of 0.5.

The depots are not shifted during crossover. Instead, the depot, which is assigned to a particular route, is always the one that has the smallest sum of distances to endpoints of the route. In the optimal solution, such depot has to be assigned to every route. A check is performed after every step, in which routes have changed, to determine the depot corresponding to the route endpoints.

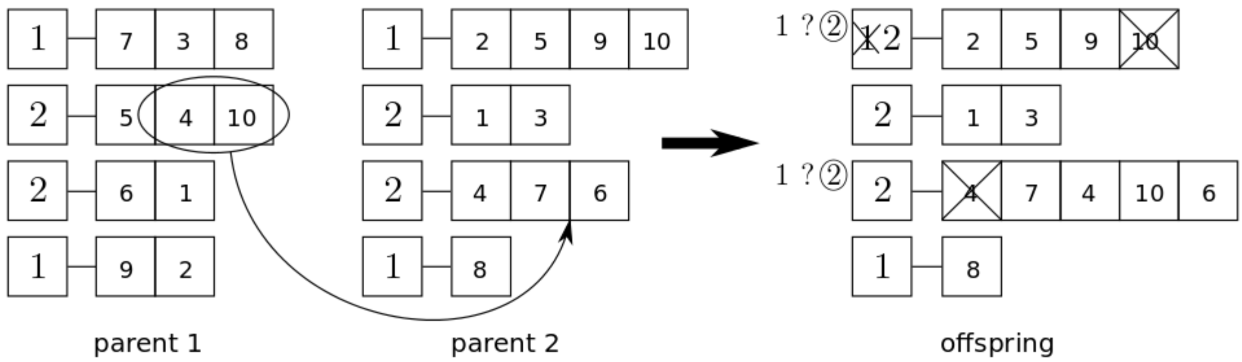

Fig. 2 Example of the crossover

Mutation. In the mutation, two random points in the solution change their places. Such mutation ensures creating feasible solution without duplications, just like in the case of the crossover.

Elitism. The elitism was included in the algorithm. In previously tested algorithms for VRP, adding elitism ensured significantly better results.

Results of the tested algorithm

Here, we summarize results of an improved and more general version of our genetic algorithm for approaching the supreme solution for route allocation in the testing problem of spare parts distribution to authorized garages of two car manufacturers in the Czech Republic.

We have tested the code with various parameter settings, which are all listed in Table 1 along with the corresponding values of the fitness function. All runs lasted over 30.000 cycles with two choices for the size of population of 30 and 50 individuals. Probabilities of mutations we set to 0.05, 0.10, or 0.15 and probabilities for crossovers are 0.70, 0.80, 0.85, 0.90 or 0.95. The minimum value for fitness function has varied from 3963 to 4248 km, which makes less than 4% spread in the results suggesting that the algorithm itself is quite robust and the particular choice of parameters (within a reasonable range) has little influence on its results.

Still, a difference of 100 km in the resulting fitness function represents significant annual savings in our particular problem. Generally, we can see that runs with larger population produce on average better results. Out of 15 runs for each population size, 9 results have come out with better fitness function value for the larger of the two populations and also the spread of fitness function is narrower for the larger population. The other two parameters (mutation and crossover probability) do not have a clear systematic influence on the results.

