Does mixed frequency vector error correction model add relevant information to exchange misalignment calculus? Evidence for United States

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Abstract

Real exchange rate is an important macroeconomic price in the economy and affects economic activity, interest rates, domestic prices, trade and investiments flows among other variables. Methodologies have been developed in empirical exchange rate misalignment studies to evaluate whether a real effective exchange is overvalued or undervalued. There is a vast body of literature on the determinants of long-term real exchange rates and on empirical strategies to implement the equilibrium norms obtained from theoretical models. This study seeks to contribute to this literature by showing that it is possible to calculate the misalignment from a mixed ointegrated vector error correction framework. An empirical exercise using United States' real exchange rate data is performed. The results suggest that the model with mixed frequency data is preferred to the models with same frequency variables.

JEL Codes: F31, C52, F37.

Key Words: Real effective exchange rate, Cointegration, Mixed Frequency.

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Introduction

An important macroeconomic price in the economy is the real exchange rate. This variable affects economic activity, interest rates, domestic prices in the economy. Thus, large movements in the real exchange rate can have important effects on those variables. The empirical and theoretical literature seeks to respond if such movements are excessive or are in line with a change in fundamentals. The literature has advanced towards a better understanding of long-term determinants of the real exchange rate. Empirical strategies can be formulated based on models that use the doctrine of Purchasing Power Parity (PPP) or based on fundamentals analysis.

There is a vast literature that seeks to obtain the best estimate for the misalignment of the real exchange rate. Basically, this debate can be divided into a search for the determinants of real exchange rate in theoretical and empirical grounds. The first issue is to discuss what are the determinants of real exchange rate from different theories. The second issue has to do with the choice of the best empirical strategy to implement a specific norm to estimate real exchange rate misalignment. This work focus on the second issue.

The main motivation for using mixed frequency models comes from the fact that real effective exchange rate and many its fundamentals can be observed at a different frequencies. Although there might be a great loss of information if the analyst opt to work with models at the lowest available frequency, this is the usual choice in the literature. By temporal aggregating the data, the analyst can incur in bias when estimating the parameters of a time series model. If the analysts opt to work with end of period data he will loose the information of higher frequency data. Working with mixed frequency data models is a natural choice.

Our goal is to estimate a mixed frequency vector error correction model (MF-VECM) an obtain an estimative of real exchange rate misalignment for United States. The MF-VECM methodology follows the research of Götz et al (2013). Our paper address the costs and benefits of using mixed frequency models to address exchange rate misalignment.

Regarding the results, we can anticipate that the mixed frequency models are superior to the low frequency models for the analyzed dataset .

The article is divided into five sections, besides this introduction. The first section briefly reviews the literature of models for the misalignment of the exchange rate. The second section presents the methodology for MF-VECM with variables in mixed frequency. The third section presents the data used in the

work, as well as the methodology used. The fourth presents the results of the work. Finally, we make concluding remarks.

1 Brief review of real exchange rate misalignment literature

The literature on real exchange rate is extensive (Froot and Rogoff, 1995). The classical doctrine and perhaps the oldest one on real exchange rate determinants is the Purchasing Power Parity (PPP). Reference to this theory can be found in classical authors. Recent studies confirm the validity of PPP for tradable goods although the adjustment towards equilibrium is quiet slow. Ahmad and Craighead (2010) obtained strong evidence of mean reversion with high half-life using a secular consumer prices index dataset for United States and United Kindgom. Their work investigates the point made by Taylor (2001) about the effects of temporal aggregation on the tests of PPP.

There is a theoretical discussion about which variables drives the real exchange rate in the long-term. An older literature goes back to Edwards (1987 and 1991) and Dornbusch (1976). The first analyzes the causes and consequences of exchange rate misalignment. The second one is the classic flexible exchange model approach under which monetary policy shocks cause deviations from PPP fundamentals.

The studies of Bilson (1979) and Mussa (1976) are also classicals. These are key references for the monetary approach to the exchange rate. Under this approach, the exchange rate would be primarily driven by the following fundamentals: the difference between domestic and foreign income and the money supply. The approach assumes that PPP and uncovered interest parity (UIP) holds continuously and the demand for money is stable in all countries. The research of Meese and Rogoff (1983) casts doubt on the explanatory power of this theory by showing that the predictions of this approach are not superior to a 'naive' forecast model for the exchange rate such as a pure random walk. Rossi (2013) shows that the random walk can be outperformed by an econometric model that uses the information based on net foreign investiment position.

Stein (1995) formulates the natural exchange rate approach (NATREX). According to the author, the equilibrium exchange is the one that equals the level of investment savings generated by economic fundamentals.

Williamson (1994) had a great impact on the exchange rate misalignment

literature. The equilibrium exchange rate for the author is the one that allows a country to sustain a desirable result in the external accounts. This is the fundamental real exchange rate approach (FRER). A more recent reference to this approach is Cline and Williamson (2007). A limitation of this approach comes from the high degree of arbitrariness and subjectivity in choosing the target of foreign accounts. The results may not be robust to different targets. In addition, this approach focuses on flows and not stocks.

