Are Cryptocurrencies Suitable for Portfolio Diversification? Cross-Country Evidence

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Abstract. In this paper, we analyze whether a representative investor, who holds a well-diversified portfolio of stocks, may benefit from investing in cryptocurrencies. Our analysis encompasses several capital markets and the four most liquid cryptos. Using country-specific stock market indices and risk free rates, our results indicate that, for most countries, cryptocurrencies fit in the tangent portfolio (maximum sharpe ratio), but not – or very little – in the minimum variance portfolio (MVP). Cryptocurrencies' returns are riskier, but its co-movements with global stock indices are close to zero, on average. Globally, we find that the optimal holding of cryptocurrencies is represented by Bitcoin (BTC) only and at a 4.6% share of the portfolio (when not considering short sales). Importantly, optimization exercises using rolling windows reveal that the optimal weight of cryptocurrencies on the tangent portfolio is extremely sensitive to the period being analyzed, and also with the premises on expected returns.

Keywords and phrases. Cryptocurrencies; Portfolio optimization; Digital assets.

JEL Classifications. G00, G11, G15

Resumo. Neste artigo, nós analisamos se um investidor representativo, que possui uma carteira bem diversificada de ações, pode se beneficiar do investimento em criptomoedas. Nossa análise engloba vários mercados de capitais e as quatro criptomoedas mais líquidas. Usando índices do mercado de ações e taxas livres de risco específicos do país, nossos resultados indicam que, para a maioria dos locais, as criptomoedas possuem pesos não-negativos na carteira tangente (máximo índice de sharpe), mas não – ou muito pouco – no portfólio de variância mínima (MVP). Os retornos das criptomoedas são mais arriscados, mas seus co-movimentos com os retornos dos índices de ações mundiais são próximos de zero, em média. Globalmente, nossos testes sugerem

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que a alocação ótima de cryptoativos é representada pelo Bitcoin (BTC) somente, com um peso de 4,6% na carteira (quando não são permitidas vendas a descoberto). Importante, nossos exercícios de otimização utilizando janelas móveis revelam que o peso ideal das criptocorrências no portfólio de tangentes é extremamente sensível ao período que está sendo analisado e também às premissas sobre os retornos esperados.
Palavras-chave. Criptomoedas; Otimização de carteira; Ativos digitais.

Classificação JEL. G00, G11, G15

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1 Introduction

Cryptocurrencies represent a recurring discussion topic among academics, professionals and general public. This growing and proliferating market – a subset of the so called market of "digital assets" – offers appealing features, such as operating independently of central banks (decentralization) and using encryption techniques to verify transactions and regulate total supply. The numbers of the market are quite impressive: as of May 31st, 2019, the market capitalization for bitcoin (BTC) only has reached US$149.0 billions (CoinMarketCap, 2019).[1] Taken together, there are now estimated 2,213 cryptocurrencies with a market capitalization of US$267.2 billions (CoinMarketCap, 2019).

Though appealing, cryptocurrencies face huge challenges to become popular to a larger audience. There are several reasons for that. One is that even industry experts find it difficult to evaluate the intrinsic value of these assets. For example, a recent report from JP Morgan conclude that Bitcoin "has surged beyond its "intrinsic value"" (Bloomberg, 2019). While inherently difficult to estimate, the authors of the report came to this conclusion by treating Bitcoin as a commodity and calculating its "cost of production" using inputs such as estimated computational power, electricity expense and hardware energy efficiency. A second important reason that repel investors from investing in those assets are fraudulent events registered in the cryptocurrencies' ecosystem (in Brazil, see, e.g., a pyramid scam that defrauded $200 millions on May, 2019) (Cointelegraph, 2019b). Finally, one could point out the ongoing challenge to regulate cryptocurrencies transactions, specially in emerging markets – in Brazil, for example, the President of the Chamber of Deputies has only recently ordered to establish a commission to consider cryptocurrency regulation in the country (Cointelegraph, 2019a).

From a finance perspective, cryptocurrencies have been the subject of several recent academic studies, exploiting and connecting subjects such as price drivers (Liu & Tsyvinski, 2018), suspicious activity and price manipulation (Gandal *et al.* , 2018), and mean/variance analysis in the context of portfolio analysis (Gangwal, 2016, Liu & Tsyvinski, 2018). In this paper, we seek to contribute to the debate on the role of cryptocurrencies on diversifying portfolios of "traditional assets", proxied by a well diversified portfolio of stocks.[2] We built on modern finance theory to identify whether, under a set of assumptions, including cryptocurrencies on a well diversified portfolio of country-specific stock portfolios may increase risk-adjusted expected returns. Our focus is on different developed and developing economies, totaling 22 countries and four cryptocurrencies – Bitcoin (BTC), Ripple (XRP), Ethereum (ETH), and Litecoin (LTC) – in our final data set.

Specifically, we estimate the marginal effect of adding different cryptocurrencies to different portfolios composed by traditional assets. The information on daily prices of crytocurrencies and stock market indices come from Yahoo Finance, and goes up to May 31st, 2019. Our goal is to identify how much of a portfolio should be held in cryptocurrencies (if any) in order to optimize risk-adjusted expected returns, under different sets of assumptions. In other words, we estimate

[1] BTC was traded in this particular day at a market price of US$ 8,454.81.

[2] Traditional assets oppose digital assets. As examples, we can cite stocks, corporate bonds, real estate, traditional currencies, gold, etc.

efficient mean-variance portfolios of traditional and non-traditional assets, giving flexibility on the underlying assumptions of the models (e.g., expected returns, standard deviation of returns and variance-covariance matrix).

As a proxy for well-diversified portfolios of risky assets, we consider original investments in a global stock market index, such as: S&P 500 (U.S.), Canada S&P/TSX 60 (Canada), Dax (Germany), FTSE100 (U.K.), CAC 40 (France), BEL 20 (Belgium), Australia ASX (Australia), Nikkei 225 (Japan), Shanghai SE Composite Index (China), Taiwan TSEC 50 Index (Taiwan), Hang Seng Index (Hong Kong), Mumbai Sensex (India), Jacarta Composite Index (Indonesia), MOEX (Russia), Ibovespa (Brazil), Santiago Index IPSA (Chile), IPC (Mexico), and Merval (Argentina).

Our results indicate that cryptocurrencies may enhance the performance of global stock portfolios by allowing higher risk-adjusted returns. However, this seems to be more due to higher average returns rather than risk reduction. Also, allowing for short positions result in increasing BTC weight and in the addition of ETH to the long position, in contrast to the minimum variance portfolio, which results in near zero weights under both assumptions. Moreover, we find evidence of time-varying optimal weights for each asset. Even though bitcoin's presence in optimal portfolio is more frequent than other cryptos, its weight is highly unstable over time and full dominance over other digital assets is not verified.

