Problem Set 3

Question 1

Lifetime of a light bulb T is given by $P(T > t) = e^{-t/3}$ for all $t \geq 0$. The bulb has lasted x years, so the conditional probability that it will last at most $x+2$ years is given by the conditional probability equation.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = \int_x^{x+2} e^{-t/3} dt = -\frac{1}{3} e^{-t/3} \Big|_x^{x+2} = \frac{1}{3} e^{-x/3} - \frac{1}{3} e^{-(x+2)/3}$$

$$P(B) = \int_0^x e^{-t/3} dt = -\frac{1}{3} e^{-t/3} \Big|_0^x = -\frac{1}{3} e^{-x/3} + \frac{1}{3} e^{-0/3} = \frac{1}{3} - \frac{1}{3} e^{-x/3}$$

$$P(A|B) = \frac{\frac{1}{3} e^{-x/3} - \frac{1}{3} e^{-(x+2)/3}}{\frac{1}{3} - \frac{1}{3} e^{-x/3}} = \frac{e^{-x/3} - e^{-x/3} e^{-2/3}}{1 - e^{-x/3}}$$

Question 2

Part A

The probability that both children are female, assuming the first is female, is given by the conditional probability equation $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.25}{.5} = .5$$

There is a probability of .5 that given the first child is female that the second will also be female.

Part B

Here the conditional probability equation is used again, but the order of children no longer matters.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.25}{.75} = .333$$

There is a probability of .333 that given as least one child is female that the other will also be female.

Part C

$$P(\text{ff} \mid \geq \text{Katie}) = \frac{P(\text{ff} \cap \geq \text{Katie})}{P(\geq \text{Katie})} =$$

Question 3

Part A

Part B

Part C

Question 4

Part A

The set $\mathcal{X}$ of possible values for X is given by $S = \{1, 2, 4, 8, 16, 32, ...\}$ (or by 2^x where x can take all integers starting at 0).

Part B

The PMF of X is given by $p_X(x) = P(X = x)$.

Part C

Part D

Part E

Question 5

Part A

$$\begin{cases} \\ 0 & \text{x ¡ -2} \\ .2 & \text{-2} \leq x < -1 \end{cases}$$

$.1 - 1 \leq x < 0.40 \leq$

Part B

Part C

Question 6

Part A

Part B

Part C

Part D

Part E