Employee Profit Sharing and Labor Extraction in a Classical Model of Distribution and Growth

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Abstract: This paper sets forth a classical model of economic growth in which the distribution of income features the possibility of profit sharing with workers, as firms choose periodically between two labor-extraction compensation strategies. Firms choose to compensate workers with a base wage or a share of profits on top of this base wage. In accordance with robust empirical evidence, labor productivity in profit-sharing firms is higher than labor productivity in non-sharing firms. The frequency distribution of labor-extraction compensation strategies and average labor productivity in the population of firms is evolutionarily time-varying. The paper derives two main results which carry relevant theoretical and policy implications. First, heterogeneity in labor-extraction compensation strategies across firms can be a stable long-run equilibrium. Second, in the long-run equilibrium, a higher frequency of profit-sharing firms can yield a higher net profit share in income and thereby higher rates of net profit and economic growth.

Keywords: profit sharing, income distribution, economic growth, evolutionary dynamics.

Resumo: Este artigo propõe um modelo clássico de crescimento econômico em que a distribuição de renda conta com a possibilidade de compartilhamento de lucros com os trabalhadores, pois as firmas escolhem periodicamente entre duas estratégias de remuneração para extração de trabalho. As firmas escolhem remunerar os trabalhadores com um salário base ou com uma participação nos lucros em adição a esse salário base. Conforme evidência empírica robusta, a produtividade do trabalho é maior nas firmas que adotam compartilhamento de lucros que naquelas que não o adotam. A distribuição de frequência das estratégias de remuneração para extração de trabalho e da produtividade média do trabalho na população de firmas varia ao longo do tempo de maneira evolucionária. O artigo deriva dois principais resultados com relevantes implicações teóricas e de política pública. Primeiro, a heterogeneidade nas estratégias de remuneração para extração de trabalho entre as firmas pode ser um equilíbrio de longo prazo estável. Segundo, no equilíbrio de longo prazo, uma maior frequência de firmas que adotam compartilhamento de lucros pode gerar uma maior parcela líquida dos lucros na renda e, por meio disso, de taxas mais elevadas de lucro líquido e crescimento econômico.

Palavras-chave: compartilhamento de lucros, distribuição de renda, crescimento econômico, dinâmica evolucionária.

1. <u>Introduction</u>

Employee profit sharing has experienced an increasing (even if fluctuating) popularity in several advanced economies in the last few decades (D'Art and Turner, 2004, Kruse et al., 2010). Meanwhile, from a longer-term perspective, there have been rising and falling waves of interest in employee profit-sharing schemes since their inception in the 19th century (Mitchell et al.,1990, D'Art and Turner, 2006, Blasi, et al., 2013).

The main motivation behind a firm's offering of profit sharing to workers is that connecting workers' earnings to the profit performance of the firm is believed in theory to induce workers to increase commitment, effort and other attitudes leading to their higher productivity. Indeed, attitude surveys find that employers and employees usually believe that profit sharing helps improve firm performance in several dimensions (Weitzman and Kruse, 1990, and Blasi et al., 2010).

Other compensation mechanisms through which workers' earnings depend on the performance of the firm (or work group) are gain sharing, employee ownership and stock options. Profit-sharing plans themselves vary considerably, and some major ways in which they differ concern what is shared (e.g., total profits or profits above some target or threshold level), how and when sharing is made (e.g., in cash or company stocks, in a deferred or non-deferred way) and with whom sharing is made (e.g., directly to workers or to some workers' retirement or pension plan). Yet, there is survey evidence that non-deferred and in cash profit sharing is ranked first by employees as a motivation device (Blasi et al., 2010).

There is considerable empirical evidence that profit sharing raises labor productivity. Although the estimated magnitude of the productivity gain varies from study to study, it is often non-negligible. Weitzman and Kruse (1990) apply meta-analysis to sixteen studies to find that the productivity gain of profit sharing is positive at infinitesimal significance levels. Doucouliagos (1995) applies meta-analysis to forty-three studies to find that profit sharing is positively associated with productivity. Cahuc and Dormont (1997) use French data to find that profit sharing firms outperform other firms in productivity and profitability. Conyon and Freeman (2004), using UK data, find that profit-sharing firms tend to outperform other firms in productivity and financial performance. D'Art and Turner (2004) use data for 11 European countries to find that profit sharing is positively associated with productivity and profitability, while Kim (1998), using U.S. data, finds that albeit profit sharing raises labor productivity, profits do not rise, as gains from profit sharing are cancelled out by increased labor costs. In a field, quasi-experimental study, Peterson and Luthans (2006) randomly assigned profit-sharing plans at three of twenty-one establishments within a U.S. firm, finding that labor productivity and profits rose in the profit-sharing establishments relative to the control group.

Also motivated by the above empirical evidence, this paper sets forth a classical model of economic growth, in which the distribution of income can feature profit sharing with workers. In attempting to extract labor from labor power more effectively, firms can behave heterogeneously as regards the choice of compensation strategy. Firms choose to compensate workers with either only a conventional base wage or a share of profits on top of this conventional base wage. In line with the empirical evidence reported above, workers in profit-sharing firms have a higher productivity than their counterparts in non-sharing firms. Meanwhile, any observed heterogeneity in labor-extraction compensation strategies across firms and the resulting additional productivity gain that accrues to profit-sharing firms are not parametric constants, but rather co-evolve endogenously towards the long-run as driven by an evolutionary process. Therefore, our model is well fitted to explore possible explanations for evidence such as that U.K. firms switch modes of employee compensation frequently, with the gross changes in modes (which include profit sharing) being far more numerous than the net changes (which suggests that firms have trouble

optimizing and that the transaction costs of switching are relatively low) (Bryson and Freeman, 2010).

Moreover, we explore the implications of the coupled dynamics of the distribution of labor-extraction compensation strategies across firms and the additional productivity gain that accrues to profit-sharing firms for the distribution of aggregate income between profits and wages and thereby capital accumulation and economic growth. Arguably, given the essential role of the functional distribution of income in the classical tradition, it is fitting to explore the implications of profit sharing for capital accumulation and economic growth in a model that conforms to essential tenets of this tradition.[1]

The remainder of this paper is organized as follows. Section 2 describes the structure of the model and investigates its behavior in the short run. Section 3 explores the coupled evolutionary dynamics of the frequency distribution of labor-extraction compensation modes across firms and the additional productivity gain that accrues to profit-sharing firms. This section also investigates the implications of such coupled evolutionary dynamics for income distribution and thereby capital accumulation and economic growth in the long run. The final section summarizes the main conclusions reached along the way.

2. Structure of the model and its behavior in the short run

The model economy is closed and without government activities, producing a single (homogeneous) good for both investment and consumption purposes. Production is carried out by a large (and fixed) population of firms, which combine two (physically homogeneous) factors of production, capital and labor, by means of a fixed-coefficient technology. Firms produce (and hire labor) without facing realization (or effective demand) constraints, thereby being able to sell profitably all its output production at the prevailing price level. However, the model is cast in real terms.

In attempting to extract labor from labor power more effectively, an individual firm chooses periodically between two worker compensation strategies: it compensates workers with either only a conventional (or subsistence) base wage v (*non-sharing labor-extraction compensation strategy*) or with a conventional base wage and a share of profits $\delta \in (0,1) \subset \mathbb{R}$ (*profit-sharing labor-extraction compensation strategy*). In a given period there is a proportion $\lambda \in [0,1] \subset \mathbb{R}$ of profit-sharing (or type *s*) firms, while the remaining proportion, $1-\lambda$, is composed by non-sharing (or type *n*) firms. In accordance with the empirical evidence on profit sharing reported in the preceding section, a profit-sharing firm is willing to play such labor-extraction compensation strategy because the resulting labor productivity gain is strictly higher than otherwise. Productivity is homogeneous across workers in firms playing a given worker compensation strategy. Since the conventional base wage is assumed to be the same under both labor-extraction compensation strategies, a worker hired by a sharing firm does receive a higher

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total compensation than a worker hired by a non-sharing firm along the economically meaningful domain given by strictly positive profits for sharing firms.[2]

To keep focus on the dynamics of the distribution of labor-extraction compensation strategies and its implications for the functional distribution of income and thereby economic growth, we simplify matters by assuming that the conventional base wage, v, and the profit-sharing coefficient, δ, remain constant over time. The distribution of labor-extraction compensation strategies across firms, $(\lambda, 1-\lambda)$, which is given in the short run as a result from previous dynamics, changes beyond the short run according to an evolutionary dynamic (the replicator dynamic). In the short run, for given values of the conventional base wage, profit-sharing coefficient, productivity differential, distribution of labor-extraction strategies and, therefore, individual and average profit shares, the short-run equilibrium value of the rate of economic growth is determined. As the economy evolves towards the long-run, the co-evolution of the distribution of labor-extraction strategies and the labor productivity differential, by leading to changes in the average share of profits, causes changes in the short-run equilibrium value of the rate of economic growth.

