# A Multi-Agent Computational Model for Brazilian Stock Market:
## The "Gap Value" Channel of Monetary Policy Transmission Mechanism

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**Área da Anpec:** Área 4 – Macroeconomia, Economia Monetária e Finanças
**Resumo**: Este artigo apresenta a estrutura e os resultados de um modelo computacional baseado em vários agentes heterogêneos que mede os efeitos das mudanças nas taxas de juros básica e esperada sobre a série do valor intrínseco da carteira teórica do Ibovespa. Isto possibilita *insights* originais sobre o mecanismo de transmissão da política monetária sobre o valor intrínseco da carteira teórica do Ibovespa no período de 2008/2010. Os resultados do modelo quantificam bolhas no mercado de capitais (*bullmarket*) e também situações de *bearmarket*. O modelo também estima uma série semanal de hiato de valor para o Ibovespa. O hiato de valor (GV) é um novo índice de *value investing* proposto pelos autores. Duas conclusões do modelo: (i) o mecanismo de transmissão da política de juros do Copom afetará o GV do Ibovespa negativamente somente quando as companhias que fazem parte deste índice, apresentarem geração agregada líquida de caixa negativa; e (ii) as decisões do COPOM de aumentar (diminuir) a taxa Selic resultarão em aumentos (reduções) no GV do Ibovespa.
**Palavras-chave**: economia computacional baseada no agente, índice de hiato de valor, mercado de ações e mecanismo de transmissão da política monetária.
**Códigos do JEL**: C63, E44, E52.
**Abstract:** This article presents the structure and the results of a computational model based on several heterogeneous agents that measures the effects of changing basic and expected interest rates on the series of the intrinsic value of Ibovespa`s (Sao Paulo Stock Exchange Index) theoretical portfolio. This may give original insights on the effects of the transmission mechanism of monetary policy on this series in the period of 2008/2010. The model results quantify both bull market and bear market. The model also estimates a weekly time series of gap values of the Ibovespa. Gap value (GV) is a new index of value investing proposed by the authors. Two conclusions of the model are: (i) COPOM's decisions to increase (reduce) the benchmark interest rate will invariably result in increased (reduced) Ibovespa`s GV; and (ii) the transmission mechanism of monetary policy will affect Ibovespa`s GV negatively only when the companies that make up this index have, aggregated negative net cash generation[3].
**Keywords**: agent-based computational economics (ABCE), gap value index, stock market and monetary policy transmission mechanism.
**JELCodes**: C63, E44, E52.

## 1. Introduction

This paper introduces a computer-based, multi-agent financial model (henceforth, "computational model" or "model") that uses stock valuation techniques and computer simulation to estimate two time series: one with intrinsic values for Bovespa (Sao Paulo Stock Exchange) Index Theoretical Portfolio and another with a proposal of a index of value investing, named Ibovespa`s gap value.

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Section 2 describes the computer-based model and its five modules. It also introduces the notion of "gap value"[4], GV, which intends to be an index to detect and measure financial market bubbles and circumstances under which Ibovespa (Bovespa Index) market value is higher than its intrinsic value (bull market) or the other way round (bear market). Section 3 presents the corollaries of the model. Section 4 reports the results of a graph analysis of Ibovespa series of intrinsic value elasticity and GV elasticity compared to a series of benchmark and expected interest rates. It particularly highlights the events that have influenced the series behavior. Section 5 provides final remarks.

## 2. Model Development

The model includes four modules as described in the following subsections.

### 2.1. Module 1 – Ibovespa companies' cash flow

The model, developed using the software packages *Mathematica* 7[5] and Excel 2007, builds on the discounted cash flow technique (DCF). According to Damodaran (2012), leveraged companies are best assessed on the basis of their free cash flow to the firm (FCFF)[6]. FCFF is derived from EBIT (earnings before interest and taxes). Table 1 shows how this process occurs, including their meaning and data source. The cash flows included in the model were extracted from 816 accounting statements[7] from 2008 through 2010. The cash flows for the two subsequent years (the 2011-2012 biennium) were estimated using the least squares method[8].

Table 1 – Accounts used for cash flow calculation

| Symbol | FCFF item | Meaning | Data source in period t and previous periods |
|---|---|---|---|
| (=) | $NS_t$ | Net Sales Revenue in period t | Income Statement for period t |
| (-) | $SC_t$ | Sales costs in t | Income Statement for period t |
| (-) | $OE_t$ | Operating Expenses in t | Income Statement for period t |
| (=) | $EBIT_t$ | Earning before income and taxes in period t | |
| (+) | Depreciation/Amortization | Depreciation/Amortization in t | Cash Flow Statement for period t |
| (-) | Income tax (IT) + social charges | Income Tax (IT) + Social Charges in t | Income Statement for period t |
| (=) | Operational cash generation | Operational Cash Generation in t | |
| (-) | Permanent current investiments (Working Capital) | Permanent Current Investments (Working Capital) in t | Income Statement and Balance Sheet for period t |
| = | $FCFF_t$ | Free cash flow to the firm for period t | |

Source: Adapted from Damodaran (2012), Abrams (2001) and Stewart III (1991). The methodology was adapted according to current accounting standards in Brazil.

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[4]In the rest of text, this term wil be called GV.

[5]The use of this software relies on its power to work with symbolic mathematics (or literal algebra). Calculations using symbolic mathematics are common in models of macro and microeconomics as well as in computer-based models.

[6]Leveraged companies are those companies that use debt to finance their operations. They are not necessarily very debt-laden companies, but companies with any given burden of debt ( Damodaran 2002  and Bisciari, Durré & Nyssen 2003).

[7]These data are the standardized financial statements available from the Brazilian Stock Market Regulatory Comission (Comissão de Valores Mobiliários/CVM) website.

[8]Excel 2007 was used to calculate the cash flows, and function PROJ.LIN was used to calculate the least squares.

## 2.2. Module 2 – Inter-temporal discounting model

This model includes the time horizon of the cash flow projections to be discounted. Before projecting a company's cash flow, it is important to decide how many periods or stages will be included for cash flow in perpetuity. In accordance with Cuberthson & Nitzsche (2004: 21) and Stewart III (1991: 97), a number of stages can be identified in a business growth to maturity[9], depending on the characteristics of each company. As one of the goals of the model is to build a series of weekly intrinsic values for the companies making up the Bovespa Index, they will be assumed to have two stages for the sake of simplicity. In this assumption, the first stage corresponds to the short term, while the second stage refers to an infinite time horizon (i.e., in perpetuity)[10]. The formula for the first stage[11] is:

$$IV_A = \frac{FCFF_1}{(1+i)^1} \quad (1)$$

Where: $IV_A$ stands for the intrinsic value in the first stage, $FCFF_1$ is the cash flow in the present period, and it refers to the benchmark/base interest rate as determined by COPOM (Monetary Policy Committee, Central Bank of Brazil). All these variables are calculated on a weekly basis. The intrinsic value in stage 1 is equal to the sum of results of a simple discount formula in a given period, where the basic interest rate $i$ [12] substitutes every company's weighted average cost of capital (WACC), a variable commonly adopted in finance and valuation techniques[13]. The second stage assumes that the net cash flow result is in perpetuity[14]. The inter-temporal discounting formula in the second stage is as follows:

$$IV_B = \frac{FCFF_3 / (i_e - g_2)}{(1+i_e)^2} \quad (2)$$

Where: $IV_B$ stands for the intrinsic value in the second stage, $FCFF_3$ refers to the free cash flow to the firm in the third period (t+2), $i_e$ is the expected basic interest rate, $g_2$ corresponds to the growth rate in the second stage. All these variables are calculated on a weekly basis. As shown, discounting formula (2) includes an expected growth rate ($g_2$) and an expected interest rate ($i_e$)[15]. Rate $g_2$ is a geometric mean including the per share profit variation in period 2001-2010 plus an investors' confidence component (Ψ). An animal spirit variable ( Ψ ) stands for the stockholders' willingness to invest. This variable corresponds to an adjusted average rate of return on equity in the 1996-2010 period and a rate of variation obtained from the exam of the technical analysis

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[9]Growth to maturity: period of transition and adjustment of a company's return and risk.

[10]According to Bodie, Kane & Marcus (2003) and Damodaran (2011), two-stage valuation models adopt the simplifying hypothesis that the companies will not experience any extraordinary changes in the upcoming years, yet they will experience some changes to their capital structure, and growth and productivity patterns. Therefore, the ideal is to project an adjustment period (the first stage).The dividend policy will not be definitive before the second stage, which includes the perpetuity. The first stage is expected to have a distinct level of growth with higher levels of net investments, both in tangible assets and working capital.

[11]For a discussion of this formula, see Damodaran (2011: 301), McKinsey & Co. (2000: 76) and Stickney, Weil & Schipper (2009: 143).

[12]It corresponds to the basic Selic interest rate as defined by the Monetary Policy Committee. Selic is the acronym of Custody and Settlement Special System (Sistema Especial de Liquidação e Custódia).