Faruqee (1995) incorporates issues related to the evolution of stocks and constructs a model which allows for interaction between flows and stocks. Thus, there must be a stable relationship between real exchange rate and net foreign asset position between residents and non-residents. This is called behavioral real exchange rate approach (BRER). The model is extended by Alberola, Cervero et al. (1999).

Kubota (2009) uses a model with a representative agent who maximizes intertemporal consumption and accumulates capital. This study indicates that the real exchange rate is a function of terms of trade, net external position and relative productivity of tradable and non-tradable sectors. This approach seeks to reduce the degree of subjectivity in the estimation of the exchange rate misalignment. Thus, she establishes a link between the real exchange rate and a set of fundamentals derived from a theoretical model. Then, she decomposes the series of real exchange rate in transitory and permanent components using time series econometric technique.

Recently, the International Monetary Fund (IMF) started to systematically disseminate the results of staff research effort to measure the exchange rate misalignment in several countries members of the Fund.The External Balance Assessment (EBA) methodology, developed by IMF's Research Department, is based on two panel estimations: for current account and real effective exchange rate (REER) indices [1]. The basic idea is that the REER can be written as a function of the output gap, real interest rate differential and factors that may affect saving, investment, current account, capital flows and changes in foreign currency reserves. The explanatory variables included in the EBA model are: commodity terms of trade, trade openness, share of administered prices, VIX[2],

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[1] Full description of the methodology, data and routines are available at http://www.imf.org/external/np/spr/2013/esr/)

[2] Chicago Board Options Exchange Market Volatility Index

share of own currency in world reserves, financial home bias, population growth, expected GDP growth over the next 5 years, productivity and changes in foreign reserves. Policy-related regressors are also included: health expenditure to GDP, foreign exchange interventions, real short-term interest rate differential, private credit to GDP and capital controls. Most of the variables described are relative to country's trade partners using the same weights as in the REER calculation and/or interacted with capital account openness and some variables are lagged to control for endogeneity. Sample data covers 40 countries and the period of 1990-2010. The model includes countries fixed effects. In order to guarantee multilateral consistency of the results the exchange rate misalignment must be adjusted.

Given the results of the estimation, the "Total REER Gap" can be calculated by the sum of the regression residual with the "Total Policy Gap". The policy gap is a measure of a cyclical gap (over a benchmark) on six policy areas: fiscal balance, capital controls, social spending, foreign exchange market intervention, financial policies and monetary policy. The gap is calculated by the difference of the actual level of the variable and their "desirable" level, times the value of the estimated coefficient. The "desirables" levels are supplied by each IMF's countries desks. The next section discusses the cointegration structure with mixed frequency.

2 A mixed-frequency VECM

The notation used in this work follows the literature on the subject, some key references are Clements and Galvão (2007, 2009), Götz, Hecq and Urbain (2012a, 2012b, 2013) and Ghysels and Miller (2013). Let us start from a two variables mixed-frequency system (but it can be easily extended to larger dimensions), where y_t is the low-frequency variable, and $x_{t-i/m}^{(m)}$ the high frequency variables with m high frequency observations per low-frequency period t. In a year/quarter-example, m=4 and the value of i indicates the specific quarter under consideration, ranging from first quarter $\left(x_{t-3/m}^{(m)}\right)$ until fourth quarter $\left(x_t^{(m)}\right)$. In a straightforward notation, $x_{t-m/m}^{(m)} = x_{t-1}^{(m)}$. L denotes the low-frequency lag operator, i.e., $Ly_t = y_{t-1}$ or $Lx_{t-i/m}^{(m)} = x_{t-1-i/m}^{(m)}$, whereas L_m denotes the high-frequency lag operator, i.e., $L_m x_{t-i/m}^{(m)} = x_{t-i/m-1/m}^{(m)} = x_{t-(i+1)/m}^{(m)}$. Similarly, the same logic is applied to the difference operator, $\triangle$ e $\triangle_m$. Note that $L_m x_{t-(m-1)/m}^{(m)} = x_{t-1}^{(m)}$ and $\triangle_m x_{t-(m-1)/m}^{(m)} = x_{t-(m-1)/m}^{(m)} -$

Notation	$t = 2012,\ m = 4$
$x^{(m)}_{t+1-(m-1)/m} = x^{(4)}_{t+1/4}$	$x^{(4)}_{2012,Q1}$
$x^{(m)}_t$	$x^{(4)}_{2011,Q4}$
$x^{(m)}_{t-1/m}$	$x^{(4)}_{2011,Q3}$
$\vdots$	$\vdots$
$x^{(m)}_{t-(m-1)/m} = x^{(4)}_{t-3/4}$	$x^{(4)}_{2011,Q1}$
$x^{(m)}_{t-m/m} = L_m x^{(4)}_{t-3/4} = x^{(4)}_{t-1}$	$x^{(4)}_{2010,Q4}$
$x^{(m)}_{t-1-1/m}$	$x^{(4)}_{2010,Q3}$

$x^{(m)}_{t-1}$, by the same reasoning. The table illustrates the notation for a year/quarter-example.

