

# Is Petrobras Options Market Efficient? A Study Using the Delta-Gamma Neutral Strategy

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**ABSTRACT.** This study aims to determine if the Petrobras options market is efficient in its weak form. To do so, an attempt is made to systematically profit from the firm's preferred stock and respective buy options using the delta-gamma neutral strategy. To simulate the strategy as it would be used in the real world, we built buy books every five minutes considering all buy and sell orders sent for both the underlying asset and its options. The strategy is implemented when we observe distortions among the implied volatilities of the options. The results show evidence that the Petrobras options market is not efficient, since in 371 day-order transactions, with an average investment of R\$ 81,000 and an average holding time of one hour and thirteen minutes, the average return was 0.49% – which corresponds to more than 1,600% of the highest interbank interest rate for the period, with 85% of the options strategies effected being profitable.

**KEYWORDS.** options, market efficiency, implied volatility, delta-gamma neutral strategy

JEL Classification: G13

## 1 - INTRODUCTION

Similar to countries with more developed financial markets, the Brazilian market also includes traders who operate by detecting distortions among options prices. These distortions are observed by comparing the implied volatility of an option, estimated by an options pricing model, with the volatility estimated by another model, or when comparing two or more implied volatilities. However, if participants can accrue systematic profits from this type of transaction, the options market is not efficient, according to the definition of Fama (1970).

This work aims to verify whether the options market for Petrobras preferred stock is efficient in the weak form when attempting to earn profits by exploiting distortions in the prices of call options. The possible distortions are detected by comparing the implied volatilities of the options with a base volatility estimated by the EWMA model<sup>1</sup>. The model used to obtain the implied volatilities is Black & Scholes (B-S) (Black & Scholes, 1973).

The strategy used is the delta-gamma neutral, i.e., buying relatively inexpensive calls (with lower implied volatility), and selling relatively expensive calls (with higher implied volatility) of the same underlying asset and maturity, and simultaneously buying or selling the stock in an amount that makes the Greek deltas and gammas be worth zero<sup>2</sup>. If the implied volatilities tend to the same value, it is possible to garner a profit when liquidating the position.

Among the variables used for pricing a stock option, volatility is the only one that is not directly observed in the market. Thus, saying that an option is poorly priced is the same as saying that the volatility used to price the option is incorrect. We use the delta-gamma neutral strategy to try to earn profits from the distortions in volatilities (and, therefore, the prices of options), thus protecting ourselves from potential fluctuations in the price of the underlying stock (since the delta and gamma of the strategy are null). It is known that when hedging only against the price of the underlying stock, the position continues to be exposed to factors other than volatility, such as time to maturity and the interest rate. However, the influence of these variables on the position tends to be extremely low, since as options are bought and sold (much of the effect generated by one of the options in the position is offset by another) and positions held in this study are very short term since they are day orders<sup>3</sup>.

In order to properly simulate actual market situations, thereby avoiding any semblance of doubt as to the success/failure of the position, the database used contains information for all (buy and sell) orders sent and executed during the period October

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<sup>1</sup> *Exponential weighted moving average*, also known in the econometric literature as *integrated GARCH* (1,1) – IGARCH (1,1) – with null constant. This model is widely used in the financial market. See, for example, Riskmetrics™ (1996).

<sup>2</sup> Delta and gamma are the first and second order measures of sensitivity of options in relation to the price of the underlying stock.

<sup>3</sup> In particular, the impact of the Greek theta is practically nil due to the short term positions used in this work.

2012 to March 2013<sup>4</sup>. From this base, the buy-sell book is made up of the underlying stock and all options every five minutes. The book is composed of the prices and quantities of buy and sell orders. The five-minute period is verified objectively if the distortion of volatilities justifies the execution of the order. Based on the buy-sell book, we use the bid prices whenever selling a financial instrument and ask prices whenever buying<sup>5</sup>. We also use the bid and ask prices in the calculation of implied volatilities and Greek deltas and gammas.

The B-S model is the standard options pricing model among market professionals and information systems. Once all the variables for pricing are known, except for volatility, the model provides a simple way to link the volatility of the underlying stock to its option price. Thus, when using the B-S formula, the market, instead of negotiating the option directly based on price, often trades in terms of volatility.

However, professional traders do not use the model as it was originally developed. If the trader thinks the model is correct, the volatility used would be the same for all options with the same underlying stock and maturity (Rubinstein, 1994). What is observed in the market is that the implied volatility is different for options with different strike prices, which is called volatility skew or volatility smile<sup>6</sup>. Figure 1 shows an example of volatility asymmetry in the Brazilian options market to buy Petrobras PN on January 26, 2016.

= Insert Figure 1 about here =

There is an extensive literature on the systematic biases associated with the B-S model: the underpricing of out-of-the-money options (Black, 1975; Gultekin et al, 1982), the underpricing of low volatility stock options (Gultekin et al, 1982; Whaley, 1982), the sub-pricing of options with short time to maturity (Black, 1975; Whaley, 1982), and the convex shape of the graph of implied volatility curve in relation to the exercise price (Rubinstein, 1985; Sheikh, 1991; Derman & Kani, 1994; Duan, 1996).