[13]The underlying hypothesis is that the companies making up t the Ibovespa theoretical portfolio have the Selic rate as a proxy of capital cost. This is a useful simplifying hypothesis, as it allows for aggregating all Ibovespa companies without performing individual accounting calculations, which would be laborious and make the model unfeasible. This hypothesis is realistic, as most companies making up the Ibovespa borrow funds from BNDES[1] at lower rates than the COPOM-defined Selic rate. Therefore, the major Ibovespa companies' WACC is assumed not to be distant from $i_b$.

[14] The second stage is the period in which the investment will yield returns at percentages quite close to that of the minimum return rate demanded by the capital providers (creditors and stockholders).

[15] The source for the expected Selic rate ($i_e$) is the Focus Report, which is published by the Central Bank of Brazil on a weekly basis.

trend graphics[16]. Each variable has a weighting of 0.5. It was also used to calibrate the model for the weekly changes in the investors' confidence about the company's stocks. The animal spirit ( $\Psi$ ) is incorporated into the $g_2$ calculation. By including inter-temporal discounting rates ($i_b$ and $i_e$), the model integrates the financial data provided by the cash flows, a variable of monetary policy (the Selic rate defined by COPOM), and the decision rules guiding the behavior of a representative agent. Indeed, this computer-based financial model is also multi-agent-based, and both inter-temporal discounting rates can be understood as minimum attractive rates for the multiple agents making up in the model.

### 2.3. Module 3 – IV and GV in Ibovespa theoretical portfolio

This module targets the IV and GV of both companies and the Ibovespa theoretical portfolio. In general terms, the module is composed of the sum of intrinsic values in both stages. The weekly intrinsic value of a company is, therefore[17]:

$$IV_A + IV_B = \sum_{t=1}^{n} \frac{FCFF_t}{(1+i)^t} + \frac{FCFF_{n+1}/(i_e - g_n)}{(1+i_e)^2} \quad (3)$$

Following the model methodology, 68 intrinsic values are generated (building on the discounted cash flows).[18]

The concept of GV, as introduced in this paper, is aimed as an index that predicts financial market bubbles as well as bull market and bear market conditions. The GV is determined using a weekly time series that comprises the difference between intrinsic values (as shown in Module 2) and market values (number of shares x share price on the cash market) that make up the Ibovespa theoretical portfolio. Equation (5) is a matrix equation that calculates the GV:

$$GV_{(68\times1)} = \begin{bmatrix} GV_1^1 \\ GV_1^2 \\ \vdots \\ GV_1^{68} \end{bmatrix}_{(68\times1)} = \begin{bmatrix} MV_1^1 \\ MV_1^2 \\ \vdots \\ MV_1^{68} \end{bmatrix}_{(68\times1)} - \begin{bmatrix} IV_1^1 \\ IV_1^2 \\ \vdots \\ IV_1^{68} \end{bmatrix}_{(68\times1)} \quad (5)$$

Where: $GV_{(68\times156)}$ is the gap value, $MV_{(68\times156)}$ is the market value of the 68 shares in the Ibovespa theoretical portfolio, and $IV_{(68\times156)}$ is the intrinsic value. All these variables are calculated for 1 week, as represented in the vector columns.

The results are placed in a matrix, where the rows represent the weeks and the columns represent the weekly intrinsic values. Subtracting values of every row vector gives the GV. The equation below shows the same in a more aggregate form:

$$GV_{(68\times156)} = \begin{bmatrix} GV_1^1 & GV_2^1 & \dots & GV_{156}^1 \\ GV_1^2 & GV_2^2 & \dots & GV_{156}^2 \\ \vdots & \vdots & \ddots & \vdots \\ GV_1^{68} & GV_2^{68} & \dots & GV_{156}^{68} \end{bmatrix}_{(68\times156)} = \begin{bmatrix} MV_1^1 & MV_2^1 & \dots & MV_{156}^1 \\ MV_1^2 & MV_2^2 & \dots & MV_{156}^2 \\ \vdots & \vdots & \ddots & \vdots \\ MV_1^{68} & MV_2^{68} & \dots & MV_{156}^{68} \end{bmatrix}_{(68\times156)} - \begin{bmatrix} IV_1^1 & IV_2^1 & \dots & IV_{156}^1 \\ IV_1^2 & IV_2^2 & \dots & IV_{156}^2 \\ \vdots & \vdots & \ddots & \vdots \\ IV_1^{68} & IV_2^{68} & \dots & IV_{156}^{68} \end{bmatrix}_{(68\times156)}$$

Stressing the result of the GV matrix and using an alternative notation gives:

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[16] Sources: several editions of the Exame Magazine's annual list of the greatest companies in Brazil, for the first variable, and Economatica System for Investment Analysis, for the second.

[17] General formula based on Damodaran (2012).

[18] The model generates, therefore, 68 weekly time series of intrinsic values of the shares making up the Ibovespa`s theoretical portfolio. These weekly series include 52 observations each. The results are placed in a matrix, where the rows represent the weeks and the columns represent the weekly intrinsic values. Summing up the values in each row vector gives the weekly intrinsic value of the Ibovespa´s theoretical portfolio. It will become clearer in the description of the following module of the model.

$$GV_{(68x156)} = MV_{(68x156)} - IV_{(68x156)} \qquad (6)$$

Where:

$$GV_{(68\times156)} = \begin{bmatrix} MV_1^1 - IV_1^1 & MV_2^1 - IV_2^1 & \dots & MV_{156}^1 - IV_{156}^1 \\ MV_1^2 - IV_1^2 & MV_2^2 - IV_2^2 & \dots & MV_{156}^2 - IV_{156}^2 \\ \vdots & \vdots & \ddots & \vdots \\ MV_1^{68} - IV_1^{68} & MV_2^{68} - IV_2^{68} & \dots & MV_{156}^{68} - IV_{156}^{68} \end{bmatrix}_{(68\times156)} \qquad (7)$$

For the sake of easy reading of the matrix above, the second column is omitted in order to obtain:

$$GV_{(68\times156)} = \begin{bmatrix} MV_1^1 - \sum_{t=1}^{1}\frac{FCFF_1^1}{(1+i_{b,1})^1}+\frac{FCFF_3^1/(i_{e,1}-g_{e,1,1})}{(1+i_{e,1})^2} & MV_2^1 - \sum_{t=1}^{1}\frac{FCFF_1^1}{(1+i_{b,2})^1}+\frac{FCFF_3^1/(i_{e,2}-g_{e,2,1})}{(1+i_{e,2})^2} & \dots & MV_{156}^1 - \sum_{t=1}^{1}\frac{FCFF_1^1}{(1+i_{b,156})^1}+\frac{FCFF_3^1/(i_{e,156}-g_{e,156,1})}{(1+i_{e,156})^2} \\ MV_1^2 - \sum_{t=1}^{1}\frac{FCFF_1^2}{(1+i_{b,1})^1}+\frac{FCFF_3^2/(i_{e,1}-g_{e,1,2})}{(1+i_{e,1})^2} & MV_2^2 - \sum_{t=1}^{1}\frac{FCFF_1^2}{(1+i_{b,2})^1}+\frac{FCFF_3^2/(i_{e,2}-g_{e,2,2})}{(1+i_{e,2})^2} & \dots & MV_{156}^2 - \sum_{t=1}^{1}\frac{FCFF_1^2}{(1+i_{b,156})^1}+\frac{FCFF_3^2/(i_{e,156}-g_{e,156,2})}{(1+i_{e,156})^2} \\ \vdots & \vdots & \ddots & \vdots \\ MV_1^{68} - \sum_{t=1}^{1}\frac{FCFF_1^{68}}{(1+i_{b,1})^1}+\frac{FCFF_3^{68}/(i_{e,1}-g_{e,1,68})}{(1+i_{e,1})^2} & MV_2^{68} - \sum_{t=1}^{1}\frac{FCFF_1^{68}}{(1+i_{b,2})^1}+\frac{FCFF_3^{68}/(i_{e,2}-g_{e,2,68})}{(1+i_{e,2})^2} & \dots & MV_{156}^{68} - \sum_{t=1}^{1}\frac{FCFF_1^{68}}{(1+i_{b,156})^1}+\frac{FCFF_3^{68}/(i_{e,156}-g_{e,156,68})}{(1+i_{e,156})^2} \end{bmatrix}_{(68\times156)} \qquad (8)$$

