Exponential growth equation – in real life

Newcomb, HR. 1940. Ring-necked pheasant studies on Protection Island in the Strait of Juan de Fuca, Washington. MS thesis. Oregon State University.

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(Name) (Degree) (Major)

Date Thesis presented--April 21, 1940

Title---Ring-Necked Pheasant Studies on Protection Island----
----------in the Strait of Juan de Fuca, Washington-----------

Video

Photo credit: Andy Vernon

Photo credit: Lukasz Lukasik

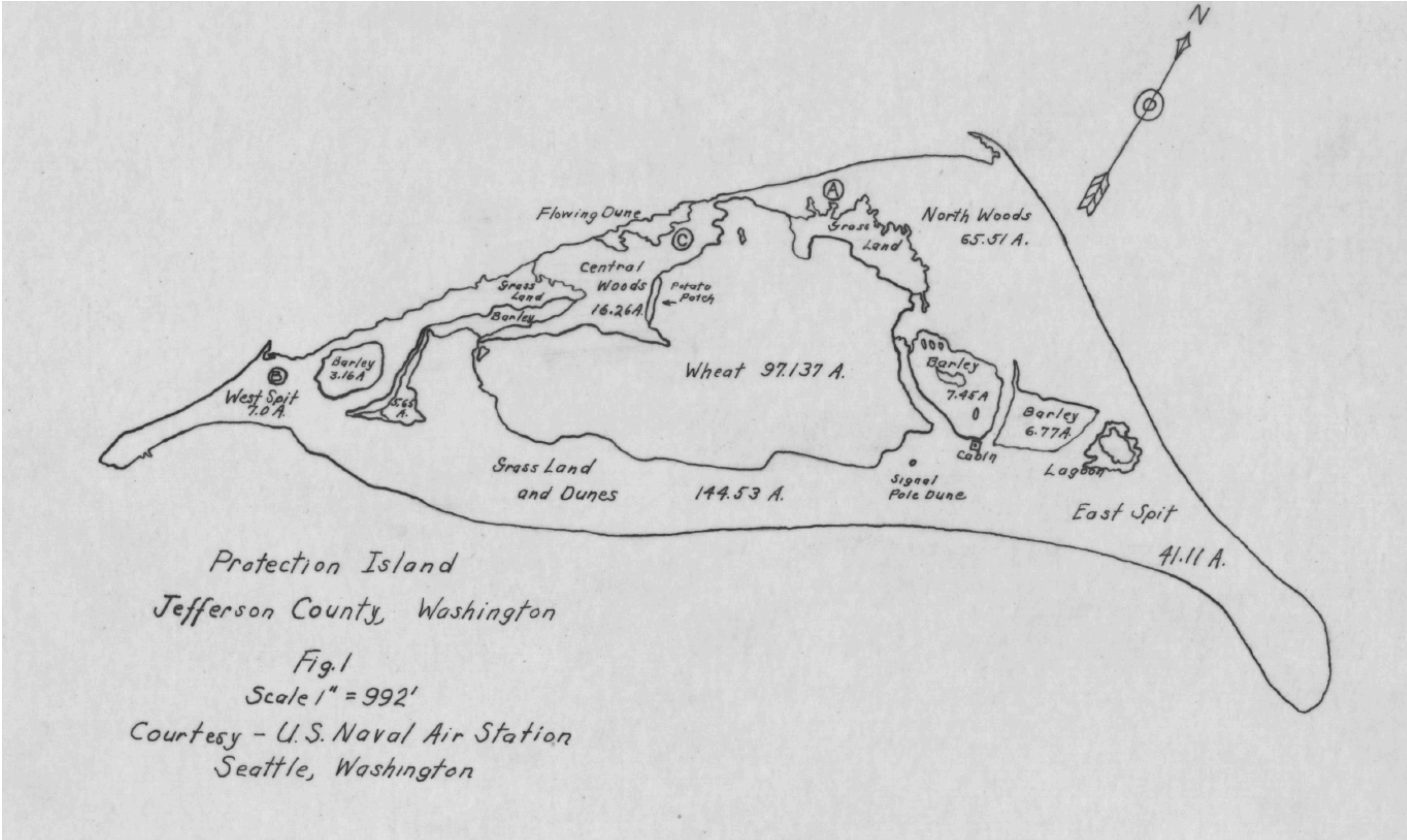

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Fig. 15

Typical owl kill
Young female pheasant
found August 17, 1939

Fig. 16

Typical pheasant
feeding on potatoes
July 29, 1939

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a. In May 1937, 10 pheasants were introduced to the island. Before the next breeding season there were 35.
b. November 10, 1938 a census estimated 110 pheasants.
c. October 13, 1939 a census estimated 400 pheasants.
d. Pheasant chicks are born during the summer.