Because volatility is what sets the price of the option, the adjustment of the volatility parameter is a way to circumvent the shortcomings of B-S model assumptions. The model can be wrong when using a certain volatility, but can generate correct prices if a different volatility is used, even if the B-S assumptions are not satisfied. This other volatility may be different from the real volatility. By varying the volatility in the B-S model with the exercise price, traders are implicitly assigning a non-lognormal

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<sup>4</sup> In April 2013, there was a change in the layout of the file that contains the orders generated by the Brazilian stock exchange, which makes the proper construction of the book impossible (more details in section 2).

<sup>5</sup> ‘Bid’ is the word referring to the highest offer price in the buy book, while ‘ask’ refers to the lowest offer in the sell book.

<sup>6</sup> The better known expression, *volatility smile*, is used when the graph of implied volatility of the formula of B-S against the strike price is shaped like a U (the implied volatility is lower for options *at the money* and gets progressively larger when the option moves *in the money* or *out of the money*. This form is more commonly seen for options on foreign currencies, but is also sometimes found for stock options. In most cases for stock options (mainly in the US market), the implied volatility decreases the higher the exercise price.

distribution to stock prices (Derman & Kani, 1994)<sup>7</sup>. Normally, volatility asymmetry generates an implied distribution probability which assigns a higher probability to the left tail (higher probability for negative outliers) and lower probability to the right tail than the lognormal distribution (the risk of disaster would be higher than as expected by the log-normal distribution)<sup>8</sup>. Furthermore, the implied probability around the mean is greater.

If the options market were efficient, the asymmetry of volatility attributed to options of the same underlying stock and maturity would mean there was no possibility of systematic gains. Therefore, this paper attempts to answer the following question: is there any distortion in the asymmetry of volatility in the market for Petrobras options?

The results are evidence that the Brazilian options market is not efficient in the weak form, since it was possible to earn systematic profits using the delta-gamma neutral strategy. Considering a minimum distance between volatilities of 20%, the 371 day-trade transactions, with an average investment of R\$ 81,000 and average holding time of one hour and thirteen minutes, had an average return of 0.49% – which corresponds to more than 1,600% of the highest interbank interest rate for the period, with 85% of the trades being profitable. For other minimum distances, the result was similar.

There are several papers in the options literature related to this work. Fuchs and Lemgruber (2001) test, with a daily database, whether the options market for Telebras PN (the most liquid options in the Brazilian market in the study period) was efficient. To perform this test, they constantly simulate delta-neutral transactions bought or sold in volatility. The authors conclude that the Telebras options market was not efficient since the strategy showed systematic gains. Barbedo and Lemgruber (2008) state, for these same options, that when the bid-ask spreads are considered, the systematic profits of such trading strategies with delta-neutral portfolios disappear. In these studies, the hedge is made only against 1st order variations of the asset underlying the options; and adjustments to the portfolios are frequent, thus decreasing any income by the difference between the buy price and the sell price. In our work, in making a hedge against 1st and 2nd order variations of the asset, we do not need to make frequent adjustments to the portfolios (Raju, 2012). Another problem of these works is that they do not use intraday data, only closing data.

Still on delta neutral portfolios, Figlewski (1989) addresses certain problems of performing this type of transaction. One of the author's conclusions is that the issue of indivisibility of assets affects the hedge ratio of the portfolio (i.e., ratio of the number of options to shares that make up the portfolio), especially in cases where the underlying asset is expensive or the quantities traded are small. In this paper, this problem also exists and is taken into account, since we are working with the effective buy-sell book.

Ederington and Guan (2001) reject the hypothesis that the implied volatilities are constants using S&P500 futures options. The authors use the closing prices of calls and puts to calculate implied volatilities using the Black model (1976) and assemble delta-

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<sup>7</sup> One B-S assumption is that the stock prices have a log-normal distribution.

<sup>8</sup> See, for example, Gabaix (2012), Gourio (2012) and Wachter (2013) for models that incorporate disaster risk to explain stock price volatility. Seo and Wachter (2015) explain why the implied volatilities for options depend on exercise prices using a pricing model with stochastic disaster risk.

gamma neutral portfolios<sup>9</sup>. The results show that investors can obtain considerable profits if transaction costs are not taken into account; however, maintaining delta neutrality requires frequent adjustment of the portfolio and these costs end up absorbing profits. In this work we minimize this problem by working only with day orders. The authors argue that not all of the composition of the volatility smile should be attributed to shortcomings in the Black model.

Our work contributes to the literature on asset pricing by showing that Petrobras options in the Brazilian market, although extremely liquid, are not being correctly priced. In this sense, the work is similar to Ederington and Guan (2001), but with two major differences. The first is the fact that we use intraday data (prices and volumes) instead of just closing prices, which makes the experiment much closer to reality. The second is the fact that we use only day order transactions, which minimizes the requirement to frequently adjust the portfolio for the hedge to be perfect.

The paper is organized as follows. Section 2 describes the data used, the study methodology, and an example of applications of the strategy. Section 3 presents the results, and in Section 4 we provide some final comments.