Formally, we define the productivity differential as:

(1) $$\alpha \equiv \frac{a_s}{a_n},$$

where $a_i = X_i / L_i$ denotes labor productivity in firms of type $i = s, n$, X_i is total output of firms of type $i = s, n$, and L_i is total employment of firms type $i = s, n$. For simplicity, we normalize labor productivity in non-sharing firms, a_n, to one, so that we obtain $a_s = \alpha > 1$ in (1). As it turns out, profit-sharing firms can be intuitively described as firms willing to bet on the prospect of attaining a productivity differential, α, which is high enough to allow them to have a lower unit labor cost than non-sharing firms. At the end of the day a sharing firm expects such productivity differential to be high enough to allow it to have a higher *net* profit rate (i.e., net of shared profits) than a non-sharing firm. However, as explored in the next section, the resulting labor productivity differential may fall short of the level required for the profit-sharing bet to prove successful.

Using (1), the total profits of sharing and non-sharing firms are, respectively:

(2) $$R_s \equiv X_s - vL_s = \left(1 - \frac{v}{\alpha}\right)X_s;$$

and:

(3) $$R_n \equiv X_n - vL_n = (1-v)X_n,$$

which requires further assuming that $v < a_n = 1 < a_s$. Using (2) and (3), the shares of profit in output of sharing and non-sharing firms in the short-run are given, respectively, by:

(4) $$\pi_s \equiv \frac{R_s}{X_s} = 1 - \frac{v}{\alpha};$$

and:

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[2] Empirical evidence shows that profit sharing has a meaningful effect on worker total compensation (Kruse et al., 2010). In fact, Capelli and Neumark (2004) find that total labor costs exclusive of the sharing component do not fall significantly in pre/post comparisons of firms that adopt profit sharing. This suggests that profit sharing tends to come on top of, rather than in place of, a base wage.

(5)
$$\pi_n \equiv \frac{R_n}{X_n} = 1 - v.$$

The profit share expression in (4) denotes the proportion of gross profits in the output of sharing firms, since an exogenously given fraction of such profits, given by $\delta \in (0,1) \subset \mathbb{R}$, is shared with workers. Using (4), the net unit return of sharing firms in the short-run is given by:

(6)
$$\pi_s^c \equiv \frac{(1-\delta)R_s}{X_s} = (1-\delta)\left(1 - \frac{v}{\alpha}\right),$$

Using (5) and (6), the short-run value of the average net profit share can be expressed as:

(7)
$$\pi = \lambda\pi_s^c + (1-\lambda)\pi_n = \lambda(1-\delta)\left(1 - \frac{v}{\alpha}\right) + (1-\lambda)(1-v).$$

As regards capital productivity, to keep focus on the main issue of the effect of profit sharing on the rate of economic growth, we simplify matters by assuming that the maximum output that can be produced by fully utilizing one unit of capital, k, is an exogenously given constant common to all firms:

(8)
$$k_s = k_n = k,$$

where $k_i = X_i / K_i$ is the output to capital ratio (and K_i is the total capital stock) of firms of type $i = s, n$.

Using (2)-(5) and (8), the (gross) profit rates of sharing and non-sharing firms in the short run can then be expressed as follows:

(9)
$$r_s \equiv \frac{R_s}{K_s} = \left(1 - \frac{v}{\alpha}\right)\frac{X_s}{K_s} = k\pi_s;$$

and:

(10)
$$r_n \equiv \frac{R_n}{K_n} = (1-v)\frac{X_n}{K_n} = k\pi_n.$$

Using (6) and (9), the net profit rate of sharing firms in the short run is then given by:

(11)
$$r_s^c \equiv \frac{(1-\delta)R_s}{K_s} = (1-\delta)\left(1 - \frac{v}{\alpha}\right)\frac{X_s}{K_s} = k\pi_s^c.$$

Therefore, using (10) and (11), the short run value of the average net profit rate can be expressed as:

(12)
$$r^c = \lambda r_s^c + (1-\lambda)r_n = k\left[\lambda(1-\delta)\left(1 - \frac{v}{\alpha}\right) + (1-\lambda)(1-v)\right].$$

Therefore, a comparison between (7) and (12) shows that the average net profit share and the average net profit rate always move in the same direction.

As assumed earlier, the population of firms, the conventional base wage, v, the labor productivity in non-sharing firms, a_n, the profit-sharing coefficient, δ, and the output to capital ratio which is common to all firms, k, all remain constant over time. The short run period t is defined as a time frame in which the aggregate capital stock, K_t, the labor supply, N_t, the

productivity differential of profit-sharing firms, $a_{s,t} = \alpha_t$, the frequency distribution of labor-extraction compensation strategies across firms, λ_t, and therefore the distribution of income as measured by the average net profit share, π_t^c, can all be taken as predetermined by previous dynamics.[3]

The economy is inhabited by two classes, capitalists who own the firms and workers. Workers provide labor and earn a base wage income, when they work for non-sharing firms. Workers in sharing firms also receive a share of the latter's profit income, which is the entire surplus over the respective base wage bill. We assume that workers' total compensation is all spent on consumption, while capitalists homogeneously save a fraction, $\gamma \in (0,1) \subset \mathbb{R}$, of their net profit income. As we further assume that capitalists save in order to fully finance their investment decisions, it follows that all income not consumed is automatically invested. Therefore, individual firms are able to sell all their production at the prevailing price level, so that aggregate demand will adjust to remove any excess aggregate (and individual) supply in the goods market. In short-run goods market equilibrium, therefore, it follows that aggregate investment, I_t, is identically equal to net aggregate savings, S_t.

Using (8)-(11), the net aggregate savings as a proportion of the capital stock at period t can be expressed as follows:

$$(13) \quad \frac{S_t}{K_t} = \gamma \left[\frac{(1-\delta)R_{s,t} + R_{n,t}}{K_t} \right] = \gamma \left[\lambda_t k_s \pi_{s,t}^c + (1-\lambda_t)k_n \pi_n \right] = \gamma k \left[\lambda_t \pi_{s,t}^c + (1-\lambda_t)\pi_n \right].$$

Thus, by substituting (13) in the goods market equilibrium identity given by $I_t / K_t \equiv S_t / K_t$, and assuming for simplicity that the capital stock does not depreciate, we obtain the short-run equilibrium growth rate:

$$(14) \quad g_t^*(\alpha_t, \lambda_t) = \gamma k \left[\lambda_t \pi_{s,t}^c (\alpha_t, \lambda_t) + (1-\lambda_t)\pi_n \right].$$

Note that the short-run equilibrium growth rate depends on parametric constants along with the labor productivity differential, α_t, and the frequency distribution of labor-extraction compensation strategies, λ_t, which are predetermined in the short run and co-evolve in the transition from the short to the long run (as described in the next section). Expectedly, in light of the classical nature of the macroeconomic dynamic, the short-run equilibrium growth rate varies positively with the common saving propensity of capitalists and the average profit share in income. This is so because economic growth is driven by capital accumulation from the aggregate saving of capitalists.

