

HETEROGENEOUS EXPECTATIONS IN GENERAL EQUILIBRIUM MODEL WITH COLLATERAL AND MONEY

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Resumo:

Este artigo analisa o impacto da política monetária não-convencional aplicada pelo Banco Central americano durante a crise de 2008-2010 em economias com agentes otimistas/pessimistas. Os resultados numéricos insinuam que agentes otimistas tendem a ser ativos na restrição do tipo *leverage* e os pessimistas tendem a ser ativos na restrição do tipo *short sale*. Além disto, a presença de otimistas potencializa os efeitos da política não-convencional aumentando a região de melhoria de Pareto.

Palavras-chave: equilíbrio geral, otimismo, política monetária não-convencional

Abstract:

This paper analyzes the impact of the unconventional monetary policy applied by the US Federal Reserve during the 2008-2010 crisis in economies with optimistic/pessimistic agents. The numerical results suggests that optimistic agents tend to be leverage constrained and pessimistics tend to be short sale constrained. Furthermore, the presence of optimistic agents potentiates the effects of the unconventional monetary policy by increasing the region of Pareto improvement.

Keywords: general equilibrium, optimism, unconventional monetary policy

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1 Introduction

The last major economic crisis of 2008-2010 has boosted new efforts in the economic literature in order to understand its causes and consequences, but it especially driven interest in new ways of regulation to prevent crisis or to mitigate its effects. Indeed, since the usual regulatory means, such as the standard interest rate, showed ineffective during the crisis, the authorities ventured with new policies whose effects were initially unknown, such as the Troubled Asset Relief Program (TARP) implemented by the US Treasury.

In recent literature, Geanakoplos (2009) analyzed the role of optimistic agents in a general equilibrium model with collateral and two or three periods, modeling crisis as the occurrence of a bad state in the subsequent period and associating the behavior of the agents in a crisis cycle with the level of leverage of the assets. A quotation of this paper is presented below:

“To reverse the crash once it has happened requires reversing the three causes. [...] Third, the lost buying power of the bankrupt leveraged optimists must be replaced. This might entail bailing out crucial players or injecting optimistic capital into the financial system.” (Geanakoplos, 2009,p. 4)

The study of optimism in a context of crisis is still of interest in literature, as the recent unpublished work of Tsomocos and Yan (2016), in which a variant of Geanakoplos’ model with three period and bayesian update is analyzed, shows. Finally, the work of Araujo et al. (2015) presents a thorough analysis of the unconventional monetary policy used by the US Federal Reserve during the crisis. This policy involved the direct purchase of durable assets by the Central Bank in order to complement the usual monetary policy. The authors used a model in the same framework of Geanakoplos but included money and the unconventional regulation. However, all the analysis is done in economies with agents with homogeneous beliefs.

In this paper the concept of optimism of Geanakoplos is combined with new regulatory policy analyzed by Araujo et al. (2015) in order to give an overview of the effect of this new policy in economies with heterogeneous beliefs. Thus, the basic methodology adopted here is to use the Araujo et al. (2015) model as basic landmark, introduce heterogeneous belief to the agents, produce numerical examples and then analyze the results.

The aim is to assess the impact, effects and relations between the unconventional monetary policy and different expectations of the agents. The analysis shows that the collateral constraint of the agents are key to understand the relation of the beliefs and the unconventional policy. In terms of transference of wealth to the states of the next period, the collateral constraint is divided into two constraints: short sale constraint and leverage constraint. The examples numerically implemented suggests that the optimistic agents tend to be leverage constrained and the pessimistic short sale constrained. Also, the presence of optimistic agents in the economy *potentiates* the good effects of the unconventional monetary policy by increasing the Pareto improvement region.

The remainder of this work is organized as follows: section 2 presents the general equilibrium model with collateral, money and unconventional monetary policy with its main theoretical properties; section 3 presents the numerical results and finally section 4 concludes. The appendix shows the equations and inequations used to compute equilibrium.

2 Model

2.1 Unconventional Monetary Policy

The main theoretical structure of the model in Araujo et al. (2015) is a general equilibrium model with financial markets and collateral from Geanakoplos and Zame (2014). The main novelty of their model, in comparison with the standard and well known general equilibrium model with incomplete markets, is the collateral associated with each financial contract. The intuition is that the institutions behind the financial contracts, like the legal enforcement structure, impose that the agents should back their promises with a collateral when issuing an asset. In the general equilibrium with incomplete markets model the promises made by the agents when issuing an asset should be fulfilled. In this case, those institutions and the law enforcement are supposed to perfectly oblige all agents to deliver exactly what was promised. In the collateral model this unreal hypothesis of perfect law enforcement is relaxed and the agents are allowed to default. If a promise is made today, tomorrow the agent is allowed to choose what is better to deliver: the promise made or the collateral. The agent always chooses to deliver what is worth least, so the real delivery of the asset in the next period is the minimum between the value of what was promised and the value of the collateral used to back it. Conceptually, this is a literature of lack of commitment or commitment limited by the collateral.

The process of backing the promises with a collateral imposes a friction in the financial markets, which is translated into the model as a new constraint in the household's budget set, called collateral constraint. It says that the agent must hold at least the amount of collateral associated with all assets issued by him in the first period. This new constraint has some intuitive implications on the inefficiency of the equilibrium. Indeed, in a general equilibrium with incomplete markets model some desirable allocations of the economy may demand a transfer of wealth not allowed by the financial structure, i.e., an allocation outside the span of the return of the assets. In the presence of collateralized promises, the scarcity of the collateral may impose additional restriction on the possible allocations on the economy in the sense that some allocations *inside* the span may not be reachable due to a high amount of collateral demanded to back the corresponding financial position.

Formally, the basic framework is the general equilibrium with collateral *and* money. It has two periods with one state in the first period and $S \in \mathbb{N}$ states in the second period. The symbol $S^* = S + 1$ will denote all states in the economy. There are $H \in \mathbb{N}$ agents, $L \in \mathbb{N}$ goods and $J \in \mathbb{N}$ assets in the economy. The preferences of the agents are incorporated in their utility functions $u^h(\cdot)$.

In the analysis developed here, as well as in Araujo et al. (2015), the amount of good will be restricted to three, being goods 1 and 2 perishable and good 3 durable. The agents obtain utility only from goods 1 and 2. The durable is used either as a collateral to the assets issued or to enjoy its service. In this sense, the durable good in this economy acts like a risky asset, paying its value $p_{s3}x_{03}$ in each state of the second period, which typically will be different in each state. Each

household face the following maximization problem.¹

$$\begin{aligned}
& \max_{x^h, \psi^h, \varphi^h, \mu^h, x_3^h \geq 0} u^h(x^h) \\
& \text{s.t.} \\
& p_0 \cdot (x_0^h - e_0^h) - p_{02}x_{03}^h + q \cdot (\psi^h - \varphi^h) + \mu^h - m^h \leq 0 \\
& \sum_{l=1}^2 p_{sl}(x_{sl}^h - e_{sl}^h) + p_{s3}(x_{s3}^h - e_{s3}^h - x_{03}^h) - p_{s2}x_{s3}^h - \sum_j (\psi_j^h - \varphi_j^h) \cdot \min\{1, p_{s3}C_j\} + \\
& + \theta^h(1+i)[(p_{03} - p_{02})\omega e_{03} + m] - (1+i)\mu^h \leq 0 \quad \forall s = 1, \dots, S \\
& x_0^h - \sum_j \varphi_j^h C_j \geq -\delta^h
\end{aligned}$$

where $e_{0,3} = \sum_h e_{0,3}^h$ and $m = \sum_h m^h$.²

The good one is an ordinary perishable good, such as food, and the good 2 is also perishable but in a special way, it is the service of a durable good, represented by good 3. So, in this model the durable is split into two goods, one is perishable and gives utility to the agents and the other one is the durable itself, which carries value over time but yields no utility to the agents. An example would be a house. The owner of a house can rent it to some other person. The agent actually enjoying the house and taking utility from it is the one paying the rent, not necessarily the owner. Even if one lives in its own house, it is theoretically possible to split this relation making him pay a rent to himself. That is why the term $-p_{s2}x_{s3}^h$, $s = 1, \dots, S$, appears in the budget set. Once the agent chooses the amount of durable x_{s3}^h in state s , he will be able to use this durable to rent and therefore receive the value $-p_{s2}x_{s3}^h$.