Matrix (8) can also be partitioned in the following row-vectors:

$$GV_{(1\times156)}^1 = \left[ MV_1^1 - \sum_{t=1}^{1}\frac{FCFF_1^1}{(1+i_{b,1})^1}+\frac{FCFF_3^1/(i_{e,1}-g_{e,1,1})}{(1+i_{e,1})^2} \quad \dots \quad MV_{156}^{68} - \sum_{t=1}^{1}\frac{FCFF_1^1}{(1+i_{b,156})^1}+\frac{FCFF_3^1/(i_{e,156}-g_{e,156,68})}{(1+i_{e,156})^2} \right]_{(1\times156)}$$

$$GV_{(1\times156)}^2 = \left[ MV_1^2 - \sum_{t=1}^{1}\frac{FCFF_1^2}{(1+i_{b,1})^1}+\frac{FCFF_3^2/(i_{e,1}-g_{e,1,2})}{(1+i_{e,1})^2} \quad \dots \quad MV_{156}^{68} - \sum_{t=1}^{1}\frac{FCFF_1^1}{(1+i_{b,156})^1}+\frac{FCFF_3^1/(i_{e,156}-g_{e,156,68})}{(1+i_{e,156})^2} \right]_{(1\times156)}^2 \qquad (9)$$

$$\vdots$$

$$GV_{(1\times156)}^{68} = \left[ MV_1^{68} - \sum_{t=1}^{1}\frac{FCFF_1^{68}}{(1+i_{b,1})^1}+\frac{FCFF_3^{68}/(i_{e,1}-g_{e,1,68})}{(1+i_{e,1})^2} \quad \dots \quad MV_{156}^{68} - \sum_{t=1}^{1}\frac{FCFF_1^1}{(1+i_{b,156})^1}+\frac{FCFF_3^1/(i_{e,156}-g_{e,156,68})}{(1+i_{e,156})^2} \right]_{(1\times156)}$$

Summing up the row vectors in the matrix system (9) – which represent the 68 weekly time series of GVs of all stocks in the Ibovespa theoretical portfolio – gives the aggregate weekly time series of GVs as follows:

$$\sum_{t=1}^{68} GV_{(1\times156)} = \left[ GV_1^1 + \dots + GV_1^{68} \quad GV_2^1 + \dots + GV_2^{68} \quad \dots \quad GV_{156}^1 + \dots + GV_{156}^{68} \right]_{(1\times156)} \qquad (10)$$

This series refers to the 156 weeks in the 2008-2010 triennium.

Returning to the notion of GV and considering the system of row vector in (9) gives:

$$\sum_{t=1}^{68} GV_{(1\times156)} = \left[ (MV_1^1 - IV_1^1) + \dots + (MV_1^{68} - IV_1^{68}) \quad \dots \quad (MV_{156}^1 - IV_{156}^1) + \dots + (MV_{156}^{68} - IV_{156}^{68}) \right] \qquad (11)$$

## 2.4. Module 4 – gap value elasticity in relation to benchmark and expected interest rates

GV elasticities are deduced from equations (8) using software package Mathematica 7[19] as follows.

Differentiating (8) in relation to $i_b$, which is a weekly benchmark interest rate[20], gives the matrix equation (12), which shows the several elasticity values of the intrinsic values of every share in the Ibovespa theoretical portfolio in relation to COPOM's benchmark interest rate. Elasticity (12) herein determines the effect of, for instance, an 1-percentage-point variation on the intrinsic value of a share in the Bovespa Index.

$$\frac{\partial GV_{(68\times156)}}{\partial i_b} = \begin{bmatrix} MV_1^1 + \dfrac{FCFF_1^1}{(1+i_{b,1})^2} & \cdots & \cdots & MV_{156}^1 + \dfrac{FCFF_1^1}{(1+i_{b,156})^2} \\ MV_1^2 + \dfrac{FCFF_1^2}{(1+i_{b,1})^2} & \cdots & \cdots & MV_{156}^2 + \dfrac{FCFF_1^2}{(1+i_{b,156})^2} \\ \vdots & \vdots & \ddots & \vdots \\ MV_1^{68} + \dfrac{FCFF_1^{68}}{(1+i_{b,1})^2} & \cdots & \cdots & MV_{156}^{68} + \dfrac{FCFF_1^{68}}{(1+i_{b,156})^2} \end{bmatrix}_{(68\times156)} \quad (12)$$

Partitioning matrix (12) gives the row values that represent the elasticity curve of all Ibovespa`s stocks in relation to the several levels of interest rates determined by COPOM throughout 156 weeks (in the 2008-2010 triennium). Summing up these row vectors gives the series of Ibovespa`s GV elasticity in relation to the Selic rate, the first objective of this paper. The vector calculations are as follows:

$$\frac{\partial GV_{(1\times156)}}{\partial i_b} = \left[ \left( MV_1^1 + \frac{FCFF_1^1}{(1+i_{b,1})^2} + ... + MV_1^{68} + \frac{FCFF_1^{68}}{(1+i_{b,1})^2} \right) \quad \cdots \quad \left( MV_{156}^1 + \frac{FCFF_1^1}{(1+i_{b,156})^2} + ... + MV_{156}^{68} + \frac{FCFF_1^{68}}{(1+i_{b,156})^2} \right) \right] \quad (13)$$

After that, matrix (8) is differentiated in relation to the expected interest rate. The calculations, similar to those provided before, give the series of GV for every Ibovespa`s stock in relation to the expected interest rate, as well as the series of GV of the Ibovespa`s theoretical portfolio as a whole in relation to this very variable (expected interest rate). Therefore, result (14) below attains the second objective of this paper.

$$\frac{\partial GV_{(68\times156)}}{\partial i_e} = \begin{bmatrix} MV_1^1 + \dfrac{FCFF_1^1}{(1+i_{e,1})^2\cdot(i_{e,1}-g_{e,1,1})^2} + \dfrac{2FCFF_3^1}{(1+i_{e,1})^3\cdot(i_{e,1}-g_{e,1,1})} & \cdots \cdots & MV_{156}^1 + \dfrac{FCFF_1^1}{(1+i_{e,156})^2\cdot(i_{e,156}-g_{e,156,1})^2} + \dfrac{2FCFF_3^1}{(1+i_{e,156})^3\cdot(i_e-g_{e,156,1})} \\ MV_1^2 + \dfrac{FCFF_1^2}{(1+i_{e,1})^2\cdot(i_{e,1}-g_{e,1,2})^2} + \dfrac{2FCFF_3^2}{(1+i_{e,1})^3\cdot(i_{e,1}-g_{e,1,2})} & \cdots \cdots & MV_{156}^2 + \dfrac{FCFF_1^2}{(1+i_{e,156})^2\cdot(i_{e,156}-g_{e,156,2})^2} + \dfrac{2FCFF_3^2}{(1+i_{e,156})^3\cdot(i_e-g_{e,156,2})} \\ \vdots & \vdots \ddots & \vdots \\ MV_1^{68} + \dfrac{FCFF_1^{68}}{(1+i_{e,1})^2\cdot(i_{e,1}-g_{e,1,68})^2} + \dfrac{2FCFF_3^{68}}{(1+i_{e,1})^3\cdot(i_{e,1}-g_{e,1,68})} & \cdots \cdots & MV_{156}^{68} + \dfrac{FCFF_1^{68}}{(1+i_{e,156})^2\cdot(i_{e,156}-g_{e,156,68})^2} + \dfrac{2FCFF_3^{68}}{(1+i_{ee,156})^3\cdot(i_e-g_{e,156,68})} \end{bmatrix}_{(68\times156)} \quad (14)$$

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[19]For the use of Mathematica 7, see Shingareva & Lizárraga-Celaya (2009) and Blachman (1992). For examples applied to Economics and Finance, the most relevant references are Stinespring (2002) and Varian (1993). Kendrick, Mercado & Amman (2005) also provide some examples of using the software.
[20]Selic rate as defined by COPOM.

Partitioning matrix (12) gives the row values that represent the elasticity curve of all Ibovespa stocks in relation to the several levels of interest rates determined by COPOM thought 156 weeks (in the 2008-2010 triennium). Now let each expression contained in the matrix (14) cells be $GV_w^s$, where $s$ stands for the number of stocks in the Ibovespa`s theoretical portfolio (listed in alphabetical order), and w stands for the week to which a given share's market value and intrinsic value estimation correspond. From that follows:

$$\frac{\partial GV_{(1\times156)}}{\partial i_{b,w}} = \left[ \frac{\partial GV_1^1}{\partial i_{b,1}} \quad \frac{\partial GV_2^1}{\partial i_{b,2}} \quad \cdots \quad \frac{\partial GV_{156}^1}{\partial i_{b,156}} \right]$$

$$\frac{\partial GV_{(1\times156)}}{\partial i_{b,w}} = \left[ \frac{\partial GV_1^2}{\partial i_{b,1}} \quad \frac{\partial GV_2^2}{\partial i_{b,2}} \quad \cdots \quad \frac{\partial GV_{156}^2}{\partial i_{b,156}} \right] \quad (15)$$

$$\vdots$$

$$\frac{\partial GV_{(1\times156)}}{\partial i_{b,w}} = \left[ \frac{\partial GV_1^{68}}{\partial i_{b,1}} \quad \frac{\partial GV_2^{68}}{\partial i_{b,2}} \quad \cdots \quad \frac{\partial GV_{156}^{68}}{\partial i_{b,156}} \right]$$

