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Research Paper

Evaluating the performance of the skewed distributions to forecast value-at-risk in the global financial crisis

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ABSTRACT

This paper evaluates the performance of several skewed and symmetric distributions by modeling the tail behavior of daily returns and forecasting value-at-risk (VaR). First, we use some goodness-of-fit tests to analyze which distribution best fits the data. The comparisons in terms of VaR are carried out by examining the accuracy of the VaR estimate and minimizing the loss function from the points of view of the regulator and the firm. The results show that the skewed distributions outperform the normal and Student t (ST) distributions in fitting portfolio returns. Following a two-stage selection process, whereby we initially ensure that the distributions provide accurate VaR estimates, and focusing on the firm's loss function, we can conclude that skewed distributions outperform the normal and ST distributions in forecasting VaR.

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From the point of view of the regulator, the superiority of the skewed distributions related to ST is not evident. As the firms are free to choose the model they use to forecast VaR, in practice, skewed distributions will be used more frequently.

Keywords: value-at-risk (VaR); parametric model; skewness generalized t (SGT) distribution; generalized autoregressive conditional heteroscedasticity (GARCH) model; risk management; loss function.

1 INTRODUCTION

A primary tool for financial risk assessment is value-at-risk (VaR). This is defined as the maximum loss expected from a portfolio of assets over a certain holding period at a given confidence level. Since the Basel Committee on Banking Supervision at the Bank for International Settlements requires financial institutions to meet capital requirements on the basis of VaR estimates, allowing them to use internal models for VaR calculations, this measurement has become a basic market risk management tool for financial institutions.

Despite VaR's conceptual simplicity, its calculation can be rather complex. Many approaches have been developed to forecast VaR: nonparametric approaches, eg, historical simulation; semi-parametrics approaches, eg, extreme value theory and the dynamic quantile regression conditional autoregressive VaR (CaViaR) model (Engle and Manganelli 2004); and parametric approaches, eg, RiskMetrics (JP Morgan 1996).

The parametric approach is one of the most-used by financial institutions. This approach usually assumes that the asset returns follow a normal distribution, which is an assumption that simplifies the computation of VaR considerably. However, it is inconsistent with the empirical evidence of asset returns, which finds that the distribution of asset returns is skewed, fat tailed and peaked around the mean (see Bollerslev 1987). This implies that extreme events are much more likely to occur in practice than would be predicted by the symmetric, thinner tailed normal distribution. Further, the normality assumption can produce VaR estimates that are inappropriate measures of the true risk faced by financial institutions.

Since the Student t (ST) distribution has fatter tails than the normal one, this distribution has been commonly used in finance and risk management, particularly to model conditional asset returns (Bollerslev 1987). The empirical evidence of this distribution performance in estimating VaR is ambiguous. Some papers show that the ST distribution performs better than the normal distribution (see Abad and Benito 2013; Orhan and Köksal 2012; Polanski and Stoja 2010), while other papers report that the ST distribution overestimates the proportion of exceptions (see Angelidis *et al* 2007; Guermat and Harris 2002).

The ST distribution can often successfully account for the excess kurtosis found in common asset returns, but this distribution does not capture the skewness of the returns. Taking this into account, one direction for research in risk management involves searching for other distribution functions that capture this characteristic. The skewness Student t distribution (SSD) of Hansen (1994), the exponential generalized beta of the second kind (EGB2) of McDonald and Xu (1995), the generalized error distribution (GED) of Nelson (1991), the skewness generalized t (SGT) distribution of Theodossiou (1998), the skewness error generalized distribution (SGED) of Theodossiou (2001) and the inverse hyperbolic sign (IHS) of Johnson (1949) are the most used in VaR literature. Some applications of skewness distributions to forecast VaR can be found in Chen *et al* (2012), Polanski and Stoja (2010), Bali and Theodossiou (2008), Bali *et al* (2008), Haas *et al* (2004), Zhang and Cheng (2005), Haas (2009), Ausín and Galeano (2007), Xu and Wirjanto (2010) and Kuester *et al* (2006). Chen *et al* (2012) compared the ability of a normal distribution, an ST distribution, an SSD and a GED to forecast VaR. In this comparison, the SSD and GED provide the best results. Polanski and Stoja (2010) compared the normal, ST, SGT and EGB2 distributions and found that only the latter two distributions provide accurate VaR estimates. Bali and Theodossiou (2008) compared a normal distribution with the SGT distribution and showed that the SGT provided a more accurate VaR estimate.

In this paper, we carry out a comprehensive comparison of the aforementioned skewed distributions: SSD, SGT, SGED and IHS. In addition, we include both the normal and ST distributions. This comparison is performed in the following two stages. First, we compare the distributions in statistical terms to determine which is the best for fitting financial returns. Then, we compare the distributions in terms of VaR, in order to select which is best for forecasting VaR.

The main differences with the previous literature are as follows.

(1) We consider a larger number of skewed distributions.

(2) The comparison in statistical terms is made using a large battery of tests (ie, the likelihood ratio, chi-square (Chi2) and Kolmogorov–Smirnov (KS) tests); the papers aforementioned only used the likelihood ratio test.

(3) In order to carry out the comparison in terms of VaR, we evaluate the results on the basis of two criteria: (i) the accuracy of VaR and (ii) the minimization of two loss functions that reflect the concerns of the financial regulator and the firm (Sarma *et al* 2003).

In the next section, we present the methodology used to estimate VaR and summarize the statistical tests and loss functions used to evaluate VaR estimates. In Section 3, we present the data. The results of the comparison in statistical terms and in terms of VaR are presented in Sections 4 and 5, respectively. Section 6 includes our main conclusions.

2 METHODOLOGY

According to Jorion (2001), VaR is defined as the worst expected loss over a given horizon under normal market conditions at a given level of confidence. VaR is thus a conditional quantile of the asset return distribution. Let $r_1, r_2, r_3, \dots, r_n$ be identically distributed independent random variables representing the financial returns. Use $F(r)$ to denote the cumulative distribution function (cdf), $F(r) = \Pr(r_t < r \mid \Omega_{t-1})$, conditionally on the information set Ω_{t-1} that is available at time $t-1$. Assume that r_t follows the stochastic process $r_t = \mu + \varepsilon_t$, where $\varepsilon_t = z_t \sigma_t$, $z_t \sim \text{iid}(0, 1)$, μ is the conditional mean and σ_t is the conditional standard deviation of returns. The VaR with a given probability $\alpha \in (0, 1)$, denoted by $\text{VaR}(\alpha)$, is defined as the α quantile of the probability distribution of financial returns:

$$F(\text{VaR}(\alpha)) = \Pr(r_t < \text{VaR}(\alpha)) = \alpha.$$

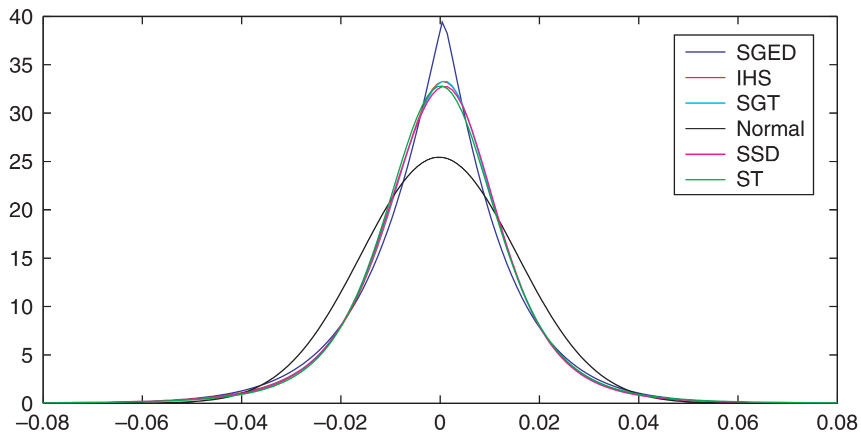
Under the framework of the parametric techniques (see Jorion 2001), the conditional VaR estimate can be calculated as $\text{VaR}_t = \mu_t + \hat{\sigma}_t k_\alpha$, where μ_t represents the conditional mean, which we assume is 0, $\hat{\sigma}_t$ is the conditional standard deviation and k_α denotes the corresponding quantile of the distribution of the standardized returns at a given confidence level $1-\alpha$.[1,2] Following other papers dedicated to comparing the performance of skewness distributions in the framework of the parametric method, in this paper, the conditional volatility has been estimated below the ST distribution (see Polansky and Stoja 2010).[3]

[1] In case of the skewed distributions, the k_α value is a function of the skewness and kurtosis parameters.