A reasonable hypothesis on the endowments is $e_{s2}^h = 0$ for all s and h , because an endowment of service would be economically meaningless. Furthermore, in this model there will be no endowment of durable in the first period, that is, $e_{s3}^h = 0$ for $s = 1, 2$ and for all h . The durable in the first period will be only those carried from $s = 0$. From the first order conditions (FOC) it is true that $p_{s2} = p_{s3}$ for $s = 1, 2$ in equilibrium.³ Thus, the problem faced by the agent can be written in a

¹The initial model presented here is slightly different from the one presented in Araujo et al. (2015) because in their paper they chose to ignore the variables x_{03}^h , x_{13}^h and x_{23}^h , even in the definition of equilibrium, because they are not actually important for the equilibrium since they give no utility for the agents. Indeed, in this model there is multiplicity of equilibria because the amount of durable held by each agent can change and still be compatible with the same equilibrium, that is, the same equilibrium prices and the same amount of x^h that affects the agent's utility. The x_{s3}^h acts much more as another asset, just like assets j .

²Note about the notation: the symbol \cdot represents the inner product and the subindex of the variables refers to states and goods (x_{sl}^h, p_{sl}) .

³In the next paragraphs the differentiability hypothesis of $u^h(\cdot)$ will be stated, as well as the strict convexity and strong monotonicity. In this setting, the first order condition of x_{s3}^h in the first period ($s = 1, 2$) imply that $\mu_s^h(p_{s3} - p_{s2}) = 0$ for all h , all $s \in S$. Since $\mu_s^h > 0$, then $p_{s3} = p_{s2}$. Recall that μ_s^h is the marginal variation of the value function $v^h(c)$ of the agent's optimization problem (indirect utility function) due to a marginal relaxation of the s budget constraint. In other words, $\mu_s^h \in \partial v^h(c)$, where ∂ is the supergradient of value function $v^h(c)$ and c is a vector of constants in the constraints. Since $u^h(\cdot)$ is strongly monotone, a collateral relaxation of the s budget constraint would lead to an increase in $v(c)$, hence $\mu_s^h > 0$.

more simple way:

$$\begin{aligned}
& \max_{x^h, \psi^h, \varphi^h, \mu^h, x_3^h \geq 0} u^h(x^h) \\
& \text{s.t.} \\
& p_{01}(x_{01}^h - e_{01}^h) + p_{02}(x_{02}^h - x_{03}^h) + p_{03}(x_{03}^h - e_{03}^h) + q \cdot (\psi^h - \varphi^h) + \mu^h - m^h \leq 0 \\
& p_{s1}(x_{s1}^h - e_{s1}^h) + p_{s2}(x_{s2}^h - x_{03}^h) - \sum_j (\psi_j^h - \varphi_j^h) \min\{1, p_{s3}C_j\} + \\
& + \theta^h(1+i)[(p_{03} - p_{02})\omega e_{03} + m] - \theta^h p_{s3}\omega e_{03} - (1+i)\mu^h \leq 0 \\
& x_0^h \geq \sum_j \varphi_j^h C_j
\end{aligned}$$

One of the distinctive feature of this model is the presence of an equivalent of money, which can be thought as a risk-free bond paying $1 + i$ non-contingent in the second period for each unit of bond hold. At state zero the agents have an endowment m^h of money and chooses μ^h . Any agent holding μ^h units of money at $s = 0$ will receive $(1 + i)\mu^h$ at any state s of period 1. The Central Bank is present in the model, not as a maximizer agent, but as an accounting equality that must hold. When he does the non-conventional monetary policy he buys an amount $\omega e_{0,3}$ of durable at the price $p_{03} - p_{02}$ and therefore increases the amount of money in a total of $(p_{03} - p_{02})\omega e_{03}$. The term ω represents the fraction of durable bought by the Central Bank (CB). Since the CB cannot buy more than the aggregate amount of durable, ω should be between 0 and 1. Hence, the total amount of money in the economy is $M = m + (p_{03} - p_{02})\omega e_{03}$, composed by the total endowment of money m plus the money issued by the CB. Note that $p_{03} - p_{02}$ is the price of the durable out of the rental income that will be received by the CB while he owns this good. All this money should be redeemed by the CB at period 1 to clean the economy. This operation may result in loss or profit to the CB, but it will be anyway distributed across the households as lump-sum transfers with proportions θ^h . In period 1 the amount M of money will value $(i + 1)M$ and the amount ωe_{03} of durable bought by CB must be returned to the market for the value $p_{s2}\omega e_{03}$ (recall that $p_{s2} = p_{s3}$). Thus, the net lump-sum tax obligation of agent h in period 1 should be $\theta^h[(1 + i)M - p_{s2}\omega e_{03}]$ in state s . The non-conventional monetary policy can therefore be summed up in setting/changing the parameter ω in view of desired objectives.

The financial market of this economy follows the standard general equilibrium model with incomplete markets and collateral. The last inequality is the collateral constraint. In this model, the financial assets are nominal, that is, asset j promises to deliver the one unit non-contingent and demands the issuer a collateral $C_j \in \mathbb{R}^L$ to back it. The actual delivery of asset j in state $s = 1, \dots, S$, however, will be $\min\{1, p_s C_j\}$. When the agents delivers $p_{s,3}C_j$ in state s it is said that they default.⁴ Therefore, the assets differ from each other only in the amount of collateral used to back them. The agents are free to choose the amount of collateral requirement of the assets actually traded. In this sense, the collateral requirement is determined by the market. This is called

⁴Note that given an asset j with its collateral C_j , the default is *not* an agent's decision. The condition to default is purely determined by market conditions, that is, the determination of p_{s3} . This is why either all agents default or no agent default.

endogenous collateral in the literature. It can be proved⁵ that the agents trade at most S assets, with collateral requirement defined by:

$$C_j = 1/p_{j3} \text{ with } j = 1, \dots, S$$

Based on this information, it is possible to rank the states by the durable price so that $C_j = 1/p_{j3} < 1/p_{j+1,3} = C_{j+1}$. Note that, with this ranking, the asset 1 has the lowest collateral requirement, given by C_1 , which gives default in all states, and the asset S has the highest collateral requirement, given by C_S , which never defaults, that is, always pays the promise in every state. Hence, from now on the asset 1 will be called *suprime asset* and the asset S will be called *prime asset*.

Next follows the definition of equilibrium of this economy.

Definition 1. Let $(u^h(\cdot), e^h)$ be the economy defined previously with monetary specification $(i, \{p_{s1}\}_{s \in S})$. The equilibrium for this economy is a vector $((x^*, \{x_{s3}^*\}_{s \in S^*}, \psi^*, \varphi^*), \mu^*, p^*, q^*)$ consistent with the monetary policy specification such that:

- (i) $(x^{h*}, \{x_{s3}^{h*}\}_{s \in S^*}, \psi^{h*}, \varphi^{h*})$ solves the optimization problem above given prices (p^*, q^*) for all h ;
- (ii) $\sum_{h=1}^H x_{01}^{h*} = \sum_{h=1}^H e_{01}^h$;
- (iii) $\sum_{h=1}^H x_{02}^{h*} = \sum_{h=1}^H e_{03}^h$;
- (iv) $\sum_{h=1}^H x_{03}^{h*} = \sum_{h=1}^H e_{03}^h$;
- (v) $\sum_{h=1}^H x_{s1}^{h*} = \sum_{h=1}^H e_{s1}^h$ for $s \in S$;
- (vi) $\sum_{h=1}^H x_{sl}^{h*} = \sum_{h=1}^H e_{03}^h$ for $s \in S$ and $l = 2, 3$;
- (vii) $\sum_{h=1}^H (\psi^{h*} - \varphi^{h*}) = 0$;
- (viii) $\sum_{h=1}^H \mu^{h*} = m$

The item (iv) is justified by the fact that when CB buys durable in $s = 0$, he sticks with his position and do not trade the durable bought anymore. However, the service of the durable is still tradeable because the durable is still in the economy. Analogously, the item (viii) of the definition of the equilibrium requires that the aggregate amount of money chosen by the agents in $s = 0$ equal the total amount of money available at this state.