Summing up the row vectors of system (15) gives the series of Ibovespa`s gap value elasticity in relation to the expected Selic rate, the second objective of this paper. This gives the series as follows:

$$\frac{\partial GV_{(1\times156)}}{\partial i_{b,w}} = \left[ \left( \frac{\partial GV_1^1}{\partial i_{b,1}} + \frac{\partial GV_1^2}{\partial i_{b,1}} + \ldots + \frac{\partial GV_1^{68}}{\partial i_{b,1}} \right) \quad \cdots \quad \left( \frac{\partial GV_{156}^1}{\partial i_{b,156}} + \frac{\partial GV_{156}^2}{\partial i_{b,156}} + \ldots + \frac{\partial GV_{156}^{68}}{\partial i_{b,156}} \right) \right] \quad (16)$$

Analogously, a number of vector calculations finally gives the series of Ibovespa`s gap value elasticity in relation to the expected interest rates throughout the 156 weeks included in the analysis:

$$\frac{\partial GV_{(1\times156)}}{\partial i_{e,w}} = \left[ \left( \frac{\partial GV_1^1}{\partial i_{e,1}} + \frac{\partial GV_1^2}{\partial i_{e,1}} + \ldots + \frac{\partial GV_1^{68}}{\partial i_{e,1}} \right) \quad \cdots \quad \left( \frac{\partial GV_{156}^1}{\partial i_{e,156}} + \frac{\partial GV_{156}^2}{\partial i_{e,156}} + \ldots + \frac{\partial GV_{156}^{68}}{\partial i_{e,156}} \right) \right] \quad (17)$$

Where the cells correspond to:

$$\frac{\partial GV_1^1}{\partial i_{e,1}} = MV_1^1 + \frac{FCFF_1^1}{(1+i_{e,1})^2 \cdot (i_{e,1} - g_{e,1,1})^2} + \frac{2FCFF_3^1}{(1+i_{e,1})^3 \cdot (i_{e,1} - g_{e,1,1})}$$

$$\frac{\partial GV_1^1}{\partial i_{e,1}} = MV_1^2 + \frac{FCFF_1^2}{(1+i_{e,1})^2 \cdot (i_{e,1} - g_{e,1,2})^2} + \frac{2FCFF_3^2}{(1+i_{e,1})^3 \cdot (i_{e,1} - g_{e,1,2})} \quad (18)$$

and so on and so forth until:

$$\frac{\partial GV_{156}^{68}}{\partial i_{e,156}} = MV_{156}^{68} + \frac{FCFF_1^{68}}{(1+i_{e,156})^2 \cdot (i_{e,156} - g_{e,156,68})^2} + \frac{2FCFF_3^{68}}{(1+i_{e,156})^3 \cdot (i_e - g_{e,156,68})}$$

Where $g_{e,w,s}$ is the expected growth for week $w$ by the company's shareholder$s$, $i_{b,w}$is the benchmark interest rate (Selic) in week $w$, and $i_{e,w}$is the expected interest rate for week $w$.

## 3. Corollaries of the model

The GV results, particularly from equation (5) onwards, fall in three categories: downside, upside, and balance (IV and MV are equal). These categories are shown in Table 2.

As Table 2 shows, the GV elasticity in relation to the benchmark interest rate (Selic rate defined by COPOM) is negative only in B; it is positive in all other conditions.

This means that *the transmission mechanism of monetary policy will affect Ibovespa`s GV negatively only when the companies that make up this index have, altogether, negative net cash generation.*

Otherwise, *coeteris paribus*, *COPOM's decisions to increase (reduce) the benchmark interest rate will invariably result in increased (reduced) Ibovespa`s GV.*

Table 2 – Corollaries of the model

| **Possible interactions between GV and share market conditions** | | |
|---|---|---|
| Condition | Possible results | Conclusion |
| 1 | $GV_{(i \times j)} > 0 \Leftrightarrow MV_{(i \times j)} > IV_{(i \times j)}$ and $GV \to \infty \Rightarrow \Pi \to \infty$ | Ibovespa's potential of losses increases: bull market, downside, or speculative bubble condition. |
| 2 | $GV_{(i \times j)} < 0 \Leftrightarrow IV_{(i \times j)} > MV_{(i \times j)}$ and $GV \to -\infty \Rightarrow \Pi \to -\infty$ | Ibovespa's potential of profits increases: bear market, upside, or share market downturn condition. |
| 3 | $GV_{(i \times j)} \cong 0 \Leftrightarrow IV_{(i \times j)} \cong MV_{(i \times j)}$ and $GV \to 0 \Rightarrow \Pi \to 0$ | Condition of intrinsic value balance across the shares in the Ibovespa theoretical portfolio.No profit nor loss potential. |
| **Possible conditions between Ibovespa`s cash flow, GV and GV elasticity in relation to the effects of changes to the benchmark interest rate –through transmission mechanism – on the stocks in the Ibovespa`s theoretical portfolio.** | | |
| A | *MV >IV* (downside), and *FCFF< 0* (negative cash flow) | $\dfrac{\partial GV}{\partial i_b} > 0$ |
| B | *MV < IV* (upside), and *FCFF< 0* (negative cash flow) | $\dfrac{\partial GV}{\partial i_b} < 0$ |
| C | *MV >IV* (downside), and *FCFF> 0* (positive cash flow) | $\dfrac{\partial GV}{\partial i_b} > 0$ |
| D | MV < IV (upside), and FCFF< 0 (positive cash flow) | $\dfrac{\partial GV}{\partial i_b} > 0$ |

Source: Elaborated by the authors

## 4. Results of the model simulations: graph analysis of years 2008 through 2010

This section provides the results of the macroeconomic, computer-based, agent-based financial model for the 2008-2010 triennium[21]. Each subsection analyses the events of each year of the triennium that influenced the elasticity curve of Ibovespa`s IV and GV in relation to changes to the benchmark and expected interest rates. The simulations were carried out on Excel

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[21]The events related to the monetary policy and the financial market evolution in the triennium under scrutiny are based on the Central Bank of Brazil's annual reports and on several editions of the *ConjunturaEconômica*magazine and the *Valor Econômico* newspaper.

2007 by calibrating the growth rate parameters ($g_2$) in vector (11) equations. This series is calculated on a weekly basis, and refers to the 156 weeks in the triennium.[22]

All the following subsections provide graph analysis for the computer-based simulations. The subsections are divided according to the year to which they refer. The weekly series correspond to the following row vectors: (1) IV – intrinsic value vector for the Ibovespa`s theoretical portfolio, which is the sum of the 52 weekly time series of the intrinsic values estimated by the computer-based model for the 68 stocks in the Ibovespa`s theoretical portfolio[23]; (2) MV – market value vector for the Ibovespa`s theoretical portfolio[24]; (3) Selic defined by COPOM short-term interest rate, represented as effective interest rates that were weekly compounded; and (4) Expected interest rate – containing expected values as published by the Central Bank of Brazil in the Focus Report for the respective weeks of the period under scrutiny.

## 4.1. Effect of the interest rates (benchmark and expected) on the Ibovespa`s IV and GV in 2008

The increased expected inflation (referred to by the Central Bank of Brazil as a "benign scenario" deriving from excess of aggregate demand over supply) led COPOM to interrupt in April 2008 the policy of progressively mild interest rates that started in September 2005.As a result, the benchmark interest rate goal, which was 11.25% since September 2007, increased 50 basis points (bps) in April 2008, and suffered other increases upon three other COPOM meetings. It eventually reached 13.75% p.a. in December 2008.The ex-ante real interest rate, calculated by the Central Bank of Brazil for a period of one year after surveying analysts in the private sector,[25] increased throughout the first nine months of 2008.In October, the expectations for the benchmark interest rates had reduction tendency, which led to a rate of 7.2% p.a. by the end of the year (0.7 percentage points higher than that registered for the same month in 2007). COPOM's contractionary monetary policy measures in the beginning of 2008 continued the decisions of previous meetings in 2007, in which the committee members assessed that the international economic conjuncture and the expanding domestic economic activity required some prudential measures from the monetary policy maker. The Committee eventually decided to keep the Selic rate at 11.25% p.a., without bias, and opted for monitoring the macroeconomic conjecture and thus gain time to deliberate about potential changes to the monetary policy in subsequent meetings[26]. As the expected inflation was increasing, COPOM increased the Selic rate in 50 bps in two meetings. Assuming that risks of increasing prices still existed in July, COPOM decided to increase the Selic rate to 13% p.a., without bias. The committee still believed in a tendency of increasing prices in September (the already mentioned "benign scenario"). The committee, thus, opted for a new increase on 75 bps in the Selic rate, without bias, even in a scenario of international economic recession (see Graph 1).In October and December, COPOM finally noticed the effects of the economic slowdown and the increased economic uncertainty resulting from the international financial crisis that was also affecting

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[22]The series builds on the analysis of 816 standardized financial statements included in the Brazilian Comission databank. The data are available at: <http://www.cvm.gov.br/>.