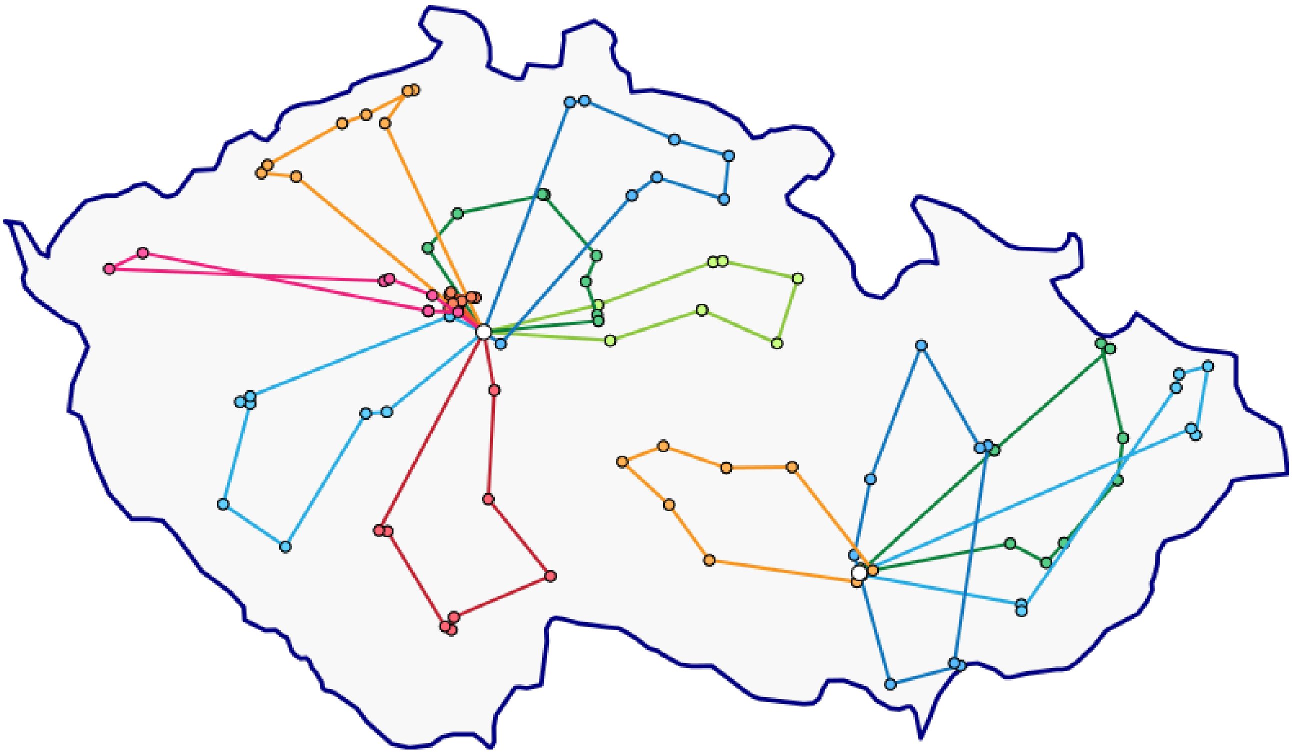

Fig. 3 The best solution displayed in the map of the Czech Republic

The absolute best result of our improved algorithm (3930 km) is very close to the best result of its previous version (3925 km) that used static allocation of customers to depots. This seem to be due to the specific geographic distribution of customers with respect to depot locations, which does not give much alternatives in customer assignment - there are only few customers that are within a good reach of both depots. The current best solution is shown in Fig. 3.

We can conclude that the size of the population has more significant effect and the results vary in more narrow range, which is probably the most important condition for practical use of the algorithm. It suggests that larger data sets require creating populations of more individuals and also executing of more cycles to ensure the convergence of the algorithm.

Table 1 Results of the testing - minimum fitness function values based on different parameters

population size 30		Mutation probability		
		0.05	0.10	0.15
crossover probability	0.70	4151.0	4114.0	4058.0
	0.80	3992.0	4025.0	4248.0
	0.85	4011.0	4110.0	4031.0
	0.90	4221.0	4147.0	4029.0
	0.95	4090.0	4011.0	3963.0

population size 50		Mutation probability		
		0.05	0.10	0.15
crossover probability	0.70	4117.0	4029.0	4058.0
	0.80	4104.0	4052.0	4141.0
	0.85	4092.0	4173.0	3990.0
	0.90	4083.0	3977.0	4069.0
	0.95	4006.0	4038.0	4157.0

Conclusions

In this paper, we propose a solution for the optimal distribution routes for garages supply in the Czech Republic of two main car manufacturers. We have made a step further in our research of genetic algorithms for VRP and we have extended the algorithm to Multi-Depot problem. Proposed algorithm brings similar results to the problem as its previous version, which handled the depots independently. This outcome seems to be due to specific customer distribution in the road network and we would like to address this topic in our future research.

Acknowledgements

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