Our paper connects to a wide set of recent studies in portfolio selection with cryptocurrencies (Baur *et al.* , 2018, Guesmi *et al.* , 2019, Kajtazi & Moro, 2019, Liu, 2019, Liu & Tsyvinski, 2018). However, most attention is given to US and China assets, and inquiries into the contributions of digital assets to diversification for emerging markets assets are still incipient. To the best of our knowledge, this is the first attempt to widely test – for both developed and developing markets – whether adding digital assets to a portfolio of traditional assets may enhance the risk/reward relationship.

Besides trying to offer empirical evidence on the question "*how much should I hold of cryptocurrencies in my portfolio?*", there are other important side contributions of the paper. For example, one is to show the correlation matrix across cryptocurrencies and among cryptocurrencies and traditional assets in different countries. We also check for stability in the optimal weights in both the tangent and the global minimum variance (GMV) portfolios.

The rest of the paper is structured as follows. In section 2, we present a brief discussion of the related literature. In section 3, we show how cryptocurrencies fit in the traditional mean-variance framework of portfolio optimization. Section 4 describes the data we use. In section 5, we present the results of the analyses. Finally, our concluding remarks are exposed on section 6.

2 A short review on cryptocurrencies and portfolio selection

The 2007-2008 financial crisis uncovered the fragility of modern financial and monetary systems where trust in governments, banks and other financial institutions played a central role for transactions in the system. In the aftermath of the crisis, a whitepaper written by Satoshi Nakamoto launched the basis for a "peer-to-peer electronic cash-system" called Bitcoin (Nakamoto, 2008). This system is based on cryptographic proof instead of trust, so that transactions between any two individuals could dispense a third trusted party. A public ledger, known as the blockchain, keeps record on credits and debts for each transaction and is publicly available as an open source code, so that legitimacy of transactions can be checked by the community. Only after verification a transaction is recorded on the blockchain. This process requires computers to solve complex mathematical puzzles, so that verification can be done and another block added to the blockchain, during a procedure called mining. For this service, miners receive a reward in the form of virtual money, the bitcoin, that is added to the network's circulation. Thus a potential new system of payments with a genuine currency was raised, backed by data security science and, more importantly, without a centralized control from government authorities.

Since the birth of bitcoin, many other blockchains and their respective cryptocurrencies – also called altcoins - emerged, based on forks of the original bitcoin's blockchain source code.[3]. As of 20th July, 2019, the number of cryptocurrencies circulating was over 2500 and continues to grow [4]. But are cryptocurrencies really currencies or do they play another role? Even though Nakamoto's definition of a "*peer-to-peer electronic cash system*" implies bitcoin's mainly use as a currency, this is not a consensus in the literature. Economists usually state that a typical currency must satisfy three functions: medium of exchange, unit of account and store of value. Several authors have argued that, so far, bitcoin and it's pairs lack at least one of them.

For instance, Baur *et al.* (2018) analyse the Bitcoin public ledger for 2011-2013 and find that most of bitcoins are held by investors, while a minority find it's use as currency. Besides, they argue that were bitcoin mainly used to pay for goods and services, it would compete with fiat currencies and influence their value. As such, Liu & Tsyvinski (2018) find that major currencies, like Australian Dollar, Canadian Dollar, Euro, Singaporean Dollar, and UK Pound co-move, but exposures of cryptocurrencies to these currencies are small and not statistically significant. Also, despite some medium of exchange characteristics, Dyhrberg (2016) classifies bitcoin as being "somewhere between a currency and a commodity". At the same time, Yermack (2013) reminds that retail good prices quoted in bitcoin depart from integer numbers, often leading to fractions which require multiple decimal places with leading zeros, such as 0.01694 BTC, 0.00529 BTC, etc, which is a non-standard practice that may confuse sellers and buyers. Together with high observed volatility, these features pose a big barrier to using bitcoin — or other cryptos – as a unit of account. Finally, many examples of security breaches, such as hacker attacks and theft (BBC (2019))) or even price manipulation (Gandal *et al.* , 2018) cast some serious doubt on the store of value function for cryptocurrencies.

On the other hand, empirical findings suggest that cryptocurrencies may better suit as an asset class ((Baur *et al.* , 2018)). These assets present, in general, very particular characteristics, such as high returns, high volatility, high kurtosis, presence of autocorrelation, time-varying return predictability and low correlation with other asset's returns (Bariviera *et al.* (2017), Liu & Tsyvinski (2018), Baur *et al.* (2018), Urquhart & Zhang (2019)). Skewness, however, have been found to be positve (Liu & Tsyvinski (2018)) and negative (Baur *et al.* (2018)).

More recently, an increasing body of the literature study their role in portfolio diversifica-

[3]We stress that Ripple was created in 2004, but it's contemporary version relies on some of bitcoin's principles. See < https://www.americanbanker.com/news/disruptor-chris-larsen-returns-with-a-bitcoin-like-payment-system >

[4]See < https://www.coinlore.com/all_coins >

tion. For instance, Liu (2019) apply different portfolio selection models, conducting a large set of robustness checks to analyze portfolio performance considering 10 cryptocurrencies. They point out that diversification among cryptocurrencies can enhance performance under the Sharpe ratio and utility criteria. However, optimal portfolios are often outperformed by the naïve equal weights portfolio and estimation error in parameters of returns distribution may offset gains of diversification. Kajtazi & Moro (2019) use a mean-CVar approach to study bitcoin diversification role in different markets — USA, China and Europe -– finding converging results across markets for performance improvement, more due to increase in returns rather than reduction of risk. These conclusions, hold mainly for long-only and naïve portfolios, in conformity to other findings in the literature, such as Platanakis *et al.* (2018) and DeMiguel *et al.* (2007). Under a different approach, Guesmi *et al.* (2019) use a set of time series models to analyze joint dynamics of bitcoin and different financial assets and conclude that investors may benefit from hedging and diversification gains adding bitcoin to their portfolios.

It is important to note that research in this field is expected to increase as cryptocurrencies present changing characteristics over time, as they move from infancy (Urquhart, 2016) to maturity. Besides, authorities seek to reverse cryptocurrencies' lack of regulation, which may lead these assets to present new empirical features in the future. Thus, even stylized facts presented in the literature may not be fully settled yet.