[23]This series estimates builds on the cash flows of the 68 theoretical portfolio stocks. The preparation of such cash flows builds on the analysis of 272 accounting statements of these companies (four statements for each company).

[24]Source: BMF&Bovespa's website, available at: <http://www.bmfbovespa.com.br/home.aspx?idioma=pt-br>.

[25]This survey data are included in the Focus Report, published weekly by the Central Bank of Brazil.These data are deflated by the IPCA and correspond to the series of expected real interest rates as described in the following graphs.

[26]In April and June, however, the COPOM members assessed that the inflationary pressures -- that were initially localized -- could pose risks of increasing inflation. This risk was aggravated by the tight aggregate demand and the factor market, besides potential supply constraints in important sectors.

several sectors in the Brazilian economy (especially the credit channels) as well as the consumers' and entrepreneurs' confidence.[27]As for the dynamics of monetary aggregates, a reduction of 2.3% was registered for the average daily balance of narrowly defined money supply (M1).[28]M1 velocity of money remained relatively stable, and the monetary base (as measured according to the average daily balances) increased 1.5% in the year. The Central Bank of Brazil assumed other expansionary pressures on the monetary supply, which led to an increase of 933 million Brazilian Reais in the monetary base (using the criterion of available balance by the end of the period). The monetary authority, therefore, decided to implement changes to the rules of compulsory deposits, which translated into: reductions in rates, increases in the amounts to be deducted from callable reserves, changes in earnings on reserve requirements, together with discounts on amounts to be deposited as an incentive to negotiation of assets among medium- and small-sized financial institutions.[29]

Graph 1 synthetizes the explanations provided so far and shows the negative effects of having increased real interest rates (benchmark and expected) on the intrinsic value of the Ibovespa theoretical portfolio. Obviously, the Ibovespa intrinsic value was also affected by other variables that are beyond the scope of the model such as:1) the default of the three mentioned financial institutions, and worsening animal spirits among the entrepreneurs in the productive sector; 2) reduced volume of foreign investments in shares (Graph 2); 3) the devaluated real exchange rate's effect on the companies making up the Ibovespa theoretical portfolio; and 4) the remarkably reduced international trade flows.

Graph 1 – Computer-based weekly series of variation effects of expected and short-term interest rates on the intrinsic values in the Ibovespa theoretical portfolio in 2008
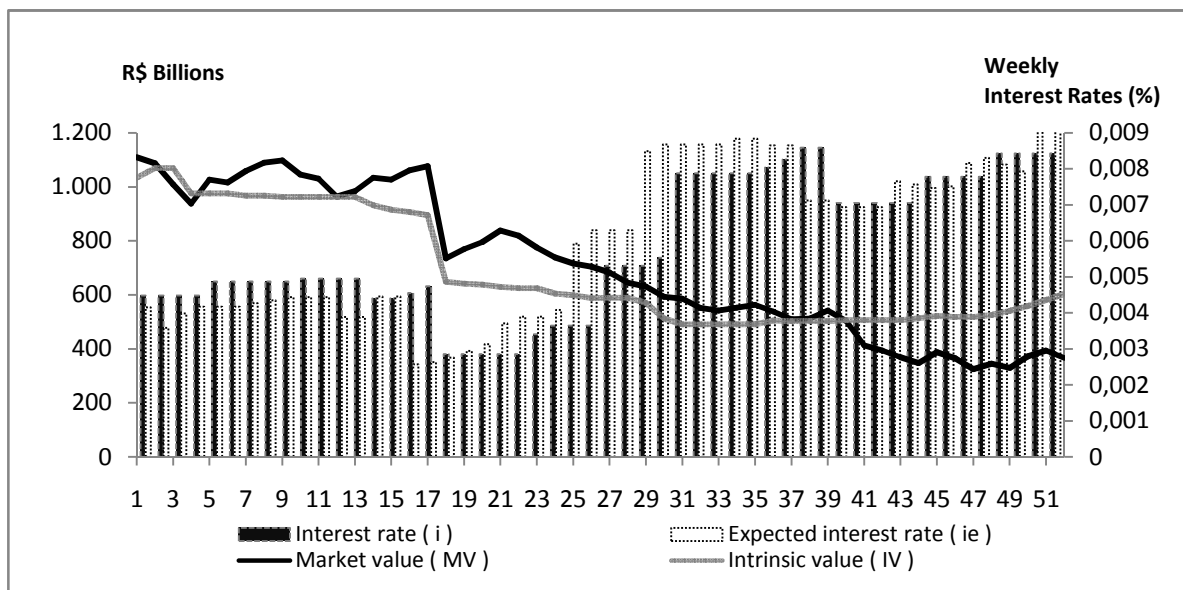


Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Ibovespa theoretical portfolio in fiscal year 2008; Central Bank of Brazil's Focus Report; BMF&Bovespa;  Monetary Policy Committee (COPOM).

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[27]The COPOM analysis pointed to the need of preventing inflationary pressures because the families' rates of consumption growth were still accelerated despite the higher stability of the commodity prices.Therefore, the committee kept the Selic rate at 13.75% p.a., without bias, in both meetings.It trusted that the reduced liquidity in the international market and the expectation changes would help the transmission mechanisms of monetary policy and inhibit price increases (Graph 1).

[28]Considering non-seasonally adjusted and IPCA-deflated data.

[29]These changes to the rules, implemented from September 24, 2008 to January 19, 2009, caused a compulsory reserve reduction of 99.8 billion Brazilian reais.

As for the Ibovespa performance, it is worth noting the Petrobras[30] stock split that took place in the seventh week of the year[31]. In addition the period from January from May witnessed a number of repeated records of increased index values, which reached 73,516 point on May 20th[32]. From late May to the end of year, the Ibovespa experienced a sharp drop (of 42.1% in relation to the prevailing level by the end of 2007) and increased volatility. [33]

Graph 2 shows that the GV is positive until the 40th week and is negatively correlated with the real interest rates (benchmark and expected), as mentioned in the previous section. [34]From then on, the GV is negative, reflecting Lehman Brothers', Fannie Mae's , and Freddie Mac's collapse.

This moment of structural collapse in the GV series evinces that the intrinsic value of the Ibovespa`s theoretical portfolio declined at a fast pace and started a progressive upside (tendency of profit)[35].

Graph 2 – Computer-based weekly series of variation effects of expected and short-term interest rates on the GV ($GV = MV – IV$) in the Ibovespa theoretical portfolio in 2008
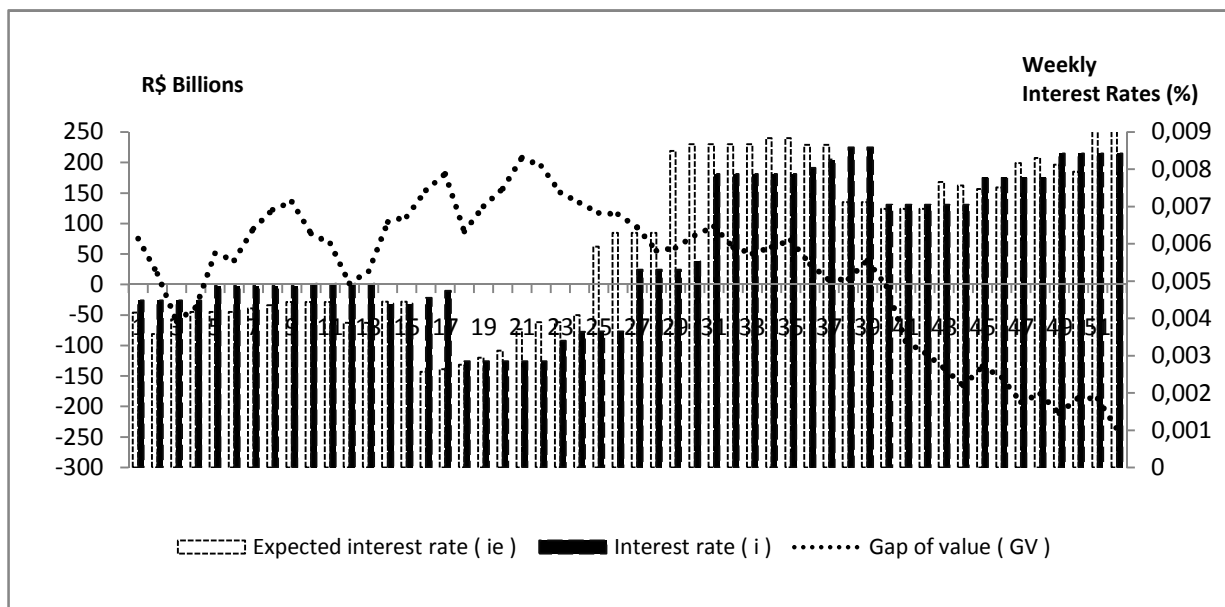


Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Ibovespa theoretical portfolio in fiscal year 2008; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee (COPOM).

The result herein described shows the model's power to detect and measure speculative bubbles in the stock market, as well as to identify when they emerge and disappear (Graph 2)[36].