Consider a vector that includes the variables of high frequency, i.e., $X^{(m)}_t = \left(x^{(m)}_t, x^{(m)}_{t-1/m}, \ldots, x^{(m)}_{t-(m-1)/m}\right)'$. Ghysels (2012) starts from a VAR(p):

$$Z_t = \Gamma_1 Z_{t-1} + \ldots + \Gamma_p Z_{t-p} + \varepsilon_t \tag{1}$$

where $Z_t = \left(y'_t, X^{(m)'}_t\right)'$ and $\varepsilon_t \sim N(0, I_{m+1})$. Observations of high frequency are added stacked in the regression with the low frequency variable. That is, if the variable y is annual and x is a quarterly variable, the regression includes together one year with the inclusion of the variable y_t and four quarters with the inclusion of x_t, $x_{t-1/4}$, $x_{t-2/4}$ and $x_{t-3/4}$.

But assuming that the series in Z_t are I(1) and that there is cointegration between variables, estimating (1) in first difference will generate misspecified model. According to Götz, Hecq and Urbain (2013), we can rewrite (1) like the VECM representation such that

$$\triangle Z_t = \widetilde{\Gamma}_1 \triangle Z_{t-1} + \ldots + \widetilde{\Gamma}_{p-1} \triangle Z_{t-p-1} + \Pi Z_{t-1} + \varepsilon_t \tag{2}$$

where $\widetilde{\Gamma}_i = - \sum_{k=i+1}^{p} \Gamma_k$, $i = 1, \ldots, p-1$ and $\Pi = - \left(\sum_{j=1}^{p} \Gamma_j\right) = \alpha\beta'$ with $rank(\Pi) = (r_0 + r_1) = m$.

In this case, there is a difference in the *rank* of the matrix Π with the variables at the same frequency and with mixed frequency. r_0 denotes a prespecified

cointegration vectors (not genuine cointegration) because the first difference of the high-frequency I(1) variables is stationary. r_1 refers to the additional long-run relationship between the two different variables. If there is cointegration (true cointegration) between the variables, y is cointegrated with one of $x's$ of high-frequency. That's not important which of these high-frequency $x's$ is used to the cointegration relationship, but one has to be used (see Gotz et al, 2012a). In our case, we model the cointegration using the end-of-period observation of the high-frequency variable (i.e., $x_t^{(m)}$) and assuming m=4, i.e., the high frequency variable is quarterly and low frequency variable is annual.

Disregarding the short term, we can write the mixed frequency structure in the VECM framework as[3],

$$
\begin{pmatrix} \triangle y_t \\ \triangle x_t^{(4)} \\ \triangle x_{t-1/4}^{(4)} \\ \triangle x_{t-2/4}^{(4)} \\ \triangle x_{t-3/4}^{(4)} \end{pmatrix} = \alpha \begin{pmatrix} 1 & \theta & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1}^{(4)} \\ x_{t-1-1/4}^{(4)} \\ x_{t-1-2/4}^{(4)} \\ x_{t-1-3/4}^{(4)} \end{pmatrix}
$$

$$
+ \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t}^{(4)} \\ \varepsilon_{2,t-1/4}^{(4)} \\ \varepsilon_{2,t-2/4}^{(4)} \\ \varepsilon_{2,t-3/4}^{(4)} \end{pmatrix} \qquad (3)
$$

and $\Pi = \alpha \begin{pmatrix} 1 & \theta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$ which can be rewritten like

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[3] A cointegration alternative in mixed frequency is CoMIDAS (cointegrating mixed data sampling) of Miller (2013). However, CoMIDAS is an ADL (Autoregressive Distributed Lag) uni-equational model.

$$\Pi = \alpha\beta' = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} & \alpha_{54} \end{pmatrix} \begin{pmatrix} 1 & \theta & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

If there isn't cointegration between y and x, we have only the presence of "not genuine" cointegration relationships (the first difference of x being stationary or prespecified cointegration relationships) leading to a matrix Π such

$$\Pi = \alpha\beta' = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} \end{pmatrix} \begin{pmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

The presence of cointegration relationship between y and x will be tested using Horvath and Watson (1995) approach. This test is used when some of the cointegration vectors are prespecified. The prespecified cointegration vector improve the power results of test of unknown cointegration relationships according to Horvath and Watson (1995). This test has the null hypothesis $rank(\Pi) = r_0$, i.e., without the relationship between y and x, against the alternative hypothesis that $rank(\Pi) = r_0 + r_1$. This cointegration test is a likelihood ratio test, where the unknown cointegration vector between y and x is estimated under the alternative hypothesis. The likelihood ratio statistic is given by

$$LR = 2\left(l_{HA} - l_{Ho}\right)$$

where l_{HA} and l_{Ho} are the log-likelihood function evaluated under the alternative and the null hypothesis, respectively. The critical values of the test are available in Horvath and Watson (1995).