3. Behavior of the model in the long run

In the long run we assume that the short-run equilibrium value of the rate of economic growth is always attained, with the economy moving towards the long run due to changes in the aggregate stock of capital, K, coming from changes in the individual stocks of capital, K_s and K_n, the supply of available labor, N, the labor productivity differential, α, and the frequency distribution of labor-extraction compensation strategies, λ. However, as the maximum output that can be produced by fully utilizing one unit of capital is an exogenously given constant k

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[3] As in the next section we explore the behavior of the economy in the transition from the short to the long run, thereafter we attach a subscript t to the short-run value of all variables (be they endogenous or predetermined).

common to all firms, it follows from (8) that $K_{s,t}/K_{n,t}=k$. Meanwhile, to sharpen focus on the coupled dynamics of the frequency distribution of labor-extraction compensation strategies and the productivity differential as state variables, we further assume that the supply of available labor grows endogenously at the same rate as the aggregate capital stock.[4]

Let us then start by deriving the dynamics of the productivity differential. At a given (short-run) period t there is a fraction $\lambda_t \in [0,1] \subset \mathbb{R}$ of the population of firms, which may vary from one period to the next one, adopting the profit-sharing labor-extraction strategy. The complementary fraction $1-\lambda_t$ is made up of firms that pay only the conventional base wage. Let $\overline{y}_t \equiv \lambda_t y_{s,t} + (1-\lambda_t)y_n$ be the average earnings of workers at period t, where $y_{s,t} \equiv v + \delta R_{s,t}/L_{s,t}$ and $y_n \equiv v$ are the earnings of a worker in a profit-sharing firm and a worker in a non-sharing firm in period t, respectively. Therefore, the differential between the higher earnings and the average earnings can be written as $y_{s,t}-\overline{y}_t = (1-\lambda_t)\delta R_{s,t}/L_{s,t}$ for all $\lambda_t \in [0,1] \subset \mathbb{R}$. In accordance with the empirical evidence reported earlier, we assume that the extent to which productivity in profit-sharing firms is greater than productivity in non-sharing firms varies positively with the relative earnings differential given by $y_{s,t}-\overline{y}_t$. Formally, we consider the following labor-extraction differential function:

(15) $\qquad \alpha_{t+1} = f(y_{s,t}-\overline{y}_t) = f\left((1-\lambda_t)\delta R_{s,t}/L_{s,t}\right),$

where $f'(\cdot) > 0$ and $f''(\cdot) < 0$ for all earnings differential $(y_{s,t}-\overline{y}_t) \subset \mathbb{R}$. Moreover, we assume that $\lim_{(y_s-\overline{y})\to\infty} f'(y_s-\overline{y}) = 0$. Therefore, in accordance with the empirical evidence that the labor productivity gains arising from profit sharing are not unlimited, the labor-extraction differential in (15) not only rises at a decreasing rate, but also tends to become insignificant for very large values of the relative earnings differential.[5] We can use (2) to re-write (15) as follows:

(15-a) $\qquad \alpha_{t+1} = f\left(\delta(1-\lambda_t)(\alpha_t - v)\right).$

As re-written in (15-a), the labor-extraction differential function has an intuitive interpretation. Given that α_t is the output per worker of a profit-sharing firm and v is the respective unit cost of labor, it follows that $\alpha_t - v$ is the profit per worker of a profit-sharing firm and $\delta(\alpha_t - v)$ is the amount of profit per worker of a profit-sharing firm which is shared with its workers. Therefore, given α_t and λ_t, the next-period labor-extraction differential varies positively with the profit-sharing coefficient and negatively with the conventional base wage. Meanwhile, given α_t, v and δ, the next-period labor-extraction differential increases with

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[4] Consequently, the constancy of the productivity differential and the distribution of labor-extraction compensation strategies in the long-run equilibrium guarantee the constancy of both the average labor productivity and the average employment rate. In fact, using (1) and (8)-(9), the average employment rate in the short-run equilibrium is $e_t^* \equiv \dfrac{L_{s,t}^* + L_{n,t}^*}{N_t} = \left[\dfrac{\lambda_t}{\alpha_t} + (1-\lambda_t)\right]\dfrac{K_t}{N_t}k$, where the expression in square brackets represents the weighted average of the inverse of the individual productivities $a_{s,t} = \alpha_t$ and $a_{n,t} = 1$.

[5] In the meta-analyses in Weitzman and Kruse (1990) and Doucouliagos (1995), the size of the estimated effect of profit sharing on labor productivity is usually on the order of 3 to 7 percent.

$1-\lambda_t$, the proportion of firms playing the non-sharing strategy in a given period, which is an indicator of the prospects of not receiving any shared profits in the next period. Since an increase in $1-\lambda_t$ acts as an incentive on workers in profit-sharing firms in the next period to yield a higher productivity differential, it follows that $\delta(1-\lambda_t)(\alpha_t-v)$ reflects how valuable it is to work for a profit-sharing firm.

In fact, note from (15-a) that $\partial\alpha_{t+1}/\partial\lambda_t=-\delta(\alpha_t-v)f'\big(\delta(1-\lambda_t)(\alpha_t-v)\big)<0$ for all $\lambda_t\in[0,1]\subset\mathbb{R}$ and for any $\alpha_t>v$ (recall that the latter condition was assumed earlier to ensure strictly positive gross profits for profit-sharing firms in (2)). The greater the proportion of sharing firms in a given period, the smaller the labor-extraction differential between sharing and non-sharing firms in the next period. One firm's decision to play the profit-sharing labor-extraction strategy in a given period, by reducing $\delta(1-\lambda_t)(\alpha_t-v)$ for a given α_t and making it less valuable to workers to be employed by a sharing firm in the next period, has a negative payoff externality on all other sharing firms. Thus, there is strategic substitutability in the firms' choice of labor-extraction compensation mechanism. Meanwhile, if all firms choose to play the profit-sharing labor-extraction strategy $(\lambda_t=1)$, the relative earnings differential given by $y_{s,t}-\overline{y}_t=\delta(1-\lambda_t)(\alpha_t-v)$ vanishes. In this case with all firms playing the profit-sharing strategy, given that labor productivity becomes uniform across all firms, and should be higher than the average labor productivity when all firms pay only the conventional base wage (recall that the productivity in non-sharing firms was normalized to one), we further assume that $f(0)>1$. Conversely, if all firms choose to play the non-sharing strategy $(\lambda_t=0)$, the potential relative earnings differential given by $y_{s,t}-\overline{y}_t=\delta(\alpha_t-v)$ takes its maximum value. In this case, a non-sharing firm which switches compensation to the profit-sharing mode is able to reap the largest possible additional labor-extraction gain, since $f\big(\delta(\alpha_t-v)\big)>f\big(\delta(1-\lambda_t)(\alpha_t-v)\big)$ for all $\lambda_t\in(0,1]\subset\mathbb{R}$ and for any $\alpha_t>v$. In order to better convey the substance of all these properties of the labor-extraction differential function (15-a), they are displayed in Figure 1.
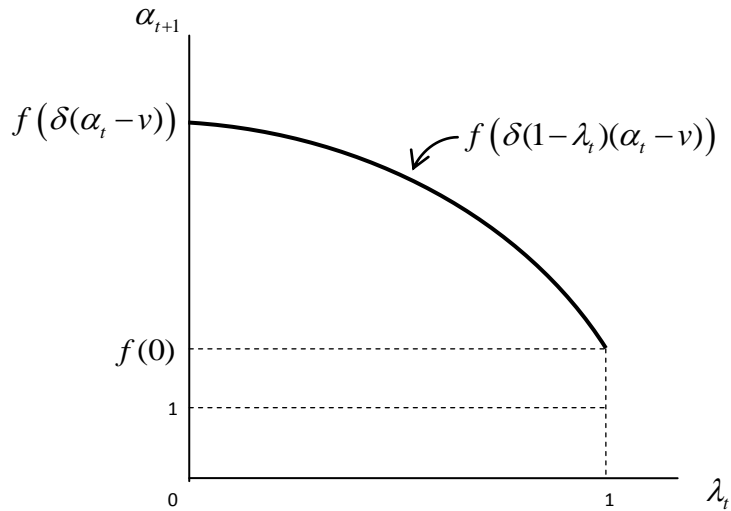
Figure 1. *Labor-extraction differential function*

The following intuitive rationales can be proposed for the additional labor-extraction gain in (15). First, the average earnings can be seen by a worker as a conventional estimate of her outside option or fallback position. Consequently, workers who receive a share of profits in

addition to a conventional base wage generate an extra productivity gain (relatively to the productivity gain they would offer if remunerated with only a conventional base wage) which increases with the excess of the higher earnings over their outside option or fallback position. Second, the average earnings can be perceived by a worker as a norm-based reference point against which a compensation package featuring a conventional base wage and shared profits should be compared when she is deciding how much above-normal productivity to provide in return. Therefore, above-average earnings could be seen by workers as warranting the offer of above-normal levels of productivity. In fact, Blasi, Kruse and Freeman (2010) propose an interesting rationale for profit-sharing compensation based on reciprocity and gift exchange (as these notions are articulated in Akerlof (1982)): a "gift" of higher worker compensation through profit sharing raises worker morale, and workers reciprocate with a "gift" of greater productivity. More generally, a "gift" of profit-sharing compensation on top of a conventional base wage can be seen as helping to create and reinforce a sense of shared interests and the value of a reciprocal relationship. Alternatively, the norm-based reference point offered by the average earnings can be interpreted as reflecting workers' earnings expectation under uncertainty. Thus, a worker compensation package that features a conventional base wage and shared profits is greeted as a pleasant surprise which warrants the offer of above-normal levels of productivity.

