**Nonlinearities in Brazilian Yield Curve**

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**Abstract**

This article investigates the effect of macroeconomic conditions on the term structure represented by the spreads of 1, 5 and 10 years term bonds. The results indicate the presence of strong nonlinearities that may compromise the effectiveness of monetary policy. On the other hand the effects of key macroeconomic variables, such as interest rate, output, inflation, and exchange rates, in normal times, on the spreads, are consistent with expectations of coherent monetary policies.

JEL: E31, E43, E50

Keywords: inflation, yield curve, monetary policy

**1. Introduction**

The conduct of monetary policy is a recurring theme in studies and economic center of important theoretical divisions in the history of economic thought.

Most of the modern monetary policy models are based on the assumption that the economy presents a stable term structure. This is a key element for the coherent monetary policy based on the control of the short interest rate.

In some cases, the monetary authorities seek to create an effective and direct communication with the agents who work in the financial market, to reduce the uncertainty of its effect on interest rates for short-term and provide a clear understanding of the trajectory of interest rates short, medium and long term. The hope is that the interest rates on short-term, which is the instrument of monetary policy, have the capacity to affect the long-term rate and thus affect aggregate demand.

It has been the case in Brazilian economy that the risk of default due to confidence and external demand shocks has raised the long term interest rate independently of monetary policy. This is noteworthy during the currency crisis of the year 1999 and election cycle expectations in the year 2002. In both cases the short run interest rates jumped substantially without much effect on the long run rates. During these events there is an indication that monetary policy was endogenous, i.e. the long term may have affected the short term and not the other way around.

The aim of this paper is to examine the role of macroeconomic variables in determining the yield curve of the Brazilian economy.

Considering that the economy has suffered considerable confidence and external demand shocks in recent decades the econometric methodology employed will consider the presence of non-linear relationships between the variables.

This article is divided into four sections, besides this introduction, section 2 motivates the work showing the relationship between macroeconomic variables and interest rates in the financial market, section 3 presents the smooth transition regression model of the behavior of Brazilian interest rates and, finally, section 4 presents the main conclusions.

**2. Relationship between macroeconomic variables and interest rates in the financial market**

The study of the term structure of interest rates used to focus solely on the behavior of demand and supply of bonds in the financial market, as an example we can mention Merton (1973, p.163) and Vasicek (1977).

A new line of research attempts to identify the macroeconomic forces that affect the movement of term structure. It analyzes, the role of the monetary authority as influencing effectively and efficiently market expectations about the present and future trajectory of interest, such as Diebold, Rudebusch and Aruoba (2006).

To clarify the statement of the preceding paragraphs, note that possible explanations for the yield curve, particularly for movements in long-term rates, have opposite implications for the conduct of monetary policy. These explanations are, on the one hand a traditional analysis of the term structure of interest centered on the risk premium and, secondly, a new chain that cares about economic conditions and their effect on the interest rate path.

The possibility of substitution between bonds with different maturities ensures arbitrage condition according to which the rate of return on these bonds is equal. This condition is called by Campbell (1995) as the pure expectations hypothesis. Accordingly the rate of return of a bond of n periods maturity is equal to the average of current and expected rates of bonds of maturities of one period.



Where  corresponds to the rate of return of n periods bond, is for the rate of return of a one period bond. The spread is the difference between the long-term interest rate and current short-term interest rate as follows:



If the market expects the rate of return on short-term securities rise in the near future, then the long-term rate will tend to rise. The difference between the long-term rate and the short rate, i.e. the spread, can be considered as an approximation of the slope of the yield curve. This means that the slope of the yield curve depends on the behavior of short-term rates expected. The opposite occurs when it is expected that the rate of return will be reduced in the short term future.

It can be seen therefore that the role of expectations is key to determining the rate of long-term return and consequently the spread.

In addition to the average of the short term of interest there is an additional element to explain the long-term rate. The long-term bonds, as long as their face value is only available over the future, are subject to risks, such as default and inflation. Thus the yield of these bonds can embed a risk premium. This is represented as the factor .

The objective of this work is to explain the behavior of spread by macroeconomic conditions, through their effects on the short-term interest rates and on the risk premium.

