

INTRO TO ISOMORPHISMS AND AUTOMORPHISMS IN FLAGCALC

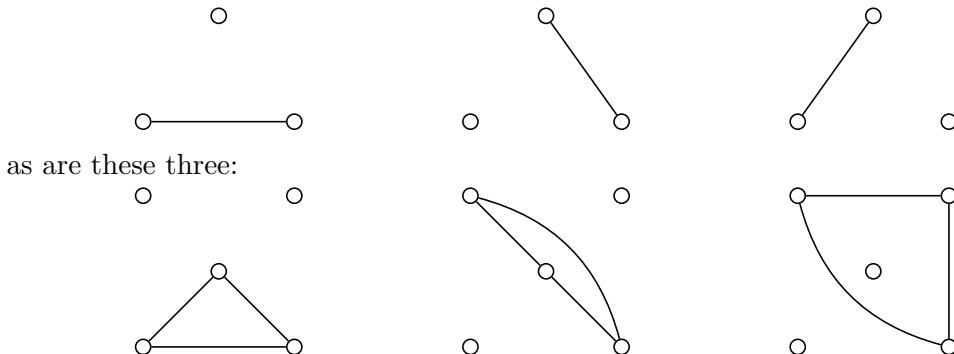
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FlagCalc has a “help” feature: invoke the command `./flagcalc -h`. This is the most up-to-date record of features¹. Here, we explore the `-f` feature and the `-i` feature².

1. ISOMORPHIC GRAPHS

Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are “isomorphic” if there exists a bijection between the vertices $\sigma : V_1 \rightarrow V_2$ such that $(a, b) \in E_1$ if and only if $(\sigma(a), \sigma(b)) \in E_2$. In particular, isomorphic graphs have the same number of vertices.

These three graphs are isomorphic:



It is an exercise in itself to work out examples of isomorphic graphs: the game is in finding a visually appealing way to display, say, the complete bipartite graph $K_{2,2}$ on four vertices, and then finding another way of looking at that graph: these two graphs are isomorphic

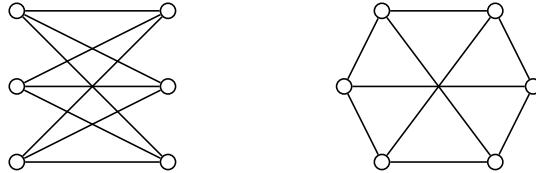
Date: December 1, 2025.

¹Missing from `-h` are the many “first order quantifiers” available, such as `FORALL`, `EXISTS`, `SUM`, `COUNT`, `MIN`, etc. These will be documented soon, and a crash course after reading the present paper would consist simply of looking at the shell scripts in the folder `scripts`.

²These two features are “early” in the evolution of the flagcalc ecosystem, and while quite powerful, there are some changes on the future roadmap: for example, while `-a ... sorted` computes one measure (query) per isomorphism class, it would be nice in the case of a measure like `Chigreedyt` and `Chigreedyp` to check how these vary within an isomorphism class (that is, most queries/measures are equivalent within any given set of isomorphic graphs, but the greedy chromatic number algorithm can produce varied output depending on how the vertices are ordered). Quick to note: notwithstanding this roadmap, there is already the ability to compute within the isomorphism class by using `-a ... all` and manually comparing within an isomorphism class, as demonstrated later in this paper.



Or the complete bipartite graph $K_{3,3}$ on six vertices:



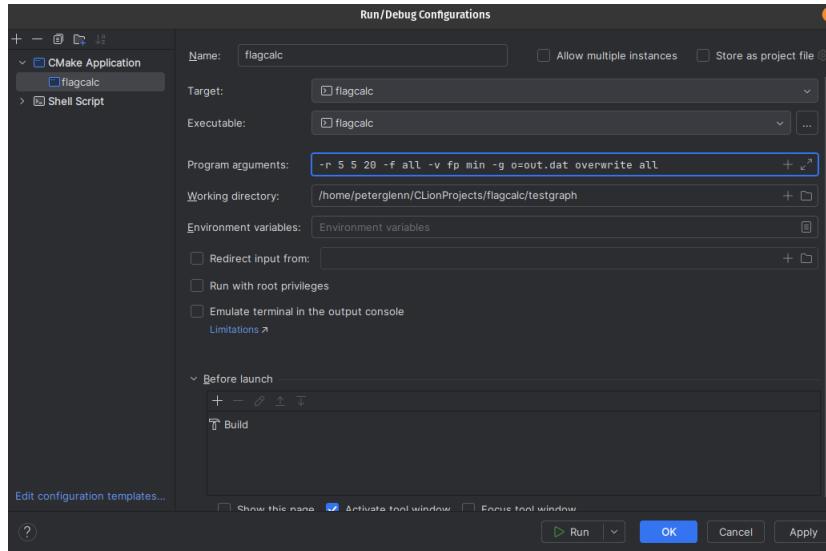
2. ISOMORPHISMS WITH FLAGCALC

One basic exercise is to pick a small vertex count, say, five, and request, say, twenty random graphs with an average edge count of $0.5\binom{5}{2} = 5$; then invoke flagcalc with the command-line options `-f all`; we add the optional “verbosity” level of `fp min`:

```
/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -r 5 5 20 -f all -v fp min
CMPFINGERPRINTS CMPFINGERPRINTS21:
Ordered, with pairwise results: GRAPH14 < GRAPH10 < GRAPH4 < GRAPH7 == GRAPH16 < GRAPH11 < GRAPH3 == GRAPH8 <
GRAPH6 == GRAPH12 < GRAPH2 < GRAPH19 < GRAPH13 < GRAPH0 < GRAPH17 == GRAPH1 == GRAPH9 < GRAPH18 == GRAPH15 < GRAPH5
Some fingerprints MATCH and some DON'T MATCH: 6 adjacent pairs out of 19 match
Process finished with exit code 0
```

Please note that `-f` asks flagcalc to linearly order graphs according to their “fingerprint” (not a term in the literature; use `-v fp` or `-v fp FpMin` (in either case, omit `min`) to see examples of a graph’s “fingerprint”), and so the isomorphic graphs work out to appear adjacent (“`==`”) to each other in the ordering shown in the output above.