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[30] Brazilian distributor and marketer of petroleum derivatives and biofuel.

[31]This explains the inflexion of both intrinsic and market value curves in the period, as Petrobras is the company is the greatest weight in the Ibovespa portfolio.

[32]This was due to the positive effects derived from the upgrade of sovereign debt rating to investment grade that was granted by two risk rating agencies: Standard & Poor's , and Fitch Ratings.

[33]Therefore, the greatest value (that of May 20th) represented the bull market apex of a period that started in year 2005 and experienced high Ibovespa returns.

[3434]That is, increased in both rates are associated with reductions in IV and MV, and vice-versa. However, GV variations depend on the IV elasticity, and on MV elasticity to both rates, as shown in matrix equations (12) and (14).Therefore, GV reduced significantly in 2008 as these rates increased from the 25th week to the end of the year.

[35]A characteristic phenomenon of fundamental analyses based on the notion of value investing in periods of bear market.

[36]Results of this nature are of special interest for policy makers (especially those concerned with making and executing the monetary policy), financial executives, and the representative agent of the computer-based model.

### 4.3. Effect of the interest rates (benchmark and expected) on the Ibovespa intrinsic value and GV in 2009

The quarterly growth rates of real GDP in 2009 showed that only the service sector was not affected by the financial crisis, which, as mentioned before, had its epicenter in August 2008. The industrial and agricultural GDPs reduced in the first three quarters of the year, and only improved in the last.

The consequences of the world crisis worsened from the last quarter 2008, when the domestic economic activity faced strong recession in response to the reduced credit availability, worsened expectations, and increased levels of risk aversion. The external scenario changed at a fast pace, and the Central Bank of Brazil changed the contractionary orientation of the monetary policy, and reduced 500 bps of the base Selic rate. In late 2008, COPOM interrupted a series of four consecutive rate increases, which had started in the April meeting of that year and translated into a 250-basis-point increase of the Selic rate. In October and December, however, COPOM took the new scenario described before and unanimously took the decision to keep the Selic rate at 13.75% p.a., without bias.

In the first meetings of 2009 (in January and March), COPOM decided to reduce the Selic rate in respectively 100 and 150 basis points. The decision built on the assumption that a strong inhibition of the expected inflation and the reduced economic activity would make IPCA (Extended National Consumer Price Index) follow a trajectory that would be consistent with the inflation targets determined by the National Monetary Committee (CMN). In March, COPOM understood that the reduced levels of economic activity was ensuring a level of idle capacity that could refrain the risks of worsening demand inflation despite the effects of adjusting the balance of payments and persisting with transmission mechanisms in some contracts. In the third and fourth meetings (April and June), the committee promoted two reductions of 100 basis points in the base Selic rates. The committee recognized that such measures were needed to gradually recover the industrial production rates, industry capacity utilization rates, employment rates, and both investors' and consumers' confidence. In the following meeting, in July, COPOM reduced the Selic rate in 50 bps to 8.75% p.a.; it kept this rate in the subsequent meetings, in September, October, and December, assuming that a more prudential attitude was needed.[37]

As for the monetary aggregates, the expansionary monetary policy implemented in mid-20008 to remediate the effects of the intense contraction of credit both internally and externally produced an actual increase[38] of 5.7% in M1. Despite this increase, M1[39] velocity of money remained relatively stable in the year. The average daily balances of the monetary base increased 14.9% in the year due to increases to its components (the average balance of issued currency increased 14.3%, and bank reserves grew 16.8%[40]). In 2009, monetary aggregates M2, M3 and M4 enlarges respectively 8.6%, 15.5%, and 16.1%. Graph 3 synthetizes these data and relate them to the time series of Ibovespa`s IV`s that was obtained using the model simulations.

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They help policy makers and financial executives measure and detect speculative bubbles in the share market. They also inform the model representative agent's decision: if the GV is declining and the agent trust in a positive future scenario for the Ibovespa tendency in the long term, s/he will opt for increasing investments in this index; if the value is increasing, however, s/he may decide to keep his/her levels of investments or sell shares if this increase magnitude is great enough to yield him/her the desired profit.

[37]The real expected interest rates (or ex-ante defined) declined from January to July in the year under scrutiny[1].In the interest derivatives market, the interfinancial swap deposit contracts (DI) x 360-day pre-DI swap registered decreasing rates in the first semester 2009, dropping to the lowest rate of 8.97 p.a. in July. A reversal took place in this downward trend in the expected interest rates throughout the second semester of the year. By the end of December, the rates of the negotiated contracts was 10.46% p.a., a level 171 bps lower than that registered in December 2008.

[38]Considering non-seasonally adjusted and IPCA-deflationed data.

[39]Defined as the relation between the GDP at current values and M1 average balance.

[40]This increase was due to the reduced rate of reserve requirements on demand resources that became effective in October 2008.

As for the capital market dynamics, the international crisis depreciated asset prices and declined the number of negotiations in the international financial markets. The number of new issues in the capital market dropped, especially in the first semester of 2009.In the second semester, however, the volume of share supplies, debentures, and promissory notes recovered in the respective primary markets, enough to stimulate positive expectation to 2010. The Ibovespa ended 2008 82.7% higher at 68,588 points. [41]In 2009, the dollar-based index variation was positive in 145.2% (this includes both Ibovespa variation and the sharp decline of the real exchange rate)[42].

Graph 3 – Computer-based weekly series of variation effects of expected and short-term interest rates on the intrinsic values in the Ibovespa theoretical portfolio in 2009
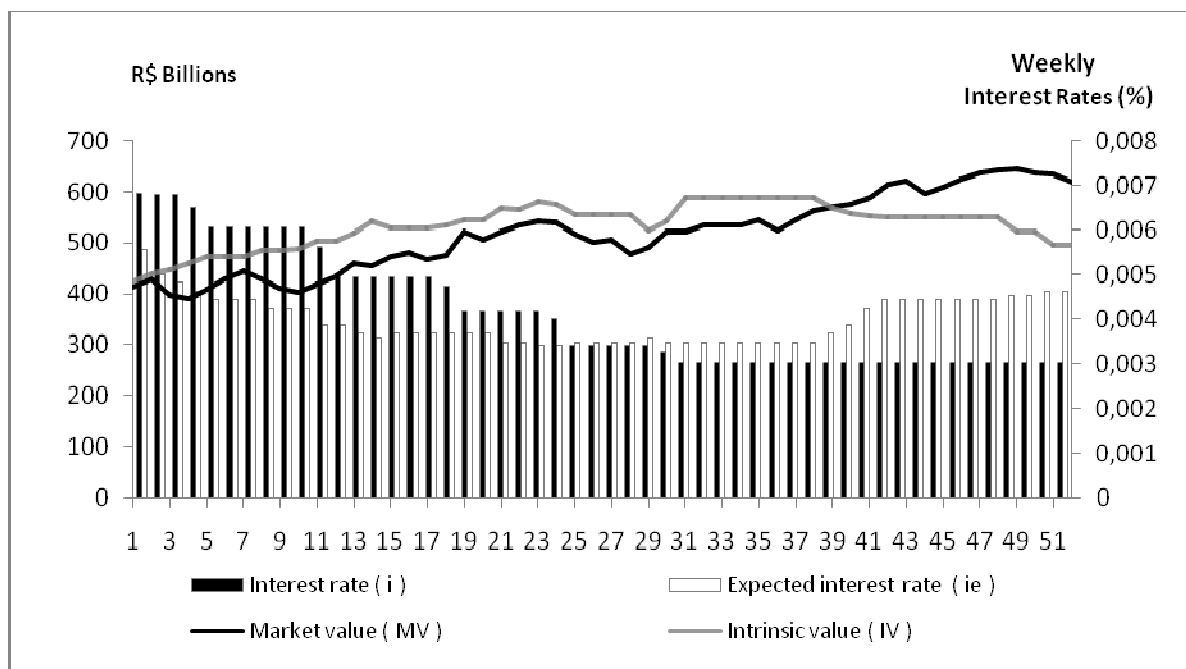


Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Ibovespa theoretical portfolio in fiscal year 2008; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee (COPOM).

The second quarter 2009 saw a recovery of the Brazilian balance of trade. In this period, the government also took countercyclical fiscal policy measures – i.e., reducing the tax on industrialized products (IPI) for durable consumer goods. The period also witnessed increased public expenditures and increased volume of bank credit to the private sector. Graph 4[43] shows that these circumstances in general led to a MV and IV recovery. It also shows that the GV

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[41]The strong volatility in January and February 2009 gave way to a an increase period that lasted up to June, when it reached 54,486 points.

[42]For comparison purposes, three events are worth highlighting in that year:1) the increases of 18.8% and 43.9% in the Dow Jones and Nasdaq indexes, respectively; 2) the increase of 69.8% (in relation to 2008) in the marked value of the companies listed in Sao Paulo Stock Exchange (Bovespa); 3) the average daily trading volume peaked 5.2 billion Brazilian reais (yet a reduction of 4.1% in relation to 2008).Stock funds, in turn, benefited from this scenario, presenting an annual return of 46.8% and an increase of 49.3 in consolidated net equity.