[2] The squared daily returns, as employed by generalized autoregressive conditional heteroscedasticity (GARCH) models, are not the most efficient measure of daily volatility. The recent literature has focused on the realized volatility and daily stock ranges. These are also known to be more efficient measures of return volatility than daily returns, since they employ all price changes during the day. Chou *et al* (2009) present a thorough review of range-based models. Lin *et al* (2012) propose a nonlinear smooth transition conditional autoregressive range model for capturing smooth volatility asymmetries in international financial stock markets. However, we only consider GARCH models because the database is daily based.

[3] Although considering the skewness distribution for the estimation of the conditional volatility of the financial returns may be interesting for future research, at this stage we dismiss this option. This is because the standard statistic programs do not include the estimation of these models, so practitioners will not be able to use them.

FIGURE 1 Density functions.



The considered distributions. The data used in the graphs is obtained from the Nikkei index. The sample spans from January 3, 2000 to November 30, 2012.

Finally, once the variance has been calculated, we estimate the distributions of the standardized returns under each of the considered distribution functions: normal, ST, SGT, SGED, SSD and IHS. Table 1 shows the density function of these skewed distributions. Their graphs for the Nikkei index are shown in Figure 1.

In the first stage, before the calculation of the VaR, we compare the distributions in statistical terms. To do this, we use a likelihood test, in order to compare the fit of two models, and two goodness-of-fit tests, the chi-square of Pearson (1900) and the KS test (Kolmogorov 1933; Smirnov 1939; Massey 1951), in order to determine whether a sample can be considered as a draw sample from a given specified distribution. The KS test is based on the maximum difference between an empirical and a hypothetical cumulative distribution function. The Chi2 test is based on the probability distribution function. It performs by grouping the data into bins and calculating the observed and expected counts for those bins.

In the second stage, we calculate the VaR and evaluate the accuracy of the VaR estimate under these distributions. We have an exception when $r_{t+1} < \mathrm{VaR}(\alpha)$; in this case, the exception indicator variable (I_{t+1}) is equal to 1 (0 in other cases).

A common criterion to compare VaR models is the rate of violation (VRate), defined as the proportion of exception over the forecast period. The ratio VRate/α should be close to 1. Thus, models with $\mathrm{VRate}/\alpha \approx 1$ are better.

When $\mathrm{VRate}/\alpha < 1$ ($\mathrm{VRate} < \alpha$), risk and potential loss estimates are conservative; alternatively, when $\mathrm{VRate}/\alpha > 1$ ($\mathrm{VRate} > \alpha$), financial institutions may not

TABLE 1 Density functions. [Table continues on next page.]

Distribution	Formulation	Restrictions
SDD of Hansen (1994)	$f(z_t \mid v, \eta) = \begin{cases} bc\left[1 + \dfrac{1}{\eta - 2}\left(\dfrac{bz_t + \alpha}{1 - \eta}\right)^2\right]^{-(\eta+1)/2} & \text{if } z_t < -\left(\dfrac{a}{b}\right) \\ bc\left[1 + \dfrac{1}{\eta - 2}\left(\dfrac{bz_t + \alpha}{1 + \eta}\right)^2\right]^{-(\eta+1)/2} & \text{if } z_t \geqslant -\left(\dfrac{a}{b}\right) \end{cases}$ $a = 4\lambda c\left(\dfrac{\eta - 2}{\eta - 1}\right), \quad b^2 = 1 + 3\lambda^2 - a^2,$ $c = \dfrac{\Gamma((\eta+1)/2)}{\sqrt{\pi((\eta-2)\Gamma(\eta/2))}}, \quad z_t = (r_t - \mu_t)/\sigma$	$\lvert\lambda\rvert < 1$, $\eta > 2$
SGED of Theodossiou (2001)	$f(z_t \mid \lambda, k) = \dfrac{C}{\sigma}\exp\left(-\dfrac{\lvert z_t + \delta\rvert^k}{(1 + \mathrm{sign}(z_t + \delta)\lambda)^k \theta^k}\right),$ $z_t = (r_t - \mu_t)/\sigma_t, \quad C = k/(2\theta\Gamma(1/k)), \quad \delta = 2\lambda A S(\lambda)^{-1},$ $\theta = \Gamma(1/k)^{0.5}\Gamma(3/k)^{-0.5}S(\lambda)^{-1}, \quad S(\lambda) = \sqrt{1 + 3\lambda^2 - 4A^2\lambda^2}$	$\lvert\lambda\rvert < 1$ skewed parameter, k = kurtosis parameter

TABLE 1 Continued.

Distribution	Formulation	Restrictions
SGED of Theodossiou (1998)	$f(z_t \mid \lambda, \eta, k) C \left(1 + \frac{\lvert z_t + \delta \rvert^k}{((\eta+1)/k)(1 + \operatorname{sign}(z_t + \delta)\lambda)^k \theta^k}\right)^{-(\eta+1)/k}$, $C = 0.5k \left(\frac{\eta+1}{k}\right)^{-1/k} B\left(\frac{\eta}{k}, \frac{1}{k}\right)^{-1} \theta^{-1}, \qquad \theta = \frac{1}{\sqrt{g - p^2}}$, $p = 2\lambda B\left(\frac{\eta}{k}, \frac{1}{k}\right)^{-1} \left(\frac{\eta+1}{k}\right)^{1/k} B\left(\frac{\eta-1}{k}, \frac{3}{k}\right)$, $g = (1 + 3\lambda^2) B\left(\frac{\eta}{k}, \frac{1}{k}\right)^{-1} \left(\frac{\eta+1}{k}\right)^{1/k} B\left(\frac{\eta-1}{k}, \frac{3}{k}\right), \qquad \delta = p\theta$	$\lvert\lambda\rvert < 1$ skewed parameter, $\eta > 2$ tail thickness parameter, $k > 0$ peakedness parameter, δ Pearson's skewness, $z_t = (r_t - \mu_t)/\sigma_t$
IHS of Johnson (1949)	$\mathrm{IHS}(z_t \mid \lambda, k) = -\frac{k}{\sqrt{2\pi(\theta^2 + (z_t + \delta)^2)}}$ $\times \exp\left(-\frac{k^2}{2}(\ln(z_t + \delta) + \sqrt{\theta^2 + (z_t + \delta)^2}) - (\lambda + \ln(\theta))^2\right)$, $\theta = 1/\sigma_w, \qquad \delta = \mu_w/\sigma_w$, $\sigma_w = 0.5(\exp(2\lambda + k^{-2}) + \exp(-2\lambda + k^{-2}) + 2)^{0.5}(\exp(k^{-2}) - 1)$	μ_w mean, σ_w standard deviation, $w = \sinh(\lambda + x/k)$, x standard normal variable

In all of these distributions, z represents the standardized returns.

allocate sufficient capital to cover likely future losses. As in Gerlach *et al* (2011), conservative rates are preferred for models in which VRate/α is equidistant from 1. Following Gerlach *et al* (2011), we evaluate the accuracy of the VaR estimates focusing on those ratios.