3 Diversification with Cryptocurrencies

In this section, we briefly discuss how different cryptocurrencies may help a prospective investor to diversify her portfolio of risky and/or riskless assets. Thus, to calculate the expected return of the portfolio, we just sum the expected return of each security considering its weights in the portfolio (p):

$$R_p = (R_1 w_1 + R_2 w_2 + ... + R_n w_n) \tag{1}$$

Where $i = 1, ..., n$ indexes individual securities. As first noted by Markowitz (1952), the standard deviation of a portfolio is not the weighted average of the standard deviations of the securities that are in the portfolio.[5] Instead, a portfolio's SD is given by:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1\rho_{1,2}\sigma_1\sigma_2} \tag{2}$$

Where Equation 2 is suitable to a portfolio composed by two securities. Since $Cov[R_1 R_2] = \sigma_1\sigma_2\rho_{12}$, one could substitute $Cov[R_1, R_2]$ by $\rho_{12}\sigma_1\sigma_2, [R_1, R_2]$ on Eq. 2.

The two securities case is a good start to diagnose how diversification affects the riskiness of a portfolio. However, we know that adding more securities non perfectly correlated with the securities in the portfolio may optimize the risk-adjusted returns of a portfolio. Thus, we can generalize the riskiness of a portfolio in the context of multiple securities:

[5] Actually, the SD of a portfolio equals the weighted average of the standard deviations of its securities in the particular case where $\rho i, j = 1$, for all i, j. (i.e., where the correlation among all securities in the portfolio are equal to +1.)

$$\sigma^2 = E[(R_p - \mu_p)^2] \quad (3)$$
$$= E[(w_1(R_1 - \mu_1) + w_2(R_2 - \mu_2) + ... + w_n(R_n - \mu_n))^2] \quad (4)$$

Which, in turn, can be represented by:

$$E[w_i w_j (R_i - \mu_i)(R_j - \mu_j)] = w_i w_j Cov[R_i, R_j] \quad (5)$$
$$= w_i w_j \sigma_i \sigma_j \rho_i j \quad (6)$$

Importantly, the portfolio's variance is the weighted sum of all the variances and co-variances. The matrix notation may be useful to think of the generalization of σ_p to a portfolio composed by n securities:

$$\sigma_p^2 = \sum \begin{bmatrix} w_1^2\sigma_1^2 & w_1w_2\sigma_{12} & \dots & w_1w_n\sigma_{1n} \\ w_2w_1\sigma_{21} & w_2^2\sigma_2^2 & \dots & w_2w_n\sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_nw_1\sigma_{n1} & w_nw_2\sigma_{n2} & \dots & w_n^2\sigma_n^2 \end{bmatrix} \quad (7)$$

From Eq. 7, we can note that there are n variances and $n^2 - n$ covariances as terms of the portfolio's variance. Thus, as n (number of securities) increases, the covariances dominate portfolio variances – that's where the mathematical approach come together to the common sense that diversifying is important to minimize risk.

Specifically, we consider the following linear optimization problem:

$$\text{maximize} \quad \frac{\mu_{\text{p}} - \mu_{\text{riskfree}}}{\sigma_{\text{p}}} \quad (8)$$
$$\text{subject to} \quad w_1 \geq 0, w_2 \geq 0, ..., w_n \geq 0, \quad (9)$$
$$w_1 + w_2 + ... + w_n = 1. \quad (10)$$

Where Eq. 8 refers to the Sharpe Ratio, and Eq. 9 and Eq. 10 refer to the no short selling constraint and the no leveraging constraint, respectively. Optimal portfolios without these constraints can also be achieved. Our baseline analyses use the no short selling condition. However, on Section 5.3, we also allow for short sales on both stock market indices and cryptocurrencies.

4 Data and Basic Characteristics

We start our data set by downloading adjusted prices and trade volume from Yahoo Finance, from July, 2010 (start of BTC trading) to May, 2019. The assets we collect data are both worldwide stock market indices (see list on https://finance.yahoo.com/world-indices/) and cryptocurrencies (see https://finance.yahoo.com/cryptocurrencies). We set the frequency of data as weekly, as in Liu & Tsyvinski (2018).[6] Returns are calculated using natural logarithms. To avoid estimating the parameters using different periods (e.g., while BTC started to trade on July 16th, 2010, time series on ETH starts only on August 6th, 2015) we only keep observations (rows) with no missing data. Thus, all data refers to the same time span.

Specifically, our original sample covers 25 worldwide stock market indices and the 15 cryptocurrencies with higher market capitalization on June 2019. Because some of the most traded crypocurrencies started transacting only in late 2017 (like Binance Coin, Tether, Stellar, EOS, TRON, ChainLink, Bitcoin Cash, and Cardano), we only keep cryptocurrencies with history of data starting at least in 2015. After this procedure, our list of 15 cryptos reduces to 4: Bitcoin (BTC), Ripple (XRP), Ethereum (ETH) and Litecoin (LTC). We do a similar procedure with stock market data: we drop CASE30 (Egypt), TA125.TA (Israel) and JN0U_JO (South Africa) because number of missing observations was higher than 75% of the sample ranging from August, 2015 (month where the last crypto started to trade, ETH) to May, 2019. Thus, our final sample covers 22 stock market indices and 4 cryptocurrencies, summing up 139 weeks, as Table 1 shows.

Furthermore, because our mean-variance framework includes a risk-free asset, we match our final data set with risk-free estimates from Fernandez *et al.* (2019), who surveys the Risk-Free Rate (RF) and the Market Risk Premium (MRP) used in 2019 for 69 countries.[7] Finally, we use the Stata user-writen package "*mvport*" to estimate the optimal weights across assets (Dorantes, 2016).

The descriptive statistics are shown in Table 2. Consistently with most of the literature, we find that cryptocurrencies present much higher returns and volatility than stock market indices. Also, most indices show negative skewness coefficients, while three of the four analyzed cryptocurrencies have positive skewness, with the exception of bitcoin. In terms of mean-variance analysis, while the maximum stock index return in the sample is 0.4% a week, crypto's returns are at least twice as large. At the same time, while standard deviation of stock returns range from 4.8% to 1.3%, the less volatile cryptoasset (BTC) shows 11% standard deviation. Among cryptocurrencies, these descriptive statistics also vary. For instance, we notice that ripple (XRP) has the highest mean return but it's median return is negative. Also, it is twice as much riskier than bitcoin, in terms of standard deviation. Finally, all digital assets considered present weak – and sometimes negative – correlation with stock market indices, suggesting they may be useful for diversification.

