

For Back Savers, let x_C be the number of Collegiate backpacks produced per week and x_M be the number of Mini backpacks produced per week. The goal is to maximize weekly profit 32 dollars per Collegiate and 24 dollars per Mini, subject to limits on nylon, labor, and weekly sales caps. Nylon availability of 5000 square feet with usage of 3 and 2 square feet per unit gives $3x_C + 2x_M \leq 5000$. Labor availability is $35 \text{ workers} \times 40 \text{ hours/week} \times 60 \text{ minutes/hour} = 84,000 \text{ minutes}$; production uses 45 and 40 minutes per unit, so $45x_C + 40x_M \leq 84,000$. Sales forecasts impose $x_C \leq 1000$ and $x_M \leq 1200$. Nonnegativity applies. The full LP is:

Maximize

$$z = 32 x_C + 24 x_M$$

Subject to

$$\begin{aligned} 3 x_C + 2 x_M &\leq 5000 \\ 45 x_C + 40 x_M &\leq 84,000 \\ x_C &\leq 1000 \\ x_M &\leq 1200 \\ x_C, x_M &\geq 0. \end{aligned}$$

For the Weigelt Corporation, let x_{ij} denote the number of units of size j produced per day at plant i , where $i \in \{1,2,3\}$ for plants 1–3 and $j \in \{L, M, S\}$ for large, medium, and small. Profits per unit are 420, 360, and 300 dollars for large, medium, and small, respectively. Each plant has a unit-capacity limit in units/day (750, 900, 450), and a storage-space limit per day (13,000, 12,000, 5,000 square feet). Storage use per unit is 20, 15, and 12 square feet for large, medium, and small. Market demand caps total daily sales at 900 large, 1,200 medium, and 750 small. To reflect management's requirement that all plants use the same percentage of their excess capacity, introduce a utilization variable u with $0 \leq u \leq 1$ and constrain each plant's total output to be no more than u times that plant's unit-capacity. The model is:

Maximize

$$Z = 420(x_{1L} + x_{2L} + x_{3L}) + 360(x_{1M} + x_{2M} + x_{3M}) + 300(x_{1S} + x_{2S} + x_{3S})$$

Subject to plant-capacity with common utilization

$$\begin{aligned} x_{1L} + x_{1M} + x_{1S} &\leq 750 u \\ x_{2L} + x_{2M} + x_{2S} &\leq 900 u \\ x_{3L} + x_{3M} + x_{3S} &\leq 450 u \\ 0 &\leq u \leq 1 \end{aligned}$$

Subject to plant storage

$$20 x_{1L} + 15 x_{1M} + 12 x_{1S} \leq 13,000$$

$$20 x_{2L} + 15 x_{2M} + 12 x_{2S} \leq 12,000$$

$$20 x_{3L} + 15 x_{3M} + 12 x_{3S} \leq 5,000$$

Subject to market demand

$$x_{1L} + x_{2L} + x_{3L} \leq 900$$

$$x_{1M} + x_{2M} + x_{3M} \leq 1,200$$

$$x_{1S} + x_{2S} + x_{3S} \leq 750$$

And nonnegativity

$$x_{ij} \geq 0 \text{ for all } i, j.$$