## 2 – SAMPLE AND METHODOLOGY

The study uses a database of the stock Petrobras PN (PETR4) and respective options, because of their high liquidity in the study period. The files that make up the database were obtained via FTP (File Transfer Protocol) from BM&FBovespa<sup>10</sup>. We used six different types of files: (1) offer to buy the underlying asset (PETR4); (2) offer to sell the underlying asset; (3) trades of the underlying asset, (4) offers to buy options, (5) offer to sell options and (6) executed trades of options. Among the options, we worked only with calls with the shortest maturity for each date examined date<sup>11,12</sup>.

The files relating to offers have two different layouts, one in use until March 31, 2013, and one in use from April 1, 2013 to date. This change in layout hid the information about orders that had been modified, which is indispensable for carrying out the work<sup>13</sup>. Thus, the study was restricted to information from prior to April 2013. We used as sample data the data from the last six months available, i.e., from October 1, 2012 to March 31, 2013. Figure 2 shows that the period studied shows a volatility close to that of the PETR4 historical average.

= **Insert Figure 2 about here** =

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<sup>9</sup> The Black model is the version of the B-S model used to price futures options.

<sup>10</sup> <ftp://ftp.bmf.com.br/marketdata>

<sup>11</sup> As described in Raju (2012), the transaction is also feasible with sell options (puts); however, these were not included in this study because, in general, they did not have sufficient liquidity in the Brazilian market during the study period.

<sup>12</sup> The options with the shortest maturity are usually those with the highest liquidity.

<sup>13</sup> More specifically, the information that was removed is the "date and time of offer modification" and "number of modified offer."

From the six types of files in the database, we assemble books of the underlying and options to simulate delta-gamma neutral transactions. We could build the book every second, for example. However, our sample would be too broad and we would have to decrease the period studied due to the heavy computer resources needed. Therefore, we opted for a five-minute interval to build bid books. Because the trading session lasts seven hours, each sample day has 83 books (i.e., information on pricing and buy/sell quantities of PETR4 and PETR4 options)<sup>14</sup>.

To compose books, we used the information from orders submitted and trades. The files of the orders sent classify them as 'accepted,' 'deleted,' 'frozen,' 'canceled,' 'fully executed,' 'modified,' 'stop loss,' 'void' and 'rejected'. We used, however, only the orders classified as "accepted", "fully executed," and "modified" for the construction of the sell books of the underlying asset and its options. Another important point concerns the validity of the order. Our study only looked at day orders, i.e., orders that could remain open for more than one day were disregarded<sup>15</sup>.

Most orders sent to the exchange are modified (usually several times). Thus, they are considered based on their initial characteristics (asset, price and quantity) only until the time of their adjustment. Thereafter, the order based on the initial aspects ceases to exist, being replaced by the new order (be it with a different price or a different quantity).

The trading files also include three parameters: 'traded,' 'offer voided,' and "void parameter." Of these, only the records classified as "traded" influenced the construction of the offer books.

Having, every five minutes, information about the best bids and asks for PETR4 and all of its calls for the shortest maturity (and their respective amounts), is necessary to calculate two implied volatilities for each option — one for the bid price and one for the ask — also every five minutes. To calculate these volatilities using the B-S model, the following are needed: the values of bids and asks, the strike price, the time to maturity, the day's interbank interest rate (taken from the CETIP website)<sup>16,17</sup> and the price of the underlying asset (considered as the average value between its bid and ask at the exact moment in which the implied volatility of the option is calculated).

To assess whether the transaction should be completed, all that needs to be done is to calculate the EWMA volatility<sup>18</sup>. Therefore, the daily closing price time series of the underlying asset are used, adjusted for dividends and splits, up to the day before the one on which the transaction is to be conducted. The last value to compose the series is

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<sup>14</sup> The first book of the day is done five minutes after the start of trading, the last, five minutes before the closing bell.

<sup>15</sup> These simplifications only bring more restrictions to the transaction, i.e., less likelihood of the gamma-delta neutral strategy being successful.

<sup>16</sup> [www.cetip.com.br](http://www.cetip.com.br)

<sup>17</sup> The rate that should be used as the risk-free interest rate in a pricing model is the fixed rate to maturity of the option. However, because the difference is small (since the options are short-term), there is no harm in using the interbank rate.

<sup>18</sup> We chose the EWMA model to be the central volatility of this strategy because it is a volatility estimation methodology widely used in the financial market. Furthermore, comparison of EWMA using daily returns with implied volatility is simple, because to calculate the volatility for a period  $t$  all that is necessary is to multiply the EWMA volatility by  $\sqrt{t}$ . The EWMA used in this work is the standard one, with decay parameter equal to 0.94.

the average price between the bid and ask at the evaluated time. The value of the exponential decay parameter of the model is 0.94, as suggested, for daily data, in the RiskMetrics™ (1996) risk manual.

With this information (implied volatilities of the underlying asset for options calculated using the bid and ask and the EWMA volatility of the underlying asset), the analysis as to whether to enter into the transaction can be carried out. The position is bought if, simultaneously, a) implied volatility of an option calculated using its ask price (VIMP1) is at least X% less than the estimated EWMA volatility (EWMA) and; b) the implied volatility of the other option calculated based on the bid price (VIMP2) is at least X% higher than the estimated EWMA volatility<sup>19</sup>. The distance X is arbitrary. Therefore, to minimize this variable selection bias, the study was carried out with X assuming values 20% to 50%, with intervals of 5 percentage points<sup>20</sup>. After the position goes live, the difference between VIMP2 and VIMP1 continues to be calculated. This number is the basis for closing out the position<sup>21</sup>.