5 Results

We show here the hypothetical results of a representative investor that holds the country's stock market index as her portfolio[8] and analyzes the possibility of incorporating cryptocurrencies.

[6] In the unavoidable trade-off between sample size and noisy of data, we concluded that weekly is an appropriate choice rather than daily (too noisy) or monthly (too few observations).

[7] The measure we use in the analysis is the median of the risk-free rate for each country.

[8] Even though stock market indices act just as references and are non-negotiable, one could mimic its performance by, e.g., buying and holding an ETF (BOVA11, in the Brazilian case).

Table 1: Stock market indices and cryptocurrencies used in the analysis

Ticker	**Index**	**Country**
	Panel A - Stock Market Indexes	
GSPC	S&P500	US
GSPTSE	S&P/TSX Composite Index	Canada
FTSE	FTSE 100	UK
GDAXI	DAX Performance Index	Germany
FCHI	CAC 40	France
BFX	BEL 20	Belgium
IMOEX.ME	MCX	Russia
BVSP	Ibovespa	Brazil
IPSA	S&P/CLX IPSA	Chile
MXX	IPC MEXICO	Mexico
MERV	Merval	Argentina
N225	Nikkei 225	Japan
HSCE	Hang Seng China Enterprises	China
HSI	Hang Seng Index	Hong Kong
KS11	KOSPI Index	South Korea
TWII	TSEC Weighted Index	Taiwan
KLSE	FTSE Bursa Malaysia KLCI	Malaysia
JKSE	Jakarta Composite Index	Indonesia
STI	Straits Times Index	Singapore
BSESN	BSE SENSEX	India
AXJO	S&P/ASX 200 Index	Australia
NZ50	S&P/NZX 50	New Zealand
	Panel B - Cryptocurrencies	
BTC-USD	Bitcoin	-
XRP-USD	Ripple	-
ETH-USD	Ethereum	-
LTC-USD	Litecoin	-

Source: authors' elaboration.

We assume the investor maximizes risk-adjusted expected returns.[9] We use weekly returns from January 2014 to May 2019 to calculate μ, σ, ρ, and Cov.

[9]This is the classical assumption arising from the Efficient Market Hypothesis (EMH) and applied to modern finance theory and models, such as the CAPM and the APT.

Table 2: Descriptive statistics of weekly returns, full sample

Ticker	Mean	p50	SD	Min	Max	Skewness	Kurtosis	N
Panel A - Stock Market Indexes								
GSPC	0.002	0.003	0.017	-0.074	0.046	-0.939	6.270	139
GSPTSE	0.001	0.002	0.016	-0.048	0.054	-0.257	4.607	139
FTSE	0.001	0.002	0.018	-0.049	0.069	0.057	4.623	139
GDAXI	0.002	0.004	0.023	-0.082	0.064	-0.645	4.582	139
FCHI	0.002	0.002	0.022	-0.068	0.055	-0.355	3.712	139
BFX	0.001	0.003	0.020	-0.070	0.040	-0.491	3.439	139
IMOEX.ME	0.004	0.003	0.021	-0.064	0.066	0.129	3.257	139
BVSP	0.004	0.005	0.033	-0.105	0.166	0.592	6.675	139
IPSA	0.002	0.002	0.018	-0.046	0.109	1.296	10.311	139
MXX	0.000	0.000	0.022	-0.078	0.065	-0.081	4.539	139
MERV	0.004	0.006	0.048	-0.151	0.161	-0.301	4.505	139
N225	0.002	0.003	0.026	-0.085	0.088	-0.330	4.675	139
HSCE	0.002	0.000	0.030	-0.073	0.078	-0.055	2.921	139
HSI	0.002	0.003	0.024	-0.054	0.056	-0.234	2.901	139
KS11	0.001	0.003	0.018	-0.080	0.045	-0.731	5.774	139
TWII	0.002	0.004	0.019	-0.050	0.041	-0.410	3.287	139
KLSE	0.000	0.000	0.013	-0.044	0.041	-0.218	4.465	139
JKSE	0.001	0.003	0.017	-0.047	0.054	-0.134	3.411	139
STI	0.001	0.002	0.018	-0.045	0.068	0.310	4.209	139
BSESN	0.003	0.004	0.019	-0.051	0.062	0.020	3.223	139
AXJO	0.001	0.001	0.016	-0.040	0.052	0.256	3.769	139
NZ50	0.003	0.003	0.013	-0.035	0.037	-0.114	3.091	139
Panel B - Cryptocurrencies								
BTC	0.014	0.017	0.111	-0.482	0.311	-0.588	5.783	139
XRP	0.016	-0.018	0.236	-0.624	1.27	2.776	15.68	139
ETH	0.008	-0.006	0.195	-0.521	0.687	0.543	4.9	139
LTC	0.019	0.005	0.161	-0.367	0.771	1.743	9.494	139

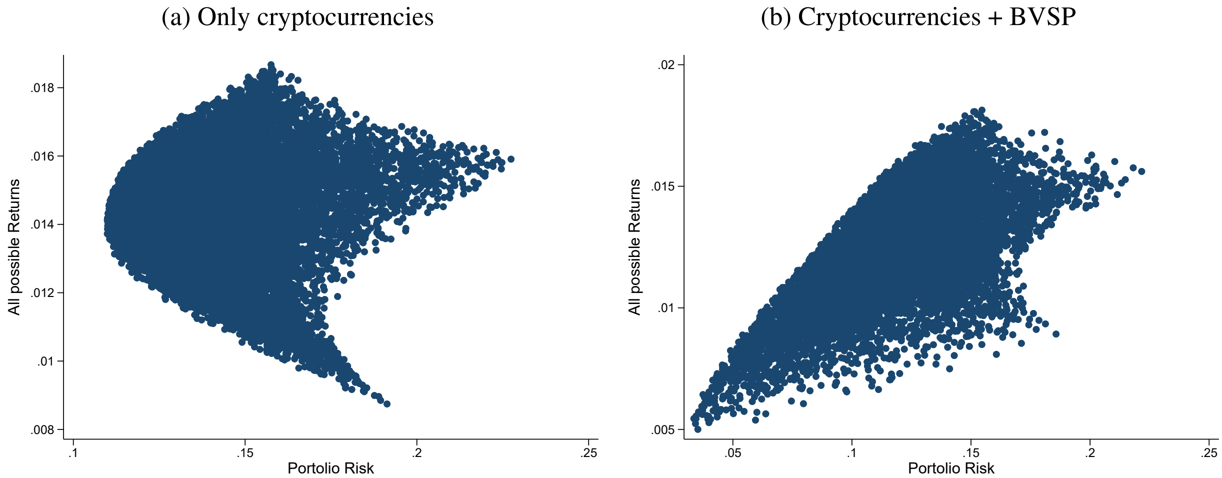
Source: authors' elaboration.