Now, this doesn’t print out the graphs unless we change the verbosity level to `-v fp fpnone graphs`. And this is a fine way to get the material for study. An alternate invocation uses the `-g o=out.dat overwrite all` option to save the graphs to a file.



Producing

```
/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -r 5 5 20 -f all -v fp min -g o=out.dat overwrite all
CMPFINGERPRINTS CMPFINGERPRINTS21:
Ordered, with pairwise results: GRAPH4 == GRAPH8 < GRAPH7 < GRAPH3 < GRAPH11 < GRAPH10 == GRAPH14 == GRAPH12 == GRAPH9 == GRAPH0 < GRAPH5 == GRAPH16 == GRAPH19 == GRAPH6 < GRAPH15 == GRAPH13 < GRAPH2 < GRAPH1 < GRAPH17 < GRAPH18
Some fingerprints MATCH and some DON'T MATCH: 9 adjacent pairs out of 19 match
Process finished with exit code 0
```

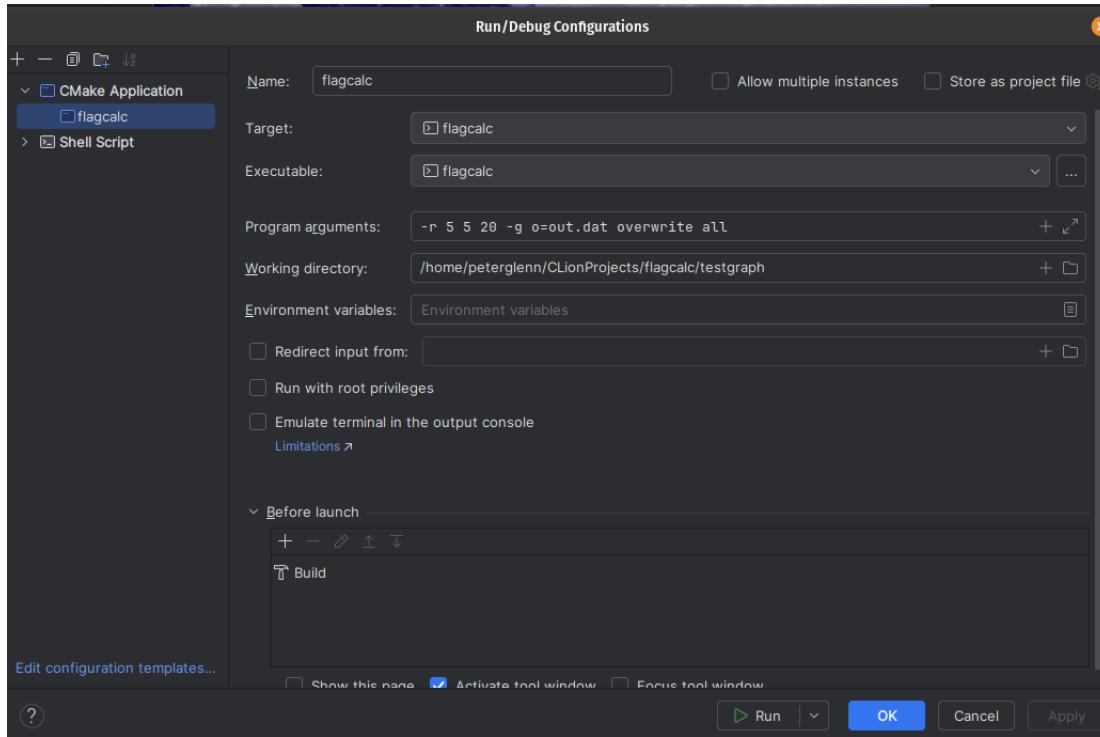
which matches to (only the first few graphs in the out.dat file are displayed here):

```

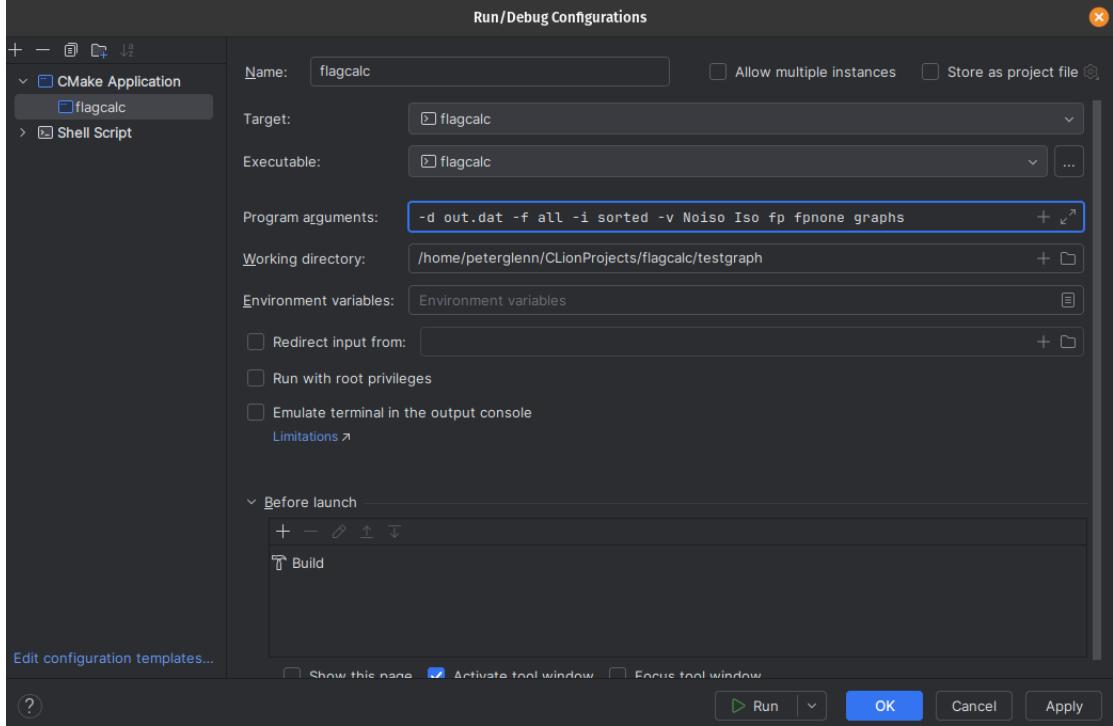
1  /* #name=GRAPH4
2  * #FP=0
3  */
4  a b c d e END
5  ae
6  bc be
7  ce
8  END
9
10 /* #name=GRAPH8
11 * #FP=1
12 */
13 a b c d e END
14 ac
15 bc be
16 ce
17 END
18
19 /* #name=GRAPH7
20 * #FP=1
21 */
22 a b c d e END
23 ab ae
24 bc bd be
25 END
26
27 /* #name=GRAPH3
28 * #FP=1
29 */
30 a b c d e END
31 ab ac ad
32 cd
33 de
34 END
35
36 /* #name=GRAPH11
37 * #FP=1
38 */
39 a b c d e END
40 ac ad ae
41 bc
42 de
43 END