In addition, we formally test the accuracy of the VaR estimates by using four standard tests: unconditional and conditional coverage tests, the backtesting criterion (BTC) and the dynamic quantile (DQ) test. Kupiec (1995) shows that the unconditional coverage test has as a null hypothesis $\hat{\alpha} = \alpha$, with a likelihood ratio statistic

$$\text{LR}_{\text{UC}} = 2[\log(\hat{\alpha}^x(1-\hat{\alpha})^{N-x}) - \log(\alpha^x(1-\alpha)^{N-x})],$$

which follows an asymptotic $\chi^2(1)$ distribution. A similar test for the significance of the deviation of $\hat{\alpha}$ from α is the BTC statistic $Z = (N\hat{\alpha} - N\alpha)/\sqrt{N\alpha(1-\alpha)}$, which follows an asymptotic $N(0,1)$ distribution. The conditional coverage test (Christoffersen 1998) jointly examines if the percentage of exceptions is statistically equal to that expected and the serial independence of I_{t+1}. The likelihood ratio statistic of the conditional coverage test is $\text{LR}_{\text{CC}} = \text{LR}_{\text{UC}} + \text{LR}_{\text{ind}}$, which is asymptotically distributed $\chi^2(2)$, and the LR_{ind} statistic is the likelihood ratio statistic for the hypothesis of serial independence against first-order Markov dependence. Finally, the dynamic quantile test proposed by Engle and Manganelli (2004) examines whether the exception indicator is uncorrelated with any variable that belongs to the information set $\varOmega_{t-1}$ available when the VaR was calculated. This is a Wald test of the hypothesis that all slopes are zero in a regression of the exception indicator variable on a constant, five lags and the VaR.

Additionally, we evaluate the magnitude of the losses experienced. The model that minimizes the total loss is preferred to other models. For this purpose, we have considered two loss functions: the regulator loss function and the firm's loss function.[4] Lopez (1998, 1999) proposed a loss function that reflects the utility function of a regulator. In this specification, the magnitude loss function assigns a quadratic specification when the observed portfolio losses exceed the VaR estimate. Thus, we penalize only when an exception occurs according to the following quadratic specification:

$$\text{RLF}_t = \begin{cases} (\text{VaR}_t - r_t)^2 & \text{if } r_t < \text{VaR}_t, \\ 0 & \text{otherwise.} \end{cases}$$

[4] One strand of the literature has proposed a lot of loss functions. However, Abad *et al* (2014) show that the VaR model that minimizes the total losses is robust within both groups of loss functions but differs across firms' and supervisors' loss functions. Therefore, we consider only two loss functions: one function designed by regulators and the other designed by risk managers.

This loss function gives higher scores when failures take place and considers the magnitude of the failure. In addition, the quadratic term ensures that large failures are penalized more than small failures.

But firms use VaR in internal risk management and, in this case, there is a conflict between the goal of safety and the goal of profit maximization. A too-high VaR forces the firm to hold too much capital, imposing the opportunity cost of capital upon the firm. Taking this into account, Sarma *et al* (2003) define the firm's loss function as follows:

$$\mathrm{FLF}_t = \begin{cases} (\mathrm{VaR}_t - r_t)^2 & \text{if } r_t < \mathrm{VaR}_t, \\ -\beta \mathrm{VaR}_t & \text{otherwise,} \end{cases}$$

with β being the opportunity cost of capital.

3 DATA

The data consists of closing daily returns on nine composite indexes from January 1, 2000 to November 30, 2012 (around 3250 observations). The indexes are Nikkei (Japan), Hang Seng (Hong Kong), Tel Aviv (100) (Israel), Merval (Argentina), Standard & Poor's 500 (S&P 500) and Dow Jones (United States), FTSE 100 (United Kingdom), CAC 40 (France) and IBEX 35 (Spain). The data was extracted from the Bloomberg database. The computation of the indexes' returns (r_t) is based on the formula $r_t = \ln(I_t) - \ln(I_{t-1})$, where I_t is the value of the stock market index for period t.

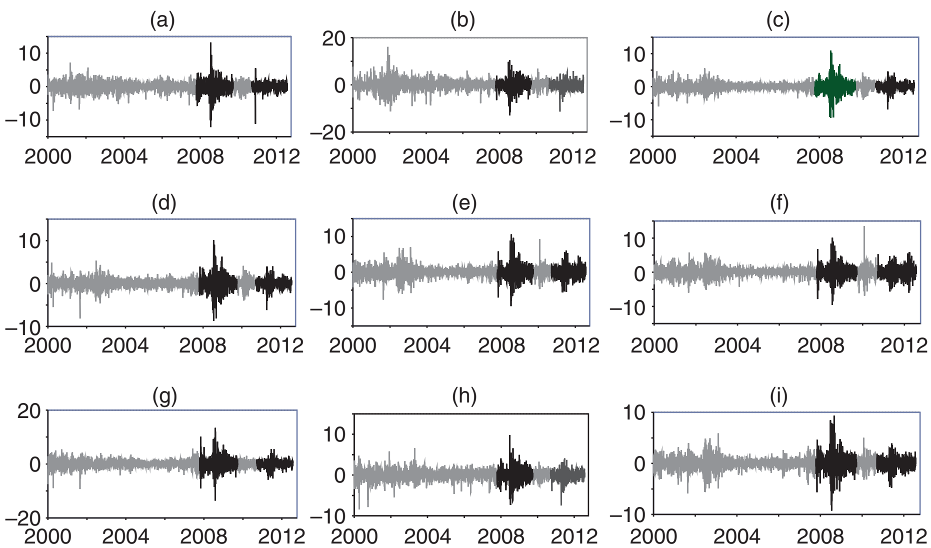
Figure 2 shows the daily returns and Table 2 provides basic descriptive statistics for them. For each index, the unconditional mean of daily returns is very close to 0. The unconditional standard deviation is especially high for Merval (2.14). For the rest of stock index returns, the standard deviation moves between 1.27 (Dow Jones) and 1.63 (Hang Seng). Going back to Figure 2, we can see that the fluctuation range of the returns is not constant, which means that the variance of these returns changes over time.

In order to gain some insight, we adopt the volatility measure proposed by Franses and van Dijk (2000), wherein the volatility of returns is defined as

$$V_t = (r_t - E(r_t^2 \mid \Omega_{t-1}))^2,$$

where Ω_{t-1} is the information set at time $t-1$. Figure 3 presents V_t as "volatilities". The volatility of the series was high during the early 2000s, especially in the Merval index. From 2001 to 2002, the conditional volatility of Merval was almost one point higher than the whole period, and even greater than those volatilities shown from 2008 to 2009. This corresponds to the Argentine economic crisis (1999–2002), which

FIGURE 2 Stock index returns.



The daily evolution of returns of nine indexes from January 3, 2000 to November 30, 2012. (a) Nikkei. (b) Merval. (c) S&P 500. (d) Dow Jones. (e) CAC 40. (f) IBEX 35. (g) Hang Seng. (h) Tel Aviv. (i) FTSE 100. *Source:* Bloomberg.

was a major downturn in Argentina's economy.[5] The period from 2003 to early 2007 was very quiet. The financial market tensions started in August 2007, and they were followed by a global financial and economic crisis, which led to a significant rise in the volatility of returns. This increase was especially important after August 2008, coinciding with the fall of Lehman Brothers. From 2008 to 2009, the volatilities of S&P 500, Nikkei and IBEX 35, measured by the standard deviation of returns, were 2.42, 2.20 and 2.10, respectively. In the case of S&P 500, the standard deviation was almost one point higher than the standard deviation of the whole period 2000–2012 (1.57). A similar increase can be observed in all indexes. In the last two years of the sample, we observe a more stable period than during the financial crisis.