In the following subsection the main properties of this model is presented.

2.2 Theoretical properties

The most important theoretical properties of the model is presented below. The proofs can be found in Araujo et al. (2015). An asset j is called “inessential” when the same allocation and prices of equilibrium can be obtained with another portfolio such that $\psi_j^h = \varphi_j^h = 0$ for all h , that is, as if the market of asset j were closed. The concept of inessentiability translates the idea that the economy

⁵For more details on this result see Araujo et al. (2012) and Araujo et al. (2015).

has non-financial objects that mimics the role of some financial asset. The main role of the financial assets is to give the agents the opportunity to transfer wealth between states. The durable in this model, for example, since does not give utility to the agents, in some sense also has this property. Holding one unit of durable in $s = 0$ means receiving p_{s3} in state s of period 1. The money also has similar property. The next two lemmas shows the conditions in which the subprime and the prime are inessential in this model.

Lemma 1. *Suppose $((x^*, \{x_{s,3}^*\}_{s \in S^*}, \psi^*, \varphi^*), \mu^*, p^*, q^*)$ is an equilibrium to the economy previously defined. Then:*

- (a) *If asset 1 is transacted, then $q_1^* = (p_{0,3}^* - p_{0,2}^*)C_1$*
- (b) *If asset S is transacted, then $q_S^* = \frac{1}{1+i}$*
- (c) *Asset 1 is inessential if, and only if, $x_{0,3}^{*h} \geq \varphi_1^{*h} C_1$ for all agents*
- (d) *If one of the following items is satisfied:*
 - $\mu^{*h} \geq \varphi_S^{*h} \frac{1}{1+i}$ *for all agents*
 - $q_S^* > \frac{1}{1+i}$

then asset S is inessential

In the numerical analysis the model will be simplified and restricted for two states in period 1 ($S = 2$), with subjective probability of occurrence given by p_s^h , and the same utility function for all agents:

$$U^h(x^h) = u(x_{01}^h, x_{02}^h) + p_1^h u(x_{11}^h, x_{12}^h) + p_2^h u(x_{21}^h, x_{22}^h)$$

where $x^h = (x_{01}^h, x_{02}^h, x_{11}^h, x_{12}^h, x_{21}^h, x_{22}^h)$ and $u(\cdot, \cdot)$ is a function that does not depend on the states or on the agents. This is done to simplify the analysis and make the agent's choice on the demand depend exclusively on the endowment distribution. Hence, different demand choices would be due different endowment position. As a consequence, since there are only two states in period 2, there are also only two financial assets, prime and subprime.

The technical hypothesis over $u(\cdot, \cdot)$ are: *continuity, strictly increasing, strictly concave and homothetic*. These hypotheses and the first order conditions of the optimization problem of the

agents imply that $\frac{x_{s1}^h}{x_{s2}^h}$ is uniquely determined by $\frac{p_{s2}}{p_{s1}}$ due to following equality⁶:

$$\frac{\partial_2 u(1, r(p_{s2}/p_{s1}))}{\partial_1 u(1, r(p_{s2}/p_{s1}))} = \frac{p_{s2}}{p_{s1}}$$

Once the fraction $\frac{x_{s1}^h}{x_{s2}^h}$ is uniquely determined as a function of $\frac{p_{s2}}{p_{s1}}$ and is equal to every agent h , because they are subject to the same utility function $u(\cdot, \cdot)$ in every state, then in equilibrium the relative prices $\frac{p_{s2}}{p_{s1}}$ must be determined from the aggregate endowment of the economy⁷ through the following equation;

$$\frac{\partial_2 u(1, \frac{e_{s2}}{e_{s1}})}{\partial_1 u(1, \frac{e_{s2}}{e_{s1}})} = \frac{p_{s2}}{p_{s1}}$$

where $e_{sl} = \sum_h e_{sl}^h$ for $l = 1, 2$.

Considering this discussion, the first order conditions implies that $p_{s2} = p_{s3}$ for all $s = 1, \dots, S$. Additionally, the Central Bank can guarantee the value of the money in the first period, thus he can normalize the prices in state $s = 1, \dots, S$ by fixing $p_{s1} = 1$ for all $s = 1, \dots, S$. Since, as argued above, p_{s2}/p_{s1} is fixed for all s , then p_{s2} is also known and there is no price left to determine in the first period. In $s = 0$ the price of the money is not normalized and also the first order conditions does not imply anymore that $p_{02} = p_{03}$. Therefore, the only prices in the economy left to be determined is p_{01} and p_{03} because p_{02}/p_{01} is still fixed in the state 0.

In the standard Arrow-Debreu model the wealth of the agents is completely determined by the value of their endowments. In models with financial markets and the possibility to transfer wealth between states, like the one presented here, the wealth available at state s is determined not only by the endowments but also by the vector of transferences chosen by the agent h . These transferences are the possibility that agents have to carry resources from one state to another, between states of period 1, from period 1 to state 0 or the other way around. As previously mentioned, the agent must use some instrument (financial or not) that in some way allows this transference. It can be a financial asset, that promises resources in each state of period 1 or it can be a non-financial object, such as the durable, that “survives” until the period 1 and in this way carry its value to its owner. The agent’s optimization problem of this model can be rewritten in a more simple way in terms of

⁶From the appendix it can be seen that the first order condition for $l = 1, 2$ in $s = 0$ is $\partial_{0l} u^h(x^h) - \mu_0^h p_{0l} + \sum_{s=1}^S \mu_s^h \sum_{l'} Y_{s,l,l'} p_{sl'} + \text{col} \mu_l^h = 0 \quad \forall h \implies \partial_{0l} u^h(x^h) - \mu_0^h p_{0l} = 0$, because $Y_{s,l,l'} = \text{col} \mu_l^h = 0$ for $l = 1, 2$, and for $s = 1, \dots, S$ the first order condition becomes $\partial_{sl} u^h(x^h) - \mu_s^h p_{sl} + x_{sl}^h \mu_{sl}^h = 0 \implies \partial_{sl} u^h(x^h) - \mu_s^h p_{sl} = 0$ because $x_{sl}^h \mu_{sl}^h = 0$ for $l = 1, 2$ for at least one agent due to market clearing (therefore $x_{s3}^h > 0$ for some h). Therefore, the equality $\frac{\partial_2 u(x^h)}{\partial_1 u(x^h)} = \frac{p_{s2}}{p_{s1}}$ holds. The uniqueness can be formalized in the following way. If another $\frac{\bar{x}_{s1}^h}{\bar{x}_{s2}^h} \neq \frac{x_{s1}^h}{x_{s2}^h}$ exists, each point $(\bar{x}_{s1}^h, \bar{x}_{s2}^h)$ and (x_{s1}^h, x_{s2}^h) belongs to a different ray passing through the origin. If, without loss of generality, $u(\bar{x}_{s1}^h, \bar{x}_{s2}^h) > u(x_{s1}^h, x_{s2}^h)$, one can use a standard argument with continuity and strongly increasing to conclude that exists $\bar{t} \in \mathbb{R}_+$ such that $u(\bar{x}_{s1}^h, \bar{x}_{s2}^h) = u(\bar{t}x_{s1}^h, \bar{t}x_{s2}^h)$. The homothetic property implies that $\partial_i u(\cdot, \cdot)$ is homogeneous of zero degree, therefore $\frac{\partial_2 u(1, \frac{\bar{x}_{s1}^h}{\bar{x}_{s2}^h})}{\partial_1 u(1, \frac{\bar{x}_{s1}^h}{\bar{x}_{s2}^h})} = \frac{\partial_2 u(1, \frac{\bar{t}x_{s1}^h}{\bar{t}x_{s2}^h})}{\partial_1 u(1, \frac{\bar{t}x_{s1}^h}{\bar{t}x_{s2}^h})} = \frac{p_{s2}}{p_{s1}}$. The same marginal utility of substitution at different points over the same indifference curve is incompatible with the strict concavity.