Stock and Watson (1989), Dombrosky and Haubrich (1995), Stock and Watson (2001), Hamilton and Kim (2002), used the information of the term structure to anticipate economic cycles.

Evans (1985) studied the effects of fiscal policy and showed that large deficits affect interest rates of long and short term, changing the behavior of the yield curve. In another study, Evans (1987) found that the announcement of deficit has a temporary effect on short term interest rates.

In Brazil, Rocha, Moreira and Magalhães (2002) identified the importance of foreign debt on the spread of foreign securities, Matsumura and Moreira (2005) studied the importance of macroeconomic variables in the determination of the spread.

The inclusion of macroeconomic variables followed in this work comes from Diebold, Rudebusch and Aruoba (2006) that relates the level, slope and curvature of the yield curve with macroeconomic variables: Capacity Utilization installed (UC), Selic[[1]](#footnote-1), short-term interest () and inflation rate ().

**3. Modeling the term structure of interest rates for the Brazilian economy**

The models of the term structure exhibit nonlinearity as shown by Tabak and Andrade (2001). The objective of this work differs, however, and it is not to estimate the yield curve model but assess the importance of the macroeconomic variables in the term structure represented by the spread.

The first aspect that can be observed in the behavior of the spread in the Brazilian economy is the presence of substantial and rapid changes in the term structure.



Figure 1: Spread of the Term Structure in Brazil.

The spread behavior in Brazil can be seen in Figure 1. We can observe that the term structure of interest rates in Brazil does not present always a positive slope. Abrupt changes in the spread are notorious reflecting possibly confidence and demand shocks affecting the risk of default of public bonds. Note that the spread of the 1 year, 5 years and 10 years bonds, present negative values in some years.

Thus, the Brazilian economy displays some characteristics that may indicate the presence of non-linearity (regime change) in the term structure.

The model used is based on the idea of ​​"threshold", the dependent variable is a function of the independent variables in a peculiar way: the dependent variable is described by a linear process to a certain threshold, from which the coefficient of the variables changes.

The "threshold" approach is based on Hansen (2000), which provided the possibility to split the sample and use an indicator function with observable variables. The "threshold" variable has its use linked to the division of the sample into subgroups that can be considered as classes or arrangements of economic policy.

Teräsvirta (2007) showed that nonlinear models have gained importance in macroeconomics and financial modeling. Among the categories of nonlinear models it is well known the Switching, Markov-switching and the Smooth Transition Regression (STR) models. The STR model can be seen as an evolution of switching regression model.

The typical STR model is defined as:

(1)



Where is a vector of explanatory variables that contains a vector of lags of the dependent variable and a vector of exogenous variables . Note that  is the vector of parameters of the linear part and  is the parameter vector of the non linear.



The transition function has a slope parameter , a vector of location parameters , where , and a limit . The transition function is a logistic of the general type:



(2)



Where  is a constraint for identification.

The estimated model for the Brazilian economy follows the equation (1) where

and *zt* is the vector containing the exogenous macroeconomic variables  and the lagged dependent variable .



Note that the linearity is tested against a STR model with transition variable predetermined. As the theory does not specify the transition variable, the test is repeated for each variable in the set of potential transition variables. Thus, the model STR has the property of being identified under the alternative hypothesis, instead of the null hypothesis of linearity.

When  we have the transition function  and the model is linear. Otherwise, being nonlinear, we have to choose *K* restricted to *K = 1* or *K = 2*. For *K = 1*, the parameters  change monotonically as a function of  from up to . For *K = 2*, they change symmetrically around the midpoint , where the logistic function reaches its minimum value. The minimum value is between zero and ½, reaching zero when and ½ when . The parameter  controls the tilting and  provide the location of the transition function.



Similarly to the linear models, in STR have to reduce their size by elimination of

redundant variables in .

**3.1 Estimation of the Smooth Threshold Model – STR**

The database used contains monthly data for the period August 1997 to September 2011 (169 observations). The historical series of transactions of Future Pre x DI were obtained from the BM&F and were used for the construction of the term structure of interest rates; the capacity utilization of the Brazilian industry (UCI) in the last twelve months was obtained from the National Confederation of Industry - CNI; the rate of inflation measured by the National Index of Consumer Prices (IPCA) for 12 months was obtained from the IBGE and the Selic rate was obtained from the Central Bank of Brazil. The evolution of the EMBI + Brazil (Emerging Markets Bond Index) was obtained from Bloomberg and corresponds to RBrazil.