One version of common features unaddressed in Götz et al (2013) is analyzed in our paper. In our case, we asked if there is the presence of common features that annihilate "not genuine" or predetermined cointegration relationships. The search for common feature can be important in the cases where the difference between the frequencies is high. In this case the models can suffer from the curse of dimensionality. This kind of common feature could be represented as

$$\Pi \begin{pmatrix} y_{t-1} \\ x_{t-1}^{(4)} \\ x_{t-1-1/4}^{(4)} \\ x_{t-1-2/4}^{(4)} \\ x_{t-1-3/4}^{(4)} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & C_1 \\ \alpha_{21} & C_2 \\ \alpha_{31} & C_3 \\ \alpha_{41} & C_4 \\ \alpha_{51} & C_5 \end{pmatrix} \begin{pmatrix} 1 & \theta & 0 & 0 & 0 \\ 0 & 0 & 1 & \phi_1 & \phi_2 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1}^{(4)} \\ \triangle_4 x_{t-1}^{(4)} \\ \triangle_4 x_{t-1-1/4}^{(4)} \\ \triangle_4 x_{t-1-2/4}^{(4)} \end{pmatrix}$$

$$(4)$$

where the second column of the matrix α and the second line of matrix β refer to the common feature component. We decided to normalize the coefficient for $\triangle_4 x_{t-1}^{(4)}$ in the common feature component to 1. In the next section, we detail the methodology adopted in this paper.

3 Methodology and data description

3.1 Database

In this work, we use quartely and annual data for the United States from 1971 to 2013. The real effective exchange rate and the index of consumer and of wholesale prices were collected in International Financial Statistics (IFS - IMF). The foreign trade weights were collected from the International Monetary Fund (IMF) - Direction of Trade Statistics (DOTS-IMF). The values of net foreign assets are net position of assets and liabilities of residents and non-residents. This variable was obtained using from Lane and Milesi-Ferreti (2007) and IFS - IMF. The variable used in the model is the net foreign asset divided by the GDP (Gross Domestic Product).

The index of consumer (CPI) and of wholesale prices (WPI) were used for the Balassa-Samuelson variable. The BS variable was calculated for country i by the ratio CPI_i/WPI_i of country i relative to the international ratio CPI_i^*/WPI_i^* (constructed from the weights of foreign trade previously mentioned).

3.2 Econometric methodology

This work applied the mixed frequency VECM structure, like (3). In this research, three variables are used: the real exchange rate (RER), net foreign assets divided by the GDP (denominated by NFA only) and relative price variable (hereafter referred as Balassa-Samuelson variable or just BS). The high

frequency variable (quarterly) are RER and BS, while the low frequency variable (annual) is the NFA [4]. The high frequency variables were included quarterly. They are also available at monthly frequency, but the model will have to many parameters to be estimated given the available sample size. The VECM structure with mixed frequency represented by equation (1) has the following variables:

$$\triangle Z_t = \begin{pmatrix} \triangle NFA_t \\ \triangle RER_t^{(4)} \\ \triangle RER_{t-1/4}^{(4)} \\ \triangle RER_{t-2/4}^{(4)} \\ \triangle RER_{t-3/4}^{(4)} \\ \triangle BS_t^{(4)} \\ \triangle BS_{t-1/4}^{(4)} \\ \triangle BS_{t-2/4}^{(4)} \\ \triangle BS_{t-3/4}^{(4)} \end{pmatrix}, Z_{t-1} = \begin{pmatrix} NFA_{t-1} \\ RER_{t-1}^{(4)} \\ RER_{t-1-1/4}^{(4)} \\ RER_{t-1-2/4}^{(4)} \\ RER_{t-1-3/4}^{(4)} \\ BS_{t-1}^{(4)} \\ BS_{t-1-1/4}^{(4)} \\ BS_{t-1-2/4}^{(4)} \\ BS_{t-1-3/4}^{(4)} \\ C \end{pmatrix}$$

here C is a constant, i.e., the specification of the VECM is established with restricted constant to avoid trend in the data. In this case, we have one cointegration relationship that will be estimated and 3 cointegration relationships pre-specified (we denominated as "not genuine cointegration relationships") by the quarterly variations in RER and 3 cointegration relationships pre-specified by the quarterly variations in BS. We have 7 cointegration relationships. We estimate four parameters of the cointegration relationship: the constant, the coefficient associated with the NFA and the coefficients associated with $RER_{t-1}^{(4)}$ and $BS_{t-1}^{(4)}$. But the coefficient associated with $RER_{t-1}^{(4)}$ was normalized. The VECM has no further short-term structure because it would leave the estimation with very few degrees of freedom. The appropriateness of this choice can be assessed from the specification tests, particularly analyzing the results of the residual autocorrelation tests.

Then, the matrix dimensions are 9 rows and 10 columns, in which the α matrix has dimension 9x7 (9 dependent variables and 7 cointegration relationships) and β' matrix has 10x7 (10 variables in the cointegration vector and 7

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[4]The NFA variable presents quarterly data for U.S., but only for the recent period. Therefore, we use this variable in the low frequency (annual). Another point is that we could use monthly data for BS and RER variables for estimation in mixed frequency. However, the number of parameters to be estimated increased considerably, which did not allow estimation with enough degrees of freedom in this case.

cointegration relationships).

After estimating the cointegration relationship, we intend to get the measure of real exchange rate misalignment with mixed frequency. This type of analysis is not done by Gotz et al (2013). The present work calculates the misalignment of the exchange rate based on the decomposition of Gonzalo and Granger (1995). Before, we present the simple case of the decomposition used to get a measure of real exchange rate misalignment.