⁷Indeed, if $\frac{x_{s1}^h}{x_{s2}^h} = c$ for all h , then $\sum_h x_{s1}^h = c \sum_h x_{s2}^h \implies \sum_h e_{s1}^h = c \sum_h e_{s2}^h \implies c = \frac{\sum_h e_{s1}^h}{\sum_h e_{s2}^h}$, where $c = \frac{p_{s2}}{p_{s1}}$ and the first implication is true only in equilibrium due to the market clearing.

vector of transferences of wealth. It follows the definition of transference to state s of period 1, in units of perishable:

$$y_s^h = \left(\frac{1+i}{p_{s1}} \right) \left[\mu^h + \frac{1}{1+i}(\psi_2^h - \varphi_2^h) \right] + \left(\frac{p_{s3}}{p_{s1}} \right) \left[x_{03}^h + (\psi_1^h - \varphi_1^h)C_1 \right]$$

The first part of the sum is the *effective position in cash*, that is, the total amount of the prime asset and its “equivalent”, the money. Analogously, the second part of the sum is the *effective position in risky durable*, that is, the total position in subprime and in its “equivalent”, the durable. The y_s^h represents the *purchase power* carried by agent h to state s and it can be negative, which would mean that agent h is bringing resources from state s of the first period to $s = 0$.

Since the durable is one of the objects used to define the agent’s transference vector $y^h = (y_1^h, y_2^h)$, it is plausible to think that the collateral constraint would also impose some constraint over the set of y^h available to agent h . Araujo et al. (2015) found this relationship and here this is adapted for the presence of the regulation δ^h . Next lemma summarizes this.

Lemma 2. *In the economy defined above, the following equivalence holds:*

$$x_{03}^h \geq \varphi_1^h C_1 + \varphi_2^h C_2 \iff \begin{cases} p_{21}y_2^h \leq p_{11}y_1^h \\ y_2^h \geq 0 \end{cases}$$

Since the original utility function $u^h(\cdot)$ of each agent is separable in each state, it is possible to substitute each component $u(x_s^h, x_s^h)$ by the corresponding indirect utility function $\tilde{u}(c_s^h)$, where c_s^h is the total wealth available at that state. And the total wealth available at each state depends on the choice of transferences y^h by the agents, that can increase or decrease the total wealth available at some state. Using the definition of transference of wealth y_s^h , the previous lemma, the state prices a_1, a_2 from the non-arbitrage conditions and some basic algebra, the original agent’s optimization problem can be rewritten in the following way:

$$\begin{aligned} & \max_{y_1^h, y_2^h \in \mathbb{R}} \tilde{u}(c_0^h(y^h)) + \frac{1}{2}\tilde{u}(c_1^h(y^h)) + \frac{1}{2}\tilde{u}(c_2^h(y^h)) \\ & \text{s.t.} \\ & p_{21}y_2^h \leq p_{11}y_1^h \\ & y_2^h \geq 0 \end{aligned}$$

where $c_0^h(y^h) = e^h + a_1 f_1^h + a_2 f_2^h - a_1 y_1^h - a_2 y_2^h$; $c_s^h(y^h) = g_s^h + y_s^h - \frac{\theta^h}{p_{s1}} [(1+i)(p_{03} - p_{02})\omega e_{03} - p_{12}\omega e_{03}]$; $e_{03} = \sum_h e_{03}^h$; $e^h = e_{01}^h + \frac{p_{02}}{p_{01}} e_{03}^h$; $f_s^h = (\frac{p_{s2}}{p_{s1}}) e_{03}^h + (\frac{1+i}{p_{s1}}) m^h$; $g_s^h = c_{s1}^h - \theta^h (1+i) \frac{m}{p_{s,1}}$; and $\tilde{u}(\cdot)$ is the indirect utility function of $u^h(x_{s1}^h, x_{s2}^h)$ subject to the constraint $x_{s1}^h + \frac{p_{s2}}{p_{s1}} x_{s2}^h \leq c_s^h$.

Note that the two constraints that remains are due to the collateral constraint because the budget constraint at each state are already incorporated in the indirect utility function through c_s^h . The first constraint, $p_{21}y_2^h \leq p_{11}y_1^h$ is called *short sale constraint* and the second one $y_2^h \geq 0$ is called *leverage constraint*.

However, this presentation has the disadvantage that the non-conventional monetary policy, given by ω , affects the indirect utility function. With a change in the variable it is possible to see the effect of the unconventional policy only in the constraints over the transferences. Define

$$\tilde{y}_s^h = y_s^h - \frac{\theta^h}{p_{s1}} [(1+i)(p_{03} - p_{02})\omega e_{03} - p_{s2}\omega e_{03}]$$

Finally, the agent's optimization problem becomes:

$$\begin{aligned}
& \max_{\tilde{y}_1^h, \tilde{y}_2^h \in \mathbb{R}} \tilde{u}(e^h + a_1 f_1^h + a_2 f_2^h - a_1 \tilde{y}_1^h - a_2 \tilde{y}_2^h) + \frac{1}{2} \tilde{u}(g_1^h + \tilde{y}_1^h) + \frac{1}{2} \tilde{u}(g_2^h + \tilde{y}_2^h) \\
& \text{s.t.} \\
& p_{21} \tilde{y}_2^h \leq p_{11} \tilde{y}_1^h - (p_{12} - p_{22}) \theta^h \omega e_{03} \\
& \tilde{y}_2^h \geq -\theta^h \phi(a_1, a_2) \omega e_{03}
\end{aligned}$$

where $\phi(a_1, a_2) = \frac{a_1(p_{12} - p_{22})}{a_1 p_{2,1} + a_2 p_{11}} > 0$.

Thus, the problem now depends only on the transference choices \tilde{y}^h , already considering the Central Bank non-conventional monetary policy, with all the policies and regulatory instruments explicitly appearing in the collateral constraint, so making explicit their relationship as constraints of the set of possible transfereces. Also that the commodities or asset prices are no longer important to the problem, since the terms still appearing are known. So, given state prices (a_1, a_2) , the problem is well defined and a maximizer $(\tilde{y}_1^h, \tilde{y}_2^h)$ can be found. Finally note that when CB does an unconventional monetary policy with $\omega > 0$, it tightens the short sale constraint and relaxes the leverage constraint.

Thus, the equilibrium in this reformulated problem also changes in the following way:

Definition 2. *In an economy with two states in the first period the previous properties of the utility function, given the policy $(p_{11}, p_{21}, i, \omega)$, an equilibrium is a vector of state prices and transfereces $(a_1, a_2, \tilde{y}_1^h, \tilde{y}_2^h)$ such that*

- (i) for each h , \tilde{y}^h maximizes the last optimization problem
- (ii) for each $s = 1, 2$

$$\sum_h^H \tilde{y}_s^h = \sum_h^H f_s^h$$

The terms f_s^h can be interpreted as the initial transfereces of each agent for each asset, given by their initial endowments endowments.

3 Numerical Results

In this section a particular framework of the economy previously described will be defined for the numerical analysis. There will be no calibration to represent any specific society, the objective is to offer a conceptual and pictorial description of some characteristics of the economy related to the regulation of the collateral constraint proposed. In this sense, the purpose is to conceptually analyse the effects of the regulation in economies with certain endowment distribution generically, without referring to any real distribution in particular. The analysis involves computing the equilibria of several economies, for different values of the regulatory parameter, and see the impact of this instrument on the endogenous variables and on the welfare of the agents. The computation of the equilibria follows Schommer (2013), which uses KKT conditions to characterizes the equilibrium as a

system of equations and then implements it in the software ALGENCAN⁸. Such characterization is possible because each agent's optimization problem will be convex. This program uses a Lagrangean Augmented method as described in Andreani et al. (2008).⁹

All the agentes have the same cobb-douglas utility function, but with different subjective probability:

$$u^h(x) = \sum_l \log(x_{0l}^h) + \sum_s p_s^h \sum_l \log(x_{sl}^h)$$

The aggregate endowment will remain fixed along the entire analysis: $e_0 = \sum_h e_0^h = (7, 0, 7)$, $e_1 = \sum_h e_1^h = (15, 0, 0)$, $e_2 = \sum_h e_2^h = (6, 0, 0)$. Note that state $s = 1$ can be interpreted as the good state since it has more aggregate endowment of perishable. Analogously, $s = 2$ is interpreted as the bad state.