[43]Risk rating agencies Fitch Ratings and Moody's updated the country's sovereign debt rating to investment grade in the third quarter 2009.The Brazilian economy was booming by the end of the quarter. Eventually, the forth quarter saw a remarkable increase in the Ibovespa values. Notice, however, that IV did not follow this tendency in that quarter, since the expected interest rates increased. This shows that the intrinsic value was negatively affected by the expected interest rate variations.

remained negative until the 40th week, i.e. it remained below the intrinsic value in the period, in an upside condition. This took place because of the financial crisis and the aforementioned events. From the 40th week on, however, expectations change in response to the economic recovery, expansionary factors as explained above, and award of good investment grade. The GV became thereafter positive (i.e., a downside condition).[44]

Graph 4 – Computer-based weekly series of variation effects of expected and short-term interest rates on the GV ($GV = MV – IV$) in the Ibovespa theoretical portfolio in 2009
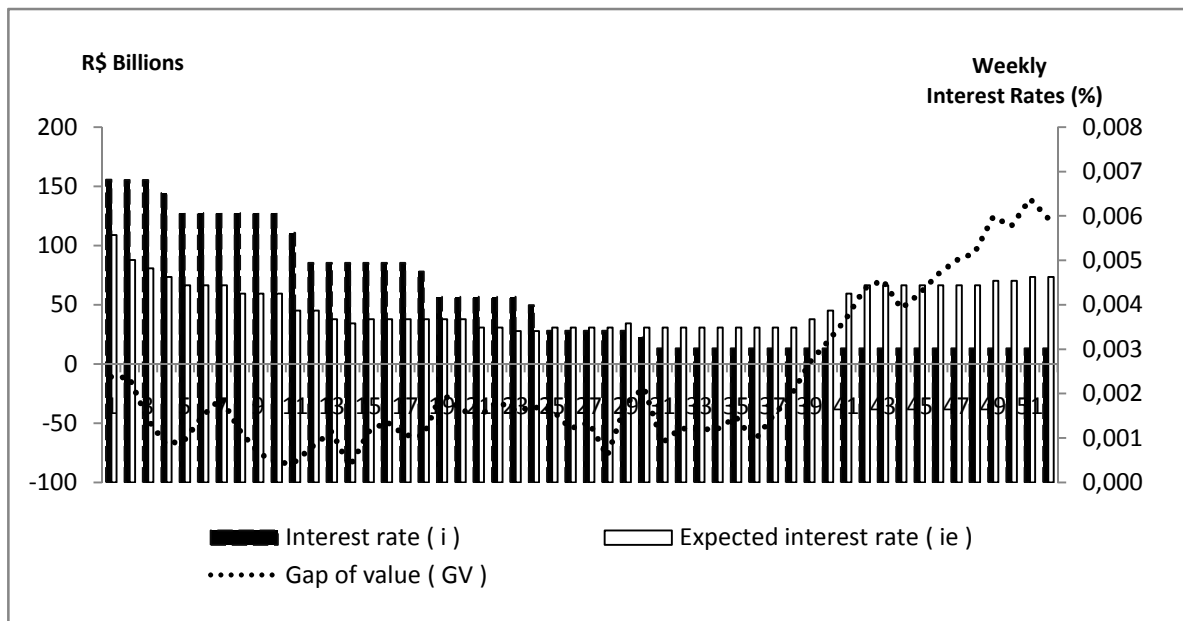


Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Ibovespa theoretical portfolio in fiscal year 2008; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee (COPOM).

## 5.3. Effect of the interest rates (benchmark and expected) on the Ibovespa intrinsic value and GV in 2010

As for the real interest rates (both benchmark and expected rates), COPOM kept the targeted Selic rate at 8.75% p.a. throughout the first quarter of 2010.In other words, the targeted base interest rate was kept at 8.75 in the two first COPOM meeting of the year, the lowest rate in a series that began in 1999. In the subsequent meetings, however, which took place in the second and third quarters, COPOM made a series of increases in the rate that summed up 200 bps. In the last three meetings of the year, the committee kept the rate at 10.75% p.a.[45]. The yearly IPCA (Extended National Consumer Price Index) reached 5.91%, which was close to the upper limit of a range of 2.5% to 6.5% that was defined as inflation target for 2010 by the National Monetary Council (CMN). As a result, the effective accumulated Selic rate was 9.7% in 2010, while the real accumulated Selic rate deflated by IPCA hit 3.7%.The expected real rate[46] jumped from 5.1% in the end of 2009 to 6.2% p.a. in the end of 2010. In the second semester, however, the

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[44]In fact, investors turned opted for variable-income assets (the foreign investments in shares increased substantially as shown in Graph 2).The GV trajectory in Graph 6 reflects this upward tendency.

[45]Such an interest policy was implemented in response to the inflationary pressures related to the increased commodity prices (from the international economy perspective) and the inconsistency between the aggregate supply growth and the expansion of domestic demand (from the domestic economy perspective).

[46]Ex-ante real interest rate as estimated by the Central Bank of Brazilian (as mentioned in the previous section).

contracts proved to be volatile and ended up negotiated at 12.03% p.a. by the end of 2010[47]. In December, CMN and the Board of Directors of the Central Bank of Brazil decided to implement some measures aimed at strengthening the financial regulation and, therefore, sustaining the stability of the financial institutions that make up the National Financial System (SFN) and ensuring a safe process of credit market expansion. These measures became known as macro-prudential regulatory measures[48]. Still, they needed to be articulated with COPOM's policy of base interest rates. Accordingly, COPOM assessed at the last meeting of the year that the economic scenario showed no major risks of aggravating the until-then benign process of demand inflation[49].

Graph 5 shows a decline in the IV and MV series in the period herein analyzed. The graph also shows a negative correlation between both series of interest rates (benchmark and expected) and the series of IV and MV.

The public offering of Petrobras` stocks reached a trading volume of 120.2 billion Brazilian Reais, a number that boosted the primary offering of debentures, promissory notes, and shares listed in the Brazilian Security Exchange Commission (CVM) in 2010.The volume increased 341% in relation to the previous year[50]. The correlation level between IV and MV was 99% in the period.

In 2010, the Brazilian economy saw a GPD growth rate of 7.5%.Such a rate had not been recorded for over two decades. Nevertheless, the worsening expectations about the US economy indexes, along with the reduced credit availability in China and the fiscal crises in important Eurozone countries (i.e., Portugal, Greece, Spain, Italy, and France) caused considerably increased uncertainty in the productive and financial market worldwide, which eventually reduced the Ibovespa companies' MV.

In the second quarter, the international scenario remained alarming and led investors who usually buy variable-income assets to focus their concern on the European fiscal crisis effects on the very Europe countries and on the Chinese economy. Such concerns reflected the fact that the export dynamics of several major Brazilian companies is strongly linked to the market growth in China, Europa and the USA – the Chinese market having increasingly gained more importance than the others in the last ten years). Foreign investments also declined in the first quarters of the year. They dropped to U$ 274 billion in the first quarter and hit U$ 622 billion in the second quarter – still quite lower than the U$ 2.99 trillion in the last quarter of 2009[51].

In addition to this downturn scenario of the world economy, the domestic conjuncture also imposed more inflationary pressure and eventually caused COPOM to pursue a cycle of increased Selic rates. In turn, the interest rates and the foreign investors' expectations caused a declining trend in the GV of the Ibovespa theoretical portfolio.

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[47]This rate was 157 bps higher than the rate registered by the end of 2009.

[48]Two of them were especially relevant:1) increased compulsory reserve requirements; and 2) increased capital requirements for operations involving credit to individuals for periods longer than 24 months.

[49]The Committee, however, opted for wait some time to contemplate the impacts of the increased Selic rates (implemented since early 2010) and the macroprudential measures (adopted in December) on the IPCA evolution. The maintenance of the Selic rate in the December meeting was based on this analysis of the committee.

[50]The market value of the companies making up the Ibovespa theoretical portfolio increased 10.1% in 2010.It reached 2.6 trillion Brazilian reais in December, while it had been 2.3 trillion in the year before. The Bovespa average daily trading volume increased 22.4% in the year, totalizing 6.4 billion Brazilian reais.

[51] Sources: Central Bank of Brazil's Focus Report and Applied Economic Research Institute Database (IPEADATA).