While the frequency distribution of labor-extraction compensation strategies is given in the short run, an individual firm revises periodically its choice of compensation strategy in a way described by the following replicator dynamic:[6]

(16) $$\lambda_{t+1} - \lambda_t = \lambda_t (r^c_{s,t} - r^c_t) = \lambda_t (1 - \lambda_t)(\pi^c_{s,t} - \pi_n)k ,$$

where $r^c_t \equiv \lambda_t r^c_{s,t} + (1 - \lambda_t)r_n$ is the average net profit rate, which is given by (12), and the latter equality is obtained using (10) and (11), so that $\pi^c_{s,t}$ and π_n are given by (5) and (6), respectively. According to the replicator dynamic in (16), the frequency of the profit-sharing labor-extraction strategy in the population of firms increases (decreases) exactly when it has above-average (below-average) payoff.

Using (5) and (6), the replicator dynamic in (16) becomes:

(16-a) $$\lambda_{t+1} = \lambda_t \left\{ 1 + (1 - \lambda_t)\left[(1 - \delta)\left(1 - \frac{v}{\alpha_t}\right) - (1 - v)\right]k \right\}.$$

Thus, the state transition of the economy is determined by the system of difference equations (15-a) and (16-a), whose state space is $\Theta = \{(\alpha_t, \lambda_t) \in \mathbb{R}^2_+ : 0 \leq \lambda_t \leq 1, \alpha_t > v\}$, as represented by the shaded area in each panel in Figure 2.

We will show that the dynamic system given by (15-a) and (16-a) has two long-run equilibria featuring survival of only one labor-extraction compensation strategy. These pure-strategy equilibria are denoted by E_1 and E_2 in the three panels in Figure 2. Moreover, we will show the possible existence of a third long-run equilibrium (denoted by E_3 in panel (b) in Figure 2), now featuring the survival of both labor-extraction compensation strategies.

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[6] The replicator dynamic can be derived from a model of (social or individual) learning as in Weibull (1995, sec. 4.4).

Note that $\lambda_{t+1} = \lambda_t = 0$ for any $t \in \{0,1,2,...\}$ satisfies (16-a) for any state $(\alpha_t, 0) \in \Theta$. Moreover, let $\alpha_{t+1} = \alpha_t = \bar{\alpha} \in (v,\infty) \subset \mathbb{R}$ for any $t \in \{0,1,2,...\}$. In this case, the difference equation in (15-a) is satisfied for any $t \in \{0,1,2,...\}$ if the following condition holds:

$$(17) \qquad \bar{\alpha} = f\big(\delta(\bar{\alpha} - v)\big).$$

We demonstrate in Appendix 1 that $\bar{\alpha} \in (v,\infty) \subset \mathbb{R}$ exists and is unique. Therefore, one of the two pure-strategy long-run equilibria of the system, E_1 in Figure 2, is given by the state $(\bar{\alpha}, 0) \subset \Theta$, which features the non-sharing labor-extraction compensation strategy as the only survivor.

Meanwhile, if $\lambda_{t+1} = \lambda_t = 1$ for any $t \in \{0,1,2,...\}$, the difference equation in (16-a) is satisfied for any state $(\alpha_t, 1) \in \Theta$ and, given (15-a), it follows that $\alpha_{t+1} = \alpha_t = f(0)$ for all $t \in \{0,1,2,...\}$. Therefore, the other pure-strategy long-run equilibrium of the system, E_2 in Figure 2, is represented by the state $(f(0),1) \subset \Theta$, which features the profit-sharing labor-extraction compensation strategy as the only survivor.

Finally, if $\lambda_{t+1} = \lambda_t = \lambda^* \in (0,1) \subset \mathbb{R}$ for any $t \in \{0,1,2,...\}$, the difference equation in (16-a) is satisfied if the individual profit shares in (5) and (6) (and hence the profit rates in (10) and (11)) are equalized. Given that the labor-extraction differential (which is equal to the labor-extraction in profit-sharing firms) is the only adjusting variable among the determinants of the individual profit shares in (5) and (6), the latter become equalized when the long-run equilibrium value of the labor-extraction differential is:

$$(18) \qquad \alpha^* = \frac{(1-\delta)v}{v - \delta},$$

where we assume that $v > \delta$. Meanwhile, given that $\alpha_{t+1} = \alpha_t = \alpha^*$ for all $t \in \{0,1,2,...\}$, the difference equation in (15-a) is satisfied if the following condition holds:

$$(19) \qquad \alpha^* = f\big(\delta(1-\lambda^*)(\alpha^* - v)\big).$$

As demonstrated in Appendix 2, there is a unique $\lambda^* \in (f(0), \bar{\alpha}) \subset \mathbb{R}$ which satisfies (19) if the following necessary and sufficient condition is satisfied:

$$(20) \qquad f(0) < \alpha^* < \bar{\alpha}.$$

Therefore, if the condition in (20) is satisfied, there exists a third long-run equilibrium given by the state $(\alpha^*, \lambda^*) \subset \Theta$ (and denoted by E_3 in panel (b) in Figure 2), which features the survival of both labor-extraction compensation strategies in the long run.

The well-defined ordering in (20), which implies and is implied by the existence and uniqueness of the mixed-strategy equilibrium, E_3, can be interpreted intuitively with recourse to Figure 3, which plots the next-period labor-extraction differential as a function of its current-period value. Note that, *ceteris paribus*, the function in (15-a) shifts down with an increase in the proportion of profit-sharing firms at period t. More precisely, this function rotates clockwise around the point $(v, f(0))$ as λ_t increases, due to the resulting squeeze in the relative earnings differential given by $y_{s,t} - \bar{y}_t$ for every $\alpha_t > v$.
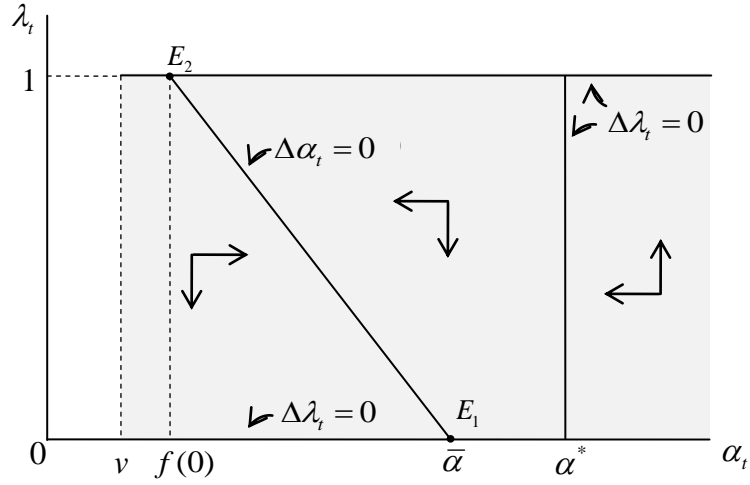
(a) *Relatively weak labor-extraction differential:* $f(0) < \bar{\alpha} < \alpha^*$
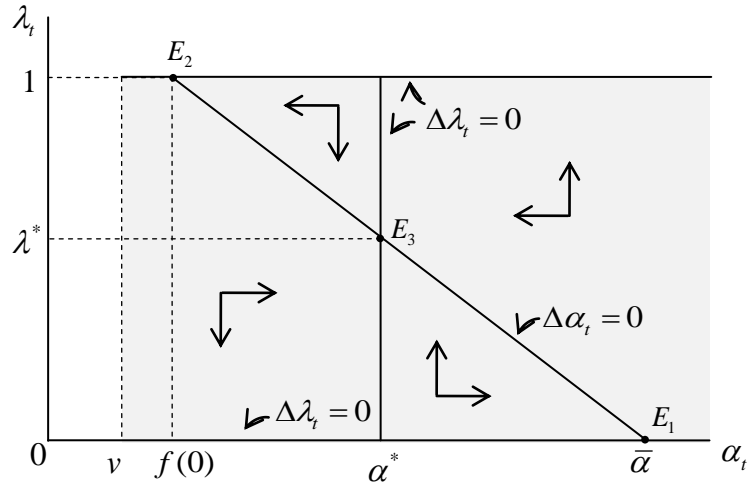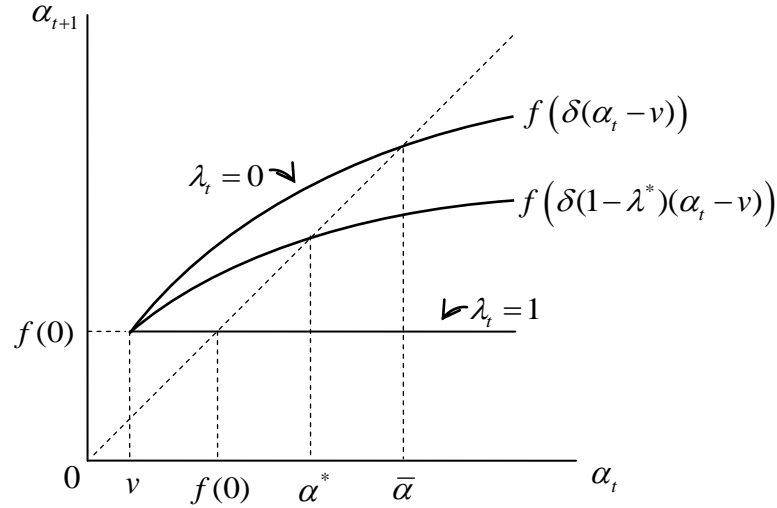