One important consequence of the logarithm in the utility function is that all equilibrium relative prices of the states $s = 1, 2$ are previously known and depends only on the proportion of aggregate endowments of the corresponding goods: $\frac{\sum_h e_{sl'}^h}{\sum_h e_{sl}^h} = \frac{p_{sl}^*}{p_{sl'}^*} = \frac{x_{sl'}^h}{x_{sl}^h} \quad \forall h$.¹⁰ Therefore, in this model:

$$\frac{p_{11}^*}{p_{12}^*} = \frac{\sum_h e_{12}^h}{\sum_h e_{11}^h} = \frac{7}{15} = 0.46666666 \quad \text{and} \quad \frac{p_{21}^*}{p_{22}^*} = \frac{\sum_h e_{22}^h}{\sum_h e_{21}^h} = \frac{7}{6} = 1.16666666$$

The next three subsections organize the presentation of the numerical results:

- Fixed endowments in the states of period 2 and varying probabilities p_s^h ;
- Fixed probabilities p_s^h and varying endowments in second period;
- Fixed endowments and fixed probabilities, changing ω ranging from 0 to 1

In the first two cases it will be analyzed the utility and the constraints of the agents, and in the last case only the utility of the agents will be analyzed.

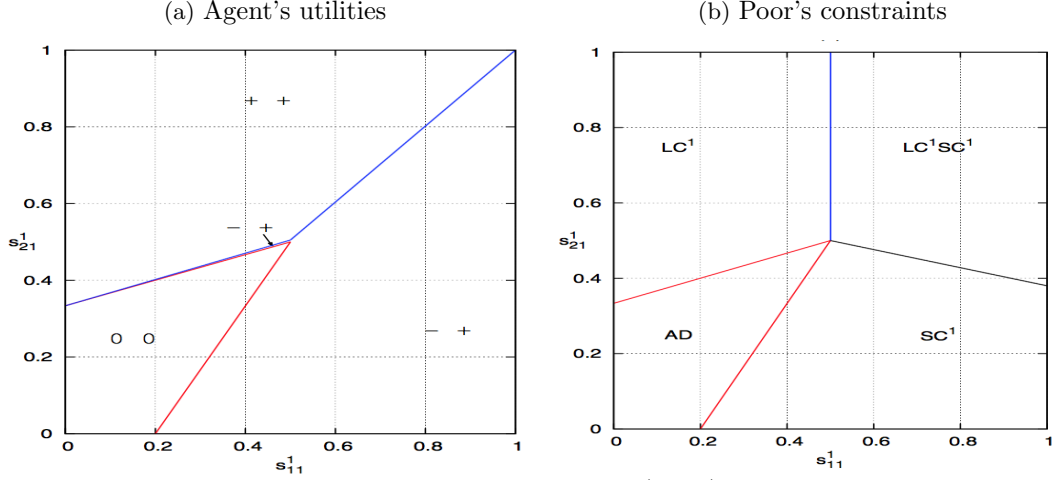
The next graphics, from Araujo et al. (2015), will be used as a reference point in the analysis of the numerical results.

⁸See TANGO project (2013).

⁹It was used a Macbook Pro with an Intel Core i7, 2.5GHz and 16 GB 1600 MHz DDR3 for the computations. The tolerance in all cases was 10^{-10} .

¹⁰Indeed, since the utility function is logarithm, it is known that $x^{*h} >> 0$ for all h . Thus, $x_{sl}^{*h} = 0 \quad \forall h, \quad \forall s \in \{2, 3\}, \quad \forall l$. By the first order condition detailed in the appendix, then $\partial_{sl} u^h(x^{*h}) - \mu_s^h p_{sl}^* = 0 \quad \forall h, \quad \forall s, \quad \forall l$. Fixing h and s , this becomes $\frac{\partial_{sl} u^h(x^{*h})}{p_{s,l}^*} = \mu_s^h \quad \forall l \implies \frac{\partial_{s,l} u^h(x^{*h})}{p_{sl}^*} = \frac{\partial_{s,l'} u^h(x^{*h})}{p_{sl'}^*} \implies \frac{\partial_{s,l} u^h(x^{*h})}{\partial_{s,l'} u^h(x^{*h})} = \frac{p_{sl}^*}{p_{sl'}^*} \quad \forall l, l'$. Using $\partial_{sl} u^h(x^{*h}) = \frac{1}{2x_{sl}^{*h}}$ it follows $\frac{x_{sl'}^{*h}}{x_{sl}^{*h}} = \frac{p_{sl}^*}{p_{sl'}^*} \implies x_{sl'}^{*h} = \frac{p_{sl}^*}{p_{sl'}^*} x_{sl}^{*h} \implies \sum_h x_{sl'}^{*h} = \frac{p_{s,l}^*}{p_{s,l'}^*} \sum_h x_{sl}^{*h} \implies \sum_h e_{sl'}^h = \frac{p_{sl}^*}{p_{sl'}^*} \sum_h e_{sl}^h$ [using market clearing] $\implies \frac{\sum_h e_{sl'}^h}{\sum_h e_{sl}^h} = \frac{p_{sl}^*}{p_{sl'}^*} = \frac{x_{sl'}^h}{x_{sl}^h} \quad \forall h$.

Figure 1: Agent's utilities and constraints with homogeneous expectations ($p_s^h = 0.5$ for all h and s)

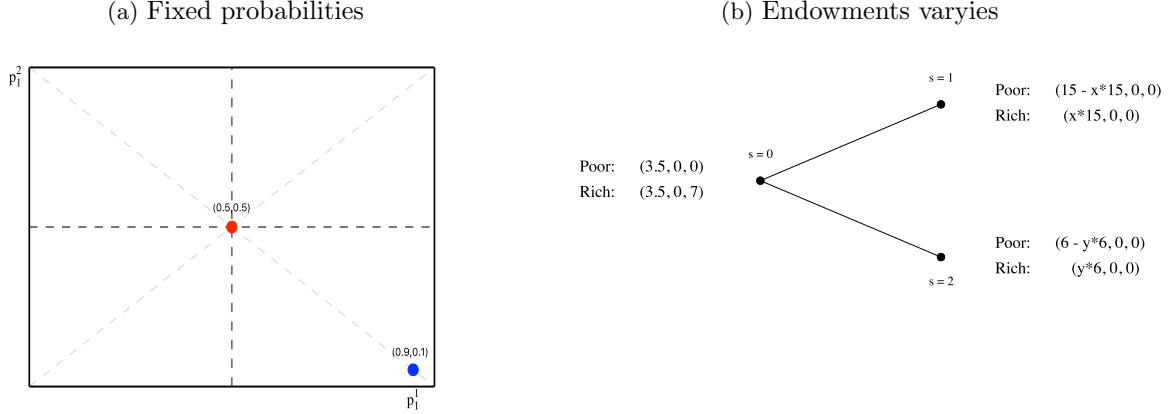


Source: Araujo et al. (2015)

where $s_{11}^1 = e_{11}^1 / \sum_h e_{11}^h \in [0, 1]$ and $s_{21}^1 = e_{21}^1 / \sum_h e_{21}^h \in [0, 1]$. In the left figure, for each economy of the box, the equilibrium was calculated first with $\omega = 0$ and second with $\omega = 0.001$. The symbol $++$ indicates that, in the economies of that region, the utility of both agents increase when ω increases. The symbol $--$, for example, indicates that agent 1 loses and agent 2 gains. In the right figure, the symbols LC^1 , SC^1 and AD indicates which constraint of the agent is binding with $\omega = 0$. Generally, LC^h means that the leverage constraint of agent h is binding, SC^h is defined in an analogous way and AD stands for "Arrow-Debreu", meaning that in those economies none of the constraints of the agents are binding and therefore the equilibrium is equivalent to an Arrow-Debreu equilibrium since the markets are complete. Note that there is Pareto improvement in the economy only when agent 1 is leverage constrained. The rich's constraints are not binding in the entire box.

