Recitation 2: Counting & Probability

1 Review

When attempting the exercises below, the following may be useful:

- If we sample without replacement and order does matter, then there are

$$\frac{n!}{(n-k)!} = n \times (n-1) \times \cdots \times (n-k+1)$$

ways to sample k items without replacement from a collection of size n. This can be computed with the help of R's `factorial(n)` function.

- If we sample without replacement and order does not matter, then there are

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

ways instead. This can be computed in R using `choose(n, k)`.

- If a population contains N_1 Type 1 members and N_2 Type 2 members. If we draw a sample of size n without replacement, then

$$P\left(\text{Sample contains } n_1 \text{ Type 1 Items}\right) = \frac{\binom{N_1}{n_1}\binom{N_2}{n-n_1}}{\binom{N_1+N_2}{n}}$$

which is called the hypergeometric probability.

2 Exercises

1. A professor is hosting a dinner party. His current wine supply includes 4 bottles of zinfandel, 5 bottles of merlot, and 6 bottles of cabernet (he only drinks red wine), all from different wineries.

   (a) If he wants to serve 3 bottles of zinfandel and serving order is important, how many ways are there to do this?

   (b) If 3 bottles of wine are to be randomly selected, and order does not matter, how many ways are there to do this?

   (c) If 3 bottles are randomly selected, what is the probability that this results in a selection consisting of 1 bottle of each variety?

   (d) If 3 bottles are randomly selected, what is the probability that they are all of the same variety?

   (e) If 3 bottles are randomly selected, what is the probability that exactly 2 of the wines are zinfandels?

   (f) Suppose now that bottles are to be selected one by one until a cabernet is found. What is the probability that it is necessary to examine at least six bottles?

2. A fair coin is flipped 10 times.
   (a) Compute the probability that heads appears 3 times.
   (b) Suppose that heads does appear 3 times. What is the probability that all three heads occurred within the first five flips?
3. Suppose we have two fair four-sided dice: one green, one red.
   (a) Describe $\mathcal{S}$, the set of all possible outcomes. (There are 16 of them.)
   (b) Suppose we define the following events:
      - A : The red die shows a 3.
      - B : The green die shows a 3.
      - C : Both dice show the same number.
      - D : The sum of the dice is 7.
      - E : The sum of the dice is even.

      Specify each event as a subset of the sample space.
   (c) Give the probability of each of the events from part (b).