Several decompositions have been proposed in the literature to decompose the series between transitory and permanent components. The permanent component is considered the fundamentals of the economy, while the transitory component represent the misalignment of the real exchange rate. In general, the decomposition takes the folowing form:

$$x_{i,t} = [c_{i\perp}(\beta_i' c_{i\perp})^{-1}\beta_i' + \beta_{i\perp}(c_i'\beta_{i\perp})^{-1}c_i]x_{i,t} \tag{5}$$

The existence of this decomposition is not always guaranteed since the matrix $c_i'\beta_{i\perp}$ may not have full rank. Gonzalo and Granger (1995) proposed $c_i = \alpha_{i\perp}$. Using the decomposition of Gonzalo and Granger, it is possible to calculate the transitory component (T_{it}) and the permament component (P_{it}) from the following equations:

$$P_{it} = \beta_i(\alpha_{i\perp}'\beta_{i\perp})^{-1}\alpha_{i\perp}' x_{i,t} \tag{6}$$

$$T_{it} = \alpha_i(\beta_i'\alpha_i)^{-1}\beta_i' x_{i,t} \tag{7}$$

This leads to a minor adaptation compared to the case with the same frequency because the estimated cointegrating relationship is the one that would lead to an estimate of the economic fundamentals. In this case, the permanent and transitory components are calculated in the same manner as in (6) and (7). But instead of the entire matrix α and β, we will use only the first column of α and β because of the comment made earlier. So, these procedures will be performed in the next section together with the specification tests of the structure of mixed frequency.

4 Results

4.1 Cointegration and specification tests and final estimation of the VECM

First, we discuss the Horvath and Watson (1995) cointegration test result. The value of the test statistic calculated is 264.96, while the critical value based on Horvath and Watson (1995) is 12.49. The null hypothesis of no cointegration between RER, NFA and BS is strongly rejected. As already aforementioned our specification choice for the model does not contain further short-term dynamics than lagged values of the variables within the same year due to sample size restrictions.

Because of the small sample, we tested the possibility of reducing the number of estimated parameters. One of the tests was to see if we could eliminate the not genuine cointegrating vectors of the model. Because the coefficients of the matrix β are given, it is necessary to restrict some of the parameters in matrix α. The likelihood ratio test was used for such test, in which the null hypothesis was the restricted estimation (no not genuine cointegration relationships)[5]. The value of the test calculated is 325.73, while the critical value based on the chi-squared distribution with v degrees of freedom (where v is the total number of parameters reduced by the restricted estimation) is 84.50.

We tested the presence of common features that annihilate the pre-determined cointegration relationships by a likelihood ratio test. The null hypothesis corresponds to the restricted estimation (that is, the model with common features), while the alternative is the model without common features. The value of the test calculated is 187.75, while the critical value based on the chi-squared distribution with v degrees of freedom is 66.77. Then, the best model does not contain common features. But the likelihood ratio test has size distortion in a small sample. In other words, the null hypothesis is rejected too often when it's true. So, we compare the information criteria between models with and without common features as an alternative measure, which is shown in Table 1. The result of three information criteria corroborates the likelihood test.

Finally, the last test seeks to answer the following question: which model is better? The model that contains mixed frequency data or that contains only lower frequency variables (in the same frequency)? The null hypothesis

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[5] Henceforth, all likelihood ratio tests are based on the confidence level of 5%.

Information Criteria	Common feature	
	Without	With
Akaike	-71,30	-68,74
Schwarz	-68,57	-67,66
Hannan-Quinn	-70,30	-68,34

Table 1: Information criteria for model with and without common feature

corresponds to the restricted estimation, that is, the model with variables in the same frequency whereas the alternative hypothesis contains mixed frequency specification. The structure of VECM for the variables in only one frequency, that is, the usual structure in which the end-of-period of the high frequency variable is nested to the mixed frequency model[6] and can be represented as:

$$\Pi \begin{pmatrix} y_{t-1} \\ x_{t-1}^{(4)} \end{pmatrix} = \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \end{pmatrix} \begin{pmatrix} 1 & \theta \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1}^{(4)} \end{pmatrix} \tag{8}$$

The value of the test calculated is 2269.3 and the proper critical value based on the chi-squared distribution with 60 degrees of freedom is 91.95. Then, the null hyptohesis is rejected in favor of mixed frequency specification. However, we also present the comparison by the information criteria due to the possible size distortion problem with the likelihood ratio test, previously mentioned. We present the information criteria for the two models in Table 2. The model with the variables in mixed frequency is the most appropriate for all information criteria, i.e., the best model would be the one with higher frequency information.

Now, we analyze the VECM with mixed frequency which was considered as the most appropriate after all tests. In equation form, the estimated cointegration relationship is given by:

$$ECM = RER_t - 13.00 - 0.19 * NFA_t + 1.81 * BS_t \tag{9}$$

The theory suggests that the BS coefficient is about 1 but our unrestricted

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[6] The variable high frequency is used as end-of-period to avoid problems caused by data aggregation. In certain contexts, the temporal aggregation can cause significant distortions (Taylor, 2001; Ghysels and Miller, 2013).