3.1 Fixed probabilities and various endowments distributions

Figure 2: Fixed probabilities and endowments varying



The endowment distribution in $s = 0$ is asymmetric on the durable, representing a moment of crisis with high inequality. Generally, since the agents are equal except for the endowments and probabilities, the endowment distribution will typically induce the roles of the agents in the financial markets. The variables x and y in the endowments of the agents indicates the methodology to create the figures. The x-axis will represent the *proportion* of perishable owned by the poor in state 1 (i.e., $x = e_{11}^1 / \sum_h e_{11}^h \in [0, 1]$) and the y-axis will represent the proportion of perishable owned by the poor at state 2 (i.e., $y = e_{21}^1 / \sum_h e_{21}^h \in [0, 1]$). The probability is fixed in $p_1^1 = 0.9$ and $p_1^2 = 0.1$, as represented in the blue dot, which means that the poor is optimistic and the rich is pessimistic. This can be interpreted as the case in which the optimistic become poor right after the crash due to frustrations from the crisis. The red dot highlight the case with homogeneous expectation which was analyzed in Araujo et al. (2015).

Figure 3 (a), for probabilities $p_1^1 = 0.9$ and $p_1^2 = 0.1$, shows that the Pareto improving $(++)$ region increases, following the increasing of the leverage constrained region of the poor agent, which is depicted in 3 (b). It suggests that the more the poor/borrower agent is optimistic, the more he becomes leverage. This numerical fact supports the intuition that optimistic agents tend to leverage more. Indeed, the optimistic agent thinks that the good state $s = 1$ is more likely to happen and, symmetrically, that the bad state is not going to occur. Therefore, he wants to bring the maximum wealth possible from the poor state $s = 2$ to the good state $s = 1$. The need to accomplish this kind of transference makes him more prone to be leverage constrained, making $y_2^1 = 0$ and *not* short sale constrained because he will probably choose $y_1^1 > y_2^1$. The constraints for the pessimistic rich agent has a symmetric analysis. Since he is pessimistic, he wants to bring wealth for the poor state and therefore will probably have $y_2^2 > 0$, thus not leverage constrained, and also $y_2^2 = y_1^2$, thus short sale constrained, because he is decreasing the wealth transfered to good state y_1^h and in contrast increasing the transference y_2^2 to the bad state. This is depicted in Figure 4 (b). Finally, in Figure 4 (a) it is shown the vector of transferences of both agents. The rich agent has higher transferences, hence his choices of transferences are depicted in the straight segment at a level higher than 5. Note

that he is short sale constrained for some chosen transferences, because they are over the line with slope 1. And the poor agent has his transferences in lower levels, most of them over the line of level 0, which is the region where the agent becomes leverage constrained.

Figure 3: Agent's utilities and constraints with $p_1^1 = 0.9$ and $p_1^2 = 0.1$

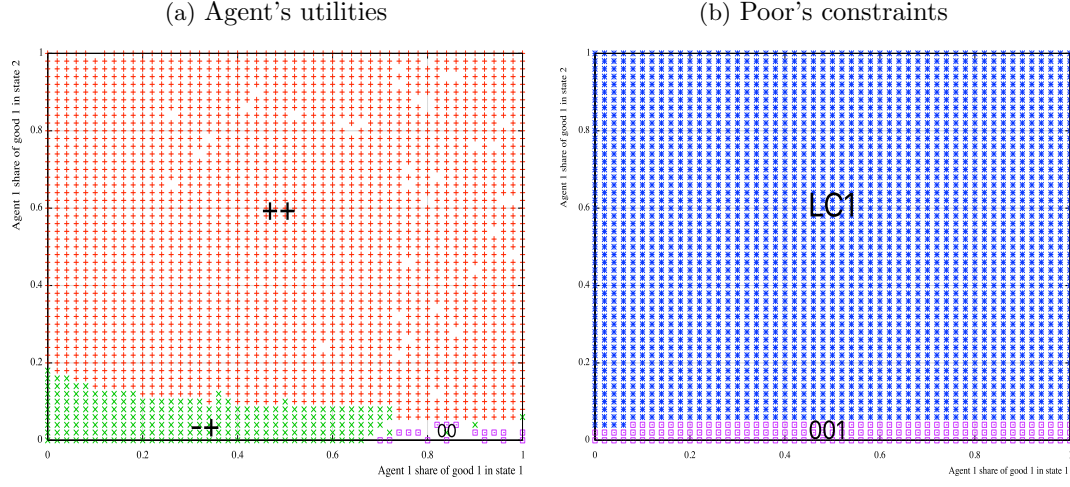
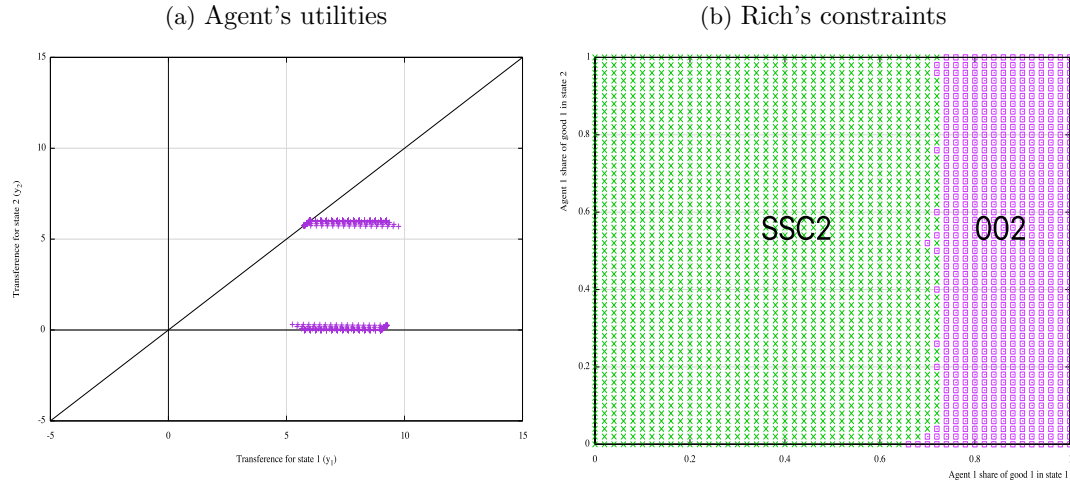


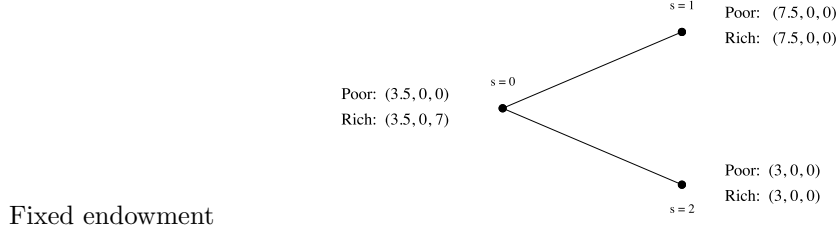
Figure 4: Agent's utilities and constraints with $p_1^1 = 0.9$ and $p_1^2 = 0.1$



3.2 Fixed endowments and various probabilities distributions

In this case the endowment distribution is fixed for the analysis of various probabilities distribution:

Figure 5: Fixed endowment



This endowment distribution is exactly in the middle of the box in figure 1, which is in the border of almost all regions. The probabilities will vary as in the box of figure 2 (a). The graphic in figure 6 (a) shows that, for this endowment distribution, there will be Pareto improvement whenever the poor is more optimistic than the rich. This result is related with figure 3 (a) because it suggests that if the poor is more optimistic than the rich, then the region of Pareto improvement will probably increase. Note that in figure 1 (a) the economy with this endowment distribution is in the frontier of the Pareto improving region, whereas in figure 3 (a), with probabilities $(p_1^1, p_1^2) = (0.9, 0.1)$, this economy is in the *interior* of the Pareto improving region.

Figure 6 (b) shows that for this endowment distribution, which is symmetric in the second period, the relative optimism is crucial to determine which constraint will be binding for the poor agent. When he is more optimistic than the rich, he will be leverage constrained, and when he is more pessimistic than the rich he will be short sale. This result reinforces the interpretation that more optimism tends to lead to leverage constrained agents and more pessimism tends to lead to short sale constrained agents.

