Applicative Functors

Hidden in plain view...

Jim Powers

Patch.com

NYC Functional Programming Meetup

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Inspiration

```haskell
buildName :: String -> Maybe String -> String -> String
buildName f m l = f ++ " " ++ (maybe "" id m) ++ " " ++ l
```

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Won't compile:

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desired :: Maybe String -> Maybe String -> Maybe String -> Maybe String
desired f m l = buildName <$> f <$> (Just m) <$> l
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buildName :: String -> Maybe String -> String -> String
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Won't compile:

```haskell
desired :: Maybe String -> Maybe String -> Maybe String -> Maybe String
desired f m l = buildName <$> f <$> (Just m) <$> l
```

Not bad:

```haskell
nice :: Maybe String -> Maybe String -> Maybe String -> Maybe String
nice f m l = do
  fn <- f
  ln <- l
  return $ buildName fn m ln
```

Inspiration

FUNCTIONAL PEARL

Applicative programming with effects

CONOR MCBRIDE
University of Nottingham

ROSS PATERSON
City University, London

Abstract

In this paper, we introduce Applicative functors—an abstract characterisation of an applicative style of effectful programming, weaker than Monads and hence more widespread. Indeed, it is the ubiquity of this programming pattern that drew us to the abstraction. We retrace our steps in this paper, introducing the applicative pattern by diverse examples, then abstracting it to define the Applicative type class and introducing a bracket notation which interprets the normal application syntax in the idiom of an Applicative functor. Further, we develop the properties of applicative functors and the generic operations they support. We close by identifying the categorical structure of applicative functors and examining their relationship both with Monads and with Arrows.

Inspiration

- Applicative?

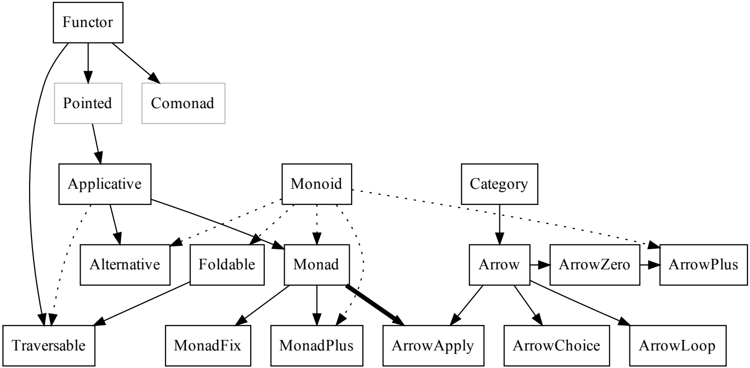
Inspiration

- Applicative?
- Functors?

Inspiration

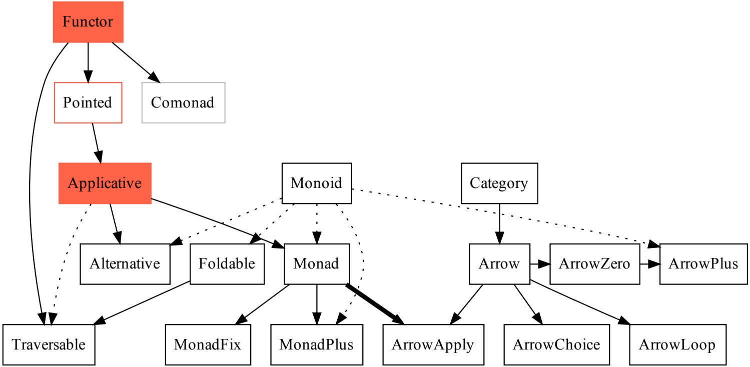
- Applicative?
- Functors?
- Effects?