Information Criteria	Model with	
	Mixed frequency	Same frequency
Akaike	-71,30	-20,13
Schwarz	-68,57	-19,88
Hannan-Quinn	-70,30	-20,04

Table 2: Information criteria for model with variables in mixed frequency and with variables in same frequency

Each of equations	Each of cointegration vectors						
	1	2	3	4	5	6	7
NFA	0.02	-0.28	0.39	-0.51	-1.00	0.05	-0.81
	(0.04)	(0.22)	(0.21)	(0.21)	(0.48)	(0.51)	(0.56)
RER-Q4	-0.09	0.38	0.10	0.69 *	0.93	0.75	-0.05
	(0.07)	(0.38)	(0.37)	(0.36)	(0.83)	(0.88)	(0.97)
RER-Q3	-0.11 *	1.04 ***	0.46	0.31	0.35	0.59	-0.68
	(0.06)	(0.33)	(0.32)	(0.31)	(0.72)	(0.77)	(0.84)
RER-Q2	-0.10 **	1.00 ***	1.44 ***	0.24	0.32	0.11	-0.31
	(0.05)	(0.28)	(0.27)	(0.27)	(0.61)	(0.65)	(0.72)
RER-Q1	-0.11 ***	1.19 ***	1.29 ***	1.40 ***	-0.10	-0.39	-0.35
	(0.03)	(0.16)	(0.16)	(0.16)	(0.36)	(0.38)	(0.42)
BS-Q4	0.01	-0.45 ***	-0.03	-0.24 **	-0.96 ***	0.22	0.29
	(0.02)	(0.13)	(0.12)	(0.12)	(0.28)	(0.29)	(0.32)
BS-Q3	0.01	-0.28 ***	-0.16	-0.09	0.47 **	0.35	0.58 **
	(0.02)	(0.11)	(0.10)	(0.10)	(0.23)	(0.24)	(0.27)
BS-Q2	-0.01	-0.17 *	-0.10	-0.12	0.72 ***	1.28 ***	0.26
	(0.02)	(0.10)	(0.10)	(0.09)	(0.22)	(0.23)	(0.26)
BS-Q1	0.02	-0.04	-0.10	-0.13	1.09 ***	0.93 ***	1.16 ***
	(0.02)	(0.08)	(0.08)	(0.08)	(0.17)	(0.18)	(0.20)

Obs.: the standard deviation of the coefficients is in parentheses.
*, ** and *** corresponds, respectively, to statistically significant at 10%, 5% and 1%.

Table 3: Estimates of Alpha Matrix

estimative is 1,81 but not not statistically different from 1[7]. The coefficient associated with the NFA is lower in the presence of mixed frequency than the estimate at the same frequency. We have obtained that an increase in net foreign assets leads to appreciation of the real exchange rate as expected.

Tables 3 shows the estimative of matrix α, the loading matrix of cointegrating relationship. The variable that responds to deviations from the long-term relationship (between NFA, RER and BS) is only RER (first, second and third quarters). The RER's speed of adjustment of the long term disequilibrium is very similar, especially for the first, second and third quarters.

Regarding the not genuine cointegration relationships, the NFA variable re-

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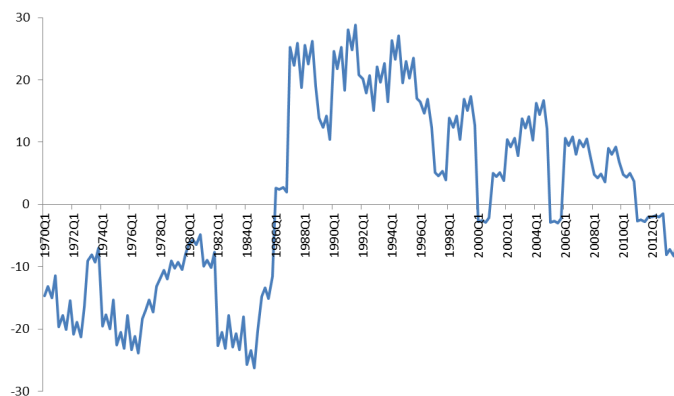[7] We did a likelihood ratio test, in which the restricted estimate is the coefficient of BS equal to -1.

Figure 1: Estimates of the Misalignment of the Real Exchange Rate with Mixed Frequency

sponds (statistically significantly) to quarterly variations in the RER of the third and second quarters and the quarterly variation in BS of the fourth quarter. The RER variable is only affected by its own lagged quarterly changes. The BS variable responds to the RER and its lagged quarterly variations. The RER and BS variables of the first and second quarters (mostly) are greatly affected by past quarterly fluctuations. But this follows from the VECM structure with mixed frequency that is used. The first and second quarters are closer (more than the other dependent variables on third and four quarters) of the fourth, third and second quarters of the previous year.