In figure 7, pane (a) shows that agent 1 is almost always constrained, either in the short sale or in the leverage, and also that agent 2 is almost always not constrained. However, note that panel (b) shows that even the rich may be short sale constrained in the cases he is extremely pessimistic, in tune with the explanations of the previous subsection.

Figure 6: Agent's utilities and constraints with $x = y = 0.5$

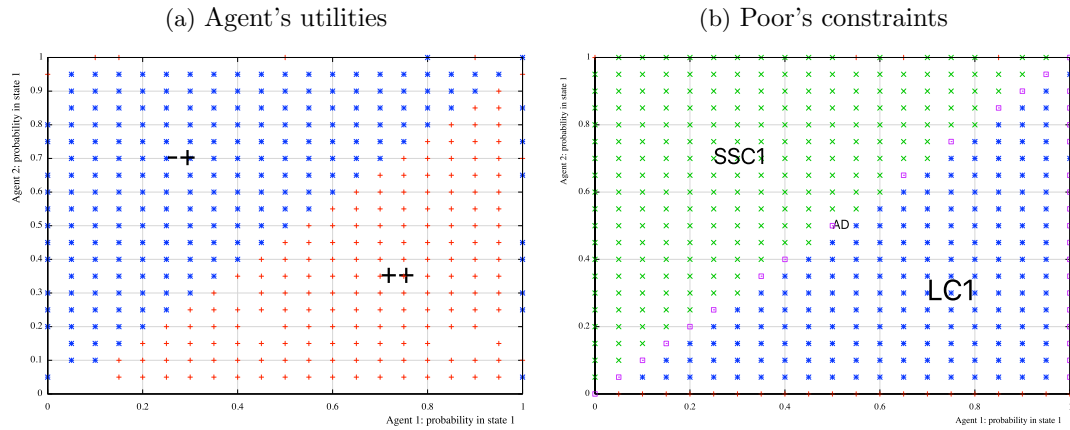
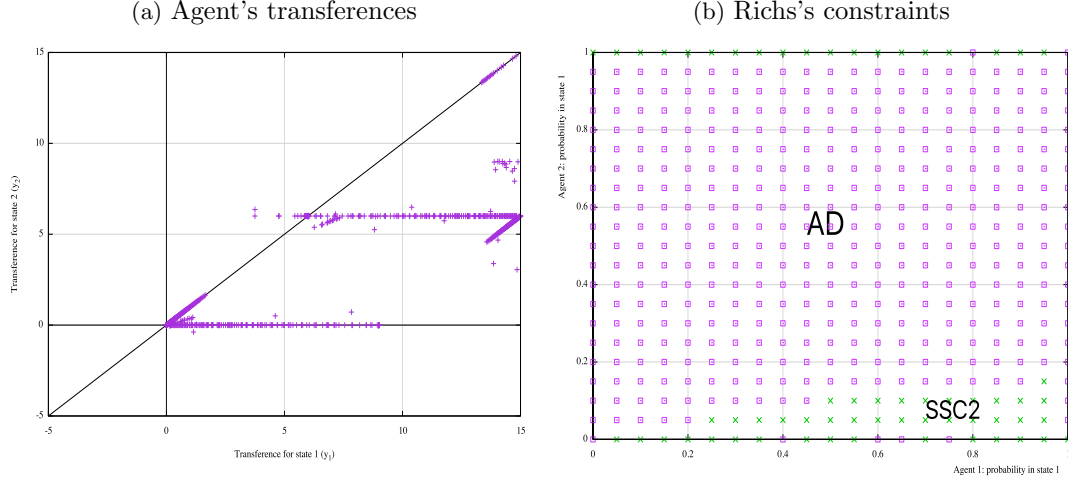


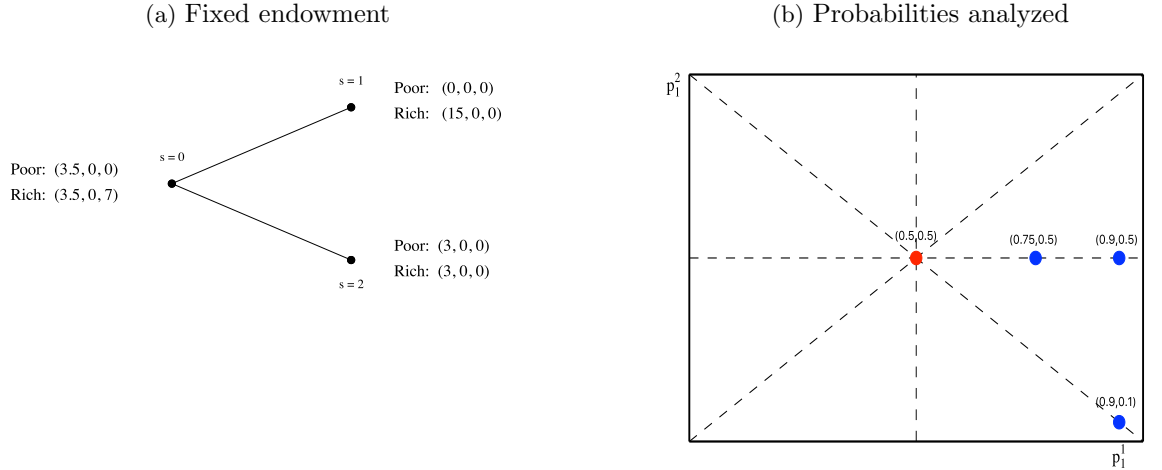
Figure 7: Agent's transferences and constraints with $x = y = 0.5$



3.3 Fixed endowment and fixed probabilities

Finally, an endowment distribution is fixed as in the picture below and three probability distributions were chosen, depicted in blue in the picture below, in order to analyze the utility of the agents for $\omega \in [0, 1]$. The case with homogeneous expectation were also computed for comparison purposes.

Figure 8: Fixed endowment and fixed probabilities

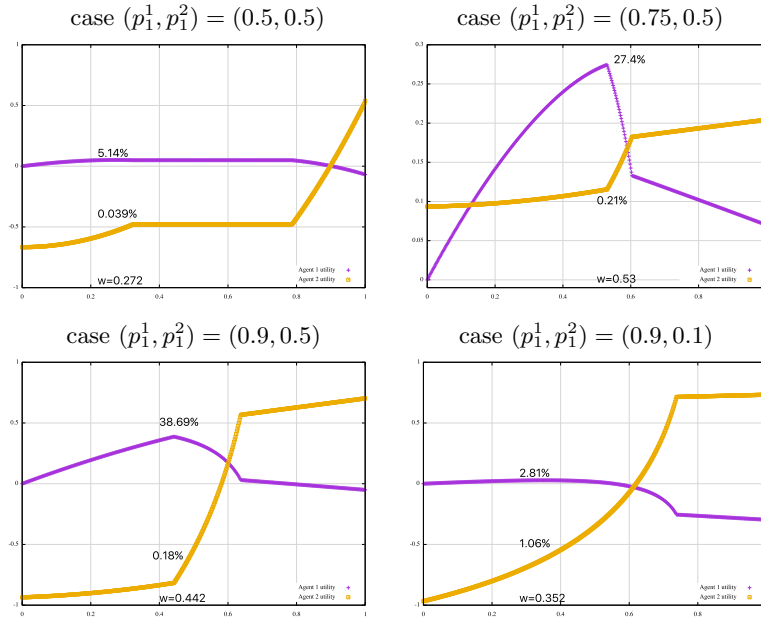


Inside each chart of figure 9 below there is the indication of the value of ω where the maximum Pareto improvement gain occurs. The percentages shows the relative gain of each agent in comparison with the initial $\omega = 0$ case. Generally speaking, figure 9 shows that the poor agent gains more when he is optimistic and the rich is neutral. When approaching $(p_1^1, p_1^2) = (1, 0.5)$, the improvement in agent's 1 utility increases, being 38,69% in the case $(0.9, 0.5)$ and only 5.14% in the

homogeneous case. The rich's gain also increases in this line, from 0.039% in the case (0.5,0.5) to 0.18% in the case (0.9,0.5). However, in the case (0.9,0.1), where the poor is optimistic and the rich pessimistic, although everyone still gains, the poor gains less than in the homogeneous case and the rich gains more than if he is neutral. It seems that the interaction between the rich pessimistic and the poor optimistic seems to cancel those high benefits of the poor of the cases (0.75,0.5) and (0.9,0.5). And this benefits the rich.