From there, we already know what options will be traded; however, the quantities involved in the structure still have to be set. To do so, we calculate, using the B-S formula, the delta and the gamma of the options (treating the price of the underlying asset as the average between the bid and ask price at the specific time and treating the volatility of the underlying asset as the implied volatility of the ask price if the option is bought, or bid price, if the option is sold). The quantities of options are such that the resulting portfolio gamma is equal to zero. However, despite the zeroing of the gamma, these quantities leave a residual delta value. In order to obtain zero delta for the structure, a specific quantity of the underlying asset is also traded. The strategy, therefore, has the following formula (for each call 1 bought):

$$C_1 + \left( \frac{\Gamma_1}{\Gamma_2} \Delta_2 - \Delta_1 \right) S - \frac{\Gamma_1}{\Gamma_2} C_2$$

All trades are day-orders, i.e. the order is opened and closed on the same day. Thus, only when there is an extremely large intra-day volatility is there a need to rebalance the portfolio, since, in addition to the delta being neutral, the gamma is also zero.

The position is closed in two cases:

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<sup>19</sup> The Implied Volatility 1 (VIMP1) is calculated with the *ask* price of the option, since it will be bought, while the Implied Volatility 2 (VIMP2) is calculated on the *bid* price of the option, since this option will be sold. The sequence to carry out the strategy is always  $VIMP1 < EWMA < VIMP2$ .

<sup>20</sup> The distance used between the volatilities to operationalize the transaction should not be too low, since the market already assigns different implied volatilities to the underlying asset for different option strikes in order to resolve the distortions generated by the assumptions of the B-S model. Thus, if the distance between the volatilities is small, there is less likelihood that the option prices really do reflect distortions.

<sup>21</sup> We could also implement the position without using a "central" volatility (in this study, EWMA volatility), i.e., taking the position when the distance between the implied volatilities is greater than a certain limit.

- 1) If the position garners a positive result, with the difference between VIMP2 and VIMP1 representing less than two-thirds of the difference calculated at the moment of opening of the transaction; or
- 2) If the day ends without the first criterion obtaining, the transaction is closed out at 16:55, the last time considered in this study, regardless of the outcome.

Finally, there are two scenarios to calculate the percentage profit of the transaction. If it does not require margin collateral (i.e., the investor necessarily making a disbursement to open the position), the profit percentage is simply the amount invested less the amount received, divided by the amount invested, as in any conventional transaction. On the other hand, if the position requires a cash outflow, it will require the investor to put up margin collateral, calculated for both the options as well as the underlying asset. For options, the margin is equal to 100% of the risk of the uncovered derivatives sold<sup>22</sup>; and, for the underlying asset, the margin was set as 130% of the uncovered derivatives sold<sup>23</sup>. In this case, we consider that the initial investment is this margin subtracted from the initial cash position. Thus, the percentage profit of the structure is the gain (amount received initially less the amount paid to exit the position) divided by the initial investment.

To illustrate investor behavior, consider the following real example. At 14:50:00 on October 1, 2012, it was observed that the PETR4 call with an exercise (strike) price of R\$ 23.50 for the shortest maturity (Option 1) had, according to the B-S model, an implied volatility of 27.47% (calculated based on ask) while, simultaneously, the call exercise price of R\$ 20.00 for the same maturity (Option 2) had an implied volatility of 58.08% (calculated on the bid). Because, at that instant, the EWMA volatility of the underlying asset was estimated at 44.89%, the time was ideal to open the position if we consider the distance between the volatilities (X, in our notation) equal to 20% or 25%, since the implied volatility of the option bought is 39% less than the EWMA volatility of the underlying asset and the implied volatility of the option sold is 29% higher than the same EWMA volatility.

The difference between the implied volatilities of the two options is 30.61 percentage points. The delta and gamma values of both options were calculated to mount the options strategy, taking into account the implied volatility in the ask price for the option to be bought and the implied volatility in the bid price of the option to be sold. From there, the exact quantities are transacted in order to achieve a portfolio gamma of zero, i.e., buy up 498 Options 1 (at a price of R\$ 0.21 per contract) and sell 1,754 Options 2 (at R\$ 2.89 each). These quantities, despite the overall zero gamma, made the portfolio delta negative. To achieve a delta of zero for the structure, 1,400 PETR4 are bought at the price of R\$ 22.67. Thus, the portfolio was put together at a cost of R\$ 26,773, according to Table 1.

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<sup>22</sup> For example, consider a position taken by buying 100 options with a strike price of 10, selling 300 options at a strike price of 12 and buying 100 of the underlying asset. In this transaction, for 300 options sold, 100 are covered with the underlying asset and 100 are covered with the options bought (since these options have an exercise price that is below the price of the options sold). Thus, the value of the margin in this case is: 100 (number of options sold uncovered) x 12 (exercise price of these options) = R \$ 1,200.

<sup>23</sup> Margin required by CBLC (Brazilian Clearing and Depository Corporation) for the position during the study period.