(b) *Relatively moderate labor-extraction differential:* $f(0) < \alpha^* < \bar{\alpha}$
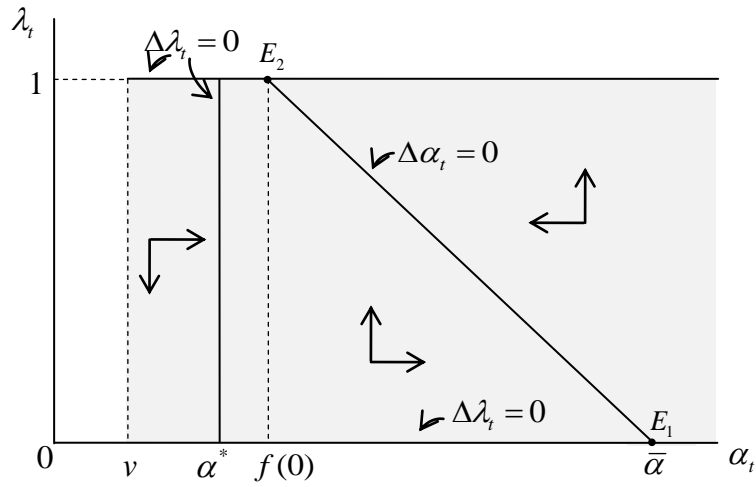
(c) *Relatively strong labor-extraction differential:* $\alpha^* < f(0) < \bar{\alpha}$

Figure 2. *Phase diagram for different magnitudes of the labor-extraction differential*

Figure 3. *Ordering of labor-extraction differentials when there is a mixed-strategy long-run equilibrium*

Let us now explore the dynamics of the system towards the long run as governed by the motion equations (15-a) and (16-a). The $\Delta\alpha_t \equiv \alpha_{t+1} - \alpha_t = 0$ isocline is the locus of all the states of the set given by $\{(\alpha_t, \lambda_t) \in \Theta : f(\delta(1-\lambda_t)(\alpha_t - v)) - \alpha_t = 0\}$. Therefore, this isocline connects the equilibrium solutions E_1 and E_2, as depicted in Figure 2. In order to know more about the $\Delta\alpha_t = 0$ isocline, we can use (15-a) to compute the following derivative:

$$(21) \qquad \frac{d\lambda_t}{d\alpha_t}\bigg|_{\Delta\alpha_t=0} = \frac{\delta(1-\lambda_t)f'(\delta(1-\lambda_t)(\alpha_t - v)) - 1}{\delta(\alpha_t - v)f'(\delta(1-\lambda_t)(\alpha_t - v))}.$$

The sign of (21) in the neighborhood of both pure-strategy long-run equilibria ($\lambda^* = 0$ and $\lambda^* = 1$) can be determined by taking the limit of (21) as the state of the system approaches each of these equilibria. These limits are given by:

$$(22) \quad \lim_{(\alpha_t,\lambda_t)\to(f(0),1)} \frac{d\lambda_t}{d\alpha_t}\bigg|_{\Delta\alpha_t=0} = \frac{-1}{\delta(f(0)-v)f'(0)} < 0 \text{ and } \lim_{(\alpha_t,\lambda_t)\to(\bar{\alpha},0)} \frac{d\lambda_t}{d\alpha_t}\bigg|_{\Delta\alpha_t=0} = \frac{\delta f'(\delta(\bar{\alpha}-v))-1}{\delta(\bar{\alpha}-v)f'(\delta(\bar{\alpha}-v))} < 0,$$

with the sign of the second limit in (22) coming from (A-2.2) in Appendix 2. The reason why in the vicinity of the extinction of one of the existing labor-extraction strategies, the higher is the labor-extraction differential, the lower is the proportion of profit-sharing firms, is that a higher relative earnings differential is then necessary to generate a higher labor-extraction differential (recall that because there is strategic substitutability in the firms' choice of labor-extraction compensation mode, the labor-extraction differential in (15) rises at a falling rate).

Meanwhile, the $\Delta\lambda_t \equiv \lambda_{t+1} - \lambda_t = 0$ isocline is the locus of all the states of the set given by $\{(\alpha_t, \lambda_t) \in \Theta : \lambda_t = 0\} \cup \{(\alpha_t, \lambda_t) \in \Theta : \lambda_t = 1\} \cup \{(\alpha_t, \lambda_t) \in \Theta : \alpha_t = \alpha^*\}$. As depicted in Figure 2, this set is represented by a semi-inverted H-shaped isocline.

From the local stability analysis carried out in Appendices 3-5, the following results emerge. In the configuration depicted in panel (a) in Figure 2, the additional labor-extraction gain

for sharing firms returned by a given relative earnings differential is relatively low. In this case, there are only two pure-strategy long-run equilibria, $E_1 = (\bar{\alpha}, 0)$, with no firm playing the profit-sharing labor-extraction strategy, and $E_2 = (f(0), 1)$, with all firms rather playing the profit-sharing labor-extraction strategy. As shown in Appendices 3 and 4, $E_2 = (f(0), 1)$ is a repulsor, while $E_1 = (\bar{\alpha}, 0)$ is an attractor.

Meanwhile, in the situation depicted in panel (c) in Figure 2, the additional labor-extraction gain for profit-sharing firms ensured by a given relative earnings differential is relatively high. Consequently, there are only two pure-strategy long-run equilibria, $E_1 = (\bar{\alpha}, 0)$, with all firms playing the non-sharing labor-extraction compensation strategy, and $E_2 = (f(0), 1)$, with all firms rather playing the sharing labor-extraction strategy. As shown in Appendices 3 and 4, while $E_2 = (f(0), 1)$ is an attractor, $E_1 = (\bar{\alpha}, 0)$ is a repulsor.

Finally, in the configuration depicted in panel (b) in Figure 2, the additional labor-extraction gain accruing to profit-sharing firms is relatively moderate. In this case, there are two pure-strategy long-run equilibria, $E_1 = (\bar{\alpha}, 0)$ and $E_2 = (f(0), 1)$, and also one mixed-strategy long-run equilibrium, $E_3 = (\alpha^*, \lambda^*)$, which features heterogeneity in labor-extraction compensation strategies in the population of firms. As shown in Appendices 2-5, while $E_1 = (\bar{\alpha}, 0)$ and $E_2 = (f(0), 1)$ are repulsors, $E_3 = (\alpha^*, \lambda^*)$ is an attractor if condition (A-5.4) in Appendix 5 is satisfied.[7]

Nonetheless, since the state space of the system is positively invariant (as shown in Appendix 6), in this configuration heterogeneity in labor-extraction compensation strategies does persist in the long run even if the mixed-strategy long-run equilibrium is not an attractor. This positive invariance implies that, if the long-run equilibrium with coexistence of both labor-extraction compensation strategies is not an attractor, the system keeps undergoing endogenous, self-sustaining and persistent fluctuations in the frequency distribution of labor-extraction compensation strategies and the average productivity, with the average levels of the income shares and economic growth persistently fluctuating as well. In fact, Mitchell et al. (1990) and D'Art and Turner (2006) find that the adoption of profit sharing schemes has tended to be cyclical in nature in many advanced countries since the origin of these schemes in the 19th century.