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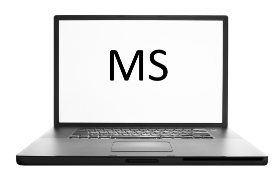
N_0: initial population size

N_t: size of the population after t years at the time of census

1. Ideally, the census points should be exactly 1 year apart. When does the census occur on Protection Island? How does this relate to the biology of the pheasant?

2. What are the values of N_0, N_1 and N_2?



Recall that for a population growing exponentially:

$$N_{t+1} = N_t + bN_t - dN_t$$

$$N_{t+1} = \lambda N_t$$

N_{t+1} : popn size next year (number)
N_t: Popn size this year (number)
b: per capita number of births each year (unitless)
d: fraction of popn that dies each year (unitless)
λ =1+ b-d: finite rate of population growth (unitless)

e. Between the 1938 and 1939 censuses, Newcomb observed that 17 out of 110 adult birds died.

3. d is fraction of popn that dies each year. What is d for the ring-tailed pheasant population on Protection Island?

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Those questions had clear answers. Interpreting the information on reproduction from Newcomb's MSc thesis is a lot more tricky – as is understanding what the parameter b actually means.

f. During the 1938 nesting season: 5.86 eggs/nest. 83.57% of eggs hatched.

g. During the 1939 nesting season: 8.73 eggs/nest. 64.58% hatched. Average number of chicks per clutch was 6.93.

h. You can assume the sex ratio is 50:50 male to female

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- b: per capita number of births each year (unitless)
- d: fraction of popn that dies each year (unitless)
- $\lambda = 1+b-d$: finite rate of population growth (unitless)

4. What is the value of b?

5. What is the value of λ?

- b: per capita number of births each year (unitless)
- d: fraction of popn that dies each year (unitless)
- $\lambda = 1+b-d$: finite rate of population growth (unitless)

4. What is the value of b? Ans: 6.93/2 = 3.465

5. What is the value of λ? Ans: 1+3.465 – 17/110 = 4.31

$$N_{t+1} = N_t + bN_t - dN_t \quad (1)$$

$$N_{t+1} = \lambda N_t \quad (2)$$

$$N_t = \lambda^t N_0 \quad (3)$$

Note: t is the time in years since the initial introduction. Each of the 3 formulas will give the same answer.

6. If the population grows exponentially, how many pheasants will there be in 1940?
7. How many pheasants will there be in 1942?
8. All of the formulas above are equivalent, but which formula is more useful? Which is easier to understand?

Some extra questions to think about at home:

9. Find the value of t when the population size equal to 1000?
10. For what values of λ does the population decrease?
11. What happens when λ=1?
12. Let td be the time when a population has doubled for its size at time t, i.e. $N_{td} = 2N_t$. Derive an expression for td.
13. When will the pheasant population have doubled from its size in 1939?

Exponential growth in discrete time

$$N_t = \lambda^t N_0 \qquad\qquad N_{t+1} = \lambda N_t$$

- The equations we have seen so far are called 'discrete time'. This equation is appropriate for populations that have regular, pulse reproduction.
- For example, pheasants reproduce once per year in the summer.
- Can you think of examples of other species that reproduce like this?

Exponential growth in continuous time

- Can you think of any species that reproduce continuously throughout the year?

- These species are better modelled with a continuous time model

Exponential growth in continuous time

Let $N(t)$ be the popn size at time t

$$\frac{dN(t)}{dt}$$

- The slope of a graph of population size versus time
- The instantaneous rate of change in the popn size, $N(t)$
- If positive, the population is increasing in size. If negative, the population is decreasing in size

Exponential growth in continuous time

$$\frac{dN(t)}{dt} = (b - d)N(t)$$

$$\frac{dN(t)}{dt} = rN(t)$$

- b: is the per capita birth *rate* (1/time)
- d: is the per capita death *rate* (1/time)
- $r = b$-d: intrinsic *rate* of natural increase (1/time)

Discrete time

$$N_{t+1} = \lambda N_t$$

- The population is increasing if $N_{t+1} > N_t$
- For what values of λ does the population size increase?