= **Insert Table 1 about here** =

Later, at 16:45:00 the same day, Option 1 had a bid price equal to R\$ 0.19, with an implied volatility 28.64%, Option 2 had an ask of R\$ 2.71 with an implied volatility 49.03%, while the best bid for PETR4 was R\$ 22.52. Thus, the distance between the new implied volatilities was 20.38 percentage points, which represents 66.57% of the initial distance, thus indicating a possibility of closing out the position, as shown in Table 2.

= **Insert Figure 2 about here** =

At these prices, the investor received R\$ 26,869, representing a profit of R\$ 96 profit, or 0.36% of the initial investment, in a position that lasted an hour and fifty-five minutes.

An important point to note relates to transaction costs. An important transaction cost, the bid-ask-spread, is being implicitly observed when we use the prices listed in the order book, instead using actual trading prices (as in most of the work on this subject). Other costs, such as the brokerage and exchange fees, were not considered in this example, or in the results of the paper. This is because, firstly, if we use the strategy exactly as used in the experiment, it would be preferably carried out through proprietary trading (where there would be only small day-trade transactions costs charged by the exchange), so that the brokerage does not absorb much of the profits. Other market participants who could employ this strategy are institutional investors, such as, for example, investment funds, which have bargaining power with the brokers. In the latter case, a negotiation between the investor and the broker is to be expected so that the broker covers fees that would otherwise render the transaction nonviable (often a percentage of profits or a fixed monthly cost). We chose not to include these transaction costs, since there is no consensus on the exact amounts practiced.

### 3 - RESULTS

As mentioned in the previous section, the options position is only taken if the implied volatilities of two options of the same maturity simultaneously have a distance greater than X% of the EWMA volatility calculated for the underlying asset (one upper and one lower). Because X is an arbitrary measure, we consider different values for this variable. The results section is divided into three parts; the first with overall results, considering the various values of X; the second with the details of the least restrictive transaction, i.e., with the variable X assuming the value of 20%; and the third with a strategy simulation using rounded quantities. By using the rounded quantities, despite the hedge becoming imperfect, the structure has the exact characteristics of an actual transaction.

### 3.1 Overall Results

Table 3 shows the overall results for each minimum distance between the volatilities for implementation of the strategy (X). As can be seen, there are almost 15 times more transactions when X is 20% (average 3.23 transactions per day) than when X is 50%. The transaction proved to be successful regardless of the value assigned to X: the lowest percentage of transactions with a profit is 66%, but the median of all options strategies with a profit for all X is 80%. Transactions with outflow of cash are the vast majority, with a median of 90% of transactions for all X.

Differently from what one might expect, the percentage of profitable transactions did not increase with the increase of X; that is, the investor being more restrictive to enter the transaction did not result in higher precision. This is because, in general, in order for the options to have an implied volatility as distant from EWMA volatility, the options need have low liquidity. Thus, even if the investor enters into the transaction, they may find it difficult to exit and face a high price to do so. Because the study is based only on day orders, the investor may have little time to close out their position, ultimately, being forced liquidate at the closing bell, regardless of the result.

= **Insert Tables 3 and 4 about here** =

When comparing the performance of the transactions separately, the positions bought (cash outflow) apparently are more successful than those sold (cash inflow). However, the present study cannot prove this, since there are very few sell transactions. Furthermore, Table 4 shows that, to the extent that the distance restriction between the volatilities is decreased (smaller values of X), the rate of success of sell transactions increases, until it approaches the profit levels achieved by bids.

### 3.2 Details of the transaction with less restriction on minimum distance between the volatilities (X equal to 20%)

As noted in Section 3.1, the transaction showed positive results regardless of the value assigned to the minimum distance restriction between the volatilities (variable X). In this section, the details of the results are presented only for a distance of 20% between implied volatilities and the base volatility (EWMA), simply because it is less restrictive and, therefore, provides a greater amount of information for analysis.

As shown in Table 5, there were 371 day-trade transactions, of which 85% were profitable. More than 92% of the transactions were made with cash outflow. The average ticket<sup>24</sup> value of the transactions (given the maximum availability on the books of the options and underlying stock), considering the absolute opening value (independent, therefore, of the transaction having been opened with cash inflow or outflow), was almost R\$ 81,000.

The average duration of a position was a little less than one hour and thirteen minutes, ranging from the shortest, of only five minutes, to the longest, of five hours

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<sup>24</sup> Name commonly used in the financial market for the average value of a trade.

and ten minutes. The average result of all transactions was a gain of 0.49%, while the median was 0.32%. For purposes of comparison, the highest daily CDI during the study period was 0.028% (i.e., the average profit is more than 1600% greater than the overnight interest rate in the period. The highest return of the series was 7.67%; the largest loss was 6.71%; and the standard deviation of returns, which presents a measure of risk for the strategy, was 1.08%.

= **Insert Table 5 about here** =

Figure 3 shows the histogram of the results of the transactions. It can be seen that the upper tail is thicker than the lower, as already suggested by comparison of the mean and median. In addition, note that there are many more above average outliers (6% of observations garnered a result of greater than 2%) than below average outliers (only 1% of the transactions accrued losses greater than 2%).