5.1 Feasible random portfolios with traditional and non-traditional assets

We begin our results by showing the feasible set of asset allocation after simulating 10,000 random portfolios. Figure 1 plots these simulations. On Subfigure 1a, which combines only cryptocurrencies (BTC, XRP, ETH and LTC), we can see that it is possible to increase expected returns (for a given level of risk) of the portfolio by diversifying across cryptos. As shown in the Appendix, the pairwise correlation across cryptos ranges from 0.38 (ETH and XRP) to 0.71

(XRP and LTC), which is not as high as the correlation among some stock market indices (for example, the correlation between DAX and CAC 40 is 0.94 in our sample). The efficient frontier – which represents all combinations of assets that offers the highest possible expected return for a given level of risk – can be seen in the northwestern points of Subfigure 1a, starting on the portfolio that minimizes risk (x axis).

Figure 1: Feasible random portfolios with cryptocurrencies and stock market indices, 10,000 simulations



Note: The left-side graphic includes 4 cryptocurrencies: BTC, XRP, ETH and LTC. The right-side graphic includes the previous four assets + BVSP (Ibovespa, Brazil).

Subfigure 1b shows the 10,000 random portfolios composed by BTC, XRP, ETH, LTC and one stock market index: BVSP (Brazil). As we can see, the efficient frontier becomes clear as different combinations of these five assets are simulated. BVSP is just a reference: we run this analysis for each country in our sample, seeking to identify the combinations of cryptocurrencies and stock market indices (portfolios) that form the efficient frontier. Later, we also add country-specific risk free-rates so we can identify one singular point in the efficient frontier: the so called "tangent" or "market portfolio", considering investments in traditional (proxied by stocks), non-traditional (proxied by cryptoassets) assets, and the possibility to invest in a risk-free asset.

5.2 Optimization: finding country-specific tangent and global minimum variance portfolios

The core analysis of this paper is to estimate the efficient frontier and both the tangent and the minimum variance portfolio considering prospective investors that hold a well diversified portfolio of stocks in a given country. Assuming a representative investor in this condition, and expanding the universe of investable assets to cryptocurrencies (BTC, XRP, ETH, and LTC), we find the weights that optimize a certain objective function (either minimize total risk or maximize risk-adjusted returns).

Table 3 shows the results of this analysis. Panel A – "Tangent Portfolio" shows the estimated weights for the tangent portfolio. First, we can observe that, although riskier, cryptocurrencies boosted risk-adjusted returns in all countries, both because of the higher average return and the very low correlation with stock market indices. Digging deeper in the results, we can infer that BTC dominates the competing cryptocurrencies regarding its share on the portfolio that maximizes the Sharpe Ratio – it averages (p50) 32.0% (24.8%) of the optimal portfolio across countries, with non-negative weights in all estimations. LTC also appears to benefit the risk-adjusted returns, even in the presence of BTC — it seems that combining cryptocurrencies is a good idea instead of picking just one to invest. XRP only entered the portfolio in New Zealand, with a close to zero weight (0.1%). The weight of ETH on the optimal portfolios is consistently zero. As a summary, the average (p50) share of the stock market index on the tangent portfolio is 54.1% (65.1%), ranging from 0% in UK, Mexico, Malaysia and Indonesia to 82.4% in Russia, 84.6% in the US, and 93.0% in New Zealand.

Table 3: Optimal weights for the tangent and the global minimum variance portfolios, per country

This table summarizes the results of the country-level optimization. We consider a portfolio of well diversified stocks (proxied by the local stock market index) and four cryptocurrencies: BTC, XRP, ETH and LTC. "SMI" stands for Stock Market Index. Optimal weights (w_i^*) for all securities are provided for both the tangent portfolio (Panel A) and the global minimum variance portfolio (Panel B). Sharpe Ratio and the Risk-free rate (Rf, in annual basis) are also shown.

		Panel A - Tangent Portfolio						**Panel B - Global Minimum Variance Portfolio**					
Country	Rf (year)	w^*_{SMI}	w^*_{BTC}	w^*_{XRP}	w^*_{ETH}	w^*_{LTC}	Sharpe R.	w^*_{SMI}	w^*_{BTC}	w^*_{XRP}	w^*_{ETH}	w^*_{LTC}	Sharpe R.
US	0.028	0.846	0.103	0	0	0.051	0.157	1	0	0	0	0	0.116
Canada	0.025	0.615	0.265	0	0	0.120	0.135	1	0	0	0	0	0.051
UK	0.020	0	0.701	0	0	0.299	0.133	1	0	0	0	0	0.028
Germany	0.010	0.645	0.251	0	0	0.104	0.142	1	0	0	0	0	0.076
France	0.010	0.611	0.274	0	0	0.114	0.139	1	0	0	0	0	0.069
Belgium	0.010	0.360	0.454	0	0	0.186	0.135	1	0	0	0	0	0.047
Russia	0.083	0.824	0.115	0	0	0.060	0.160	0.973	0.016	0.009	0.002	0	0.122
Brazil	0.074	0.669	0.225	0	0	0.107	0.143	0.927	0.072	0	0	0.001	0.101
Chile	0.045	0.741	0.193	0	0	0.066	0.140	0.994	0.006	0	0	0	0.079
Mexico	0.074	0	0.678	0	0	0.322	0.125	0.990	0.009	0.001	0	0	-0.044
Argentina	0.097	0.439	0.362	0	0	0.200	0.127	0.879	0.100	0	0.001	0.020	0.081
Japan	0.010	0.664	0.242	0	0	0.094	0.146	0.977	0.023	0	0	0	0.086
China	0.038	0.607	0.282	0	0	0.111	0.141	0.924	0.059	0.016	0.001	0	0.087
Hong Kong	0.021	0.740	0.197	0	0	0.063	0.154	0.955	0.039	0.006	0	0	0.107
South Korea	0.025	0.680	0.233	0	0	0.088	0.139	0.988	0.012	0	0	0	0.066
Taiwan	0.018	0.804	0.145	0	0	0.051	0.158	0.980	0.020	0	0	0	0.108
Malaysia	0.043	0	0.692	0	0	0.308	0.129	0.998	0	0.002	0	0	-0.052
Indonesia	0.070	0	0.682	0	0	0.318	0.125	0.987	0.013	0	0	0	0.012
Singapore	0.026	0.314	0.478	0	0	0.208	0.133	0.995	0.003	0.002	0	0	0.031
India	0.066	0.763	0.171	0	0	0.066	0.140	0.988	0.012	0	0	0	0.086
Australia	0.028	0.657	0.244	0	0	0.099	0.136	0.993	0	0.007	0	0	0.057
New Zealand	0.030	0.930	0.059	0.001	0	0.010	0.209	0.999	0	0	0.001	0	0.183
Mean	0.039	0.541	0.320	0.000	0	0.138	0.143	0.979	0.017	0.002	0.000	0.001	0.068
p50	0.028	0.651	0.248	0.000	0	0.105	0.140	0.991	0.007	0.000	0.000	0.000	0.078

Source: Author's elaboration.