```

Please note that once you have used `-g` to save to a file such as `out.dat`, you can replace the `-r` random source of graphs, with `-d out.dat` to read from the file named “`out.dat`”. This allows continued analyses of a fixed set of random graphs, and indeed, one can simply start the project with the following invocation:



That is, if invoked as above, then for the remainder of the work just use the file “out.dat” as the source of graphs, rather than the randomizer **-r** feature:



As an exercise now, choose an arrangement of vertices, draw each of the twenty graphs according to the `-g` output or the verbose output, and explore how they match up with those marked “`==`” (that is, isomorphic).

3. AUTOMORPHISMS WITH FLAGCALC

Next, we consider isomorphisms of a graph with itself, which are called *automorphisms*. Run `flagcalc` as above, but with the additional option `-i sorted` (placed after `-f all`) and with verbosity options `-v Noiso Iso fp min`. This produces

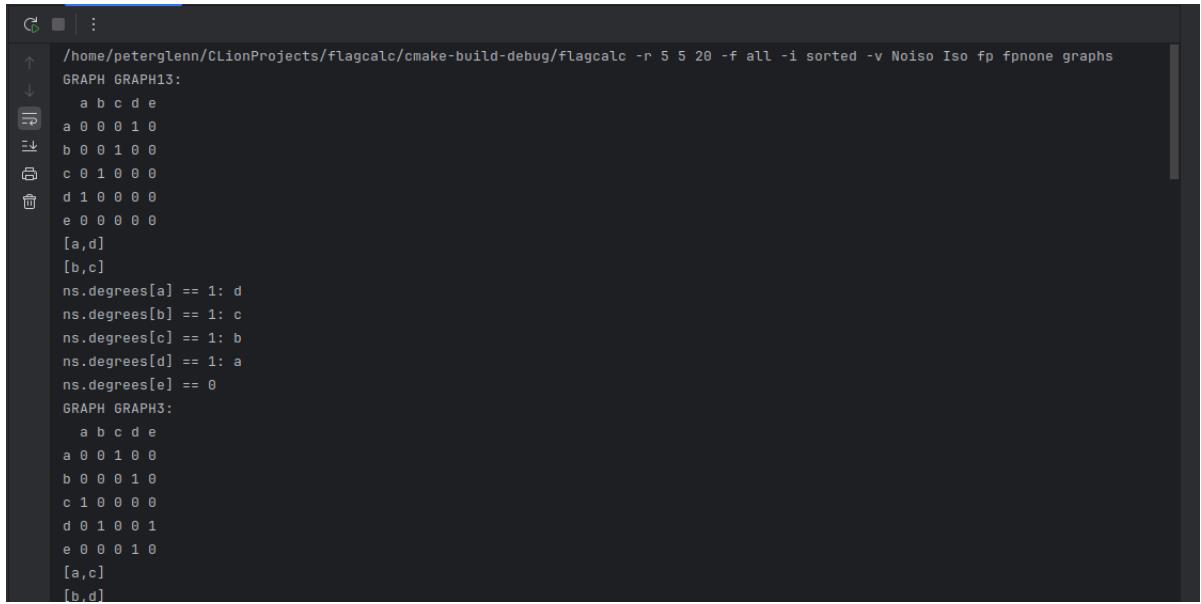
```

Run  flagcalc x
/home/peterglenn/cLionProjects/flagcalc/cmake-build-debug/flagcalc -r 5 5 20 -f all -i sorted -v Noiso Iso fp min
CMPFINGERPRINTS CMPFINGERPRINTS21:
Ordered, with pairwise results: GRAPH8 < GRAPH1 < GRAPH10 == GRAPH7 < GRAPH15 == GRAPH19 < GRAPH12 < GRAPH13 == GRAPH0 < GRAPH2
    < GRAPH4 == GRAPH17 < GRAPH11 == GRAPH5 < GRAPH6 < GRAPH18 == GRAPH9 < GRAPH16 < GRAPH14 < GRAPH3
Some fingerprints MATCH and some DON'T MATCH: 6 adjacent pairs out of 19 match
GRAPHISOS GRAPHISOS23:
Total number of isomorphisms == 8
GRAPHISOS GRAPHISOS24:
Total number of isomorphisms == 4
GRAPHISOS GRAPHISOS25:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS26:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS27:
Total number of isomorphisms == 4
GRAPHISOS GRAPHISOS28:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS29:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS30:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS31:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS32:
Total number of isomorphisms == 6
GRAPHISOS GRAPHISOS33:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS34:
Total number of isomorphisms == 12
GRAPHISOS GRAPHISOS35:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS36:
Total number of isomorphisms == 4
Process finished with exit code 0

```

Please note that for each equivalence class of isomorphic graphs this outputs the number of automorphisms of the graph (convince yourself that if two graphs are isomorphic, then their automorphism counts respectively are equal).