Another important point concerns the dispersion of profits and losses. While the vast majority of the losses is concentrated close to the 0% return (up to 0.25% negative return), the profits are more distributed, with 33% of the observations between 0% and 0.25% profit; 32% between 0.25% and 0.75% profit; and 22% of the observations with profits greater than 0.75%.

As noted earlier, the only transaction cost observed in the calculation of the results is the bid-ask-spread. Because the day-trade exchange and broker fees are small relative to the average profit of 0.49%, or almost R\$ 400,00 in absolute amounts, we believe that these fees do not invalidate the findings of the study.

= **Insert Figure 3 about here** =

### 3.3 Simulation of delta-gamma neutral strategies with quantities rounded for value multiples of 100

Despite all the above results being based on strictly true trade values to buy and sell PETR4 and its options, the fact of not rounding the quantities to maintain the perfect hedge (i.e., such that the structure have no delta or gamma residual), makes the transaction feasible only in theoretical terms. Therefore, this section is dedicated to presenting a variation in the methodology so that the structure can actually be put into practice.

The quantities of the transactions were rounded to value multiples of 100, since this value is the standard lot for both options as well as the underlying asset. However, as evidenced in Figlewski (1989), the volume of the transaction is too small, the indivisibility of the assets in the portfolio may materially affect the ratio of the hedge. To try to minimize this effect, we consider, for transactions with rounded amounts, only structures opening with a minimum of R\$ 80,000.

The results show that, considering this cutoff in the sample, the fact of rounding quantities, thereby rendering the hedge not exactly perfect, does not result in any great loss of quality in the transactions. According to Table 6, 119 transactions were made, 84% of which turned a profit. Note that there are no transactions with cash inflows. The average ticket was greater than R\$ 207,000. The mean duration of positions was slightly more than one hour eleven minutes, the shortest being only five minutes and the longest,

five hours and ten minutes. The average yield was 0.40%; The highest profit was 3.36% and the greatest loss, just 0.25%. The standard deviation of returns was 0.55%.

**= Insert Table 6 about here =**

A histogram of the dispersion of returns is shown in Figure 4. Similar to the histogram of returns with “broken” amounts, the graph of rounded amounts also shows a positive asymmetry. All of the losses presented (12% of observations) are between -0.25% and 0%, whereas the profits are much more diversified: 42% of observations are between 0% and 0.25%; 30% are between 0.25% and 0.75%; and 16% of positive results reflect a profit exceeding 0.75%.

**= Insert Figure 4 about here =**

#### 4 – FINAL COMMENTS

In this work, we see the market for Petrobras options as not efficient in the weak form using delta-gamma neutral strategies. Using only day-trade transactions, attempts were made to obtain systematic profits by observing distortions in implied volatilities derived from options, while hedging against changes in the underlying asset. Because the transactions are short term and the hedge in assets is 2nd order, there is no need to rebalance the portfolio frequently.

In order to simulate the strategy exactly as it would be used in reality, we built the books of bids for options and the underlying asset every five minutes. The results show that of the 371 transactions, the average return was 0.49%, which corresponds to more than 1,600% of the highest interbank interest rate over the period studied. These positions involve an average investment of R\$ 81,000 and average duration of one hour and thirteen minutes. Of all the transactions, 85% were profitable. Thus, we conclude, based on evidence, that the market for Petrobras options is not efficient, since it was possible to derive systematic profit from the trading strategy.

It is important to note that we did not contemplate in the returns of the positions any transaction costs relating to fees charged by a broker or the Brazilian stock market, which could absorb a share of the profits. However, because this investment can be handled by proprietary traders and does not require much cash (i.e., there is not much restriction on availability), we believe that this fact does not invalidate the findings.

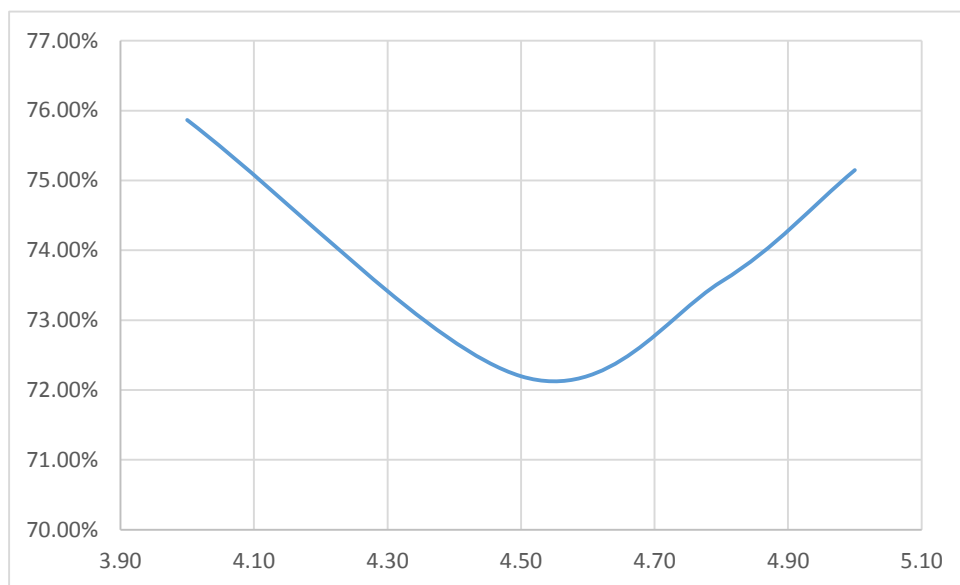
Finally, we note that other strategies could be carried out to confirm that the Petrobras options market is not efficient in the weak form. For example, one strategy that could be the continuation of this work would be to use the delta-gamma neutral itself, but combined put and call options or only with put options, given that in recent years these Petrobras options have gained substantial liquidity.