It is worth computing the long-run equilibrium values of the functional distribution of income and economic growth in each one of these three configurations as regards the relative magnitude of the additional labor-extraction gain which accrues to profit-sharing firms, as depicted in Figure 2. We can use (7) to establish:

(23) $\pi^*(\bar{\alpha}, 0) = \pi(\alpha^*, \lambda^*) = \pi_n = 1 - v$,

and:

(24) $\pi^*(f(0), 1) = \pi_s^c(f(0)) = (1 - \delta)\left[1 - \dfrac{v}{f(0)}\right]$,

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[7] As implied by the analysis in Appendix 5, the output to capital ratio, k, is likely to be a bifurcation parameter, as the long-run equilibrium with coexisting labor-extraction strategies is likely to lose (local) stability as the output to capital ratio crosses a given threshold from below. We leave this issue for future research.

where the first equality in (23) follows from the fact that $E_3 = (\alpha^*, \lambda^*)$ is defined implicitly by the condition given by $\pi_{s,t}^{c*}(\alpha_t, \lambda_t) - \pi_n = 0$ (recall that the profit share of non-sharing firms is exogenously given). In panel (a), with $\bar{\alpha} > f(0)$, we have $\pi^*(\bar{\alpha}, 0) > \pi^*(f(0), 1)$ and, consequently, using (14), $g^*(\bar{\alpha}, 0) > g^*(f(0), 1)$. In panel (b), with $\alpha^* > f(0)$, we have $\pi^*(\bar{\alpha}, 0) = \pi^*(\alpha^*, \lambda^*) > \pi^*(f(0), 1)$, with (14) yielding $g^*(\bar{\alpha}, 0) = g^*(\alpha^*, \lambda^*) > g^*(f(0), 1)$. In panel (c), with $\alpha^* < f(0)$, we have $\pi^*(\bar{\alpha}, 0) < \pi^*(f(0), 1)$, it following from (14) that $g^*(\bar{\alpha}, 0) < g^*(f(0), 1)$. Therefore, if the economy converges to a long-run equilibrium (which may not occur in panel (b), as shown in Appendix 5), the coupled evolutionary dynamics of profit-sharing adoption and labor-extraction gain takes the economy to a position in which the average net profit share, the net profit rate and the rate of economic growth are all at their highest possible long-run equilibrium levels.

Finally, note that despite the possibility of a stable long-run equilibrium outcome with strategic dualism, the evolutionary dynamic in (16) ensures that the (net) rates of profit of the coexisting labor-extraction strategies become equalized. However, in our one-sector model, profit rate equalization is brought about not by classical competition with capital mobility across sectors, but by evolutionary competition with firms' (and their capital) mobility across labor-extraction compensation strategies.

4. Conclusions

This paper is motivated by several pieces of suggestive empirical evidence. First, historically, there has been a persistent heterogeneity in worker compensation strategies across firms, and profit-sharing schemes have experienced a fluctuating popularity. Second, profit sharing raises labor productivity in the firm, and surveys find that both employers and workers usually perceive profit sharing as helping to improve firm performance in several dimensions. Third, surveys find that non-deferred and in cash profit sharing is ranked first by workers as motivation device. Fourth, firms switch mechanisms of worker compensation (which include profit sharing) frequently, with the gross changes in mechanisms being more numerous than the net changes. Fifth, profit sharing has a meaningful effect on worker total compensation, which suggests that profit sharing tends to come on top of, rather than in place of, a base wage.

Against this empirical backdrop, this paper formally models firms as periodically choosing to compensate workers with either a conventional base wage or a share of profits on top of this conventional base wage. As a result, the frequency distributions of labor-extraction compensation strategies and labor productivity in the population of firms are evolutionarily time-varying. Moreover, we explore implications of this coupled evolutionary dynamics for income distribution and economic growth.

When the additional productivity gain for profit-sharing firms is relatively low, there are two long-run equilibria, one featuring all firms sharing profits and the other with no firm sharing profits. While the former is a repulsor, the latter is an attractor. When the additional productivity gain for profit-sharing is relatively high, the economy has the same two long-run equilibria. However, the long-run equilibrium with all firms sharing profits is an attractor, while the long-run equilibrium with no firm sharing profits is a repulsor. When the additional productivity gain for profit-sharing firms is relatively moderate, there is another long-run equilibrium, now one that features heterogeneity in worker compensation modes across firms. In this case, the long-run equilibria with survival of only one strategy are repulsors, while the long-run equilibrium with coexistence of both strategies can be an attractor. However, even if the long-run equilibrium configuration with coexistence of both labor-extraction strategies is not an attractor, the economy

keeps undergoing endogenous, self-sustaining and persistent fluctuations in the distribution of labor-extraction compensation strategies and average labor productivity (which accords with the empirical evidence), with functional income distribution and economic growth persistently fluctuating as well.

Meanwhile, when there is convergence to some of the three long-run equilibria, the average net profit rate (which excludes shared profits) and the rate of economic growth are at their highest possible long-run equilibrium values. Moreover, there is profit rate equalization even in the long-run equilibrium with coexisting labor-extraction strategies. However, this profit rate equalization is brought about by evolutionary competition with firms' mobility across labor-extraction compensation strategies and not by classical competition with capital mobility across sectors.

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Appendix 1 - Existence and uniqueness of a long-run equilibrium with all firms playing the non-sharing labor-extraction strategy

Let $x = \alpha - v$ and $h(x) = f(\delta x) - v$. In order to show the existence and uniqueness of a $\bar{\alpha} \in (v, \infty) \subset \mathbb{R}$ which satisfies (17) we have to show that the function h has a unique strictly positive fixed point, i.e, there is a unique real value $\bar{x} = \bar{\alpha} - v$ such that $\bar{x} = h(\bar{x})$.

We can accomplish this by using Theorem 3 in Kennan (2001, pp. 895), which makes it possible to ascertain that h has a unique $\bar{x} \in \mathbb{R}_{++}$, if the following conditions are satisfied: (i) h is increasing; (ii) h is strictly concave; (iii) $h(0) \geq 0$; (iv) $h(a) > a$ for some $a > 0$; and (v) $h(b) < b$ for some $b > a$.

Conditions (i) and (ii) are satisfied. Since $f'(\cdot) > 0$ and $f''(\cdot) < 0$ in the entire domain of f, it follows that for all $x \in \mathbb{R}_+$ we have:

(A-1.2) $\qquad h'(x) = \delta f'(\delta x) > 0$ and $h''(x) = \delta^2 f''(\delta x) < 0$.

Therefore, the function h is strictly increasing and strictly concave.

Condition (iii) is also satisfied. Since $f(0) > 1 > v$, it follows that $h(0) = f(0) - v > 0$.

Condition (iv) is also satisfied. Given that $0 < v < 1$, it follows that $0 < 1 - v < 1$. Besides, since h is strictly increasing and $f(0) > 1$, it follows that there is a $a = 1 - v$ such that $h(1-v) = f(\delta(1-v)) - v > f(0) - v > 1 - v$.

Condition (v) is also satisfied. Recall that we have assumed that $\lim\limits_{(y_s - \bar{y}) \to \infty} f'(y_s - \bar{y}) = 0$. Hence, since $y_s - \bar{y} = \delta(\alpha - v) = \delta x$ for $\lambda = 0$, it follows that $\lim\limits_{x \to \infty}(y_s - \bar{y}) = \lim\limits_{x \to \infty} \delta x = \infty$. For $\lambda = 0$, we can deduce that $\lim\limits_{x \to \infty}\dfrac{h(x)}{x} = \lim\limits_{x \to \infty}\dfrac{f(\delta x) - v}{x} = \lim\limits_{x \to \infty} \delta f'(\delta x) = 0$, where in the last equality we have used L'Hôpital's rule. Thus, we can assert that for every $\varepsilon > 0$ there is some $M > 0$ such that for all $x > M$ we have $\left|\dfrac{h(x)}{x}\right| < \varepsilon$. This last inequality can be re-written as $h(x) < \varepsilon x$, since $h(x) = f(\delta x) - v > f(0) - v > 0$ for all $x > 0$. We can set $\varepsilon = 1$. In this case, there is some $M > 0$ such that for all $x > M$ it follows that $h(x) < x$. Thus, it is enough to choose any $x = b > Max\{M, a\}$ to obtain $h(b) < b$ and $b > a = 1 - v$.