Use either the “out.dat” file or add the additional verbosity level `graphs` to output the twenty test-graphs, so one can experiment with paper and pen on these graphs, counting automorphisms and checking that they match the output.

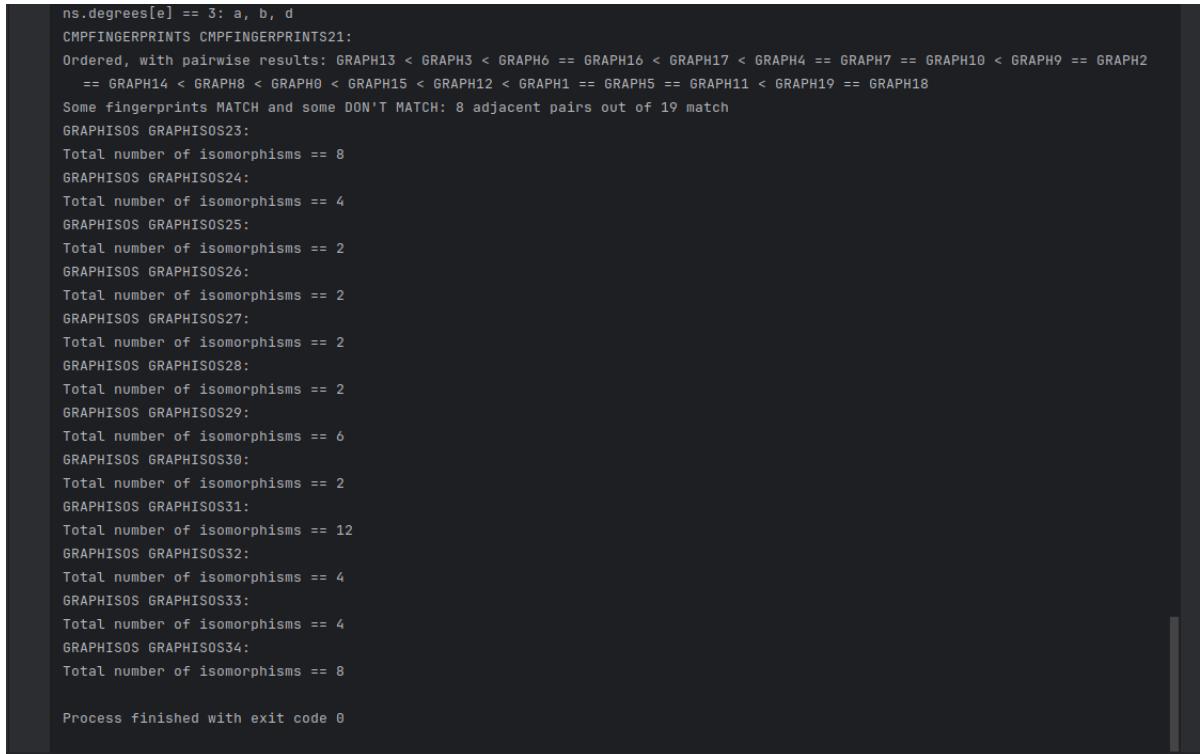


```

/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -r 5 5 20 -f all -i sorted -v Noiso Iso fp fphone graphs
GRAPH GRAPH13:
  a b c d e
  a 0 0 0 1 0
  b 0 0 1 0 0
  c 0 1 0 0 0
  d 1 0 0 0 0
  e 0 0 0 0 0
  [a,d]
  [b,c]
  ns.degrees[a] == 1: d
  ns.degrees[b] == 1: c
  ns.degrees[c] == 1: b
  ns.degrees[d] == 1: a
  ns.degrees[e] == 0
GRAPH GRAPH3:
  a b c d e
  a 0 0 1 0 0
  b 0 0 0 1 0
  c 1 0 0 0 0
  d 0 1 0 0 1
  e 0 0 0 1 0
  [a,c]
  [b,d]

```

...which continues...



```

ns.degrees[e] == 3: a, b, d
CMPFINGERPRINTS CMPFINGERPRINTS21:
Ordered, with pairwise results: GRAPH13 < GRAPH3 < GRAPH6 == GRAPH16 < GRAPH17 < GRAPH4 == GRAPH7 == GRAPH10 < GRAPH9 == GRAPH2
== GRAPH14 < GRAPH8 < GRAPH0 < GRAPH15 < GRAPH12 < GRAPH1 == GRAPH5 == GRAPH11 < GRAPH19 == GRAPH18
Some fingerprints MATCH and some DON'T MATCH: 8 adjacent pairs out of 19 match
GRAPHISOS GRAPHISOS23:
Total number of isomorphisms == 8
GRAPHISOS GRAPHISOS24:
Total number of isomorphisms == 4
GRAPHISOS GRAPHISOS25:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS26:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS27:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS28:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS29:
Total number of isomorphisms == 6
GRAPHISOS GRAPHISOS30:
Total number of isomorphisms == 2
GRAPHISOS GRAPHISOS31:
Total number of isomorphisms == 12
GRAPHISOS GRAPHISOS32:
Total number of isomorphisms == 4
GRAPHISOS GRAPHISOS33:
Total number of isomorphisms == 4
GRAPHISOS GRAPHISOS34:
Total number of isomorphisms == 8

Process finished with exit code 0

```

For example, each of the three three-vertex graphs at the opening of this paper have two automorphisms apiece. Remove the `Noiso` from `-v`, and you will see these two automorphisms spelt out:

```

/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -d std::cin -f all -i sorted -v fp rt Iso min
Using std::cin for input
ab c END END
TIME RUN TIMEDRUN1:
4.23583
CMPFINGERPRINTS CMPFINGERPRINTS2:
Ordered, with pairwise results: GRAPH0
Fingerprints MATCH: 0 adjacent pairs out of 0 match
TIME RUN TIMEDRUN3:
0.000235
GRAPHISOS GRAPHISOS4:
Total number of isomorphisms == 2
Map number 1 of 2:
c maps to c
a maps to a
b maps to b
Map number 2 of 2:
c maps to c
a maps to b
b maps to a
TIME RUN TIMEDRUN5:
0.000992

Process finished with exit code 0