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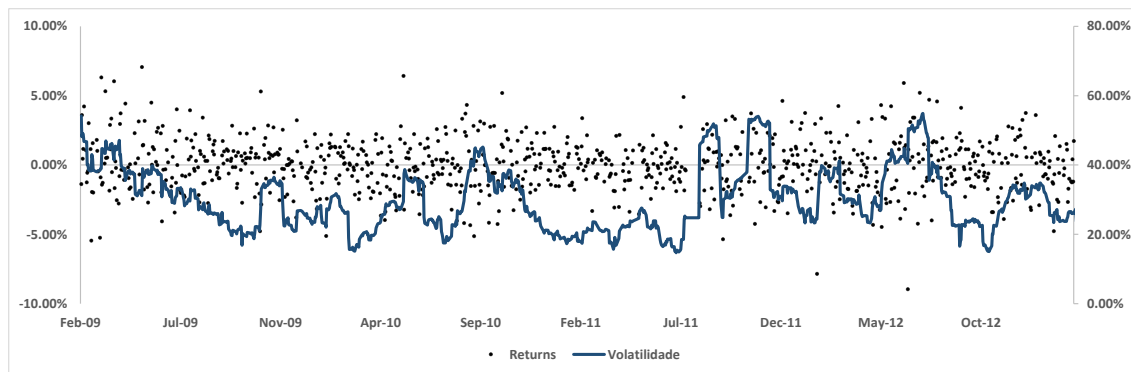
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**FIGURE 1** Asymmetry of volatility of calls of PETR4 at 14:30 on Jan. 26, 2016



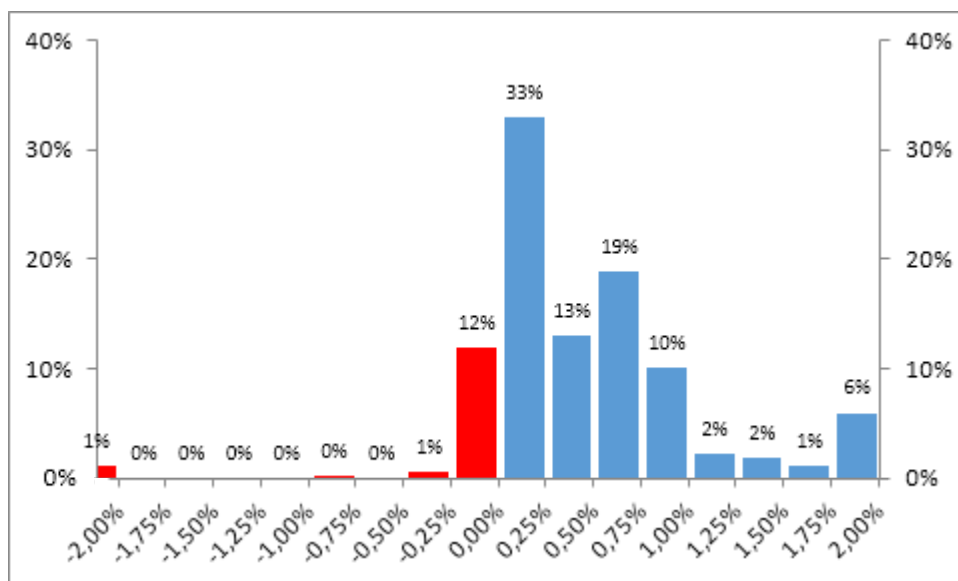
Note: The options used are PETRB21, PETRB55, PETRB26 and PETRB62, the most actively traded options in the day. Implied volatilities were calculated using the B-S model With the average price of offers to buy and sell options and the underlying asset (PETR4). The implied volatilities are plotting on the axis and the exercise prices along the abscissa