The skewness statistic is negative and significant for all of the indexes considered, except in the case of CAC 40 and IBEX 35. This means that the distribution of those returns is skewed to the left. When considering CAC 40 and IBEX 35, the skewness

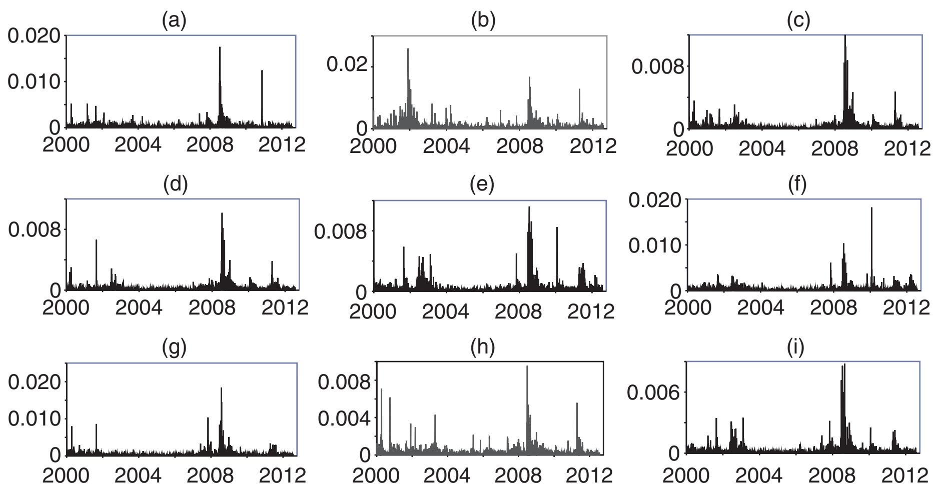
[5] It began in 1999 with a decrease of the real gross domestic product. The crisis caused the fall of the government, default on the country's foreign debt, widespread unemployment, riots, the rise of alternative currencies and the end of the peso's fixed exchange rate to the US dollar.

TABLE 2 Descriptive statistics.

Index	Mean	Median	Maximum	Minimum	SD	Skewness	Kurtosis	JB
Nikkei	−0.022	0.004	13.234	−12.111	1.568	−0.393**	9.686**	5996
						(0.044)	(0.087)	(0.001)
Hang Seng	0.008	0.044	13.407	−13.582	1.632	−0.065	10.386**	7253
						(0.043)	(0.087)	(0.001)
Tel Aviv	0.024	0.055	9.782	−8.425	1.338	−0.311**	6.945**	2107
						(0.044)	(0.087)	(0.001)
Merval	0.047	0.090	16.117	−12.952	2.140	−0.093*	7.944**	3243
						(0.043)	(0.087)	(0.001)
S&P 500	−0.001	0.050	10.957	−9.47	1.354	−0.158**	10.293**	7212
						(0.043)	(0.086)	(0.001)
Dow Jones	0.010	0.049	10.089	−8.7	1.265	−0.185**	9.372**	5515
						(0.043)	(0.086)	(0.001)
FTSE 100	−0.004	0.025	9.384	−9.266	1.301	−0.135**	8.692**	4416
						(0.043)	(0.086)	(0.001)
CAC 40	−0.015	0.019	10.595	−9.472	1.572	0.038	7.494**	2782
						(0.043)	(0.085)	(0.001)
IBEX 35	−0.012	0.060	13.484	−9.5858	1.576	0.1227**	7.8219**	3177
						(0.043)	(0.086)	(0.001)

The descriptive statistics of the daily percentage returns of Nikkei, Hang Seng, Tel Aviv, Merval, S&P 500, Dow Jones, FTSE 100, CAC 40 and IBEX 35. The sample period is from January 2, 2000 to November 30, 2012. The index return is calculated as $r_t = 100(\ln(I_t) - \ln(I_{t-1}))$, where I_t is the index level for period t. Standard errors of the skewness and excess kurtosis are calculated as $\sqrt{6/n}$ and $\sqrt{24/n}$, respectively. The Jarque–Bera (JB) statistic is distributed as the chi-square with two degrees of freedom. *, ** denote significance at the 5% and 1% levels, respectively.

FIGURE 3 Volatility of stock index returns.



The conditional volatility of daily returns. The volatility was estimated using the approach proposed by Franses and van Dijk (2000). Sample runs from January 3, 2000 to November 30, 2012. (a) Nikkei. (b) Merval. (c) S&P 500. (d) Dow Jones. (e) CAC 40. (f) IBEX 35. (g) Hang Seng. (h) Tel Aviv. (i) FTSE 100. *Source:* Bloomberg.

statistic is positive, implying that these distributions are skewed to the right; only in the case of IBEX 35 is this statistic significant at the 1% level.

For all of the indexes considered, the excess kurtosis statistic is very large and significant at the 1% level, implying that the distributions of those returns have much thicker tails than the normal distribution. Similarly, the Jarque–Bera (JB) statistic is significant, rejecting the assumption of normality. These results are in line with those obtained by Bollerslev (1987), Bali and Theodossiou (2007) and Bali *et al* (2008), among others. All of these authors find evidence that the empirical distribution of the financial return is asymmetric and exhibits a significant excess of kurtosis (fat tails and peakness).

In order to capture the nonnormal characteristics observed in our data set, we fit several skewed distributions: SGT, SGED, SSD and IHS. In this comparison, we also include the normal and symmetric ST distributions. In Table 3, we present the estimated parameters of these distributions. This provides the estimates for the mean (μ) and the standard deviation (σ) of log returns. The standard errors are in parentheses. As expected, these estimates are quite similar across distributions and do not differ much from the simple arithmetic means and standard deviations of log returns (see Table 2). The unconditional mean is close to 0 for all of the indexes, and the unconditional standard deviation moves around 1.5%, except in the case of

TABLE 3 Maximum likelihood estimates of distribution functions. [Table continues on next two pages.]

	SGT	SGED	SSD	IHS	ST	Normal
Nikkei						
μ	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
σ	0.016** (0.001)	0.015** (0.000)	0.016** (0.000)	0.015** (0.000)	0.016** (0.001)	0.016** (0.000)
λ	−0.047* (0.021)	−0.041** (0.004)	−0.048* (0.021)	−0.086 (0.032)		
η	4.766** (0.282)		4.442** (0.236)		4.404** (0.232)	
κ	1.896** (0.078)	1.133** (0.033)		1.472** (0.054)		
Hang Seng						
μ	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
σ	0.016** (0.001)	0.016** (0.000)	0.017** (0.000)	0.016** (0.000)	0.017** (0.001)	0.016** (0.000)
λ	−0.034** (0.014)	−0.031 (–)	−0.041* (0.018)	−0.067* (0.027)		
η	6.328** (0.547)		3.314** (0.100)		3.297** (0.100)	
κ	1.338** (0.044)	0.977** (0.028)		1.210 (0.033)		
Tel Aviv						
μ	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.001** (0.000)	0.000 (0.000)
σ	0.013** (0.001)	0.013** (0.000)	0.014** (0.000)	0.013** (0.000)	0.014** (0.001)	0.013** (0.000)
λ	−0.060** (0.021)	−0.052** (0.016)	−0.062** (0.021)	−0.102** (0.032)		
η	5.247** (0.365)		4.381** (0.232)		4.331** (0.228)	
κ	1.785** (0.068)	1.175** (0.035)		1.463** (0.054)		

TABLE 3 Continued.