Family Tree



Typeclassopedia, Brent Yorgey, TMR-issue13

Family Tree



Typeclassopedia, Brent Yorgey, TMR-issue13

Functor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b

(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
```

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```

Functors enable performing a computation within a context. The function does not know anything about the context it is running in.

Functor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b

(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
```

Functor Laws

```
fmap id = id

fmap (g . h) = fmap g . fmap h
```

Functor

```haskell
class Functor f where
  fmap :: (a -> b) -> f a -> f b

(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
```

Common Functor Instances

```haskell
instance Functor Maybe where
  fmap _ Nothing  = Nothing
  fmap f (Just x) = Just (f x)
```

Functor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b

(<$>) :: Functor f => (a -> b) -> f a -> f b
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Common Functor Instances

```
instance Functor Maybe where
  fmap _ Nothing  = Nothing
  fmap f (Just x) = Just (f x)
```

Example

```
mFunc :: (Show a, Show b) => (a -> b) -> Maybe a -> Maybe b
mFunc f x = f <$> x
```

Functor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b

(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
```

Common Functor Instances

```
instance Functor [] where
  fmap = map
```

Functor

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class Functor f where
  fmap :: (a -> b) -> f a -> f b

(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
```

Common Functor Instances

```
instance Functor [] where
  fmap = map
```

Example

```
lFunc :: (Show a, Show b) => (a -> b) -> [a] -> [b]
lFunc f x = f <$> x
```

Functor

```haskell
class Functor f where
  fmap :: (a -> b) -> f a -> f b

(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
```

Common Functor Instances

```haskell
instance Functor (Either b) where
  fmap f (Left l)  = Left l
  fmap f (Right r) = Right (f r)
```

Functor

```haskell
class Functor f where
  fmap :: (a -> b) -> f a -> f b

(<$>) :: Functor f => (a -> b) -> f a -> f b
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```

Common Functor Instances

```haskell
instance Functor (Either b) where
  fmap f (Left l)  = Left l
  fmap f (Right r) = Right (f r)
```

Example

```haskell
eFunc :: (Show a, Show c) => (a -> c) -> (Either [String] a) ->
(Either [String] c)
eFunc f x = f <$> x
```

Functor

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b

(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap
```

Common Functor Instances

```
instance Functor ((->) t) where
  fmap f g = f . g
```

Functor

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class Functor f where
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(<$>) = fmap
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Common Functor Instances

```haskell
instance Functor ((->) t) where
  fmap f g = f . g
```

Example

```haskell
fFunc :: (Show a, Show b, Show c) => (b -> c) -> (a -> b) -> (a -> c)
fFunc f x = f <$> x
```

Pointed

```
class Functor f => Pointed f where
  pure :: a -> f a
```

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  pure :: a -> f a
```

The **Pointed** typeclass is not represented in the Haskell standard library by itself. Conceptually, **Pointed** presents the ability to **lift** an ordinary value into a context, thus creating a **point** in that context.

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
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The **Applicative** combines that ability to create a **point** within a context, as well as the ability to apply a **lifted** function within that context.

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
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```

The **Applicative** combines that ability to create a **point** within a context, as well as the ability to apply a **lifted** function within that context.

Each context expresses how to apply the lifted function in its own particular way. Also known as its **idiom**.

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Applicative Laws

```
fmap g x = pure g <*> x
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Common Applicative Instances

```
instance Applicative Maybe where
  pure a = Just a
  Nothing <*> _ = Nothing
  _ <*> Nothing = Nothing
  (Just f) <*> (Just x) = Just (f x)
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Common Applicative Instances

```
instance Applicative Maybe where
  pure a = Just a
  Nothing <*> _ = Nothing
  _ <*> Nothing = Nothing
  (Just f) <*> (Just x) = Just (f x)
```

Example

```
mAplic :: (a -> b) -> Maybe a -> Maybe b
mAplic f x = pure f <*> x
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Common Applicative Instances

```
instance Applicative [] where
  pure a = [a]
  [] <*> _ = []
  _ <*> [] = []
  f:fs <*> l = (map f l) ++ (fs <*> l)
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Common Applicative Instances