Considering the assumptions of economic theory some results are expected. Inflation (IPCA) is expected to have a positive effect on the interest rate term structure, due to the risk of inflation embedded in the yield curve. Besides this risk premium there is yet another element which is the expected increase in short term interest rates bonds driven by the monetary policy. Both factors tend to reinforce the positive effect of inflation on the long-term interest rate.

The coefficient of the annualized overnight rate of interest (Selic), in the spread equation depends on market expectations. If market believes that the increase in the Selic rate is permanent the effect on the long term rate will be close to one and so will not affect significantly the spread. This result is crucial for the effectiveness of monetary policy. When the coefficient is equal or greater than zero it means that the Selic rate changes will affect the long-term rate in equal or greater measure. However, when an increase in the Selic is understood by the market as a temporary the effect on the long-term rate will be small and the spread will fall. In this sense it is important to analyze the sign and value of the coefficient of the Selic rate in the spread equation.

 The level of capacity utilization (UCI) is the ratio of the volume produced and the ceiling that the machines and equipment are able to produce. It can be considered as a proxy for the output gap. It is important to note that a positive sign may indicate increase in expected short term interest rates (long-term yield curve). This can be understood as the expected response of the monetary authorities that dislike inflation threats.

Something similar can happen from the devaluation of the exchange rate (Dollar)*.* An increase in the exchange rate indicates an improvement in export competitiveness leading to future expansion. Another possibility is that devaluation may signal a future tightening of liquidity. In both cases an increase of the short term interest rates is expected. This means a positive coefficient of exchange rate on spread.

To pursue the empirical research on the Brazilian term structure, based on the smooth transition regression model, STR, it is necessary to identify which is the variable responsible for regime changes and what are the signs and magnitudes of the effects of macroeconomic and financial variables to explain movements in spreads measured by 1, 5 and 10 years bonds.

STR model is estimated using the conditional maximum likelihood, through the software Multi-J.

The specification began with a linear model suggested by Diebold et al (2006) including two additional variables, the exchange rate (Dollar), which accounts for the Brazilian dependence on foreign trade, and the so called Brazil Risk (RBrazil) as proxy for dependence on international capital. RBrazil is the level of country risk measured by the EMBI + Brazil[[2]](#footnote-2), the higher is the price index of this bundle of assets the higher is the risk perception by the international financial market of the directions of the Brazilian economy.

The specification of the models was parsimonious, considered the problems of autocorrelation and heteroscedasticity in residuals, and obeyed the minimization of the Akaike information criterion (AIC), Schwarz (SC) and Hannah-Quinn (HQ).

Using the usual tests for unit root in particular the DF-GLS test of Elliott, Rothenberg and Stock, the existence of unit root was rejected for all variables except the nominal exchange rate, Brazil Risk and the rate of inflation measured by IPCA. These variables, that presented unit root, were considered in first differences.

The next step is to apply econometric tests that seek to ensure the correct specification of the model. Accordingly, the first step is to test for the existence or non-linearity of the estimated model. If linearity is rejected, the choice is the correct value *K* (*K = 1 or K = 2*). Table 1 indicates that the model is non-linear and corresponds to the smooth logistic regression model LSTR1 with transition variable defined as the RBrazil.

Table 1: Linearity Test against STR.



The grid was calculated, indicating the value of the variable range that represents the type of gradient vector and the location represented by the variable *c1* whose results are detailed in Table 2.

Table 2: Grid.



Having the specification of the model, in the case LSTR1, the transition variable *RBrazil*, the variable grid that is c {-711.00, 813.00, 30} and the function that corresponds to the range {0, 50, 10,00, 30}, it is possible to run the estimation algorithms and, through an iterative process, to estimate the nonlinear model to evaluate the behavior of the yield curve.

The tests for heteroscedasticity and autocorrelation of the residuals are shown in Tables 3 and 4.