4.2 Comparing misalignment calculated by mixed frequency VECM and by the lower frequency VECM

After we estimate the VECM with mixed frequency, we apply the decomposition of Gonzalo and Granger to generate the misalignment of the real exchange rate (transitory component). Figure 1 shows the estimates of the misalignment of the real exchange rate with the VECM in mixed frequency. Focusing on recent period, we obtain that the dollar is depreciated since the first quarter of 2011 (after accounting for the fundamentals). This movement can be seem as a side effect of U.S. unconventional monetary policy, in which the second and more comprehensive quantitative easing was announced in November 2010, but became effective in the second quarter of 2011. But the important point is that
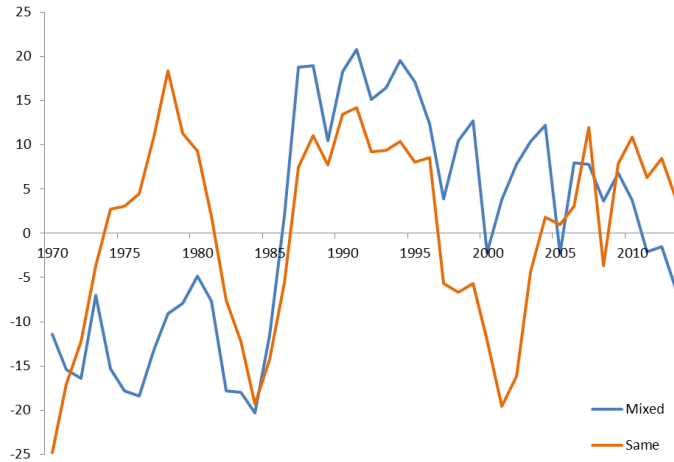
Figure 2: Comparison of estimates of misalignment of the real exchange rate obtained by the variables with mixed frequency or with the same frequency

the depreciation of the dollar became substantial from 2013.

Next, we compare the estimated misalignment of the real exchange rate obtained by the variables with mixed frequency or with the same frequency. Specifically, we will compare the annual change in the real exchange rate based on the fourth quarter, shown in Figure 2[8]. The two estimates of misalignment of the real exchange rate diverge considerably in value or even its movement in some periods. There are at least two reasons to be any difference between the two ways of calculating the misalignment. First, the estimated cointegration relationship is different, especially for the coefficient of BS. Second, the structure with mixed frequency considers the adjustment of quarterly variables (BS and RER) to calculate the misalignment. Focusing on the recent period, the estimated misalignments totally diverge from 2009 onwards. Overall, the estimated misalignment of the real exchange rate with the variables at the same frequency does not seem to change because of U.S. monetary policy to weaken the dollar.

Last, but not least, we will evaluate the two models according to the degree of adjustment[9]. We will use the RMSE (Root-Mean-Square Error) indicator as one of the ways of evaluating rival models. The RMSE measures the difference

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[8]When variables in the same frequencies are used, it is only possible to estimate the misalignment of the real exchange rate including the variation of fourth quarter (end of period).

[9]We did not forecast out of sample due to the small number of degrees of freedom that we have.
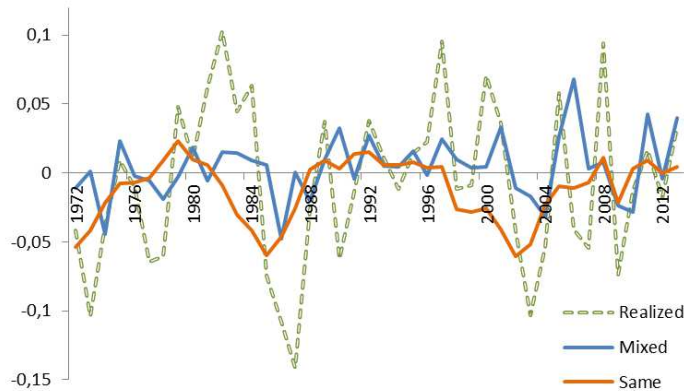
Figure 3: Comparison between models with variables in mixed frequency and in same frequency based on the realized and expected values

between values predicted by a model and realized values[10]. The smaller RMSE, the better the model. In our case, we decided to evaluate only the last observation of the RER in the fourth quarter. In this case, $RMSE = 0.0071$ of the model with the variables in mixed frequency and $RMSE = 0.0295$ of the model with the variables in the same frequency. The latter is almost three times higher than the former.

Another way to compare the two models is graphically. So, we show the expected and realized values by RER variable in the fourth quarter of each year, as shown in Figure 3. The predicted value by the model with mixed frequency seems to follow more closely the realized values for the second part of the sample, performing particularly well for the last two years of the sample.

Concluding Remarks

This paper aims to construct mixed VECM using data available atmixed frequency to get the estimate of the misalignment of the real exchange rate to the U.S. between 1971 and 2013. The structure with the variables in mixed frequency was preferred to the structure with variables at the same frequency by the information criteria and the likelihood ratio test. This mixed frequency structure was preferred even though it has a higher number of parameters to be estimated. There is evidence of cointegration between the variables from the

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[10]Consider y_t the realized value of the variable and $\hat{y}_t$ the predicted value of the variable with sample size equal to n, so $RMSE = \sqrt{\frac{\sum_{t=1}^{n}(\hat{y}_t - y_t)^2}{n}}$.

mixed frequency structure. Furthermore, there is no evidence of the presence of common features, which would reduce the number parameters to be estimated.