```
instance Applicative [] where
  pure a = [a]
  [] <*> _ = []
  _ <*> [] = []
  f:fs <*> l = (map f l) ++ (fs <*> l)
```

Example

```
lAplic :: (Show a, Show b) => (a -> b) -> [a] -> [b]
lAplic f x = pure f <*> x
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Common Applicative Instances

```
instance Applicative (Either e) where
  pure          = Right
  Left  e <*> _ = Left e
  Right f <*> r = fmap f r
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
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```

Common Applicative Instances

```
instance Applicative (Either e) where
  pure           = Right
  Left  e <*> _ = Left e
  Right f <*> r = fmap f r
```

Example

```
eAplic :: (Show a, Show c) => (a -> c) -> (Either [String] a) ->
(Either [String] c)
eAplic f x = pure f <*> x
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Common Applicative Instances

```
instance Applicative ((->) a) where
  pure = const
  (<*>) f g x = f x (g x)
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Common Applicative Instances

```
instance Applicative ((->) a) where
  pure = const
  (<*>) f g x = f x (g x)
```

Example

```
fAplic :: (Show a, Show b, Show c) => (b -> c) -> (a -> b) -> (a -> c)
fAplic f x = pure f <*> x
```

Inspiration

```haskell
buildName :: String -> Maybe String -> String -> String
buildName f m l = f ++ " " ++ (maybe "" id m) ++ " " ++ l
```

Won't compile:

```haskell
desired :: Maybe String -> Maybe String -> Maybe String -> Maybe String
desired f m l = buildName <$> f <$> (Just m) <$> l
```

Not bad:

```haskell
nice :: Maybe String -> Maybe String -> Maybe String -> Maybe String
nice f m l = do
  fn <- f
  ln <- l
  return $ buildName fn m ln
```

Inspiration

```
buildName :: String -> Maybe String -> String -> String
buildName f m l = f ++ " " ++ (maybe "" id m) ++ " " ++ l
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Won't compile:

```
desired :: Maybe String -> Maybe String -> Maybe String -> Maybe String
desired f m l = buildName <$> f <$> (Just m) <$> l
```

Not bad:

```
nice :: Maybe String -> Maybe String -> Maybe String -> Maybe String
nice f m l = do
  fn <- f
  ln <- l
  return $ buildName fn m ln
```

Sweet:

```
sweet :: Maybe String -> Maybe String -> Maybe String -> Maybe String
sweet f m l = buildName <$> f <*> (pure m) <*> l
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Applicatives can be used as a tool to remove boilerplate!

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
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```

Applicatives can be used as a tool to remove boilerplate!

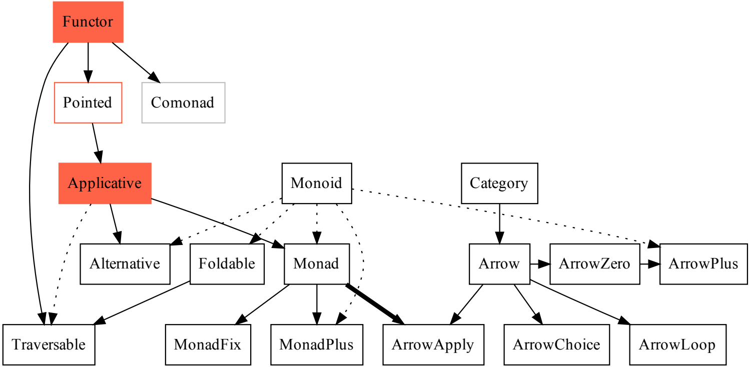
Orly? How?

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

Predictable web example!

Family Tree



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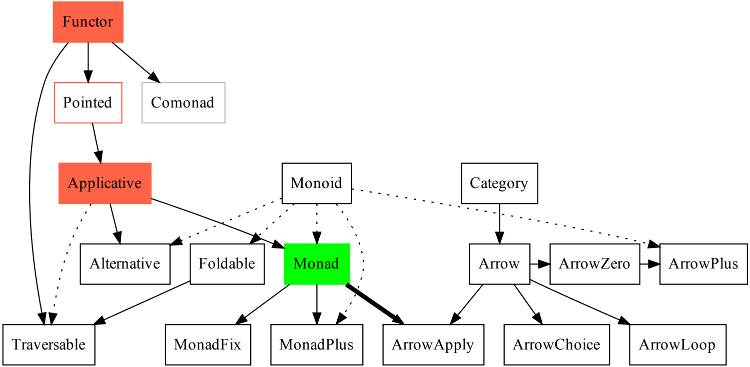

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Mo'nads!

http://twitpic.com/5j3x2t

Web Example