**FIGURE 2** PETR4 returns and volatility series for February 2009 to March 2013



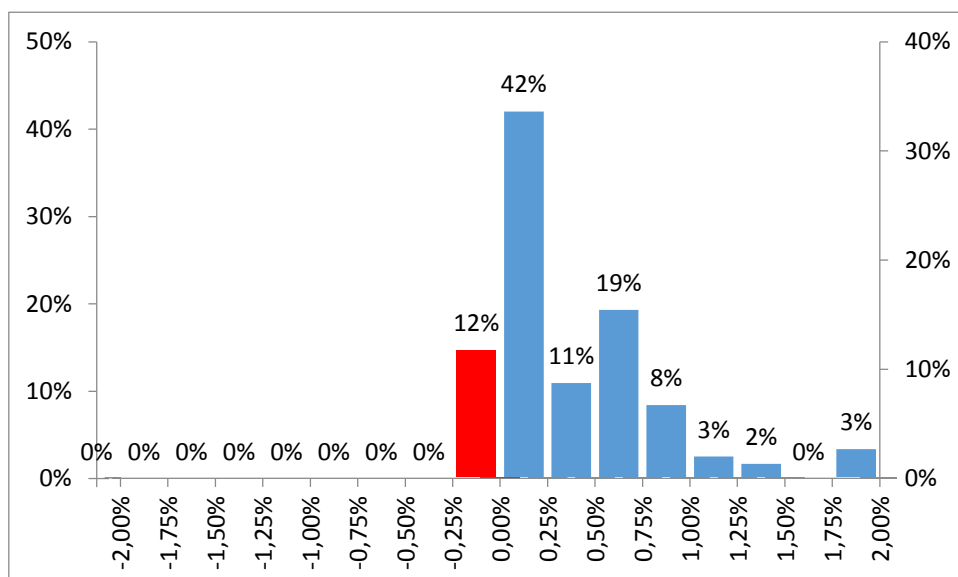
Note: The main axis of ordinates refers to PETR4 daily returns; the secondary axis reflects the annualized volatility of the stock, calculated as the standard deviation of the sampling of the last 21 daily closing returns, multiplied by the square root of 252. The shaded area represents the period considered in this work.

**FIGURE 3** Histogram of returns of delta-gamma-neutral strategies — minimum distance between the implied volatility and the base = 20



Note: The main axis of ordinates refers to PETR4 daily returns; the secondary axis reflects the annualized volatility of the stock, calculated as the standard deviation of the sampling of the last 21 daily closing returns, multiplied by the square root of 252. The shaded area represents the period considered in this work.

**FIGURE 4** Histogram of returns of the delta-gamma neutral positions with amounts rounded to multiples of 100 — minimum distance between the implied volatilities and the base = 20%



Note: On the horizontal axis are the bands of returns of the strategies, and on the ordinates are the percentage returns for each band.



**TABLE 1** Example of the gamma-delta neutral strategy with Petrobras PN (PETR4) options:  
Position entered at 14:50 on October 1, 2012

Time	Asset	Strike	Volatility	Price	Quantity
14:50:00	PETR4		44.89%	R\$ 22,67	1400
14:50:00	OPTION 1	R\$ 23,50	27.47%	R\$ 0,21	498
14:50:00	OPTION 2	R\$ 20,00	58.08%	R\$ 2,89	-1754

**TABLE 2** Example of gamma-delta neutral strategy with Petrobras PN (PETR4) options:  
Closure of Position at 16:45 on October 1, 2012

Time	Asset	Strike	Volatility	Price	Quantity
16:45:00	PETR4	-	-	R\$ 22,52	1400
16:45:00	OPTION 1	R\$ 23,50	27.47%	R\$ 0,19	498
14:45:00	OPTION 2	R\$ 20,00	58.08%	R\$ 2,71	-1754

**TABLE 3** Results for delta-gamma neutral positions by minimum distance among the volatilities to enter into strategy (X) — number of transactions; percentage of profitable operations; average number of daily transactions; and percentage of transactions with cash outflow

X	Number of transactions	Percentage profitable transactions	Average transactions per day	Percentage of transactions with cash outflow
50%	15	80%	0.13	93%
45%	35	66%	0.30	80%
40%	63	79%	0.55	84%
35%	97	79%	0.84	89%
30%	152	81%	1.32	90%
25%	249	84%	2.17	90%
20%	371	85%	3.23	92%

**TABLE 4** Results of the gamma-delta neutral options strategies with cash outflow and inflow by minimum distance between the volatilities for the executions of the strategy (X) — total number of transactions and percentage of profitable transactions

X	Transactions with cash outflow		Transactions with cash inflow	
	Total transactions	Percentage of profitable transactions	Number of transactions	Percentage of profitable transactions
50%	14	80%	1	100%
45%	28	66%	7	43%
40%	53	79%	10	60%
35%	86	79%	11	64%
30%	137	81%	15	60%
25%	225	84%	24	67%
20%	342	85%	29	72%

**TABLE 5** Descriptive statistics of the results

Statistics	
Number of transactions	371
Transactions with profit	315 (85%)
Transactions will loss	56 (15%)
Transactions with pos. cash flow	29 (8%)
Operations with neg. cash flow	342 (92%)
Average ticket	R\$ 80.980
Ave. transaction time	01:12:55
Min. transaction time	05:10:00
Max. transaction time	00:05:00
Average profitability	0.49%
Median profitability	0.32%
Max. return	7.67%
Min. return	-6.71%
1st Quartile	0.07%
3rd Quartile	0.68%
Standard Deviation	1.08%

**TABLE 6** Descriptive statistics of the results of the structure, with rounded quantities

Statistics	
Number of transactions	119
Transactions with profit	100 (84%)
Transactions will loss	19 (15%)
Transactions with pos. cash flow	0 (0%)
Operations with neg. cash flow	119 (100%)
Average ticket	R\$ 207.089
Ave. transaction time	01:11:16
Min. transaction time	05:10:00
Max. transaction time	00:05:00
Average profitability	0.40%
Median profitability	0.20%
Max. return	3.36%
Min. return	-0.25%
1st Quartile	0.04%
3rd Quartile	0.63%
Standard Deviation	0.55%