Problem Set 4

Question 1

$$f(x) = P(X = x) = \tfrac{1}{n} \qquad x = 0, 1, 2, ...(n-1)$$

Part A

$$F_X(x) = P_X(X \leq x) = \sum_{y \leq x} p_X(y)$$

$$F_X(x) = \sum_{y \leq x} \frac{1}{n} \text{ for } x = 0, 1, 2, ...(n-1)$$

$$\text{CDF, } F_X(x) = \frac{x}{n} \text{ for } x = 0, 1, 2, ...(n-1)$$

$$F_X(x) = \begin{cases} 1 & x \geq n \\ \frac{x}{n} & 0 \leq x \leq (n-1) \\ 0 & x < 0 \end{cases}$$

Part B

Question 2

Part A

Part B

Part C

Question 3

$$\text{PDF, } f(x) = \begin{cases} cx^{-6} & 1 \leq x < \infty \\ 0 & x < 1 \end{cases}$$

Part A

$$P(a \leq X \leq b) = \int_a^b f(u)du$$

$$P(1 \leq X \leq \infty) = 1$$

$$P(1 \leq X \leq \infty) = \int_1^\infty cx^{-6}dx = \frac{c}{-5}x^{-5}\Big|_1^\infty = \frac{c}{-5}\infty^{-5} - \frac{c}{-5}1^{-5}$$

$$5 = c \cdot 1^{-5} \longrightarrow c = 5$$

Part B

$$\text{CDF, } F(x) = P(X \leq x) = \int_{-\infty}^x f(u)du$$

$$F(x) = \int_{-\infty}^x 5x^{-6}dx = -x^{-5}\Big|_{-\infty}^x = -x^{-5} + (-\infty)^{-5}$$

$$F(x) = -x^{-5}$$

$$\text{CDF, } F(x) = \begin{cases} -x^{-5} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

Part C

Part D

Part E

Question 4

Since if $X < 5$, the time to departure T will be 5, and if $X > 5$ T will be X, so $T = max5, X$, with X uniformly distributed from 0 to 15.

$$E(T) = \int_0^5 5 \cdot \frac{1}{15} dx + \int_5^{15} x \cdot \frac{1}{15} dx$$

$$E(T) = \frac{x}{3}\Big|_0^5 + \frac{x^2}{30}\Big|_5^{15} = \frac{5}{3} - 0 + \frac{15^2}{30} - \frac{5^2}{30} = 8.33$$

Question 5

Gain from one share of stock in company i over the next year is X, with $i = 1, ..., 10$.
X_i are independent normal random variables (Normal(100, 196)) with $\mu = 100$, $\sigma^2 = 196$.

Part A

$$P(X_1 \geq 90) = P\Big(\frac{X_1 - 100}{14} \geq \frac{90 - 100}{14}\Big)$$

$$N(0,1) \sim \frac{X_1 - 100}{14}$$

$$\Phi\Big(\frac{90 - 100}{14}\Big) = P\Big(\frac{X_1 - 100}{14} \leq \frac{90 - 100}{14}\Big)$$

$$P(X_1 \geq 90) = 1 - P\Big(\frac{X_1 - 100}{14} \leq \frac{90 - 100}{14}\Big) = 1 - \Phi\Big(\frac{90 - 100}{14}\Big)$$

Part B

Part C

independent (P($X_1 - 2X_2 \geq 10)) so X_1 - 2X_2 \sim N(-100, 980)$

Y $\sim N(-100, 980)$

P(Y $\geq$ 10)$^{-}$100

CDF P(Y $\leq$ 1) $= \Phi(\frac{X-\mu}{\sigma}) = \Phi(\frac{Y-(-100)}{\sqrt{(980)}}) with Phi as N(0,1)$

P(Y $\geq$ 10)

P(Y-(-100)$\frac{}{\sqrt{980} \geq)}$

Question 6

Part A

$\lambda = 3$

$$Y = -2X + 2$$

with X as exp(λ)
CDF of Y, F$_Y(y)$

$$P(Y \leq y) = P(-2X + 2 \leq y) = P(X \geq \frac{-y}{2} + 1) = 1 - P(X \leq \frac{-y}{2} + 1)$$

CDF of X = P(X $\leq \frac{-y}{2} + 1)$

$$F_Y(y) = \begin{cases} 0 \\ < - - - don't actually need this CDF of X \\ 1 \end{cases}$$

Part B

Part C

Part D

FOR QUESTIONS 5 AND 6
E(X$_1 + X_2) = E(X_1) + E(X_2)$
Var(X$_1 + X_2) = Var(X_1) + Var(X_2)$
Var(5X$_1 - X_2) = 5^2 \cdot Var(X_1) + Var(X_2)$