```

Each of the three five-vertex graphs at the opening of this paper have twelve automorphisms apiece (verify this as an exercise). Many of the files named `testgraphxx.dat` include as a commented line the calculation of how many automorphisms they should have; these are fun to verify individually, e.g.

| | |
|---|--|
| 1 | <code>/* This pair of graphs should have 3!*3!*2=72 isomorphisms */</code> |
| 2 | |
| 3 | <code>a b c d e f</code> |
| 4 | <code>abc def</code> |
| 5 | <code>a b c d e f</code> |
| 6 | <code>abc def</code> |
| 7 | |

So for example in this case, compute the automorphism count using `-d testgraph14.dat -f all -i sorted -v Iso Noiso`

```

/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -d testgraph14.dat -f all -i sorted -v Iso Noiso
Opening file testgraph14.dat
GRAPHISOS GRAPHISOS5:
Total number of isomorphisms == 72
Process finished with exit code 0

```

4. EXAMPLE QUERIES: CHROMATIC NUMBER AND EDGE-CHROMATIC NUMBER

As an immediate exercise, use the `-d f="abcd=efgh" -f all -i sorted -a a=Chit` to explore chromatic number in relation to a graph being bipartite.

```

/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -d f=abcd=efgh -f all -i sorted -a a=Chit
GRAPH GRAPH0:
GRAPH0, dim==8, edgecount==16
TIMEDRUN TIMEDRUN1:
0.000738
CMPFINGERPRINTS CMPFINGERPRINTS2:
Ordered, with pairwise results: GRAPH0
Fingerprints MATCH: 0 adjacent pairs out of 0 match
TIMEDRUN TIMEDRUN3:
0.000271
GRAPHISOS GRAPHISOS4:
Total number of isomorphisms == 1152
TIMEDRUN TIMEDRUN5:
0.002189
APPLYCRITERION APPLYCRITERION6:
Average, min, max of measure Formula Chit: 2, 2, 2
TIMEDRUN TIMEDRUN7:
0.001661
TIMEDRUN TimedRunVerbosity:
5.6e-05

Process finished with exit code 0

```

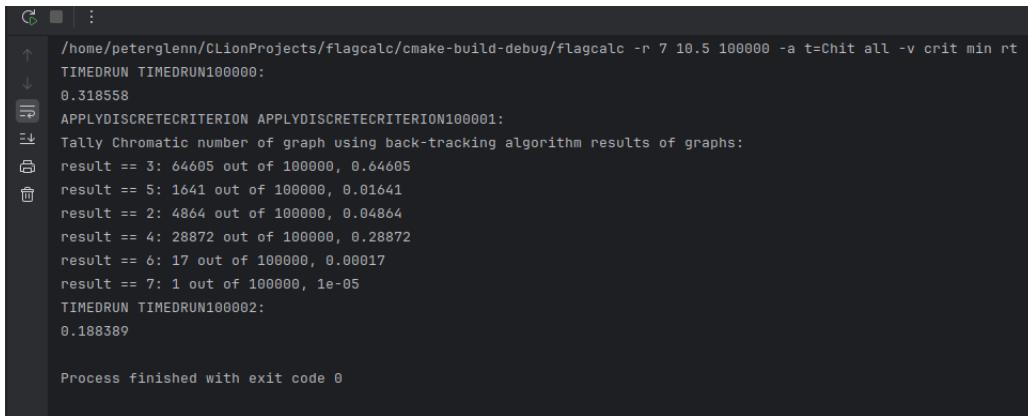
Here, the `-d f=<...>` option uses the notation `abcd=efgh` to request the complete bipartite graph $K_{4,4}$. One can also do $K_{3,4}$ with `abc=defg`, and again, try to figure out the relationship between chromatic number `Chit` and the particular complete bipartite graph under consideration.

As a next project, explore the same bipartite graphs when it comes to the tally Chiprimet (edge-chromatic number).

5. QUERIES GENERALLY

Two schools of thought exist in the way `flagcalc` is being used (as for example in the scripts under the `scripts` folder): first, since ordering by isomorphism class is slow, many invocations simply bypass `-f` and `-i` and use (often huge numbers of random graphs) the `-a` “query” feature, which is widely capable up to the limits of a weakly-typed full-featured first order querying language. Note that in this case one passes the argument `all` to `-a`, meaning please run the query on *all* graphs on the workspace, as opposed to just the most recent graph. An example is

```
./flagcalc -r 7 10.5 100000 -a t="Chit" all -v crit min rt
```



```

/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -r 7 10.5 100000 -a t=Chit all -v crit min rt
TIMEDRUN TIMEDRUN100000:
0.318558
APPLYDISCRETECRITERION APPLYDISCRETECRITERION100001:
Tally Chromatic number of graph using back-tracking algorithm results of graphs:
result == 3: 64605 out of 100000, 0.64605
result == 5: 1641 out of 100000, 0.01641
result == 2: 4864 out of 100000, 0.04864
result == 4: 28872 out of 100000, 0.28872
result == 6: 17 out of 100000, 0.00017
result == 7: 1 out of 100000, 1e-05
TIMEDRUN TIMEDRUN100002:
0.188389

Process finished with exit code 0