Graph 5 – Computer-based weekly series of variation effects of expected and short-term interest rates on the intrinsic values in the Ibovespa theoretical portfolio in 2010
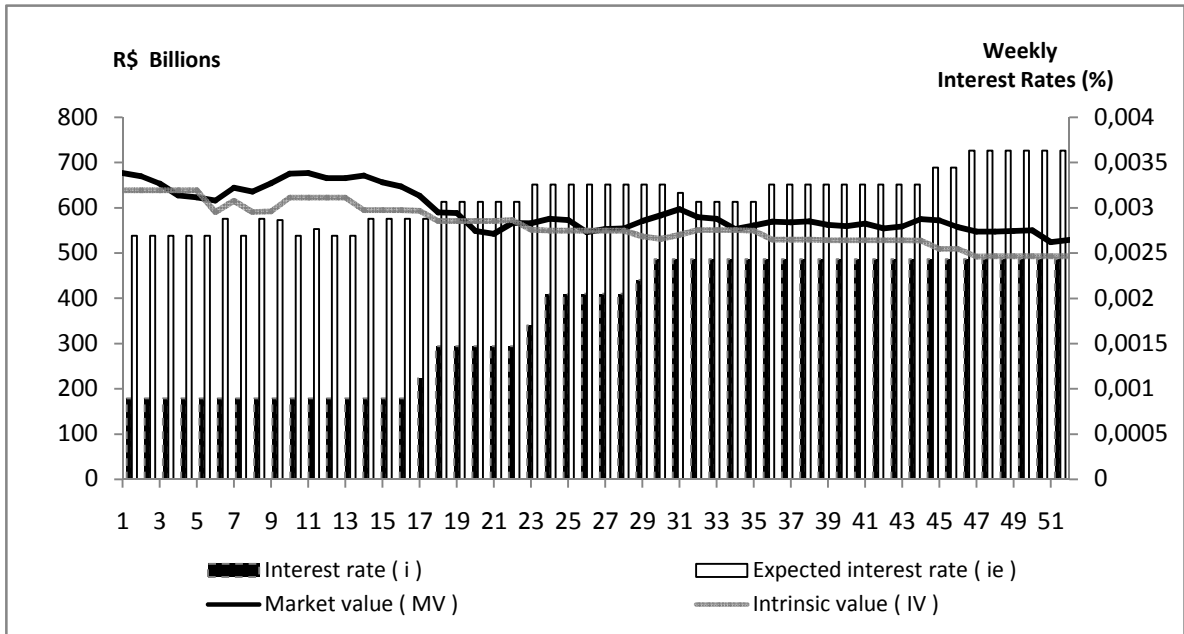


Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Ibovespa theoretical portfolio in fiscal year 2007; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee (COPOM).

Graph 6 shows the elasticity trajectory for the Ibovespa GV in relation to increases in the benchmark and expected interest rates. It also associates the dynamics of Ibovespa`s GV the major economic events of the period: the lowering of the risk ratings of Portugal's and Greece's debts in April 27th, and the expansion of the industrial and service GDP that started some weeks before April 27th.

Graph 6 – Computer-based weekly series of variation effects of expected and short-term interest rates on the GV ($GV = MV – IV$) in the Ibovespa theoretical portfolio in 2010
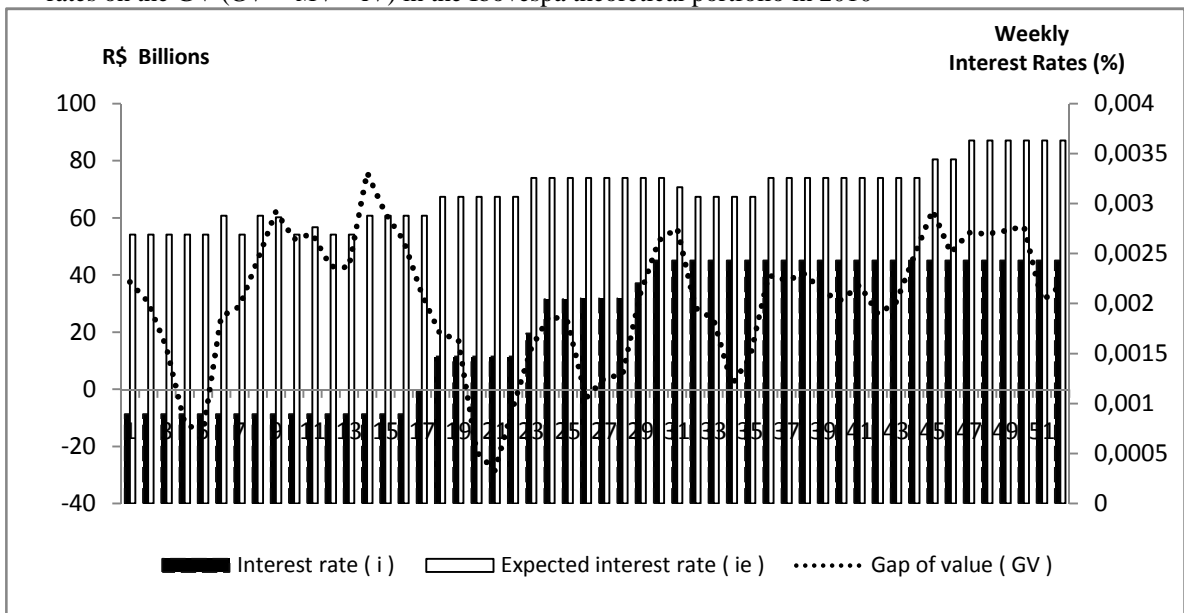


Source: Estimates as provided by the authors' computer-based model; Security Exchange Commission's standardized financial statements for the 68 companies making up the Ibovespa theoretical portfolio in fiscal year 2008; Central Bank of Brazil's Focus Report; BMF&Bovespa; Monetary Policy Committee (COPOM).

In the third quarter -- more specifically in September --, the Ibovespa reached the historical record of ca. 70,000 points.[52]The IPCA rates were indicating that the inflationary process had been restrained, and the Focus Report pointed a growth of 7% in the Brazilian economy by the end of 2010.Consequently, Brazil saw an inconsistency between its strong economic growth and the the I bovespa growth setback (60,000 points).This inconsistency was associated with the effects of the European crisis[53] and the Brazilian investors' replacement of variable-income assets with fixed-income assets. Despite this adverse scenario, the GV behavior was found to be largely positive. The higher MV compared to IV pointed to a downside condition.[54]

**6. Final remarks**

The computer-based financial model herein described is a multi-agent model and incorporates techniques from finance (the share valuation model), computational economics, and monetary economics.

The agents in the model are the shareholders of the companies making up the Ibovespa theoretical portfolio. These agents' expectations are linked to these companies' profit rates. The expected profit rates are combined – in a valuation formula, i.e., that of free cash flow to the firm – with the benchmark and expected interest rates, which serve as inter-temporal discounting rates or minimum attractive rates for negotiating the given shares)[55].

The model has first estimated the cash flows of all Ibovespa`s company in all the years of the triennium under analysis. Subsequently, it has projected the flows to years 2011 and 2012, and then discounted them to obtain intrinsic value estimates for every stock. These estimates are summed up and weighted according to their contribution share to the Ibovespa`s theoretical portfolio. The total result is the IV estimate of the Ibovespa. Finally, *the herein proposed notion of GV is deduced, this being the difference between Ibovespa's market and intrinsic values. This series of GV index is useful to detect and measure speculative bubbles in the financial market as well as bear market conditions*. The model methodology has drawn from readings of Lebaron (2006), Testfasion (2006) and Judd (2006).They all have built on the seminal work by Testfatsion & Judd (2006).Besides these texts, this paper also draws on insights of simulation and calibration techniques as described in Law & Kelton (1991), Werker & Brenner (2004), Menner (1995) and Miranda (2002).

The analysis of the model results evinces *the correlation between the series of intrinsic values and market value and the correlation between the weekly series of IV and of GV with the trajectories of the benchmark and expected interest rates*. In addition, the analysis also sought to *show the effect of exogenous factors on the series under study*. In other words, it sought to correlate the model-generated trajectories with such events as the US financial institution's default, the Petrobras` stock split, the expansionary fiscal and monetary policy adopted by the

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[52]This was an event with strong impact on Petrobras capitalization in the Bovespa, which reached ca. 30.4 billion Brazilian reais.

[53]Approximately one-third of the Bovespa activities refer to foreign investors.

[54]The GV was negative only in the period between the 9th and the 22th weeks of the year. That is, the COPOM-defined interest rates increased, while IV declined in the period; even so IV was higher than MV, indicating upside in this short period of time. This observation can be seen in Graph 9.

[55]According to Bodie, Kane & Marcus (2009), share valuation techniques can be top-down or bottom-up in fundamental analysis. The analyst uses the former when s/he first desires to assess the macroeconomic environment, then the sector in which the company operates, and finally the company's accounting statements and management quality. Inversely, s/eh uses the second technique when s/he desires to assess: first the company's accounting statements and management quality, then its sector, and finally the macroeconomic environment of the company. The model herein described used the bottom-up approached, as multi-agent computational simulations are usually used with a view to studying complexes tendencies that emerge from the micro level of the systems to their macro level.

Brazilian government, the macro-prudential decisions of market regulation, and the European financial crisis.

Another relevant conclusion of the model is *the transmission mechanism of monetary policy will affect Ibovespa`s GV negatively only when the companies that make up this index have, aggregated negative net cash generation*.

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