	SGT	SGED	SSD	IHS	ST	Normal
Merval						
μ	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.001* (0.000)	0.000 (0.000)
σ	0.022** (0.001)	0.021** (0.000)	0.023** (0.000)	0.022** (0.000)	0.023** (0.001)	0.021** (0.000)
λ	−0.043* (0.018)	−0.033** (0.002)	−0.047** (0.018)	−0.068* (0.027)		
η	4.456** (0.241)		3.083** (0.075)		3.088** (0.078)	
κ	1.531** (0.051)	0.998** (0.028)		1.171** (0.029)		
S&P 500						
μ	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
σ	0.014** (0.001)	0.013** (0.000)	0.016** (0.000)	0.014** (0.000)	0.015** (0.001)	0.014** (0.000)
λ	−0.064** (0.013)	−0.062 (–)	−0.069** (0.016)	−0.087** (0.024)		
η	5.735** (0.430)		2.760** (0.046)		2.770** (0.049)	
κ	1.239** (0.038)	0.902** (0.008)		1.079** (0.023)		
Dow Jones						
μ	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
σ	0.013** (0.001)	0.012** (0.000)	0.014** (0.000)	0.013** (0.000)	0.014** (0.001)	0.013** (0.000)
λ	−0.058** (0.017)	−0.057** (0.002)	−0.059** (0.018)	−0.088** (0.026)		
η	4.496** (0.241)		3.122** (0.078)		3.122** (0.080)	
κ	1.524** (0.051)	0.983** (0.027)		1.178** (0.029)		

TABLE 3 Continued.

	SGT	SGED	SSD	IHS	ST	Normal
FTSE 100						
μ	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
σ	0.013** (0.001)	0.013** (0.000)	0.014** (0.000)	0.013** (0.000)	0.014** (0.001)	0.013** (0.000)
λ	−0.054** (0.018)	−0.049** (0.003)	−0.056** (0.018)	−0.083** (0.027)		
η	4.273** (0.212)		3.237** (0.089)		3.231** (0.091)	
κ	1.623** (0.055)	1.015** (0.028)		1.208** (0.031)		
CAC 40						
μ	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
σ	0.016** (0.001)	0.015** (0.000)	0.016** (0.000)	0.016** (0.000)	0.016** (0.001)	0.016** (0.000)
λ	−0.062** (0.018)	−0.044* (0.021)	−0.066** (0.019)	−0.094** (0.028)		
η	4.545** (0.249)		3.540** (0.120)		3.533** (0.122)	
κ	1.673** (0.059)	1.065** (0.030)		1.277** (0.036)		
IBEX 35						
μ	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
σ	0.016** (0.001)	0.016** (0.000)	0.017** (0.000)	0.016** (0.000)	0.016** (0.001)	0.016** (0.000)
λ	−0.073** (0.017)	−0.068 (–)	−0.069** (0.018)	−0.092** (0.028)		
η	7.127** (0.717)		3.548** (0.125)		3.584** (0.132)	
κ	1.380** (0.045)	1.050** (0.030)		1.270** (0.037)		

Parameter estimates of the normal distribution, SGT distribution, SGED, SSD, IHS and ST distribution, with standard errors in parenthesis. Nine stock market returns in the period January 1, 2000–November 30, 2012. μ, σ, λ and η are the estimated mean, standard deviation, skewness parameter and tail thickness parameter; κ represents the peakness parameter. *Significance at the 5% level. **Significance at the 1% level.

Merval (2.14). As expected from the previous analysis, the Merval index is the most volatile index.

The skewness parameter λ is negative for all indexes and significant at the 1% level, which means that the distributions of these returns are skewed to the left. This result is in opposition to the preliminary evidence that suggested a symmetric distribution for CAC 40 and a skewed distribution to the right for IBEX 35.

In the case of the SGT distribution, the parameter κ mainly controls the peakness of the distribution around the mode, while the parameter η mainly controls the tails of the distribution (adjusting the tails to the extreme values). The parameter η has the degrees of freedom interpretation, as in the ST distribution. For all of the series and distributions considered, the kurtosis parameters (η and κ) are highly significant. For the SGT distribution, the value of κ is around 1.5, except for Nikkei and Tel Aviv, which are 1.89 and 1.78, respectively. The value of η is around 4.5 for Nikkei, Merval, Dow Jones, FTSE 100 and CAC 40. For the rest of the indexes, it is a little bit higher. These estimates are quite different from those of the normal distribution ($\kappa = 2$ and $\eta = \infty$), which indicates that this set of returns is characterized by excess kurtosis.

4 COMPARISON OF THE DISTRIBUTIONS IN STATISTICAL TERMS

In this section, we want to answer the following question: which distribution is the best one for fitting asset returns? The above results provide strong support for the hypothesis that stock returns are not normal. As the normal distribution is nested within the SGT, SGED and SSD distributions, we can use the loglikelihood ratio for testing the null hypothesis of normality against that of the SGT distribution, SGED or SSD. For all of the indexes considered, this statistic is quite large and significant at the 1% level, providing evidence against the normality hypothesis (see Table 4).

To evaluate which is the most adequate, we perform several kinds of test. First, as the SGT nets all of the distributions considered in this paper (except IHS), we use the likelihood ratio test to evaluate which distribution is best for fitting the data.[6] Overall, for all of the indexes considered, the likelihood statistics indicate rejection of the SGED, SSD and ST distribution in favor of the SGT distribution (see Table 4). As the IHS is not nested in the SGT distribution, we cannot conclude that the SGT distribution is the best. So, to ensure the robustness of these results, a couple of alternative tests have been used: the Chi2 and KS tests. Unlike the likelihood ratio

[6] Specifically, it gives $\eta = \infty$ for the SGED, $\kappa = 2$ for the SSD, $\lambda = 0$ and $\kappa = 2$ for the ST distribution and $\lambda = 0$, $\eta = \infty$ and $\kappa = 2$ for the normal distribution (see Hansen *et al* (2001) for a comprehensive survey on the skewed fat-tailed distributions).

test used to compare two distributions, the Chi2 and KS tests are used to examine if the asset returns' empirical distribution follows a particular theoretical distribution. The theoretical distributions considered are normal, ST, SGT, SSD, SGED and IHS. The Chi2 statistic (see Table 4) suggests that the empirical distributions of the returns can be adequately characterized using two distributions: SGT and IHS. Both distributions seem to fit the data well in eight of the nine indexes considered. For the Hang Seng, Tel Aviv and S&P 500 indexes, the SGED cannot be rejected either. However, the ST and normal distributions do not fit any index. The KS test provides similar results (see Table 4). According to this test, the empirical distributions of all of the indexes (except Nikkei) follow a SGT distribution. The IHS fits the data well in only five of the indexes (Merval, CAC 40, IBEX 35, Tel Aviv and Nikkei). The SSD fits the data well in four indexes (Merval, CAC 40, IBEX 35 and FTSE 100) and the SGED fits the data well in four indexes (Nikkei, Merval, IBEX 35 and Hang Seng). The ST distribution only fits the data well in three of the nine indexes, while the normal distribution does not do well in any index.

Taking into account the results described in this section, we can conclude that the symmetric distributions (normal and ST) do not fit financial returns well. This is in line with the previous results shown in the above sections. Among the set of skewed distributions considered in this paper, the SGT distribution seems to be the best at fitting the data, followed closely by the IHS distribution.

5 EVALUATING THE PERFORMANCE IN TERMS OF VALUE-AT-RISK

In this section, we compare the normal, ST and skewed distributions in terms of VaR. The comparison is carried out by evaluating (i) the accuracy of the VaR estimates and (ii) the losses that VaR produces. For each distribution, we use parametric approaches to forecast the VaR out-of-the-sample and one-step-ahead at the 1% and 0.25% confidence levels.

The data period is divided into a learning sample from January 1, 2000 to December 31, 2007 and a forecast sample from January 1, 2008 to the end of December 2009. We choose this forecast period because it is characterized by a high volatility all over the world; this is known in the financial literature as the global financial crisis. In Figure 2, we highlight the period analyzed in black.