```
import Data.List (lookup)
import Web.MaybeResult

type Request = [(String,String)]

required :: Request -> String -> MaybeResult String
required r k = maybe (failure $ "Required value for:"++k++" missing")
success (lookup k r)

opt :: Request -> String -> MaybeResult (Maybe String)
opt r k = success (lookup k r)
```

Web Example

```
data ResponseValue = Value String
                   | Stream [String]
                     deriving (Show)

type Response = MaybeResult ResponseValue
```

Web Example

Reader Monad

```haskell
newtype Reader r a = Reader {
  runReader :: r -> a
}

instance Monad (Reader r) where
  return a = Reader $ \_ -> a
  m >>= k  = Reader $ \r -> runReader (k (runReader m r)) r
```

```haskell
myRequired :: String -> Reader Request (MaybeResult String)
myRequired k =
  asks (doRead k) >>= return
  where
    doRead k r = required r k

myOptional :: String -> Reader Request (MaybeResult (Maybe String))
myOptional k =
  asks (doRead k) >>= return
  where
    doRead k r = opt r k
```

Web Example

```
data MaybeResult a = Failure [String]
                   | Success a
  deriving (Eq, Ord, Show, Data, Typeable)

failure :: forall a. String -> MaybeResult a
failure s = Failure [s]

success :: forall a. a -> MaybeResult a
success s = Success s
```

Web Example

```
handler :: Reader Request Response
handler = do
  first <- myRequired "first"
  middle <- myOptional "middle"
  last <- myRequired "last"
  return (fmap Value (buildName <$> first <*> middle <*> last))

browser :: Request -> String
browser r = case (runReader handler r) of
              Success (Value v) -> "Page rendered STRING: "++v
              Success (Stream vs) -> "Page rendered STREAM: "++(show vs)
              Failure v -> "BZZT! Invalid request: "++(show v)
```

Web Example

```
goodRequest = [("first","James"),("last","Powers")]

badRequest = [("first","James")]

badRequest1 = [("middle","Matthew")]

main = do
  putStrLn $ show $ browser goodRequest
  putStrLn $ show $ browser badRequest
  putStrLn $ show $ browser badRequest1
```

Web Example

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goodRequest = [("first","James"),("last","Powers")]

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main = do
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```

- => "Page rendered STRING: James Powers"

Web Example

```haskell
goodRequest = [("first","James"),("last","Powers")]

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main = do
  putStrLn $ show $ browser goodRequest
  putStrLn $ show $ browser badRequest
  putStrLn $ show $ browser badRequest1
```

- $=>$ “Page rendered STRING: James Powers”
- $=>$ “BZZT! Invalid request: [Required value for:last missing]”

Web Example

```
goodRequest = [("first","James"),("last","Powers")]

badRequest = [("first","James")]

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main = do
  putStrLn $ show $ browser goodRequest
  putStrLn $ show $ browser badRequest
  putStrLn $ show $ browser badRequest1
```

- $=>$ “Page rendered STRING: James Powers”
- $=>$ “BZZT! Invalid request: [Required value for:last missing]”
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Web Example

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goodRequest = [("first","James"),("last","Powers")]

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main = do
  putStrLn $ show $ browser goodRequest
  putStrLn $ show $ browser badRequest
  putStrLn $ show $ browser badRequest1
```

- => "Page rendered STRING: James Powers"
- => "BZZT! Invalid request: [Required value for:last missing]"
- => "BZZT! Invalid request: [Required value for:first missing]"
- => Hrm...

