

**ENGRD 2700: Basic Engineering Probability and Statistics**  
**Fall 2019**

**Homework 2**

Due Friday, September 20 at 11:59 pm. Submit to Gradescope by clicking the name of the assignment. See [https://people.orie.cornell.edu/yudong.chen/engrd2700\\_2019fa.html#homework](https://people.orie.cornell.edu/yudong.chen/engrd2700_2019fa.html#homework) for detailed submission instructions.

When completing this assignment (and all subsequent ones), keep in mind the following:

- You must complete the homework individually and independently.
  - Provide evidence for each of your answers. If a calculation involves only very minor computation then explain the computation you performed and give the results. If a calculation involves more complicated steps on many many records then hand in the calculations and formulas for the first few records only.
  - Write clearly and legibly. You are encouraged to *type* your work although you do not have to. We may deduct points if your answers are difficult to read or disorganized.
  - For questions that you answer using R, attach any code that you write, along with the relevant plots. You may use other software, but the same condition applies.
  - Submit your homework a single pdf file on Gradescope.
1. You need to give the final answer as an number, along with justification. This same rule applies to all questions in this homework that involves explicit numbers.
    - (a) How many different ways are there to divide 12 people into 2 equal-sized groups? (Hint: the naming of the groups does not matter. For example, if we pick  $\{1, 2, 3, 4, 5, 6\}$  to form a group "A" and  $\{7, 8, 9, 10, 11, 12\}$  to form group "B", it is the same as picking  $\{12, 11, 10, 9, 8, 7\}$  to be group "A" and  $\{6, 5, 4, 3, 2, 1\}$  to be group "B".)
    - (b) How many different ways are there to divide 12 people into 3 equal-size groups? (Use the same hint in (a))
    - (c) How many different ways are there to divide 12 people into 2 groups of any sizes? (A group can have zero people.)
  2. Alice and Bob want to calculate how many distinct words can be made by permuting the letters in "ENGINEER". (Gibberish such as "EEENNGIR" is counted as a word.)
    - (a) Alice computes  $\binom{8}{3} \binom{5}{2} \cdot 3! = 3360$ . Explain the reasoning for this calculation.
    - (b) Bob computes  $8!/3!/2! = 3360$ . Explain the reasoning for this calculation.
    - (c) This one is for you: How many distinct ways are there to arrange the letters in "ENGINEER" in a circular necklace?
  3. There are 36 Master of Engineering students to assign to projects this year. Suppose project teams must consist of exactly 4 people.
    - (a) How many different ways are there to construct a single project team?
    - (b) Suppose that there are 16 men and 20 women students. How many different ways are there to construct a single project team consisting of 2 men and 2 women?

4. A person can like any number of sports. We consider three sports in particular. Let  $A_1$  denote the event that a person likes football. Let  $A_2$  denote the event that a person likes basketball. Let  $A_3$  denote the event that a person has likes swimming. Suppose we have determined the following probabilities:

$$\begin{array}{lll} P(A_1) = 0.24 & P(A_2) = 0.18 & P(A_3) = 0.1 \\ P(A_1 \cup A_2) = 0.3 & P(A_1 \cup A_3) = 0.28 & P(A_2 \cup A_3) = 0.24 \\ P(A_1 \cap A_2 \cap A_3) = 0.02 \end{array}$$

- (a) Draw a Venn diagram representing the 3 events.
  - (b) What is the probability that a person does not like football?
  - (c) What is the probability that a person likes both football and basketball?
  - (d) What is the probability that a person likes football and basketball, but not swimming?
  - (e) What is the probability that a person likes at most two of the three sports?
5. An office building contains three unreliable elevators, A, B, and C, which are down 35%, 30%, and 10% of the time, respectively.
- (a) Let  $A$ ,  $B$ , and  $C$  denote the events in which elevators  $A$ ,  $B$ , and  $C$  are down, respectively. Describe the following by taking unions, intersections, and/or complements of  $A$ ,  $B$ , and  $C$ .
    - All three elevators are down.
    - Elevator A is down, but not B or C.
    - None of the elevators is down.
    - At least one elevator is down.
    - Exactly one elevator is down.
  - (b) What are the maximum and minimum possible values for  $P(A \cup B \cup C)$ ?
  - (c) You learn that  $P(A \cap B) = 0.2$ . What is the probability that either Elevator A or B is down?
  - (d) Using the information from part (c), compute  $P(A|B)$ , the probability that Elevator A is down, given that Elevator B is down.
6. (Based on a real application). A certain oil refinery keeps intermediate products in 8 tanks. There are 15 pumps of varying capacity that can be assigned to pump the intermediate products from the 8 tanks into a final-product tank.
- Question:** How many ways can you assign the 15 pumps to the 8 tanks so that each tank gets at least one pump? Not all pumps need to be assigned. (Each tank may contain a different intermediate product, so assigning e.g., pump 1 to tank 1 and pump 2 to tank 2 is different from assigning pump 2 to tank 1 and pump 1 to tank 2.)
- To answer this question, we can proceed as follows: Let  $d(k, n)$  be the number of ways to assign  $k$  pumps to  $n$  tanks in such a way that each tank gets at least one pump, but not all pumps need to be assigned.
- (a) Explain why  $d(k, n) = 0$  for  $n > k \geq 1$ .
  - (b) Explain why  $d(k, k) = k!$  for all  $k \geq 1$ .
  - (c) Explain why  $d(k, 1) = 2^k - 1$  for all  $k \geq 1$ .

- (d) Explain why, for  $k > 1$  and  $n \geq 1$ ,

$$d(k, n) = \sum_{j=1}^{k-(n-1)} \binom{k}{j} d(k-j, n-1).$$

Hint: Think which pumps can be assigned to the first tank.

- (e) Answer the **question**. Feel free to use some software, e.g., R. (Hint: you can write recursive function  $d(k, n)$  to solve this problem. The base cases would be the results we get from (a), (b), and (c) and the recursive step is the expression in (d). For recursive step you could use "for loop". Examples for recursion: <https://data-flair.training/blogs/r-recursive-function/>. Examples for "for loop": <https://www.r-bloggers.com/how-to-write-the-first-for-loop-in-r/>)
- (f) We want to determine the assignment of pumps to tanks that minimizes the time required to pump the intermediate products. Assuming we have a computer that can analyze one million assignments each second, how long in years would it take the computer to analyze all possible assignments?

P.S. It turns out that using integer programming (a technique learned in ORIE 3300), we can solve this problem almost instantaneously using off-the-shelf computer packages. Shouldn't you be planning to major in ORIE, or at least take ORIE 3300?