5.1 Backtesting

In line with Gerlach *et al* (2011), Table 5 shows the ratio VRate$/\alpha$ for $\alpha = 0.01$ and 0.0025 across all six models and nine indexes. The best model's ratio in each index is underlined, while bold font indicates that the $\mathrm{LR}_{\mathrm{UC}}$ test rejects the model at the 5% level. The results are similar for both levels, $\alpha = 1\%$ and 0.25%. Most of the normal

TABLE 4 Goodness-of-fit tests. [Table continues on next page.]

	Log-L	LR_{normal}	LR_{SGT}	Chi2	KS
Nikkei					
SGT	8920.4	463.2**	–	5.239 (0.022)**	0.031 (0.004)
SGED	8897.4	417.2**	46.0**	7.715 (0.006)	0.027 (0.021)**
SSD	8920.3	463.0**	0.2	13.448 (0.001)	0.034 (0.001)
IHS	8918.6	–	–	3.453 (0.063)*	0.029 (0.011)**
ST	8918.2	–	4.4	20.958 (0.000)	0.029 (0.008)
Normal	8688.8		–	124.218 (0.000)	0.058 (0.000)
Merval					
SGT	8016.9	612.6**	–	8.164 (0.017)**	0.019 (0.197)*
SGED	8003	584.8**	27.8**	12.318 (0.002)	0.027 (0.021)**
SSD	8012.5	603.8**	8.8*	15.965 (0.003)	0.020 (0.147)*
IHS	8017	–	–	6.005 (0.111)*	0.018 (0.260)*
ST	8010.4	–	13.0**	18.687 (0.000)	0.024 (0.053)*
Normal	7710.6	–	–	253.700 (0.000)	0.072 (0.000)
S&P 500					
SGT	9777.7	824.2**	–	14.092 (0.001)	0.028 (0.013)*
SGED	9769.2	807.2**	17.0**	8.761 (0.013)**	0.033 (0.002)
SSD	9762.2	793.2**	31.0**	35.861 (0.000)	0.038 (0.000)
IHS	9769.2	–	–	22.316 (0.000)	0.035 (0.001)
ST	9757.1	–	41.2**	33.963 (0.000)	0.037 (0.000)
Normal	9365.6	–	–	266.854 (0.000)	0.080 (0.000)
Dow Jones					
SGT	9929.7	682.6**	–	6.333 (0.042)**	0.028 (0.011)**
SGED	9914.2	651.6**	31.0**	24.553 (0.000)	0.032 (0.002)
SSD	9925.1	673.4**	9.2**	21.875 (0.000)	0.034 (0.001)
IHS	9928.4	–	–	8.647 (0.034)**	0.029 (0.007)
ST	9921.6	–	16.2**	30.360 (0.000)	0.030 (0.007)
Normal	9588.4	–	–	256.272 (0.000)	0.071 (0.000)
CAC 40					
SGT	9297.4	523.6**	–	3.209 (0.201)*	0.023 (0.067)*
SGED	9281	490.8**	32.8**	17.858 (0.000)	0.033 (0.002)
SSD	9295.3	519.4**	4.2*	7.248 (0.027)**	0.027 (0.018)**
IHS	9297.4	–	–	2.761 (0.430)*	0.022 (0.079)*
ST	9291.1	–	12.6**	38.232 (0.000)	0.025 (0.030)**
Normal	9035.6	–	–	191.314 (0.000)	0.064 (0.000)

TABLE 4 Continued.

	Log-L	LR_{normal}	LR_{SGT}	Chi2	KS
IBEX 35					
SGT	9176.8	484.2**	–	3.767 (0.052)*	0.027 (0.018)**
SGED	9169.8	470.2**	14.0**	11.509 (0.001)	0.028 (0.011)**
SSD	9167.1	464.8**	19.4**	13.293 (0.001)	0.028 (0.011)**
IHS	9170.9	–	–	7.174 (0.067)*	0.029 (0.010)**
ST	9162.4	–	28.8**	25.413 (0.000)	0.034 (0.001)
Normal	8934.7	–	–	118.562 (0.000)	0.065 (0.000)
Hang Seng					
SGT	8927.5	649.0**	–	1.543 (0.214)*	0.027 (0.020)**
SGED	8918.4	630.8**	18.2**	5.519 (0.063)*	0.029 (0.010)**
SSD	8916.3	626.6**	22.4**	9.290 (0.002)	0.037 (0.000)
IHS	8920.4	–	–	1.873 (0.392)*	0.034 (0.001)
ST	8914.6	–	25.8**	15.599 (0.000)	0.035 (0.001)
Normal	8603	–	–	23.434 (0.000)	0.072 (0.000)
Tel Aviv					
SGT	9358.2	316.8**	–	5.721 (0.057)*	0.027 (0.023)**
SGED	9343.6	332.6**	29.2**	4.288 (0.039)**	0.034 (0.002)
SSD	9357.3	360.0**	1.8	11.097 (0.004)	0.029 (0.008)
IHS	9358.6	–	–	5.878 (0.053)*	0.026 (0.024)**
ST	9354	–	8.4*	33.459 (0.000)	0.025 (0.041)**
Normal	9177.3	–	–	106.813 (0.000)	0.058 (0.000)
FTSE 100					
SGT	9857	628.2**	–	3.311 (0.191)*	0.025 (0.037)**
SGED	9839.1	592.4**	35.8**	10.540 (0.005)	0.034 (0.001)
SSD	9854.2	622.6**	5.6*	16.291 (0.000)	0.027 (0.018)**
IHS	9857.3	–	–	4.518 (0.211)*	0.027 (0.015)**
ST	9851.2	–	11.6**	25.173 (0.000)	0.029 (0.007)
Normal	9542.9	–	–	203.848 (0.000)	0.072 (0.000)

Log-L is the maximum likelihood value. LR_{normal} is the LR statistic from testing the null hypothesis that the daily returns are distributed as normal against the hypothesis that they are distributed as SGT, SGED or SSD. LR_{SGT} is the LR statistic from testing the null hypothesis of alternative distribution against the SGT. Chi2 and KS denote chi-square and Kolmogorov–Smirnov tests. Figures in parenthesis denote p-value. *Significance at the 5% level. **Significance at the 1% level.

distribution ratios are far above 1 across the nine indexes. This distribution consistently underestimates risk. The ST, SGT and IHS distributions provide the closest ratios to 1. We must highlight that SGT and IHS are the only distributions that provide ratios equal to 1 in some indexes. At the 1% level, the SSD distribution performance is the worst one, although it improves slightly at the 0.25% level.

TABLE 5 Ratio of VRate/α at $\alpha = 1\%$, 0.25%, for each VaR model across the nine stock indexes.

(a) $\alpha = 1\%$

Model	Nikkei	Merval	S&P 500	Dow Jones	CAC 40	IBEX 35	Hang Seng	Tel Aviv	FTSE 100
Normal	**2.9**	**2.2**	**3.6**	**2.8**	**2.3**	**2.2**	**1.6**	**2.6**	**3.6**
ST	1.6	0.6	1.2	1.0	1.2	1.2	0.6	0.6	2.2
SGT	1.8	1.4	1.8	1.4	1.2	1.6	1.0	1.0	1.8
IHS	1.8	1.4	1.8	1.2	1.2	1.6	1.0	1.0	1.6
SSD	1.8	1.8	2.2	1.4	1.2	1.6	1.2	1.6	2.0
SGED	1.8	1.4	1.8	1.4	1.2	1.6	1.0	1.2	2.0

(b) $\alpha = 0.25\%$

Model	Nikkei	Merval	S&P 500	Dow Jones	CAC 40	IBEX 35	Hang Seng	Tel Aviv	FTSE 100
Normal	**3.3**	**3.2**	**4.8**	**2.4**	**3.9**	**4.7**	**2.4**	**2.4**	**4.7**
ST	0.8	0.0	1.6	0.8	2.4	0.8	0.0	0.0	4.0
SGT	0.8	0.8	2.4	1.6	2.4	0.8	1.6	0.8	4.0
IHS	0.8	0.8	2.4	1.6	2.4	0.8	0.0	0.8	3.2
SSD	0.8	2.4	2.4	1.6	2.4	0.8	1.6	1.6	4.0
SGED	0.8	1.6	2.4	1.6	2.4	0.8	1.6	0.8	4.0

Underlined figures indicate closest to 1 in that index. Bold figures indicate the least favored model.