The test for absence of autocorrelation corresponds to the applied test used by Teräsvirta (1998) and corresponds to a special case of the general test Godfrey (1988). The procedure corresponds to regress the estimated residuals on the lagged ones and on the partial derivatives of the log -likelihood function with respect to the parameters of the model. The detailed results, in Table 3, show the null hypothesis of no autocorrelation of the estimated residuals of the models for spreads of 1, 5 and 10 years.

Table 3: Model Specification - Testing Absence of Correlation.



The test of Heteroscedasticity corresponds to the ARCH-LM test that represents a similar statistics to that described by Doornik and Hendry (1997), centered on a multivariate LM statistics. Table 4 shows that the null hypothesis of homoscedasticity of the residuals was not rejected for models estimated for terms of 1 and 10 years. For the term of five years, there is some probability of incurring in this type of problem.

Table 4: Model Specification - Test of Heteroscedasticity.



The proper fit of the model to estimate Spread of 1 year is suggested in figure 2, when it is shown the almost imperceptible difference between the original series and the series adjusted by linear and nonlinear components.



Figure 2: Set the STR model for the spread of 1 year.

The version of the STR model estimated is a generalization of the standard autoregressive model where the autoregressive coefficient is a logistic function, where . Note that is the smoothing parameter. For the spread of 1 year, 5 years and 10 years, values found ​​were 0.4986, 3.4501 and 4.7852, respectively, noting that the higher the value of sharper is the S shape of the transition variable.

 Analyzing the behavior of the transition function - the logistic type, it is considered a range of -800 to +1,000 points of variation in risk Brazil (RBrazil) and varying values 0 to 1​​ of the function G. It is notorious the S shape of the transition for the highest values of γ for the spreads of 5 and 10 years, as shown in Figure 3.

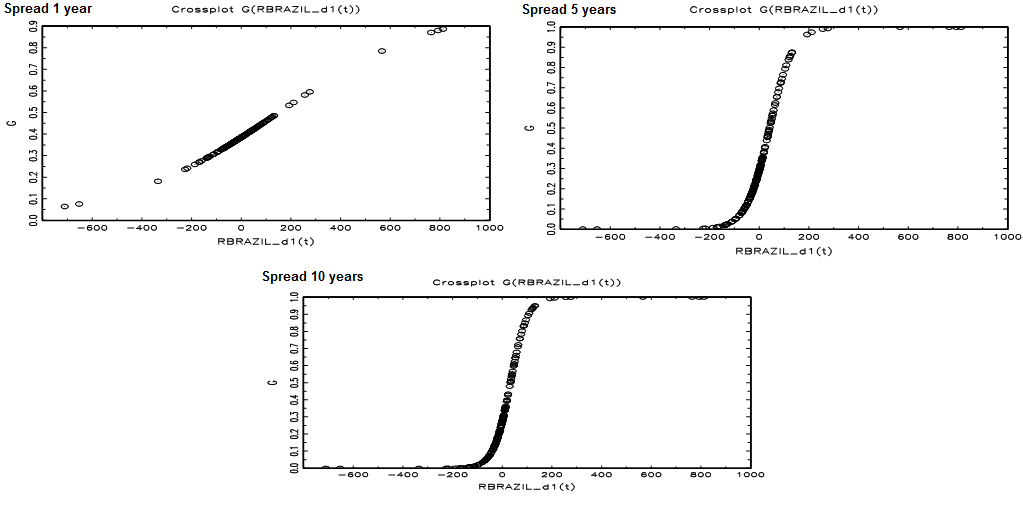


Figure 3: Values ​​of γ (gamma) in the model LSTR.

Brazilian economy has suffered in the last two decades severe shocks of confidence that affected mainly the foreign currency and long term government bonds. The variable chosen to define the transition and the threshold for the non linear model is the RBrazil.

It satisfies the tests as transition variable accordingly, as explained earlier. Note that RBrazil is measured by the weighted average of the Brazilian securities traded abroad in relation to securities of the same characteristic of the United States government and as such it is a very good proxy of country risk. It may affect directly the interest of the long term bonds; however its main importance is in explaining the main shifts of the whole term structure. The estimated thresholds of the change of the RBrazil are 150.90, 41.34 and 32.88 for the spreads of 1, 5 and 10 year bonds respectively (see Table 5 below). Its relationship with the non linear behavior of the term structure is illustrated in the Figure 4 that presents the dRBrazil (first differences of RBrazil), and the spreads of 1 and 10 year bonds.