Appendix 2 - Existence and uniqueness of a mixed-strategy long-run equilibrium (both labor-extraction compensation strategies are played across the population of firms)

Let $\phi(\alpha, \lambda) \equiv \alpha - f(\delta(1-\lambda)(\alpha - v))$. Condition (19) is satisfied if, and only if, $\phi(\alpha^*, \lambda^*) = 0$. Let us show that, given α^*, if condition (20) is satisfied, there is a unique $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\phi(\alpha^*, \lambda^*) = 0$.

Given the existence and uniqueness of the fixed point $\bar{\alpha}$ shown in Appendix 1, we can use the Index Theorem (Kehoe, 1987, p. 52) to write:

(A-2.1) $\qquad index(\bar{\alpha}) = \text{sgn}\left(1 - \dfrac{\partial f(\delta(\bar{\alpha} - v))}{\partial \alpha}\right) = 1,$

where $\mathrm{sgn}(\cdot)$ stands for the sign function. Based on this function, we can establish that:

(A-2.2)
$$\frac{\partial \phi(\bar{\alpha}, \lambda)}{\partial \alpha} = 1 - \delta f'\big(\delta(\bar{\alpha} - v)\big) > 0.$$

Therefore, given the existence and uniqueness of $\bar{\alpha}$, the graph of $\phi(\bar{\alpha}, \lambda)$ in the plane given by $\{(\alpha, \lambda) \in \mathbb{R}_+^2\}$ always crosses the 45-degree line only once and from above, as shown in Figure 3. Since it follows from (17) that $\phi(\bar{\alpha}, 0) = \bar{\alpha} - f\big(\delta(\bar{\alpha} - v)\big) = 0$, we can conclude that $\phi(\alpha, 0) = \alpha - f\big(\delta(\alpha - v)\big) < 0$ for all $\alpha \in (v, \bar{\alpha}) \subset \mathbb{R}$. For all $\alpha^* < \bar{\alpha}$, then, we can infer that:

(A-2.3)
$$\phi(\alpha^*, 0) = \alpha^* - f\big(\delta(\alpha^* - v)\big) < 0.$$

Moreover, it is straightforward that (20) implies that:

(A-2.4)
$$\phi(\alpha^*, 1) = \alpha^* - f(0) > 0.$$

Since $\phi(\alpha^*, 0) < 0$, $\phi(\alpha^*, 1) > 0$ and ϕ is continuous along the domain, we can then apply the intermediate value theorem to conclude that there is some $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\phi(\alpha^*, \lambda^*) = 0$. Moreover, given that $f'(\cdot) > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$, we have:

(A-2.5)
$$\frac{\partial \phi(\alpha^*, \lambda)}{\partial \lambda} = \delta(\alpha^* - v) f'\big(\delta(1-\lambda)(\alpha^* - v)\big) > 0,$$

for all $\lambda \in [0,1] \subset \mathbb{R}$. As a result, since the function in (A.2-5) is continuous in the closed interval $[0,1] \subset \mathbb{R}$, there is only one $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\phi(\alpha^*, \lambda^*) = 0$.

Appendix 3 - Local stability of the long-run equilibrium with all firms playing the non-sharing labor-extraction strategy

The Jacobian matrix evaluated around the equilibrium $(\bar{\alpha}, 0) \subset \Theta$ is given by:

(A-3.1)
$$J(\bar{\alpha}, 0) = \left[\begin{array}{c|c} \delta f'\big(\delta(\bar{\alpha}-v)\big) & -\delta(\bar{\alpha}-v) f'\big(\delta(\bar{\alpha}-v)\big) \\ \hline 0 & 1 + \Big[(1-\delta)\Big(1-\frac{v}{\bar{\alpha}}\Big) - (1-v) \Big] k \end{array} \right].$$

Let ξ be an eigenvalue of the Jacobian matrix (A-3.1). We can set the characteristic equation of the linearization around the equilibrium:

(A-3.2)
$$|J - \xi I| = \left| \begin{array}{c|c} a-\xi & -b \\ \hline 0 & c-\xi \end{array} \right| = \xi^2 - (a+c)\xi + ac = 0,$$

where $a \equiv \delta f'\big(\delta(\bar{\alpha}-v)\big) > 0$, $b \equiv \delta(\bar{\alpha}-v) f'\big(\delta(\bar{\alpha}-v)\big) > 0$, and $c \equiv 1 + \Big[(1-\delta)\Big(1-\frac{v}{\bar{\alpha}}\Big) - (1-v) \Big] k$. The solutions of (A-3.2) are the eigenvalues of the Jacobian matrix (A-3.1), which are given by:

(A-3.3)
$$\xi_1 = \delta f'\big(\delta(\bar{\alpha}-v)\big) \quad \text{and} \quad \xi_2 = 1 + [\pi_s^c(\bar{\alpha}) - \pi_n] k,$$

where $\pi_s^c(\bar{\alpha}) = (1-\delta)\Big(1-\frac{v}{\bar{\alpha}}\Big)$ and $\pi_n = 1-v$.

Let us investigate the absolute value of ξ_1. Given that $\delta > 0$ and $f'(\cdot) > 0$ along the domain, it follows that $\xi_1 = \delta f'\big(\delta(\bar{\alpha} - v)\big) > 0$. Also, it follows from (A-2.2) that $\xi_1 = \delta f'\big(\delta(\bar{\alpha} - v)\big) < 1$. Hence, it follows that $|\xi_1| < 1$.

Let us check the absolute value of ξ_2. We want to find under what condition(s) it follows that $-1 < \xi_2 < 1$. Per (A-3.3), it follows that $-1 < \xi_2 < 1$ obtains if, and only if, $-2 < [\pi_s^c(\bar{\alpha}) - \pi_n]k < 0$.

Since $0 < \pi_s^c(\bar{\alpha}) < 1$ and $0 < \pi_n < 1$, we obtain that $-1 < \pi_s^c(\bar{\alpha}) - \pi_n < 1$. Since the output to capital ratio typically satisfies the condition given by $0 < k < 1$ (Marquetti and Foley, 2012), it follows that $-2 < -k < [\pi_s^c(\bar{\alpha}) - \pi_n]k$. Thus, $\pi_s^c(\alpha) - \pi_n = (1-\delta)\left(1 - \dfrac{v}{\alpha}\right) - (1-v)$ is increasing in α. Given (25), we have $\pi_s^c(\alpha^*) - \pi_n = 0$. Thus, if $\bar{\alpha} < \alpha^*$ (panel (a) in Figure 2), we find that $\pi_s^c(\bar{\alpha}) - \pi_n < 0$, and hence that $[\pi_s^c(\bar{\alpha}) - \pi_n]k < 0$, which means that $|\xi_2| < 1$ if $\bar{\alpha} < \alpha^*$. But if $\bar{\alpha} > \alpha^*$ (panels (b) and (c) in Figure 2), it follows that $\pi_s^c(\bar{\alpha}) - \pi_n > 0$, and hence that $[\pi_s^c(\bar{\alpha}) - \pi_n]k > 0$, so that $|\xi_2| > 1$ if $\bar{\alpha} > \alpha^*$. This completes the demonstration that the long-run equilibrium with no firm playing the profit-sharing strategy, $(\bar{\alpha}, 0) \subset \Theta$, is an attractor if $\bar{\alpha} < \alpha^*$ (panel (a) in Figure 2) and a repulsor if $\bar{\alpha} > \alpha^*$ (panels (b) and (c) in Figure 2).

Appendix 4 - Local stability of the long-run equilibrium with all firms playing the profit-sharing labor-extraction compensation strategy

The Jacobian matrix evaluated around the equilibrium $(f(0), 1) \subset \Theta$ is given by:

$$(\text{A-4.1}) \qquad J(f(0), 1) = \left[\begin{array}{c|c} 0 & -\delta(f(0) - v)f'(0) \\ \hline 0 & 1 - \left[(1-\delta)\left(1 - \dfrac{v}{f(0)}\right) - (1-v)\right]k \end{array}\right].$$

Let ξ be an eigenvalue of the Jacobian matrix (A-4.1). We can set the characteristic equation of the linearization around this equilibrium:

$$(\text{A-4.2}) \qquad |J - \xi I| = \left|\begin{array}{c|c} -\xi & -a \\ \hline 0 & b - \xi \end{array}\right| = \xi(\xi - b) = 0,$$

with $a \equiv \delta[f(0) - v]f'(0) > 0$ and $b \equiv 1 - \left[(1-\delta)\left(1 - \dfrac{v}{f(0)}\right) - (1-v)\right]k$.

