CS 477/677 Analysis of Algorithms

Homework 6

Due November 9, 2021

1. (U & G-required) [100 points]

Suppose you are consulting for a company that manufactures PC equipment and ships it to distributors all over the country. For each of the n next weeks, they have a projected supply s_i of equipment (measured in pounds), which has to be shipped by an air freight carrier. Each week's supply can be carried by one of two air freight companies, A or B.

- Company A charges a fixed rate r per pound, so it costs $r * s_i$ to ship a week's supply (s_i)
- Company B makes contracts for a fixed amount c per week, independent of the weight. However, contracts with company B must be made in blocks of four consecutive weeks at a time.

A *schedule*, for the PC company, is a choice of air freight company (A or B) for each of the n weeks with the restriction that company B, whenever it is chosen, must be chosen for blocks of four contiguous weeks at a time. The cost of the schedule is the total amount paid to companies A and B, according to the description above.

You are asked to give a polynomial time algorithm that takes a sequence of supply values $s_1, s_2, \ldots, s_n$ and returns a schedule of minimum cost. In order to achieve this, you need to answer the following questions:

(a) [20 points] Write a recursive formula that illustrates how the **optimal value** for the cost can be computed from optimal values to subproblems. **Note:** this part should not contain any code, just the recursive formula.

Submit: the recursive formula, along with definitions and explanations on what is computed.

(b) [30 points] Write an algorithm that computes an optimal solution to this problem, based on the recurrence above. The algorithm should save in an output file the optimal values for all the subproblems as well as the optimal value for the entire problem. Implement your algorithm in C/C++ and run it on the following values:

- `r` = 1, `c` = 10, the sequence of s_i values: 11, 9, 9, 12, 12, 12, 12, 9, 9, 11.

Submit:

- The source file containing your algorithm (name the file **`cost_pb.c`** or **`cost_pb.cpp`**)
- The output file created by your algorithm (name the file **`cost_pb_out.txt`**), which contains:
  - The table with the optimal values to all subproblems (save the entire table)
  - The optimal value for the entire problem (indicate this on a separate line after the table, even if the value is found in the table above)

(c) [20 points] Update the algorithm you developed at point (b) to enable the reconstruction of the optimal solution, i.e., to **store the choices** you made when computing the optimal values for each subproblem in part (b) (which company was used in an optimal solution for shipping). Include these updates in your algorithm implementation from point (b).

Submit:

- The source file containing your algorithm (name the file **`cost_pc.c`** or **`cost_pc.cpp`**)
- The output file created by your algorithm (name the file **`cost_pc_out.txt`**), which contains the values of the table containing the additional information (choices) needed to reconstruct the optimal solution (print the entire table).

(d) [30 points] Using the additional information computed at point (c), write an algorithm that outputs which company was used for shipping in the optimal schedule. Your algorithm should save this optimal solution in an output file. Implement this algorithm in C/C+.

Submit:

- The source file containing your algorithm (name the file **`cost_pd.c`** or **`cost_pd.cpp`**)

- The output file created by your algorithm (name the file **`cost_pd_out.txt`**) that contains the optimal solution to the problem given by the numerical values in part (b).

2. (G-required) [20 points] Show that the longest simple path from a node *x* in a red-black tree to a descendant leaf has length at most twice that of the shortest simple path from node *x* to a descendant leaf.

Extra Credit

3. [20 points] Exercise 14.1-6 (page 345).