Note also that the range of Pareto improvement in ω changes depending on the beliefs of the agents. It is higher in the case (0.75,0.5), with the interval $[0, 0.53]$.

Figure 9: Utility of the agents



4 Conclusions

In this paper it was studied the relation between the heterogeneous beliefs of the agents and the unconventional monetary policy in a general equilibrium model with collateral and money. This relationship is missing in Araujo et al. (2015). The contribution is not only in filling these gaps but also advancing the study and understanding of the role of the agents with heterogeneous beliefs in moments of crisis. The role played by optimistic agents in the economy in general, and in moments of crisis in particular, is still of interest in the literature, especially after the recent financial crisis of 2009, which can be seen in works such as Geanakoplos (2009) and Tsomocos and Yan (2016).

The numerical findings suggests that the presence of optimism/pessimistic agents in the economy may affect the regulatory policy, increasing or decreasing its effects. It was found that when the borrower is optimistic the unconventional monetary policy is *more* effective in the sense that the set of economies with Pareto improvement increases. In this sense, it can be summarized saying that the optimism *potentiates* the effect of the unconventional monetary policy and in contrast, the pessimism *softens* the effect. The theoretical reason behind this is that the unconventional monetary policy can only favor with a Pareto improvement in economies with leverage constrained agents. And the optimism tends to make the agents leverage constrained.

The numerical results also showed that the optimism/pessimism affects the room for the unconventional monetary policy in the sense that it allows for Pareto improvement even for higher levels of purchase of durable by the Central Bank. And it was found that the maximum gains in the utilities happens in the region where the poor is more optimistic and the rich is neutral.

Geanankoplos (2009)

5 Appendix: details of the implementation

The complete system of equalities and inequalities implemented in ALGENCAN is depicted below.

Recall the notation: $m = \sum_h m^h$ and $e_{03} = \sum_h e_{03}^h$.

Inequality consumption $t = 0$ (collateral constraint):

$$-x_{0l}^h + \sum_j \varphi_j^h C_{lj} \leq 0 \quad \forall h, \forall l$$

Inequality to r :

$$2r_{sj} - 1 - \sum_l p_{sl} \sum_{l'} Y_{sl'l'} C_{l'j} \leq 0 \quad \forall s \in S, \forall j$$

First-order condition of x in $t = 0$:

$$\begin{aligned} \partial_{0l} u^h(x^h) - \mu_0^h p_{0l} + \sum_{s=1} \mu_s^h \sum_{l'} Y_{sl'l'} p_{sl'} + \text{col} \mu_l^h &= 0 \quad \forall h, \text{ for } l = 1, 2 \\ \implies \frac{\alpha_{01}}{x_{01}} - \mu_0^h p_{0l} + \text{col} \mu_l^h &= 0 \quad \forall h, \text{ for } l = 1, 2 \end{aligned}$$

and

$$\begin{aligned} \partial_{03} u^h(x^h) - \mu_0^h (p_{03} - p_{02}) + \sum_{s=1} \mu_s^h \sum_{l'} Y_{sl'l'} p_{sl'} + \text{col} \mu_3^h &= 0 \quad \forall h, \text{ for } l = 3 \\ \implies -\mu_0^h (p_{03} - p_{02}) + \text{col} \mu_3^h + \mu_s^h p_{s3} &= 0 \quad \forall h, \text{ for } l = 3 \end{aligned}$$

First-order condition of x in $t = 1$:

$$\begin{aligned} \partial_{sl} u^h(x^h) - \mu_s^h p_{sl} + x \mu_{sl}^h &= 0 \quad \forall h, \forall s \in S, \forall l = 1, 2 \\ \implies \frac{\text{prob}(s) \alpha_{sl}^h}{x_{sl}^h} - \mu_s^h p_{sl} &= 0 \quad \forall h, \forall s \in S, \forall l = 1, 2 \end{aligned}$$

and

$$\begin{aligned} \partial_{s3} u^h(x^h) - \mu_s^h (p_{s3} - p_{s2}) + x \mu_{s3}^h &= 0 \quad \forall h, \forall s \in S, l = 3 \\ \implies -\mu_s^h (p_{s3} - p_{s2}) &= 0 \quad \forall h, \forall s \in S, l = 3 \end{aligned}$$

Budget constraint at $t = 0$:

$$\sum_l p_{0l} (x_{0l}^h - e_{0l}^h) - p_{02} x_{03}^h + \sum_j q_j (\psi_j^h - \varphi_j^h) + \mu^h - m^h = 0 \quad \forall h$$

Budget constraint at $t = 1$:

$$\sum_l p_{sl} (x_{sl}^h - e_{sl}^h - \sum_{l'} Y_{sl'l'} x_{0l'}^h) - p_{s2} x_{s3}^h - \sum_j (\psi_j^h - \varphi_j^h) r_{sj} +$$

$$+\theta^h(1+i)[(p_{03}-p_{02})\omega e_{03}+m]-\theta^h p_{s3}\omega e_{03}-(1+i)\mu^h=0 \quad \forall h, \forall s \in S$$

First-order conditions of ψ :

$$\psi \mu_j^h + \sum_s \mu_s^h r_{sj} - \mu_0^h q_j = 0 \quad \forall h, \forall j$$

First-order conditions of φ :

$$\varphi \mu_j^h - \sum_s \mu_s^h r_{sj} + \mu_0^h q_j - \sum_l \text{col} \mu_l^h C_{lj} = 0 \quad \forall h, \forall j$$

First-order conditions of μ :

$$-\mu_0^h + \sum_{s=1}^S \mu_s^h (1+i) + \mu \mu^h = 0 \quad \forall h$$

Market clearing for x at $t = 0$:

$$\begin{aligned} \sum_h (x_{01}^h - e_{01}^h) &= 0 \quad (l = 1) \\ \sum_h (x_{02}^h - e_{03}^h) &= 0 \quad (l = 2) \\ \sum_h (x_{03}^h - (1-\omega)e_{03}^h) &= 0 \quad (l = 3) \end{aligned}$$

Market clearing for x at $t = 1$:

$$\begin{aligned} \sum_h (x_{s1}^h - e_{s1}^h) &= 0 \quad \forall s \in S, (l = 1) \\ \sum_h (x_{sl}^h - e_{s3}^h - Y_{s33} e_{03}^h) &= 0 \quad \forall s \in S, (l = 2, 3) \end{aligned}$$

Market clearing for ψ and φ :

$$\sum_h (\psi_j^h - \varphi_j^h) = 0 \quad \forall j$$

Market clearing for the money μ^h :

$$\sum_h \mu^h = m + (p_{03} - p_{02})\omega e_{03}$$

Boundary conditions:

$$\begin{aligned} x \mu_{sl}^h x_{sl}^h &= 0 \quad \forall h, \forall s, \forall l \\ \psi \mu_j^h \psi_j^h &= 0 \quad \forall h, \forall j \\ \varphi \mu_j^h \varphi_j^h &= 0 \quad \forall h, \forall j \\ \mu \mu^h \mu^h &= 0 \quad \forall h \\ \text{col} \mu_l^h (-x_{0l}^h + \sum_j \varphi_j^h C_{lj}) &= 0 \quad \forall h, \forall l \end{aligned}$$

Portfolio condition or ortogonality of ψ and φ :

$$\varphi_j^h \psi_j^h = 0 \quad \forall h, \forall j$$

Price normalization at $t = 1$:

$$p_{s1} = 1 \quad \forall s \in S$$

Equality for r_{sj} :

$$(r_{sj} - 1)(r_{sj} - \sum_l p_{sl} \sum_{l'} Y_{sl'l'} C_{l'j}) = 0 \quad \forall s \in S, \forall j$$

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