In this case, the eigenvalues of the Jacobian matrix (A.4-1) are easily computed from (A-4.2):

$$(\text{A-4.3}) \qquad \xi_1 = 0 \quad \text{and} \quad \xi_2 = b = 1 - \left[\pi_s^c((f0)) - \pi_n\right]k,$$

where $\pi_s^c(f(0)) = (1-\delta)[1 - v/f(0)]$ and $\pi_n = 1 - v$.

The local stability of $(f(0), 1) \subset \Theta$ depends on ξ_2. Given (A-4.3), it follows that $-1 < \xi_2 < 1$ obtains if, and only if, $0 < \left[\pi_s^c(f(0)) - \pi_n\right]k < 2$.

Since $0 < \pi_s^c(f(0)) < 1$ and $0 < \pi_n < 1$, it follows that $-1 < \pi_s^c(f(0)) - \pi_n < 1$. Therefore, given that it is typically the case that $0 < k < 1$, we can infer that $\left[\pi_s^c(f(0)) - \pi_n\right]k < 2$.

It can be seen that $\pi_s^c(\alpha) - \pi_n = (1-\delta)\left(1 - \dfrac{v}{\alpha}\right) - (1-v)$ is increasing in α. Given (25), we have $\pi_s^c(\alpha^*) - \pi_n = 0$. Therefore, if $f(0) < \alpha^*$ (panels (a) and (b) in Figure 2), we obtain that $\pi_s^c(f(0)) - \pi_n < 0$, and hence that $\left[\pi_s^c(f(0)) - \pi_n\right]k < 0$, which means that $|\xi_2| > 1$ if $f(0) < \alpha^*$. Meanwhile, if $f(0) > \alpha^*$ (panel (c) in Figure 2), we obtain that $\pi_s^c(f(0)) - \pi_n > 0$, and hence that $\left[\pi_s^c(f(0)) - \pi_n\right]k > 0$. Therefore, it follows that $|\xi_2| < 1$ if $f(0) > \alpha^*$. This completes the demonstration that the long-run equilibrium with all firms playing the profit-sharing labor-extraction strategy, $(f(0), 1) \subset \Theta$, is a repulsor if $f(0) < \alpha^*$ (panels (a) and (b) in Figure 2) and an attractor if $f(0) > \alpha^*$ (panel (c) in Figure 2).

Appendix 5 - Local stability of the long-run equilibrium with heterogeneity in labor-extraction compensation strategies across firms

The Jacobian matrix evaluated around the equilibrium $(\alpha^*, \lambda^*) \subset \Theta$ is given by:

$$(\text{A-5.1}) \quad J(\alpha^*, \lambda^*) = \left[\begin{array}{c|c} \delta(1-\lambda^*)f'\big(\delta(1-\lambda^*)(\alpha^*-v)\big) & -\delta(\alpha^*-v)f'\big(\delta(1-\lambda^*)(\alpha^*-v)\big) \\ \hline \lambda^*(1-\lambda^*)(1-\delta)\dfrac{v}{(\alpha^*)^2}k & 1 \end{array}\right].$$

Let ξ be an eigenvalue of the Jacobian matrix (A-5.1). We can set the characteristic equation of the linearization around the equilibrium:

$$(\text{A-5.2}) \qquad |J - \xi I| = \begin{vmatrix} a - \xi & -b \\ c & 1-\xi \end{vmatrix} = \xi^2 - (a+1)\xi + (a+bc) = 0,$$

where $\quad a \equiv \delta(1-\lambda^*)f'\big(\delta(1-\lambda^*)(\alpha^*-v)\big) > 0, \qquad b \equiv \delta(\alpha^*-v)f'\big(\delta(1-\lambda^*)(\alpha^*-v)\big) > 0, \qquad$ and

$c \equiv \lambda^*(1-\lambda^*)(1-\delta)\dfrac{v}{(\alpha^*)^2}k > 0.$

We can use the Samuelson stability conditions for a second order characteristic equation to determine under what conditions the two eigenvalues are inside the unit circle. Based on Farebrother (1973, p. 396, inequalities 2.4 and 2.5), we can establish the following set of simplified Samuelson conditions for the quadratic polynomial in (A-5.2):

$$(\text{A-5.3}) \qquad 1 + a + bc > |-(a+1)| = a+1 \quad \text{and} \quad a+bc < 1.$$

Let us prove that these conditions are satisfied if $a < 1$.

First, note that $1 + a + bc > a+1$ simplifies to $bc > 0$, which is trivially satisfied given that $b > 0$ and $c > 0$.

Meanwhile, the second inequality, $a + bc < 1$, can be expressed as follows:

$$(\text{A-5.4}) \qquad a+bc = \delta(1-\lambda^*)f'(\cdot)\left\{1+\frac{v(1-\delta)\lambda^*(\alpha^*-v)}{(\alpha^*)^2}k\right\}<1.$$

Appendix 6 - Positive invariance of the state space

We want to show that $(\alpha_t,\lambda_t)\in\Theta$ for all $t\in\{1,2...\}$ and initial condition $(\alpha_0,\lambda_0)\in\Theta$.

Let us first demonstrate that $\alpha_{t+1}>v$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$. Given (22-a) and the assumptions that $f(0)>1>v$ and $f(.)$ is strictly increasing, we can establish that $\alpha_{t+1}=f\left(\delta(1-\lambda_t)(\alpha_t-v)\right)\geq f(0)>v$ for any $\alpha_t>v$ and $0\leq\lambda_t\leq1$. By induction, we can conclude that $\alpha_{t+1}>v$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$.

Next, let us prove that $0\leq\lambda_{t+1}\leq1$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$. Let us first show that $\lambda_{t+1}\geq0$ for all $(\alpha_t,\lambda_t)\in\Theta$. As $\lambda_t\geq0$ for any $(\alpha_t,\lambda_t)\in\Theta$ and given (23-a), in order to show that $\lambda_{t+1}\geq0$ for all $(\alpha_t,\lambda_t)\in\Theta$, we need to show that for all $(\alpha_t,\lambda_t)\in\Theta$ we have:

$$(\text{A-6.1}) \qquad (1-\lambda_t)\left[(1-\delta)\left(1-\frac{v}{\alpha_t}\right)-(1-v)\right]k\geq-1.$$

We can set up the following lower and upper bounds for the profit share differential for any $\alpha_t>v$:

$$(\text{A-6.2}) \qquad -(1-v)<\pi_{s,t}^c-\pi_{n,t}=(1-\delta)\left(1-\frac{v}{\alpha_t}\right)-(1-v)\leq v-\delta.$$

From (A-6.1)-(A-6.2) and the fact that it is typically the case that $0<k<1$, we can write:

$$(\text{A-6.3}) \qquad (1-\lambda_t)(\pi_{s,t}^c-\pi_{n,t})k\geq-(1-\lambda_t)(1-v)k\geq-(1-\lambda_t)(1-v)=-1+v+(1-v)\lambda_t>-1$$

for any $(\alpha_t,\lambda_t)\in\Theta$. Hence, by induction, $\lambda_{t+1}\geq0$ for all $t\in\{0,1,2...\}$ and $(\alpha_0,\lambda_0)\in\Theta$.

Finally, let us demonstrate that $\lambda_{t+1}\leq1$ for all $(\alpha_t,\lambda_t)\in\Theta$. Given (23-a), in order to establish that $\lambda_{t+1}\leq1$ for all $(\alpha_t,\lambda_t)\in\Theta$, we need to demonstrate that:

$$(\text{A-6.4}) \qquad \lambda_t\left[1+(1-\lambda_t)(\pi_{s,t}^c-\pi_{n,t})k\right]\leq1.$$

This inequality is trivially satisfied for $\lambda_t=0$. For any $\lambda_t>0$, we can re-write (A-6.4) as:

$$(\text{A-6.5}) \qquad (\pi_{s,t}^c-\pi_{n,t})k\leq\frac{1}{\lambda_t}.$$

We can again make use of (A-6.2) and conclude that for any $\lambda_t>0$, we have:

$$(\text{A-6.6}) \qquad (\pi_{s,t}^c-\pi_{n,t})k\leq(v-\delta)k<1\leq\frac{1}{\lambda_t}.$$

This completes the proof that the state space Θ is positively invariant, so that $(\alpha_t,\lambda_t)\in\Theta$ for all $t\in\{1,2...\}$ and initial condition $(\alpha_0,\lambda_0)\in\Theta$.