TABLE 6 Summary statistics for the ratio of VRate/α at $\alpha = 1\%$, 0.25%, for each VaR model.

(a) $\alpha = 1\%$

	Mean	Median	Std (1)	1st	In top 3
Normal	**2.64**	**2.6**	**3.43**	**0**	**0**
ST	1.07	1.1	0.29	5	5
SGT	1.44	1.4	0.33	4	9
IHS	1.40	1.4	0.28	5	9
SSD	1.64	1.6	0.58	1	5
SGED	1.49	1.4	0.37	4	9

(b) $\alpha = 0.25\%$

	Mean	Median	Std (1)	1st	In top 3
ST	1.15	0.8	1.76	5	6
SGT	1.68	1.6	1.67	6	9
IHS	1.41	0.8	1.24	6	8
SSD	1.95	1.6	1.97	4	7
SGED	1.77	1.6	1.71	5	9

Underlined figures indicate the favored model, while bold figures indicate the least favored model in each column. "Std (1)" is the standard deviation in ratios from an expected value of 1. "1st" indicates the number of markets in which that model's VRate/α ratio ranked closest to 1. "In top 3" counts the number of markets in which the model's VRate/α ratio ranked among the top three models.

Following Gerlach *et al* (2011), Table 6 displays summary statistics for the VRate/α ratios for each model across the nine indexes, using the results of Table 5. The "Std (1)" column shows the standard deviation from an expected ratio of 1 (not the mean sample), while the "1st" column counts the indexes in which the model had a VRate/α ratio closest to 1. The "In top 3" column counts the indexes in which the model was ranked among the top three models based on its VRate/α ratio. The results confirm the above conclusion. The normal distribution shows a very poor performance. For both levels 1% and 0.25%, the mean VRate/α ratio is far above 1, and this distribution never ranks among the top three models. The ST, SGT and IHS distributions are the most favored across all criteria. For both levels 1% and 0.25%, the ST distribution provides the mean VRate/α ratio closest to 1. However, the IHS, which provides a mean ratio close to 1, has the smallest standard deviation from the expected ratio of 1, followed by the SGT and ST distributions. It seems that the performance of the ST distribution is more volatile. The data presented in columns four and five of Table 6 corroborates this idea. The ST distribution ranks first in five indexes; this is similar to

TABLE 7 Summary statistics for model ranks, in terms of VRate/α at $\alpha = 1\%$, 0.25%, across the nine stock indexes.

	$\alpha = 1\%$				$\alpha = 0.25\%$			
	Mean	**Median**	**Std (1)**	**Range**	**Mean**	**Median**	**Std (1)**	**Range**
Normal	**6.00**	**6.00**	**5.00**	0	**6.00**	**6.00**	**5.00**	0
ST	<u>2.44</u>	<u>1.00</u>	2.24	4	3.11	<u>3.00</u>	2.48	3.5
SGT	2.83	3.00	1.99	2.5	<u>2.78</u>	<u>3.00</u>	<u>1.91</u>	2
IHS	2.50	3.00	<u>1.72</u>	2.5	<u>2.78</u>	<u>3.00</u>	2.05	3.5
SSD	4.06	4.00	3.14	2	3.39	3.50	2.51	3
SGED	3.17	3.00	2.23	2	2.94	<u>3.00</u>	2.02	1.5

Underlined figures indicate the favored model, and bold figures indicate the least favored model in each column. "Std (1)" is the standard deviation in ranks from the value of 1.

the IHS distributions, which rank first in five and six indexes, depending on the level considered (1% or 0.25%). For both levels, this last distribution, in conjunction with the SGT distribution, ranks among the top three distributions for all nine indexes.

To help us distinguish between the best models at each quantile level, Table 7 shows summary statistics for each distribution's rank in terms of how close its VRate/α ratio is to 1 across the indexes. For ratios that are equidistant from 1, conservative ratios (of less than 1) are preferred. Table 7 displays the average, median, standard deviation (from 1) and range of the forecast ranks for each model over the nine indexes. For both levels 1% and 0.25%, normal distribution has by far the highest mean rank, equal highest median rank and by far the highest deviation in ranks away from 1 across the distributions. SSD and SGED display the next worst performances. At the 1% level, the ST distribution has the lowest mean rank, followed closely by the IHS and SGT distributions. However, the IHS and SGT distributions have the lowest standard deviation in ranks away from 1 and a lower range than the ST distribution. At the 0.25% level, the SGT and IHS distributions have the lowest mean rank and equal lowest standard deviation in ranks away from 1. For this level, the SGT distribution has the lowest range. In fact, the SGT distribution joins the IHS in ranking among the top three in each index, and thus it has a smaller range in ranks than the ST distribution.

Finally, Table 8 counts the number of rejections for each distribution over the nine indexes at the 5% level for each of the five tests considered (LR_{UC}, BTC, LR_{ind}, LR_{CC} and DQ). The accuracy tests corroborate the conclusion from Tables 5 and 6. At both levels, the normal distribution is rejected by many tests. For the ST distribution and all of the skewness distributions, the number of rejections is minimal: just one or two.

TABLE 8 Counts of model rejections for five tests across the nine stock indexes.

	$\alpha = 1\%$					$\alpha = 0.25\%$				
	UC_P	BTC	CC_P	UC_{ind}	DQ_P	UC_P	BTC	CC_P	UC_{ind}	DQ_P
Normal	**5**	**8**	**2**	0	0	**1**	**6**	0	0	**1**
ST	0	1	0	0	**1**	0	1	0	0	0
SGT	0	0	0	0	**1**	0	1	0	0	0
IHS	0	0	0	0	**1**	0	1	0	0	0
SSD	0	2	0	0	**1**	0	1	0	0	0
SGED	0	1	0	0	**1**	0	1	0	0	0

Underlined figures indicate the favored model, and bold figures indicate the least favored model in each column.

Overall, we can conclude that

(i) normal distribution performs very poorly in estimating VaR (this distribution underestimates risk in almost all indexes);

(ii) after the normal distribution, the SSD and SGED perform the worst;

(iii) the ST distribution performs very well in estimating VaR but exhibits volatile behavior (this distribution works well in some indexes and poorly in others); and

(iv) the IHS and SGT distributions outpace the ST distribution in many indexes and perform well in the cases in which the ST distribution is best.

5.2 Loss functions

In this section, we evaluate the VaR estimate in terms of the regulator loss function (Table 9) and the firm's loss function (Table 10). The results in Table 9 have been multiplied by 1000, given the small value obtained. The underlined figures represent the minimum value for this function in each case (index and confidence level).

From the regulator loss function (see Table 9), we find that the parametric approach under a normal distribution provides the highest losses, while the ST distribution provides the lowest losses, followed by the IHS and SGT distributions. Among the skewed distributions, the SSD gives the worst outcome in all cases. According to this result, we can conclude that from the point of view of the regulator the best distribution is the ST, as this distribution is the most conservative.

The problem associated with the regulator loss function is that this function does not take into account the firm's opportunity cost. So, the one model that overestimates the risk, as the ST distribution does in three of the cases, may be considered the most

TABLE 9 Magnitude of the regulatory loss function.