Regarding the global minimum variance portfolio (Panel B), we get a very different picture. For US, Canada, UK, Germany, France and Belgium, the GMVP is 100% the local stock market index. Except for Argentina (87.9%), the share of the local stock market in the GMVP is at least 92.4%. Nevertheless, small, marginal contributions to reduce risk are found in all cryptocurrencies – again, BTC leads (1.7%, on average), followed by XRP (0.2%), LTC (0.1%), and ETH (0.0%). These results suggest that cryptocurrencies have very limited effect on reducing standard deviation of a well-diversified portfolio of stocks, but can improve risk-adjusted returns significantly.[10] Kajtazi & Moro (2019) found similar results, noting that addition of bitcoin to diversified portfolios enhances performance more due to increasing returns than diminishing volatility.

5.3 Is there a global portfolio of stock market indices and crytocurrencies?

Instead of estimating country-specific efficient portfolios, we now put everything together and estimate the marginal effects of cryptoassets on a well-diversified portfolio of global stocks. The results of this exercise using all market indices in our sample and the cryptocurrencies are shown in Figure 2. Expected returns (μ) are shown on y axis, while standard deviation (σ) is shown on the x-axis. The blue line represents all the possible combinations between stock market indices and cryptocurrencies that lie in the frontier. Starting on the as west as possible asset allocation (minimum variance portfolio) in Subfigure 2a, all the combinations above this point form the efficient frontier. Importantly, the results reported on Figure 2 assume that there is no short selling (i.e., $w_1 + w_2 + ... + w_n = 100\%$, w_1, w_2, ..., w_2 >= 0 and w_1, w_2, ..., w_2 <=1).

Once we observe the tangent and the minimum variance portfolios in Figure 2, as well as the capital market line (all possible allocation between the tangent portfolio and the risk-free rate[11]), the following questions remains: which combination of assets yield the optimal and the minimum variance portfolios? What would be the results if we allow for short selling? Table 4 helps us to answer these questions.

A short position in an uncertain asset means that one is borrowing that asset, selling it at the present moment and incurring in the future obligation to buy it back and return it to the original lender. This implies that, for that asset, weights in a portfolio will be negative. As shown in Table 4, the optimal portfolio with short sales result in short positions for XRP and LTC, while remaining in long positions for BTC and ETH. We notice that in this case, BTC weight has been multiplied by four compared to the long-only scenario. In contrast, the GMV portfolio results in near zero weights for all cryptocurrencies, consistent with the long-only results.

A second and interesting approach is to include the possibility of a prospective investor to borrow and loan at the risk-free rate. In the U.S. context, it could be thought of a U.S. Treasury Bill or Bond (nearly no credit risk). In emerging markets, one approach to estimate the risk free

[10] Off course, one have to consider that we are using past returns to project expected returns. Since this is a disruptive, quickly changing technological market and the intrinsic value of cryptocurrencies is very hard to measure (if it is measurable at all), assumptions about the future based on past returns are subject to large errors.

[11] For the global portfolio, we assume the U.S. risk-free rate as proxy for the global risk-free rate.

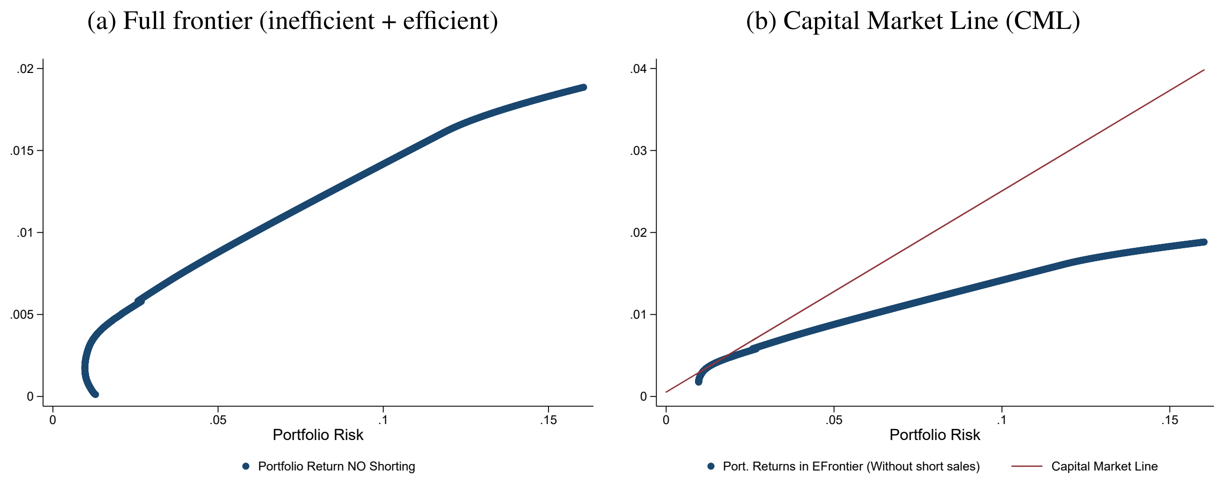
Table 4: Global optimal portfolio considering all stock market indices and crytocurrencies

This table summarizes the results of the global optimization considering twenty two stock market indices and four cryptocurrencies (BTC, XRP, ETH and LTC) all together. Optimal weights for all securities are provided for both the tangent portfolio (Panel A) and the global minimum variance portfolio (Panel B). "W/ Short Sales" means that the investor can sell the security without owning it and, at no cost and at the quantity she desires.

