Problem Set 5

Question 1

Joint density function given by $f_{X,Y}(x,y) = \begin{cases} k(2y+xy) & \text{if } 0 \le x \le 1 \text{ and } x \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$

Part A

$$\int\int f_{X,Y}(x,y)dxdy = 1$$

$$\int_0^1 \int_x^1 k(2y+xy)dydx + \int\int 0dxdy = 1$$

$$\int_0^1 \int_x^1 [2ky + kxy]dydx = 1$$

$$\int_0^1 [ky^2 + \frac{1}{2}kxy^2]\Big|_x^1 dx = 1$$

$$\int_0^1 [k - kx^2 + \frac{1}{2}kx - \frac{1}{2}kx^3]dx = 1$$

$$[kx - \frac{1}{3}kx^3 + \frac{1}{4}kx^2 - \frac{1}{8}kx^4]\Big|_0^1 = 1$$

$$k[1-0] - \frac{1}{3}[k-0] + \frac{1}{4}[k-0] - \frac{1}{8}[k-0] = 1$$

$$k - \frac{k}{3} + \frac{k}{4} - \frac{k}{8} = 1$$

$$k\frac{19}{24} = 1$$

$$k = 24/19$$

Part B

Part C

Question 2

Part A

Part B

Part C

Part D

Part E

Question 3

Part A

Part B

Part C

Question 4

Question 5

Part A

Part B

Part C

Part D

Part E