	Level (%)	Normal	ST	SGT	IHS	SSD	SGED
Nikkei	1.00	0.00338	0.00134	0.00186	**0.00176**	0.00212	0.00186
	0.25	0.00065	0.00004	0.00015	**0.00008**	0.00020	0.00015
Merval	1.00	0.00667	0.00053	0.00256	**0.00244**	0.00340	0.00251
	0.25	0.00191	0.00000	0.00013	**0.00009**	0.00039	0.00022
S&P 500	1.00	0.00617	0.00337	0.00352	0.00362	0.00393	**0.00349**
	0.25	0.00293	0.00121	0.00133	**0.00130**	0.00167	0.00137
Dow Jones	1.00	0.00220	0.00056	0.00073	**0.00065**	0.00080	0.00067
	0.25	0.00044	0.00003	**0.00004**	0.00003	0.00008	0.00006
CAC 40	1.00	0.00568	0.00462	0.00445	0.00427	0.00445	**0.00443**
	0.25	0.00282	0.00185	**0.00158**	0.00148	0.00175	0.00178
IBEX 35	1.00	0.00554	0.00308	0.00355	**0.00336**	0.00366	0.00350
	0.25	0.00274	0.00152	0.00161	**0.00158**	0.00186	0.00182
Hang Seng	1.00	0.00333	0.00048	**0.00124**	0.00127	0.00165	0.00125
	0.25	0.00062	0.00000	**0.00001**	0.00000	0.00006	0.00001
Tel Aviv	1.00	0.00150	0.00024	0.00060	**0.00054**	0.00069	0.00062
	0.25	0.00030	0.00000	**0.00000**	**0.00000**	0.00004	0.00003
FTSE 100	1.00	0.00376	0.00254	**0.00227**	0.00205	0.00228	0.00228
	0.25	0.00126	0.00056	**0.00036**	0.00029	0.00047	0.00048

The average of the loss function for each VaR model in both confidence levels. The average was multiplied by 1000. Underlined figures denote the minimum value for the average of the loss function for each index, and bold figures denote the second best.

appropriate.[7] Taking this into account, we calculate the losses from a firm's point of view.[8]

In terms of the firm's loss function (see Table 10), the normal distribution provides the lowest losses, while the ST distribution shows the highest losses. This result is coherent, since it is well known that the normal distribution underestimates risk, providing the lowest capital opportunity cost. Since the ST distribution tends to overestimate risk, the capital opportunity cost with this distribution is the highest. The magnitudes of losses obtained by all of the skewed distributions are very similar. In terms of this loss function, the best skewed distribution is the SSD. This distribution

[7] For Merval, Hang Seng and Tel Aviv, and for both levels (1% and 0.25%), the ST distribution greatly overestimates risk compared with the IHS and SGT distributions (see Table 5).

[8] In order to calculate the firm's loss function, we need to know the cost of capital. For this purpose, we have used the daily data of the interest rate of the Eurosystem monetary policy operations for the European indexes. For the rest of the indexes, we took the interest rate of the open market operations used by the Federal Reserve in the implementation of its monetary policy.

TABLE 10 Magnitude of the firm's loss function.

	Level (%)	Normal	ST	SGT	IHS	SSD	SGED
Nikkei	1.00	<u>0.00054</u>	0.00062	0.00059	0.00059	**0.00058**	0.00059
	0.25	<u>0.00066</u>	0.00080	0.00076	0.00077	**0.00074**	0.00075
Merval	1.00	<u>0.00056</u>	0.00079	0.00065	0.00066	**0.00062**	0.00066
	0.25	<u>0.00068</u>	0.00112	0.00090	0.00092	**0.00081**	0.00085
S&P 500	1.00	<u>0.00044</u>	0.00052	0.00051	0.00050	**0.00049**	0.00051
	0.25	<u>0.00054</u>	0.00066	0.00065	0.00065	**0.00062**	0.00064
Dow Jones	1.00	<u>0.00040</u>	0.00048	0.00046	0.00047	**0.00045**	0.00046
	0.25	<u>0.00050</u>	0.00062	0.00060	0.00061	**0.00058**	0.00059
CAC 40	1.00	<u>0.00111</u>	**0.00121**	0.00122	0.00123	0.00122	0.00122
	0.25	<u>0.00136</u>	0.00150	0.00153	0.00154	0.00150	**0.00150**
IBEX 35	1.00	<u>0.00109</u>	0.00132	0.00125	0.00127	**0.00124**	0.00125
	0.25	<u>0.00132</u>	0.00173	0.00167	0.00168	**0.00158**	0.00159
Hang Seng	1.00	<u>0.00062</u>	0.00080	0.00072	0.00071	**0.00069**	0.00071
	0.25	<u>0.00077</u>	0.00107	0.00092	0.00096	**0.00089**	0.00092
Tel Aviv	1.00	<u>0.00040</u>	0.00052	0.00046	0.00047	**0.00045**	0.00046
	0.25	<u>0.00050</u>	0.00069	0.00062	0.00062	**0.00058**	0.00059
FTSE 100	1.00	<u>0.00099</u>	**0.00108**	0.00111	0.00113	0.00110	0.00110
	0.25	<u>0.00122</u>	**0.00135**	0.00140	0.00143	0.00137	0.00136

The average of the loss function for each VaR model in both confidence levels. Underlined figures denote the minimum value for the average of the loss function for each index, and bold figures denote the second best.

obtains the lowest losses in seven of the nine cases. The SGT distribution, although it is not the best, works out well, giving lower losses than the ST distribution does.

Overall, following this selection process in two stages, whereby we first ensure that the distributions provide accurate VaR estimates, and focusing on the firm's loss function, we conclude that the skewed and fat-tailed distributions outperform the normal and ST distributions. From the point of view of the regulator, the superiority of the skewed distributions related to the ST distribution is not very clear.

6 CONCLUSION

This paper evaluates the performance of several skewed and symmetric distributions in modeling the tail behavior of daily returns and forecasting VaR. The skewed distributions considered are (i) the SSD of Hansen (1994), (ii) the SGED of Theodossiou (2001), (iii) the SGT distribution of Theodossiou (1998) and (iv) the IHS of Johnson (1949). The symmetric distributions are the normal and ST ones.

For this study, we have used daily returns on nine composite indexes: Nikkei (Japan), Hang Seng (Hong Kong), Tel Aviv (100) (Israel), Merval (Argentina), Standard & Poor's 500 (S&P 500) and Dow Jones (United States), FTSE 100 (United Kingdom), CAC 40 (France) and IBEX 35 (Spain). The sample used for the statistical analysis runs from January 2000 to the end of November 2012. The analysis period for forecasting VaR runs from 2008 to 2009, which is known as the global financial crisis period.

From the results presented in this paper, we can conclude that the skewness and fat-tailed distributions outperform the normal one in fitting financial returns and forecasting VaR. Among all of the skewed distributions considered in this paper, the SGT distribution of Theodossiou (1998) is the best in terms of fitting data. In terms of their ability to forecast VaR, the IHS and SGT distributions provide the most accurate VaR estimates across the indexes. Therefore, we find evidence in favor of the skewed distributions compared with the ST distribution. In statistical terms, most of these fit the data better than the ST distribution. Regarding the accuracy of the VaR estimates, the IHS and SGT distributions outperform the ST distribution, as those distributions provide less volatile results. The ST distribution performs very well in estimating VaR but exhibits more volatile behavior across indexes.

Regarding the loss function, the result depends on the kind of function used to measure the losses. From the point of view of the regulator, the ST distribution is the best at forecasting VaR, as this distribution provides a more conservative VaR estimate. However, from the point of view of the firm, the skewed distributions outperform the ST distribution, since the latter distribution tends to raise the firm's capital cost. As companies are free to choose the model they use to forecast VaR, it is clear that they will prefer the skewed distributions.

DECLARATION OF INTEREST

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