We obtained that an increase in net foreign assets leads to appreciation of the real exchange rate in the long term as expected by theory. The variable that responds to deviations from the long-term relationship (between NFA, RER and BS) is only RER (first, second and third quarters). This is a important fact to use the model to address exchange rate misalignment. The speed of adjustment of the long term disequilibrium is very similar, especially for the first, second and third quarters.

Regarding the not genuine cointegration relationships, the NFA variable responds (statistically significantly) to quarterly variations in the RER of the third and second quarters and the quarterly variation in BS of the fourth quarter. The high frequency variables of the first and second quarters (mostly) are greatly affected by past quarterly.

After we estimate the VECM with mixed frequency, we apply the decomposition of Gonzalo and Granger to generate the misalignment of the real exchange rate (transitory component). Focusing on the recent period, we obtain that the dollar is undervalued since the first quarter of 2011. This movement can be seen as a side effect of United States unconventional monetary policy.

References

AHMAD, Y., and W. D. CRAIGHEAD (2010): "Temporal Aggregation and Purchasing Power Parity Persistence," Journal of International Money and Finance, In Press, Accepted Manuscript.

ALBEROLA, E., S. CERVERO, H. LOPEZ, and A. UBIDE (1999): "Global Equilibrium Exchange Rate: Euro, Dolar, 'Ins', 'Outs' and Other Major Currencies in a Panel Cointegration Framework," Washington: IMF

BILSON, J. F. (1979): "Recent Developments in Monetary Models of Exchange Rate Determination," IMF Staff Paper, 26, 201-223.

CLEMENTS, M. P., and A. B. GALVÃO (2007), "Marcoeconomic Forecasting with Mixed Frequency Data: Forecasting U.S. Output Growth", Working Papers 616, Queen Mary, University of London, School of Economics and Finance.

CLEMENTS, M. P., and A. B. GALVÃO (2009), "Forecasting U.S. Output Growth Using Leading Indicators: An Appraisal Using MIDAS Models", Journal of Applied Econometrics, 24(7):1187-1206.

CLINE, W. R., and J. WILLIAMSON (2007): "Estimates of the Equilibrium Exchange Rate of the Renminbi: Is There a Consensus and, If Not, Why Not?," Washington: Peter Institute for International Economics.

DORNBUSCH, R. (1976): "Expectations and Exchange Rate Dynamics,"

Journal of Political Economy, 84, 1161-1176.

EDWARDS, S. (1987): "Exchange Rate Misaligment in Developing Countries," NBER working paper, 442.

FARUQEE, H. (1995): "Long-Run Determinants of the Real Exchange Rate: A Stock Flow Perspective," IMF Staff Paper, 42, 80-107.

GHYSELS, E. AND J. I. MILLER (2013): "Testing for Cointegration with Temporally Aggregated and Mixed-frequency Time Series", Working paper.

GONZALO, J., and C. W. J. GRANGER (1995): "Estimation of Common Long-Memory Components in Cointegrated Systems," Journal of business and Economics Statistics, 13.

GÖTZ, T., HECQ A., and URBAIN, J.-P. (2012a), "Forecasting Mixed-Frequency Time Series With ECM-MIDAS Models", Maastricht University METEOR Research Memorandum 12/012.

GÖTZ, T., HECQ A., and URBAIN, J.-P. (2012b), "Real-Time Forecast Density Combinations (Forecasting U.S. GDP Growth using Mixed-Frequency Data)", Maastricht University METEOR Research Memorandum 12/021.

GÖTZ, T.B., HECQ, A., URBAIN, J. (2013): "Testing for Common Cycles in Non-Stationary VARs with Varied Frecquency Data", Maastricht University working paper.

HORVATH, M. T. K., and WATSON, M. W. (1995), "Testing for Cointegration when some of the Cointegrating Vectors are Prespecied", Econometric Theory, 11, 984-1014.

KUBOTA, M. (2009): "Real Exchange Rate Misaligments," York: University of York, 201.

MEESE, R. A., and K. ROGOFF (1983): "Empirical Exchange Models of the Seventies: Do They Fi out of the Sample?," Journal of International Economics, 14, 3-24.

MILLER, J.I. (2013): "Mixed-frequency Cointegrating Regressions with Parsimonious Distributed Lag Structures," Journal of Financial Econometrics.

MUSSA, M. (1976): "The Exchange Rate, the Balance of Payments and Monetary Policy under a Regime of Controlled Floating," Scadinavian Journal Of Economics, 78, 228-248.

STEIN, J. (1995): "The Fundamental Determinants of the Real Exchange Rate of the U.S. Dollar Relative to Other G-7 Currencies," IMF Working Paper 95-81.

TAYLOR, A. M. (2001): "Potential Pitfalls for the Purchasing-Power-Parity Puzzle? Sampling and Specification Biases in Mean-Reversion Tests of the Law of One Price," Econometrica, 69, 473-498.

WILLIAMSON, J. (1994): Estimating Equilibrium Exchange Rates. Washington D.C. IIE.