Figure 4: Risk Brazil vs. Spread.

**3.2 Main Results of the Econometric Experiment**

The estimation of the spread of the term structure of interest rates support the conclusion of Diebold, Rudebusch and Aruoba (2004) and it indicates that macroeconomic variables have significant explanatory power for the volatility of the spread of the term structure of interest rates observed in the Brazilian financial market.

The importance of the spread of the previous period in the formation of the spread of the current period is positive and significant in all three terms analyzed (1, 5 and 10 years), as can be seen in the linear part of the estimates described in Table 5. The coefficient reduces by half when the term goes from 1 to 5 and 10 years, being 1.34, 0.77 and 0.72 respectively. It means that the memory of short term maturity (1 year) tends to increase volatility while longer term maturities (5 and 10 years) tend to smooth the spreads.



Table 5: Model STR Estimate.

The short term interest rate Selic rate presented significant negative coefficient, as observed in the linear part, an indication of a temporary impact on CDI's financial market. However, in the non-linear estimation, the signal is positive, indicating that the increase in the Selic rate has a positive effect on long term rates. In other words, the negative effect on the spread of the Selic is cushioned and may even become positive in the periods that the Brazil risk exceeds the threshold. This may indicate that agents expect the Selic rate will continue increasing in the future to circumvent the impact of shocks.

The exchange rate (U.S. dollars) had the expected positive sign, indicative of improved export competitiveness or devaluation involving a tightening of liquidity, both indicating increased rate of long-term (increased interest rate of short- expected term).

The analysis of the inflation rate is important because it is associated with the influence the yield curve through the inflation risk premium. The expected result of a positive effect is observed, but for longer terms the effect is reduced substantially, reflecting possibly the credibility of the Brazilian monetary authorities to control inflation. An alternative interpretation, however, is that an increase in inflation indicates that the central bank will gradually increase the rate of short-term interest. On the non-linear behavior there is a negative cushioning effect as before.

The coefficient of capacity utilization of the Brazilian industry presented a positive coefficient in the linear part, showing that the yield curve has a behavior attached to the cyclical dynamics of the Brazilian economy

Brazil risk is extremely relevant as the transition variable establishing the threshold of the non linear behavior of the Brazilian financial market. Its relevance to explain the level of the spread in the nonlinear part of the model is noteworthy. That may explain why its contribution as an additional variable within the equation is weak.

**4. Conclusion**

The study of the term structure of interest rates and the behavior of the term spread, the difference between long-term rates minus the short term, has puzzled monetary policy researchers, because not always is the expected effect of the actions of the monetary authority on long rates prevailing in the financial market.

The presence of strong nonlinearity in the term structure largely explains the pitfalls of monetary policy. That is, the lack of effectiveness of this policy in certain periods, particularly in those with non-linear behavior.

The positive correlation of inflation with the spread, is coherent with growing uncertainty, but also seems to indicate that the financial community believes in the monetary policy of the Central Bank. The same happened with the rate of capacity utilization and exchange rate.

Last but not least it is possible to consider that inflation rate, capacity utilization and exchange rate could be taken as proxies for the future expansionary cycles and in that sense the spread could be considered as a predictive factor of future expansion. This, however, shall be object of a new research.

Therefore, the Brazilian economic policymakers must watch the movements of the yield curve and analyze the information frequently, to guide the analysis and actions to be taken, to interfere in a beneficial way on the trajectory of the Brazilian economy.

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1. Interest rate is controlled by Banco Central do Brasil. [↑](#footnote-ref-1)
2. The EMBI + Brazil measure the price movement of securities from one day to the other. Its unit is the base point, i.e. 500 basis points imply that Brazilian bonds pay 5% more than the U.S., considering periodic interest payments, purchase price, redemption value and the time remaining until maturity obligations, is used by domestic and international investors [↑](#footnote-ref-2)