Problem Set 5

Question 1

Joint density function given by $f_{X,Y}(x,y) = \begin{cases} k(2y+xy) & \text{if } 0 \le x \le 1 \text{ and } x \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$

Part A

$$\int\int f_{X,Y}(x,y)dxdy = 1$$

$$\int_0^1 \int_x^1 k(2y+xy)dydx + \int\int 0dxdy = 1$$

$$\int_0^1 \int_x^1 [2ky + kxy]dydx = 1$$

$$\int_0^1 [ky^2 + \frac{1}{2}kxy^2]\Big|_x^1 dx = 1$$

$$\int_0^1 [k - kx^2 + \frac{1}{2}kx - \frac{1}{2}kx^3]dx = 1$$

$$[kx - \frac{1}{3}kx^3 + \frac{1}{4}kx^2 - \frac{1}{8}kx^4]\Big|_0^1 = 1$$

$$k[1-0] - \frac{1}{3}[k-0] + \frac{1}{4}[k-0] - \frac{1}{8}[k-0] = 1$$

$$k - \frac{k}{3} + \frac{k}{4} - \frac{k}{8} = 1$$

$$k\frac{19}{24} = 1$$

$$k = \frac{24}{19}$$

Part B

$$\text{Marginal PDF of } X \; f_X(x) = \int f_{X,Y}(x,y)dy$$

$$f_X(x) = \int_0^1 k(2y+xy)dy = k(y + \frac{1}{2}xy^2)\Big|_0^1$$

$$f_X(x) = k\Big[(1 + \frac{1}{2}x) - (0 + \frac{1}{2}0)\Big] = \frac{24}{19}[1 + \frac{1}{2}x]$$

$$f_X(x) = \frac{24}{19} + \frac{12}{19}x$$

$$\text{Marginal PDF of } X \; f_X(x) = \begin{cases} \frac{24}{19} + \frac{12}{19}x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Part C

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$f_Y(y) = \int f_{X,Y}(x,y)dx = \int_0^y k(2y+xy)dx$$

$$f_Y(y) = k(2yx + \frac{1}{2}x^2y)\Big|_0^y = \frac{24}{19}(2y^2 + \frac{1}{2}y^3)$$

$$f_{X|Y}(x|y) = \frac{\frac{24}{19}(2y+xy)}{\frac{24}{19}(2y^2+\frac{1}{2}y^3)} = \frac{2+x}{2y+\frac{1}{2}y^2)}$$

$$f_{X|Y}(x|\frac{1}{4}) = \frac{2+x}{\frac{1}{2}+\frac{1}{32}} = \frac{2+x}{\frac{17}{32}} = \frac{64+32x}{17}$$

$$f_{X|Y} = \begin{cases} \frac{64+32x}{17} & 0 \le x \le \frac{1}{4} \\ 0 & \text{otherwise} \end{cases}$$

Question 2

Part A

Part B

Part C

Part D

Part E

Question 3

Part A

Part B

Part C

Question 4

$$\text{Joint PDF given by } f_{X|Y}(x,y) = \begin{cases} 2ye^{-y(2+x)} & x,y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$f_Y(y) = \int_0^\infty 2ye^{-y(2+x)}dx = -2\int_0^\infty e^u dx \text{ with } u = -y(2+x)$$

$$f_Y(y) = -2e^u\Big|_0^\infty = -2e^{-y(2+x)}\Big|_0^\infty$$

$$f_Y(y) = -2e^{-y(2+\infty)} + 2e^{-y(2+0)} = 2e^{-2y}$$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{2ye^{-y(2+x)}}{2e^{-2y}}$$

$$f_{X|Y}(x|y) = \frac{ye^{-2y}e^{-yx}}{e^{-2y}} = ye^{-yx}$$

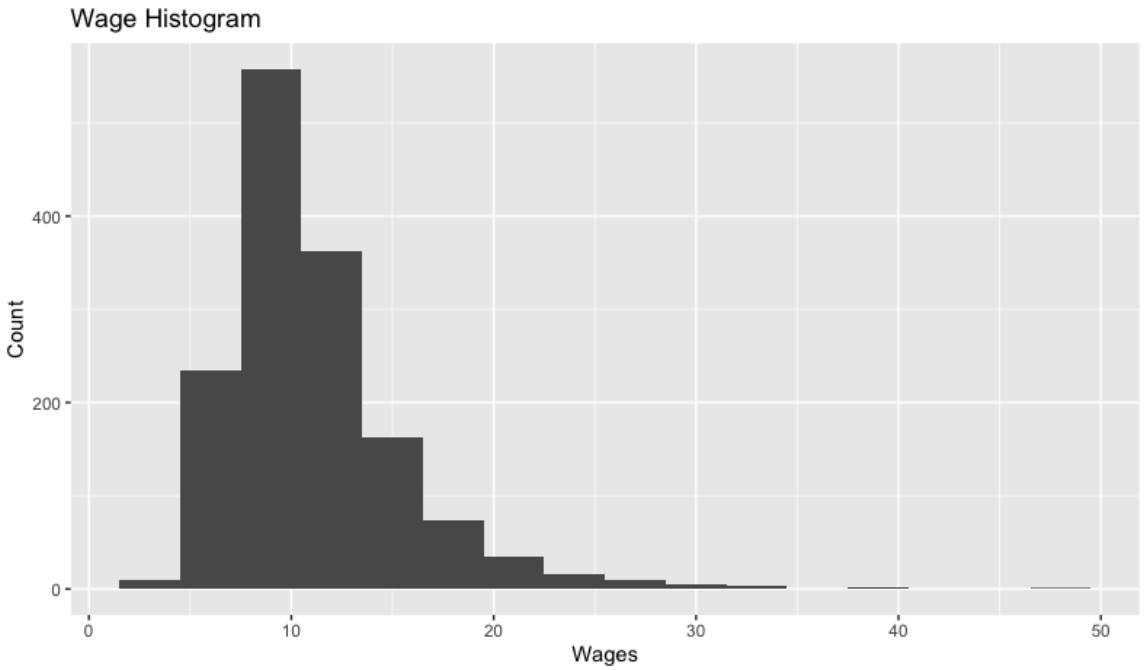
$$f_{X|Y}(x|y) = \begin{cases} ye^{-yx} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Question 5

```
1    Bwages <- read_csv("Bwages.csv")
```

Part A

```
1    ggplot(data = Bwages) + geom_histogram(mapping = aes(x = wage), binwidth = 3) +
     xlab("Wages") + ylab("Count")
```



Part B

Part C

Part D

Part E