	Panel A: Tangent Portfolio		**Panel B: GMV Portfolio**	
Country	*No Short Sales*	*W/ Short Sales*	*No Short Sales*	*W/ Short Sales*
US	0	0.665	0	0.001
Canada	0	-0.814	0	0.047
UK	0	-1.109	0	0.080
Germany	0	-0.176	0	-0.113
France	0	0.544	0	-0.199
Belgium	0	-0.747	0	0.114
Russia	0.299	1.052	0.052	0.105
Brazil	0	0.531	0	-0.093
Chile	0.063	1.058	0.039	0.150
Mexico	0	-1.351	0	-0.047
Argentina	0	-0.046	0	0.000
Japan	0	0.226	0	0.015
China	0	-0.869	0	-0.202
Hong Kong	0	1.581	0	0.169
South Korea	0	0.364	0	0.081
Taiwan	0	0.433	0	-0.062
Malaysia	0	-1.520	0.381	0.406
Indonesia	0	0.707	0.068	0.046
Singapore	0	-1.234	0	0.025
India	0	0.071	0	0.016
Australia	0	0.091	0.036	0.128
New Zealand	0.592	1.394	0.424	0.343
BTC	0.046	0.165	0	0.005
XRP	0	-0.024	0	0.002
ETH	0	0.019	0	-0.004
LTC	0	-0.012	0	-0.013
Summ	1.000	1.000	1.000	1.000
E(R)	0.004	0.017	0.002	0.001
SD	0.013	0.037	0.010	0.008
Sharpe Ratio	0.245	0.440	0.126	0.092

Source: Author's elaboration.

Figure 2: Full frontier and Capital Marker Line considering all stock market indices, all cryptocurrencies and a risk-less security



Note: The estimate considers all traditional (stock market indices) and non-traditional (cryptocurrencies) assets together ("horse race"). We consider the US risk free rate as the risk free rate for this global, efficient portfolio.

Source: Author's elaboration.

rate is to consider the yield of the country's treasury debt (say, 10 years) minus the fraction of this rate plausible linked to the country's credit risk, such as using the interest premium on Credit Default Swaps (CDS) (see, e.g., Damodaran, 2008). With a riskless asset available, the investor could not only combine investments across *risky* assets, but also with this riskless investment, according to her risk aversion.

We show the results on Figure 2. The well known Capital Market Line (CML, in maroon) represents the combinations between the riskless security and the efficient or tangent portfolio of risky assets. Any combination between the risk-free asset and the tangent portfolio is strictly preferred to investing only in risky assets.[12] The efficient portfolio is the one that gives, among all other combination between the two risky assets, the maximum Sharpe Ratio (i.e., maximum units of return per unit of risk).

At a first glance, it seems that combining a small fraction of cryptocurrencies to a well diversified portfolio of global stocks may be a good idea for an investor interested in maximizing risk-adjusted returns. Even though assuming a different model and in other context (they look only to the US market), our results seem consistent with those of Liu & Tsyvinski (2018), whom find that the optimal fraction of BTC would be 6.1%.[13] In our global portfolio, the optimal holding of cryptocurrencies for an investor that seeks to maximize the Sharpe ratio is 4.6% (BTC),

[12]Except for the tangent portfolio itself, where the investor's utility would be unaltered.

[13]Same as we do here, Liu & Tsyvinski (2018) assume in this analysis that the investor expects BTC to continue do as well as the past years.

while the larger fraction of the investment should be allocated in New Zealand (59.2%), Russia (29.9%), and Chile (6.3%).

5.4 Robustness test: how sensitive are the optimal weights to different time spams?

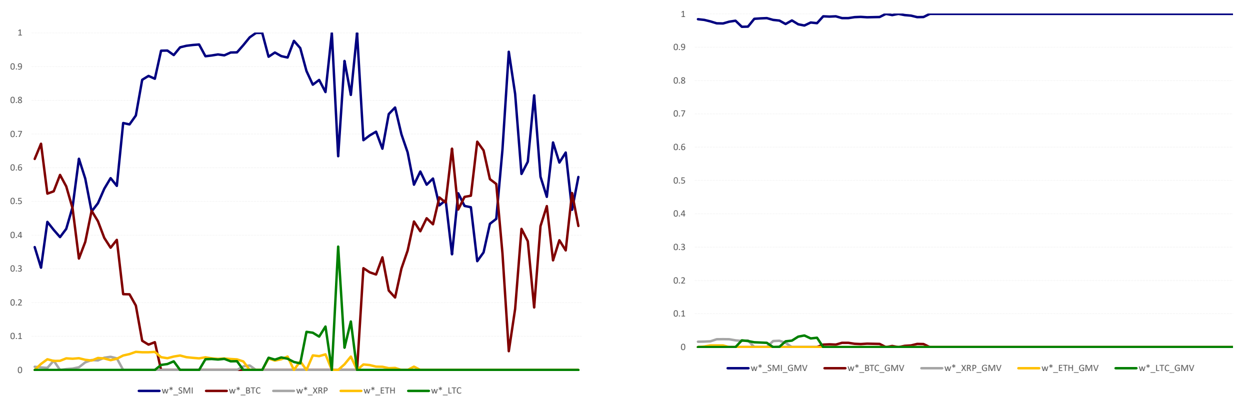
One remaining question is whether the optimal weights calculated before are sensitive (and, if so, how much) to different time spams. To answer this question, we optimize the global portfolio composed of world-wide stock portfolios (22 stock market indices) and cryptocurrenies (BTC, XRP, ETH, and LTC) considering rolling windows of 52 weeks of returns.[14] Our period initiates in the 33rd week of 2015 (2015w33) and ends in the 22nd week of 2019 (2019w22). Because we impose to have at least 52 observations to estimate the optimal weights, our first optimization refers to 2016w32.

To facilitate visualization, we reproduce in Figure 3 the optimal weights of stock market indices (summed up) and the cryptocurrencies individually. While on Subfigure 3a we plot the optimal weights for the tangent portfolio, on Subfigure 3b we present the optimal weights for the GMVP.

Figure 3: Optimal weights across rolling windows, tangent and GMV portfolios

(a) Tangent Portfolio

(b) Global Minimum Variance Portfolio



Note: This figure shows the optimal weights of stock market indices and cryptocurrencies considering rolling windows of 52 weeks of return. We consider the US risk free rate as the risk free rate for this global, efficient portfolios.

Source: Author's elaboration.

From Subfigure 3a, we can see that the optimal weights of cryptocurrencies in the tangent portfolio is indeed a lot sensitive to the sample period – it ranges from 0% to 69.7%. Furthermore,

[14]We do that to have one year of weekly returns for each estimation. Since our final sample covers 139 weekly returns, we have (139-52=) 87 sequential estimations.

which cryptocurrency marginally improves a global portfolio of stocks is also period-sensitive. BTC is the one that fits best in most rolling windows (its weight averages 25.2%), while XRP (0.3%), ETH (1.8%) and LTC (1.7%) enters the tangent portfolio sporadically.