```

The other school of thought does invoke `-f all`, and then goes right ahead to the querying with `-a`: here, the keyword `sorted` replaces `all` usually: this means, please run the query once per isomorphism class. That is, by and large our queries are invariant under automorphisms of the graph. One example is:

```
./flagcalc -r 5 5 100 -f all -a t="Chit" sorted -v crit alltally fp min rt
```

```

/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -r 5 5 100 -f all -a t=Chit sorted -v crit alltally fp min rt
TIMEDRUN TIMEDRUN100:
0.002696
CMPFINGERPRINTS CMPFINGERPRINTS101:
Ordered, with pairwise results: GRAPH93 == GRAPH89 == GRAPH10 < GRAPH3 < GRAPH75 == GRAPH92 == GRAPH23 == GRAPH36 < GRAPH64 ==
GRAPH21 == GRAPH26 < GRAPH62 == GRAPH50 < GRAPH53 == GRAPH52 == GRAPH61 == GRAPH11 < GRAPH1 < GRAPH70 == GRAPH66 == GRAPH28 <
GRAPH22 == GRAPH46 == GRAPH58 == GRAPH82 == GRAPH4 == GRAPH39 < GRAPH17 == GRAPH16 == GRAPH14 == GRAPH59 == GRAPH51 == GRAPH69
< GRAPH78 < GRAPH79 == GRAPH81 == GRAPH87 == GRAPH2 == GRAPH45 == GRAPH40 == GRAPH95 < GRAPH30 == GRAPH0 < GRAPH90 == GRAPH94
== GRAPH25 == GRAPH77 == GRAPH57 < GRAPH72 == GRAPH20 == GRAPH86 == GRAPH34 == GRAPH76 == GRAPH54 == GRAPH55 == GRAPH91 <
GRAPH67 == GRAPH85 == GRAPH96 == GRAPH97 == GRAPH18 == GRAPH41 < GRAPH19 == GRAPH47 == GRAPH48 == GRAPH9 < GRAPH8 == GRAPH68
== GRAPH35 == GRAPH84 == GRAPH65 == GRAPH24 == GRAPH12 == GRAPH56 < GRAPH32 == GRAPH13 < GRAPH37 == GRAPH74 < GRAPH71 ==
GRAPH29 == GRAPH98 == GRAPH15 < GRAPH31 < GRAPH99 < GRAPH49 == GRAPH27 == GRAPH33 == GRAPH73 == GRAPH5 == GRAPH42 == GRAPH63
== GRAPH6 == GRAPH43 < GRAPH38 == GRAPH83 == GRAPH7 < GRAPH44 == GRAPH88 == GRAPH80 < GRAPH60
Some fingerprints MATCH and some DON'T MATCH: 73 adjacent pairs out of 99 match
TIMEDRUN TIMEDRUN102:
0.001904
APPLYDISCRETECRITERION APPLYDISCRETECRITERION103:
Tally Chromatic number of graph using back-tracking algorithm results graph-by-graph:
GRAPH93: 2
GRAPH3: 3
GRAPH75: 2
GRAPH64: 2
GRAPH62: 2
GRAPH53: 3
GRAPH1: 2
GRAPH70: 3
GRAPH22: 2
GRAPH17: 2
GRAPH78: 3
GRAPH79: 2
GRAPH30: 3
GRAPH90: 3
GRAPH72: 3
GRAPH67: 2
GRAPH19: 3
GRAPH8: 3
GRAPH32: 3
GRAPH37: 3
GRAPH71: 3
GRAPH31: 2
GRAPH99: 3
GRAPH49: 3
GRAPH38: 3
GRAPH44: 4
GRAPH60: 3
Tally Chromatic number of graph using back-tracking algorithm results of graphs:
result == 2: 10 out of 27, 0.37037
result == 3: 16 out of 27, 0.592593
result == 4: 1 out of 27, 0.037037
TIMEDRUN TIMEDRUN104:
0.001663

Process finished with exit code 0

```

Additional queries can be obtained just by looking through the other options delineated by `-h`, such as `m=girthm` (graph's "girth"), `m=radiusm` (graph's "radius"), and so on. These are defined in for example Diestel, R. "Graph Theory" Fifth Edition (2017).

6. INPUTTING GRAPHS GENERALLY

Populate the workspace either with graphs from

- **-r** Choose from several randomizers (see **-h**), or default to the randomizer that chooses incidence between any two vertices as of likelihood P , where

```
-r <vertexcount> <edgecount> <numberofgraphs>
```

and $\text{edgecount} = P \cdot \binom{\text{vertexcount}}{2}$, where the parentheses are the usual binomial coefficient n -choose- k .

- **-d** Read from the filename in **-d filename.dat**, or from standard input **-d std::cin**, or “inline” graphs with **-d f="-abcda"**. Examples using **std::cin** are:

The image displays three separate terminal windows, each showing the output of a FlagCalc command. Each window has a title bar with a green icon and a close button.

Terminal 1 Output:

```
/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -d std::cin -a t=Chirprimet -v crit min rt
Using std::cin for input
abcde=fghij END END
TIMEDRUN TIMEDRUN1:
8.64884
APPLYDISCRETECRITERION APPLYDISCRETECRITERION2:
Tally Edge chromatic number of graph using back-tracking algorithm results of graphs:
result == 5: 1 out of 1, 1
TIMEDRUN TIMEDRUN3:
0.003618

Process finished with exit code 0
```

Terminal 2 Output:

```
/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -d std::cin -a t=Chirprimet -v crit min rt
Using std::cin for input
-abceda END END
TIMEDRUN TIMEDRUN1:
10.8094
APPLYDISCRETECRITERION APPLYDISCRETECRITERION2:
Tally Edge chromatic number of graph using back-tracking algorithm results of graphs:
result == 3: 1 out of 1, 1
TIMEDRUN TIMEDRUN3:
0.003487

Process finished with exit code 0
```

Terminal 3 Output:

```
/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -d std::cin -a t=Chirprimet -v crit min rt
Using std::cin for input
-abcdefa END END
TIMEDRUN TIMEDRUN1:
8.33329
APPLYDISCRETECRITERION APPLYDISCRETECRITERION2:
Tally Edge chromatic number of graph using back-tracking algorithm results of graphs:
result == 2: 1 out of 1, 1
TIMEDRUN TIMEDRUN3:
0.002198

Process finished with exit code 0
```

The “language” for inputting graphs begins with unlimited names for vertices, starting with a letter **a...z** or **A...Z** (giving 52 options) followed by any number of underscores, colons

and digits 0...9. There are additional features, such as named vertices using curly brackets; see the README.md file.

