Problem Set 9

Question 1

Data pairs: $(1,4),(2,3),(3,5),(4,10)$

Part A

$$\bar{y} = \frac{4+3+5+10}{4} = 5.5$$

$$\bar{x} = \frac{1+2+3+4}{4} = 2.5$$

$$\beta_1 = \frac{\sum_{i=1}^{4}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{4}(x_i-\bar{x})^2}$$

$$\beta_1 = \frac{(1-2.5)(4-5.5)+(2-2.5)(3-5.5)+(3-2.5)(5-5.5)+(4-2.5)(10-5.5)}{(1-2.5)^2+(2-2.5)^2+(3-2.5)^2+(4-2.5)^2} = 2$$

$$\beta_0 = \bar{y} - \beta_1\bar{x}$$

$$\beta_0 = 5.5 - 2\cdot 2.5 = .5$$

$$y = .5 + 2x$$

Part B

$$R^2 = 1 - \frac{SSE^*}{SST} = 1 - \frac{\sum_{i=1}^{4}[(y_i-\bar{y})-\beta_1(x_i-\bar{x})]^2}{\sum_{i=1}^{4}(y_i-\bar{y})^2}$$

$$R^2 = 1 - \frac{(4-(2\cdot 1)-.5)^2+(3-(2\cdot 2)-.5)^2+(5-(2\cdot 3)-.5)^2+(10-(2\cdot 4)-.5)^2}{(4-5.5)^2+(3-5.5)^2+(5-5.5)^2+(10-5.5)^2} = .69$$

Part C

$$95\%\text{ Confidence Interval } = [\beta_1 - t_{\alpha/2,n-2}\frac{\sigma}{\sqrt{s_{xx}}}, \beta_1 + t_{\alpha/2,n-2}\frac{\sigma}{\sqrt{s_{xx}}}]$$

$$\sigma = \sqrt{\frac{SSE^*}{n-2}} = \sqrt{\frac{9}{2}} = 2.12$$

$$s_{xx} = \sum_{i=1}^{4}(x_i-\bar{x})^2 = (1-2.5)^2+(2-2.5)^2+(3-2.5)^2+(4-2.5)^2 = 5$$

$$95\%\text{ Confidence Interval } = [2 - 4.303\cdot\frac{2.12}{\sqrt{5}}, 2 + 4.303\cdot\frac{2.12}{\sqrt{5}}] = [-2.08, 6.08]$$

Part D

Hypothesis test H_0 is $\beta_1 = 0$ and H_1 is $\beta_1 \neq 0$

$$t = \frac{\beta_1}{\sigma/\sqrt{s_x x}} = \frac{2}{2.12/\sqrt{5}} = 2.11$$

$$p = P(T_2 < -2.11) + P(T_2 > 2.11) = .169$$

```r
p <- 1 - pt(2.11, df = 2) + pt(-2.11, df = 2)
```

We fail to reject the null hypothesis since $.169 > .05$.

Question 2

Part A

```
quartet <- read.csv("Data/Quartet.csv")

Fit_1 = lm(quartet$y1 ~ quartet$x1)
Fit_2 = lm(quartet$y2 ~ quartet$x2)
Fit_3 = lm(quartet$y3 ~ quartet$x3)
Fit_4 = lm(quartet$y4 ~ quartet$x4)
```

The four fit lines are:
Series 1: $y = 3.0001 + .5001x$
Series 2: $y = 3.001 + .5000x$
Series 3: $y = 3.0025 + .4997x$
Series 4: $y = 3.0017 + .4999x$

Part B

```
summary(Fit_1)
summary(Fit_2)
summary(Fit_3)
summary(Fit_4)
```

Series 1: $R^2 = .6665$
Series 2: $R^2 = .6662$
Series 3: $R^2 = .6663$
Series 4: $R^2 = .6667$