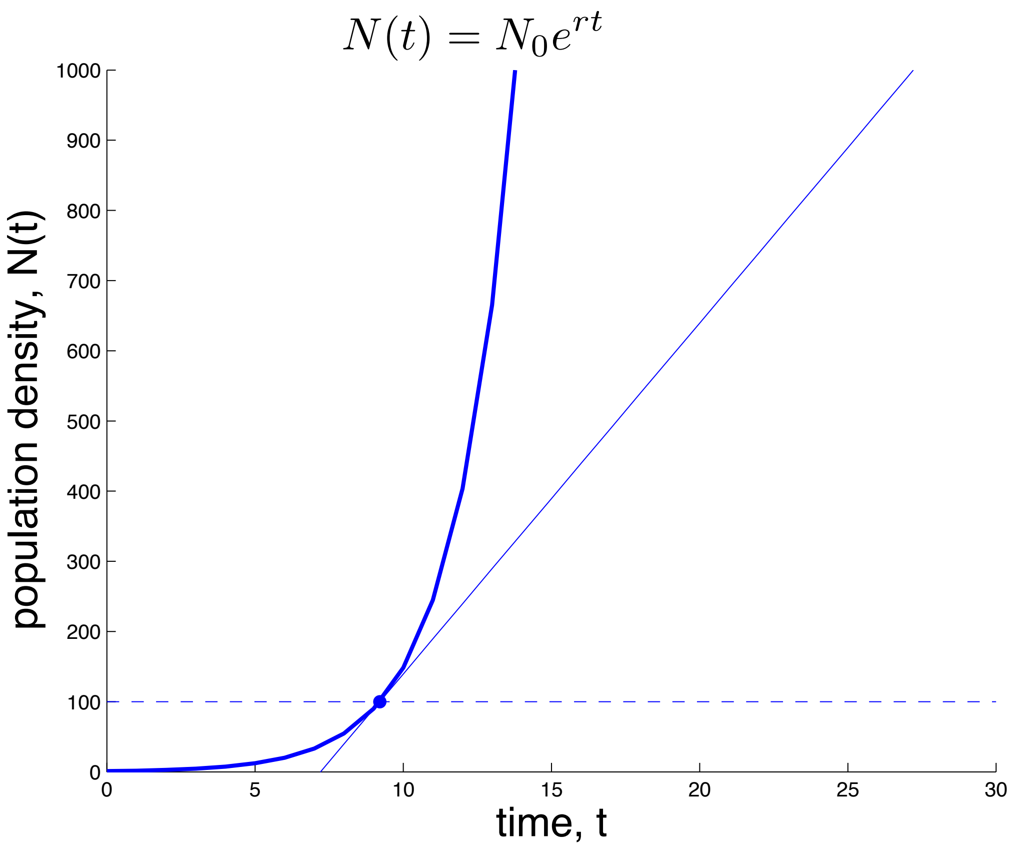
Continuous time

$$\frac{dN(t)}{dt} = rN(t)$$

- The population is increasing if $\frac{dN(t)}{dt} > 0$
- For what values of r does the population size increase?

Solving the exponential growth equation - continuous time

Continuous time exponential growth



- The population is growing exponentially with time

$$\frac{dN(t)}{dt} = rN(t)$$

- $r = 0.5$
- What is the slope of the tangent line?

Discrete time

$$N_{t+1} = \lambda N_t$$

$$N_t = \lambda^t N_0$$

1. Used for populations that have regular pulse reproduction, e.g. pheasants
2. The population is increasing if $N_{t+1} > N_t$
3. λ is non-negative (positive or zero)
4. Popn grows if $\lambda > 1$
5. $\lambda = 1+b-d$ and is called the finite rate of population growth

Continuous time

$$\frac{dN(t)}{dt} = rN(t)$$

$$N(t) = N_0 e^{rt}$$

1. Used for populations that reproduce continuously, e.g. bacteria
2. The population is increasing if $\frac{dN(t)}{dt} > 0$
3. r can be positive or negative
4. Popn grows if $r > 0$
5. $r = b-d$ and is called the intristic rate of natural increase

Summary

- Exponential growth assumes that all individuals in the population have the same chance of dying and produce the same number of offspring. These do no change over time.

- We can use the mathematical formulas to predict the size of the population in the future

Barraquand et al. 2013.
Lack of quantitative training among early career ecologists: a survey of the problem and potential solutions. PeerJ

- Survey of 937 early-career scientists who studied biology as an undergraduate
- 75% not satisfied with their understanding of mathematical models
- 75% felt the level of mathematics was "too low" in their ecology classes
- 90% wanted more mathematics classes for ecologists
- 95% wanted more statistics classes

Required reading for next class:

- 9-21 in Vandermeer, JH and Goldberg, DE. 2013. *Population Ecology: First Principles (2nd Edition)*.
- Vandermeer 2010. How poulations grow: The exponential and logistic equations. Nature Education Knowledge Project
- Levin, S. (Ed) 2009 Density dependence and single-species population dynamics in Population Ecology. Princeton Guide to Ecology. Princeton University Press.