After choosing vertex names, the old-school approach requires a space-delimited list of vertices, followed by END. The new school omits the list of vertices and simply deduces from the set of edges the names of the vertices:

```
ab bc cd da END END
```

would be a square (same as the $K_{2,2}$ that opened this paper). Equivalently, the following are also all $K_{2,2}$:

```
ac=bd END END      abcd !ac !bd END END      -abcd END END
```

Indeed, we put all four into a file named `squares.dat`, then run `flagcalc` with `-d squares.dat -f all -v fp min`.

```
/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -d squares.dat -f all -v fp min
Opening file squares.dat
CMPFINGERPRINTS CMPFINGERPRINTS5:
Ordered, with pairwise results: GRAPH1 == GRAPH2 == GRAPH3 == GRAPH0
Fingerprints MATCH: 3 adjacent pairs out of 3 match
Process finished with exit code 0
```

Additional features include tripartite graphs (`abc=def=ghi`) and radial graphs (`a+bcde` or `abc+defg`: draw these out, after asking `flagcalc` to produce their adjacency matrices).

7. AUTOMORPHISMS OF THE PLATONIC SOLIDS

The five Platonic solids are encoded for `flagcalc` in the file `platonicsolids.dat` (as an exercise, try encoding them yourself, or find varying ways to encode them).

```

1  /* #name=Tetrahedron */
2
3  abcd
4  END END
5
6
7  /* #name=Octahedron */
8
9  abc def abd bce caf
10 END END
11
12
13  /* #name=Cube */
14
15  -abcd -efghe
16  ae bf cg dh
17  END END
18
19
20  /* #name=Dodecahedron */
21
22  -abcdea
23  -afghb -bhijc -cijkl -dlmne -enofa
24  -porstp
25  op gq ir ks mt
26  END END
27
28  /* #name=Icosahedron */
29
30  abc
31  abd bcf cah
32  ahi aid bde bef cfg cgh
33  jkl
34  dij dej efk fgk ghl hil
35  END END

```

Use `-d platonic solids.dat -f all -i sorted -v Noiso Iso graphs` to compute the number of automorphisms of each Platonic solid. Use criterion (from `-h`) like `conn1c` to affirm that each Platonic solid is indeed connected (1-connected). Use the criterion `-a s="Chit==2"` with `-v crit` to reveal which Platonic solid(s) are two-colorable:

```

Run flagcalc
/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -d platonic solids.dat -a s=Chit==2 all -v crit
Opening file platonic solids.dat
APPLYBOOLEANCRITERION APPLYBOOLEANCRITERION1:
Criterion Sentence Chit==2 results of graphs:
Tetrahedron, number 1 out of 5: 0
Octahedron, number 2 out of 5: 0
Cube, number 3 out of 5: 1
Dodecahedron, number 4 out of 5: 0
Icosahedron, number 5 out of 5: 0
result == 0: 4 out of 5, 0.8
result == 1: 1 out of 5, 0.2

Process finished with exit code 0

```

Use `-a e=Cycless all` with verbosity `-v crit allsets` to list the number of cycles on each Platonic graph (be prepared for a large number). Looking ahead to the paper discussing first order queries, run

```
-d platonicsolids.dat -a z="st(SET (c IN Cycless, st(c)==dimm, c))" all -v crit
```

to compute the number of undirected Hamiltonian cycles on each graph (a Hamiltonian cycle by definition visits each vertex exactly once). `dimm` stands for “graph dimension” (a throwback, since dimension is a “tally” (aka integer) not a “measure” (aka floating point)). `st` stands for “size tally”: it is the size of the “set” found within the parentheses³.

```
/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -d platonicsolids.dat -a "z=st(SET (c IN Cycless, st(c)==dimm, c))" all -v crit
Opening file platonicsolids.dat
APPLYDISCRETECRITERION APPLYDISCRETECRITERION1:
Tally Int-valued formula st(SET (c IN Cycless, st(c)==dimm, c)) results of graphs:
Tetrahedron, number 1 out of 5: 3
Octahedron, number 2 out of 5: 16
Cube, number 3 out of 5: 6
Dodecahedron, number 4 out of 5: 30
Icosahedron, number 5 out of 5: 1280
result == 3: 1 out of 5, 0.2
result == 16: 1 out of 5, 0.2
result == 6: 1 out of 5, 0.2
result == 30: 1 out of 5, 0.2
result == 1280: 1 out of 5, 0.2

Process finished with exit code 0
```

8. OUTPUTTING GRAPHS GENERALLY

The verbosity options are not presently listed in the `-h` output or documented elsewhere; for now, here is the relevant code:

³Replacing `SET` with `SETD` speeds up some thirty percent; `SETD` means, a set that allows multiple appearances of any element (that is, no checks are performed as to whether a new element already exists in the set). It is much faster, and here it is just as correct, since the quantifier ranges once over the elements in the set named `Cycless`. Another way to get the same numbers faster is

```
-d platonicsolids.dat -a z="COUNT (c IN Cycless, st(c) == dimm)" all -v crit
```

Additionally, one can ask for the “ensemble” (French for “set”) of undirected Hamiltonian cycles itself:

```
-d platonicsolids.dat -a e="SETD (c IN Cycless, st(c)==dimm, c)" all -v crit allsets
```

Also one can use the set-valued measure `Cyclesvs` that takes one vertex as an argument, and finds all undirected cycles containing that vertex:

```
-d platonicsolids.dat -a z="COUNT (c IN Cyclesvs(0), st(c) == dimm)" all -v crit
```