Web Example

```
data MaybeResult a = Failure [String]
                   | Success a
  deriving (Eq, Ord, Show, Data, Typeable)

instance Functor MaybeResult where
  fmap _ (Failure s) = Failure s
  fmap f (Success x) = Success (f x)

instance Applicative MaybeResult where
  pure = Success
  Failure e1 <*> Failure e2 = Failure (e1 ++ e2)
  Failure e1 <*> Success _  = Failure e1
  Success _ <*> Failure e2 = Failure e2
  Success f <*> Success a  = Success (f a)

failure :: forall a. String -> MaybeResult a
failure s = Failure [s]

success :: forall a. a -> MaybeResult a
success s = Success s
```

Web Example

```
handler :: Reader Request Response
handler = do
  name <- asks buildNamehandler
  return (fmap Value name)

buildNamehandler :: Request -> MaybeResult String
buildNamehandler r = buildName <$> (required r "first") <*> (opt r
"middle") <*> (required r "last")
```

Web Example

```
goodRequest = [("first","James"),("last","Powers")]

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main = do
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main = do
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  putStrLn $ show $ browser badRequest1
```

- => “Page rendered STRING: James Powers”
- => “BZZT! Invalid request: [Required value for:last missing]”

Web Example

```haskell
goodRequest = [("first","James"),("last","Powers")]

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main = do
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```

- $=>$ "Page rendered STRING: James Powers"
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Web Example

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```

- $=>$ “Page rendered STRING: James Powers”
- $=>$ “BZZT! Invalid request: [Required value for:last missing]”
- $=>$ “BZZT! Invalid request: [Required value for:first missing,Required value for:last missing]”
- $=>$ Whoa! Now dats powah!

Applicatives vs. Monads

- Monads enable **conditional** (short-circuited) computation.

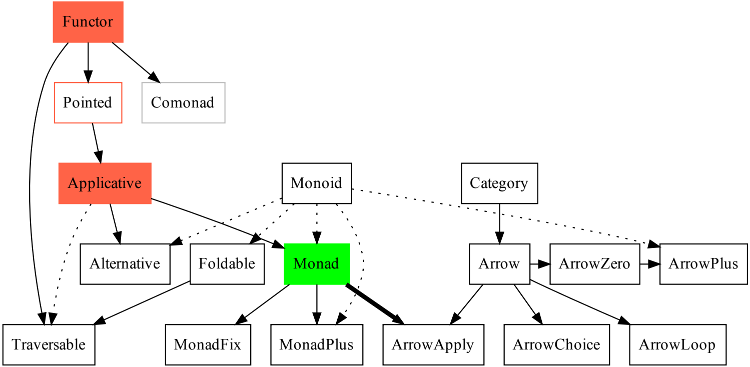
Applicatives vs. Monads

- Monads enable **conditional** (short-circuited) computation.
- Monads can be **stacked** through **monad transformers**.

Applicatives vs. Monads

- Monads enable **conditional** (short-circuited) computation.
- Monads can be **stacked** through **monad transformers**.
- Monads are much more restrictive and hence less common.

Family Tree



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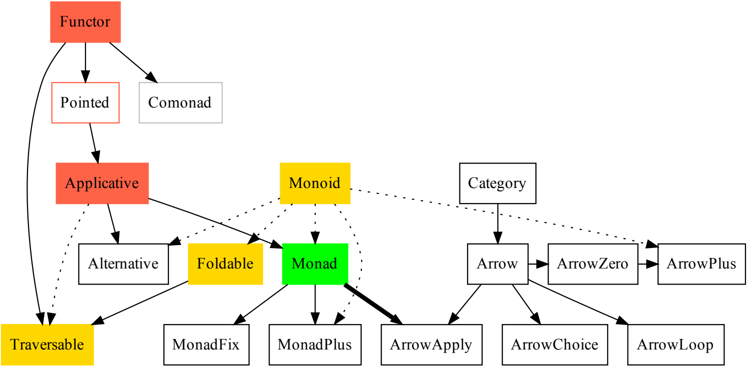

Typeclassopedia, Brent Yorgey, TMR-issue13

Monoid