Regarding the GMVP, as shown in Subfigure 3b, the sensitivity of portfolio weights to different time spams is lower. Stock market indices dominates cryptocurrencies on the rolling GMVPs. On average, the weight of cryptocurrencies for the GMVP reaches 0.7%, with the maximum being 3.9% and the median, 0%. The specific cryptocurrency also matters here: unlike in the tangent portfolio, XRP and LTC are those that enters the portfolio more often, while ETH and BTC barely appears.

6 Concluding Remarks

Using a data set of world-around stock market indices, the most liquid cryptocurrencies, and estimated risk-free rates for 22 countries, we analyze whether adding crytocurrencies to a well diversified portfolio of stocks may enhance risk/adjusted returns. Our results suggest that the answer may be yes. However, the optimal holding of cryptocurrencies depends heavily on which asset (BTC and LTC performed better than XRP and ETH) and on which stock market one considers. While cryptocurrencies fit realtively well in tangent portfolios, its marginal impacts on global minimum variance portfolios are close to zero.

Globally speaking, even a portfolio composed by stock market indices from twenty two countries might benefit from adding a small fraction of cryptcurrency. However, and very important, we find that the results of the optimization process is very sensitive to the period being analyzed. Thus, real world decisions regarding holding (or not) cryptocurrencies into a diversified portfolio of traditional assets should consider different scenarios for the future performance of these cryptoassets.

Finally, the optimization process proposed in this paper is just one possibility in the study of how traditional and non-traditional assets relate to each other. Further developments include testing the role of other classes of assets, such as housing, corporate bonds, commodities, and fiat currencies. There is also room to consider different scenarios bases on the assumptions regarding expected returns and total risk (standard deviation) for each security. For example, what if the market for cryptocurrency fails? What if it booms and future returns overcome past returns? What if these assets turns out to be less volatile? This broad application may help us to understand patterns and the how heterogeneous are the marginal effects of adding cryptocurrencies to different portfolios around the globe. Finally, one could test the effect of the growing market of derivatives of cryptocurrencies, such as the futures of Bitcoin traded at the CME (Bitcoin Futures – Ticker “BTC=F” for Jul-2019).

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Appendix I - Correlation matrix of returns

Table 5: Correlation matrix of weekly returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
(1) GSPC	1.00																									
(2) GSPTSE	0.76	1.00																								
(3) FTSE	0.67	0.67	1.00																							
(4) GDAXI	0.69	0.64	0.76	1.00																						
(5) FCHI	0.69	0.70	0.82	0.94	1.00																					
(6) BFX	0.65	0.63	0.75	0.87	0.88	1.00																				
(7) IMOEX.ME	0.48	0.50	0.46	0.52	0.54	0.45	1.00																			
(8) BVSP	0.49	0.58	0.50	0.45	0.49	0.43	0.37	1.00																		
(9) IPSA	0.41	0.47	0.47	0.46	0.48	0.43	0.26	0.54	1.00																	
(10) MXX	0.56	0.54	0.49	0.49	0.51	0.49	0.42	0.54	0.58	1.00																
(11) MERV	0.45	0.42	0.43	0.41	0.42	0.37	0.30	0.48	0.37	0.37	1.00															
(12) N225	0.60	0.55	0.58	0.69	0.72	0.71	0.41	0.37	0.30	0.37	0.35	1.00														
(13) HSCE	0.52	0.51	0.51	0.52	0.55	0.52	0.39	0.55	0.40	0.42	0.35	0.59	1.00													
(14) HSI	0.55	0.53	0.56	0.56	0.57	0.58	0.38	0.52	0.39	0.43	0.36	0.63	0.95	1.00												
(15) KS11	0.60	0.55	0.51	0.58	0.59	0.62	0.34	0.39	0.44	0.50	0.38	0.63	0.66	0.70	1.00											
(16) TWII	0.61	0.57	0.58	0.57	0.62	0.58	0.43	0.51	0.47	0.54	0.37	0.57	0.72	0.74	0.71	1.00										
(17) KLSE	0.44	0.50	0.48	0.51	0.54	0.46	0.34	0.53	0.49	0.44	0.29	0.39	0.55	0.55	0.54	0.62	1.00									
(18) JKSE	0.37	0.38	0.42	0.42	0.40	0.40	0.23	0.46	0.44	0.47	0.35	0.26	0.37	0.43	0.46	0.51	0.61	1.00								
(19) STI	0.59	0.59	0.58	0.59	0.63	0.58	0.39	0.55	0.42	0.39	0.34	0.65	0.71	0.74	0.61	0.65	0.55	0.46	1.00							
(20) BSESN	0.48	0.43	0.54	0.52	0.53	0.53	0.35	0.49	0.37	0.38	0.25	0.49	0.55	0.56	0.45	0.52	0.41	0.40	0.54	1.00						
(21) AXJO	0.56	0.63	0.60	0.63	0.64	0.55	0.43	0.42	0.32	0.36	0.27	0.51	0.46	0.43	0.38	0.41	0.38	0.26	0.54	0.52	1.00					
(22) NZ50	0.44	0.38	0.36	0.33	0.36	0.34	0.16	0.26	0.18	0.30	0.10	0.27	0.23	0.24	0.20	0.30	0.12	0.11	0.29	0.37	0.42	1.00				
(23) BTC	0.26	0.19	0.24	0.24	0.24	0.23	0.06	0.03	0.13	0.15	0.13	0.13	-0.03	0.00	0.08	0.04	0.11	0.07	0.13	0.10	0.10	0.14	1.00			
(24) XRP	0.09	0.07	0.09	0.11	0.10	0.12	-0.05	0.02	0.25	0.06	0.04	0.07	-0.12	-0.05	0.09	0.03	0.02	0.11	0.04	0.05	-0.04	0.05	0.44	1.00		
(25) ETH	0.20	0.19	0.25	0.23	0.22	0.22	0.02	0.09	0.27	0.28	0.11	0.13	-0.01	0.05	0.11	0.10	0.14	0.16	0.08	0.10	0.13	0.05	0.42	0.38	1.00	
(26) LTC	0.21	0.14	0.22	0.23	0.22	0.26	0.03	0.03	0.21	0.11	0.05	0.15	0.02	0.07	0.14	0.08	0.07	0.08	0.12	0.15	0.14	0.20	0.66	0.71	0.40	1.00

Note: This table shows the correlation matrix among returns of stock market indices and cryptocurrencies. Returns are calculated on weekly frequencies.