It is obviously much faster (fifty percent on this data). Finally, nesting quantifiers, the same output obtains as follows:

```
-d platonicsolids.dat -a z="COUNT (c IN Cyclesvs(0), FORALL (v IN V, v ELT c))" all -v crit
```

```

17
20 #define VERBOSE_CMPFINGERPRINTLEVEL "cmp"
21 #define VERBOSE_ENUMISOMORPHISMSLEVEL "enum"
22
23 #define VERBOSE_DONTLISTISOS "Noiso"
24 #define VERBOSE_LISTGRAPHS "graphs"
25 #define VERBOSE_LISTFINGERPRINTS "fp"
26 #define VERBOSE_ISOS "Iso"
27 #define VERBOSE_RUNTIMES "rt"
28 #define VERBOSE_VERBOSITYRUNTIME "vrunt"
29 #define VERBOSE_VERBOSITYFILEAPPEND "vappend"
30 #define VERBOSE_MINIMAL "min"
31 #define VERBOSE_MANTELSTHEOREM "Mantel"
32 #define VERBOSE_FINGERPRINT "Fp"
33 #define VERBOSE_SAMPLERANDOMMATCHING "srn"
34 #define VERBOSE_FPMINIMAL "FpMin"
35 #define VERBOSE_FPNONE "fpnone"
36 #define VERBOSE_APPLYCRITERION "crit"
37 #define VERBOSE_SUBOBJECT "subobj"
38 #define VERBOSE_PAIRWISEDISJOINT "pd"
39 #define VERBOSE_APPLYSSET "set"
40 #define VERBOSE_SETVERBOSE "allsets"
41 #define VERBOSE_CRITVERBOSE "allcrit"
42 #define VERBOSE_MEASVERBOSE "allmeas"
43 #define VERBOSE_TALLYVERBOSE "alltally"
44
45 #define VERBOSE_APPLYSTRING "strmeas"
46 #define VERBOSE_APPLYGRAPH "measg"
47 #define VERBOSE_RANDOMSUMMARY "randomizer"
48
49 #define VERBOSE_ALL "Noiso graphs fp Iso rt vrunt vappend min Mantel Fp srn FpMin subobj pd set allsets randomizer"
50 #define VERBOSE_DEFAULT "Noiso graphs fp Iso rt vrunt vappend min Mantel Fp srn FpMin crit rm fpnone vrunt subobj pd set allsets randomizer"
51
52 #define VERBOSE_FORDB "db"

```

Next, the `-g` feature takes `sorted` to mean one graph per isomorphism class; note the `#FP` entry in each graph's output is 1 or 0 according to whether the graph is non-isomorphic to its immediate successor (or 1 if it is the last graph). Another option, once one is using `-a`, is to pass the argument `passed` to `-g`, meaning “only output those graphs that pass (all) the boolean criteria in `-a`.

9. FURTHER THOUGHTS

- Exercise: Produce input as a file, as `f="<data>"`, or using `std::cin` for each of the six graphs that open this paper (e.g., the first one, on three vertices, is `f="a bc"`, then `f="ab c"`, and `f="a b c ac"`).
- Exercise: Produce an input file containing ten isomorphic graphs on your choice of vertex count (probably something below eight or nine).
- Exercise: Establish a table of random graphs with $P = 0.50$, that shows the runtime for `-f all` as the number of vertices increases from 2 to a reasonable upper bound (like 12, or depending on how many random graphs you decide to ask for).

```

/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -r 12 33 50 -f all -v fp rt min
TIMEDRUN TIMEDRUN50:
0.002417
CMPFINGERPRINTS CMPFINGERPRINTS51:
Ordered, with pairwise results: GRAPH6 < GRAPH29 < GRAPH26 < GRAPH48 < GRAPH23 < GRAPH24 < GRAPH11 < GRAPH38 <
GRAPH8 < GRAPH16 < GRAPH1 < GRAPH31 < GRAPH25 < GRAPH14 < GRAPH30 < GRAPH12 < GRAPH15 < GRAPH43 < GRAPH0 < GRAPH9 <
GRAPH22 < GRAPH19 < GRAPH5 < GRAPH37 < GRAPH42 < GRAPH34 < GRAPH27 < GRAPH17 < GRAPH46 < GRAPH33 < GRAPH39 <
GRAPH20 < GRAPH35 < GRAPH41 < GRAPH7 < GRAPH4 < GRAPH28 < GRAPH32 < GRAPH10 < GRAPH13 < GRAPH36 < GRAPH2 < GRAPH40 <
GRAPH45 < GRAPH44 < GRAPH47 < GRAPH3 < GRAPH18 < GRAPH49 < GRAPH21
NO fingerprints match: 0 adjacent pairs out of 49 match
TIMEDRUN TIMEDRUN52:
0.773046

Process finished with exit code 0

```

- Exercise: find examples of one isomorphism class with varying results of `Chigreedyt`. Practice constructing graphs with increasing differences between `Chigreedyt` and the `Chit`.
- Exercise: Use the graph `*abcde` (which means “the complete graph K_5 ”; equivalently, `abcde` without the asterisk) to figure out the chromatic number of complete graphs, and the number of automorphisms of K_n generally.
- Exercise: Practice computing “fingerprints” by hand, using verbosity levels `fp` or `FpMin` on one or a few input graphs (hint: “fingerprints” sort vertices by degree, then proceed to adjacent vertices, again sorting by degree).

```

/home/peterglenn/CLionProjects/flagcalc/cmake-build-debug/flagcalc -r 5 5 20 -f all -v fp FpMin
CMPFINGERPRINTS CMPFINGERPRINTS21:
fingerprint of graph GRAPH15, ordered number 1 out of 20(==):
0
0
0
1 1
1 1
fingerprint of graph GRAPH17, ordered number 2 out of 20 (<):
0
0
0
1 1
1 1
fingerprint of graph GRAPH3, ordered number 3 out of 20(==):
0
0
1 2 1
1 2 1
2 1
2 1
fingerprint of graph GRAPH18, ordered number 4 out of 20 (<):
0

```

- Another (research level) question: when working with large numbers of graphs on a given vertex count, what is the average number of equivalence classes of isomorphic graphs?