```
class Monoid a where
  mempty  :: a
  mappend :: a -> a -> a
```

Foldable

```
class Foldable t where
  fold :: Monoid m => t m -> m
  foldMap :: Monoid m => (a -> m) -> t a -> m
  foldr :: (a -> b -> b) -> b -> t a -> b
  foldl :: (a -> b -> a) -> a -> t b -> a
  foldr1 :: (a -> a -> a) -> t a -> a
  foldl1 :: (a -> a -> a) -> t a -> a
```

Must Read!

Functional Programming with Bananas, Lenses, Envelopes and Barbed Wire

Erik Meijer [*] Maarten Fokkinga [†] Ross Paterson [‡]

Abstract

We develop a calculus for lazy functional programming based on recursion operators associated with data type definitions. For these operators we derive various algebraic laws that are useful in deriving and manipulating programs. We shall show that all example functions in Bird and Wadler's "Introduction to Functional Programming" can be expressed using these operators.

All sorts of folds...

- Anamorphism

All sorts of folds...

- Anamorphism
- Apomorphism

All sorts of folds...

- Anamorphism
- Apomorphism
- Hylomorphism

All sorts of folds...

- Anamorphism
- Apomorphism
- Hylomorphism
- Paramorphism

Must Read!

The Essence of the Iterator Pattern

Jeremy Gibbons and Bruno C. d. S. Oliveira
Oxford University Computing Laboratory
Wolfson Building, Parks Road, Oxford OX1 3QD, UK
http://www.comlab.ox.ac.uk/jeremy.gibbons/
http://www.comlab.ox.ac.uk/bruno.oliveira/

Abstract

The ITERATOR pattern gives a clean interface for element-by-element access to a collection, independent of the collection's shape. Imperative iterations using the pattern have two simultaneous aspects: *mapping* and *accumulating*. Various existing functional models of iteration capture one or other of these aspects, but not both simultaneously. We argue that McBride and Paterson's *applicative functors*, and in particular the corresponding *traverse* operator, do exactly this, and therefore capture the essence of the ITERATOR pattern. Moreover, they do so in a way that nicely supports modular programming. We present some axioms for traversal, discuss modularity concerns, and illustrate with a simple example, the *wordcount* problem.

Traversable

```
class (Functor t, Foldable t) => Traversable t where
  traverse :: Applicative f => (a -> f b) -> t a -> f (t b)
  sequenceA :: Applicative f => t (f a) -> f (t a)
  mapM :: Monad m => (a -> m b) -> t a -> m (t b)
  sequence :: Monad m => t (m a) -> m (t a)
```

Traversable

```
class (Functor t, Foldable t) => Traversable t where
  traverse :: Applicative f => (a -> f b) -> t a -> f (t b)
  sequenceA :: Applicative f => t (f a) -> f (t a)
  mapM :: Monad m => (a -> m b) -> t a -> m (t b)
  sequence :: Monad m => t (m a) -> m (t a)
```

```
traverse id [Just 1, Just 2]
```

=> Just [1,2]

Traversable

```
class (Functor t, Foldable t) => Traversable t where
  traverse :: Applicative f => (a -> f b) -> t a -> f (t b)
  sequenceA :: Applicative f => t (f a) -> f (t a)
  mapM :: Monad m => (a -> m b) -> t a -> m (t b)
  sequence :: Monad m => t (m a) -> m (t a)
```

```
traverse id [Just 1, Just 2]
```

=> Just [1,2]

```
traverse id [Just 1, Nothing, Just 2]
```

=> Nothing