# ENGRD 2700: Basic Engineering Probability and Statistics
# Fall 2019

## Homework 4

Due Friday, Oct 11th by 11:59pm. Submit to Gradescope by clicking the name of the assignment. See `https://people.orie.cornell.edu/yudong.chen/engrd2700_2019fa.html#homework` for detailed submission instructions.

The same stipulations from Homework 1 (e.g., independent work, computer code, etc.) still apply.

1. Let $X$ have the following PMF:

$$f(x) = P(X = x) = \frac{1}{n}, \qquad x = 0, 1, 2, \ldots, n-1$$

   (a) Find the cumulative distribution function, $F(x)$. Recall that $F(x)$ is defined for all $x \in (-\infty, \infty)$.

   (b) Compute $E[X]$ and $\mathrm{Var}(X)$. For this problem, you may find the following facts useful:

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}, \qquad \sum_{i=0}^{n-1} i^2 = \frac{n(n-1)(2n-1)}{6}.$$

2. Suppose that a baseball game between teams A and B is tied at the end of the 9th inning. To determine a winner, extra innings will be played until one team scores more runs in an inning than the other. Suppose that the probability that A scores more than B in an inning is 0.2, the probability that B scores more than A in an inning is 0.3, and the probability that the score remains tied is 0.5. The innings are independent of each other.

   (a) What is the distribution of the number $X$ of extra innings that need to be played until a winner is determined (including the last one where one team scores more runs)? To answer this question, state the possible values of $X$ and give a formula for $P(X = k)$ for all relevant $k$.

   (b) What is the probability that at least 5 extra innings are required to determine a winner?

   (c) Eventually, one of the two teams wins. Compute the probability that B eventually wins. (Optional: You may try to use the same reasoning to compute the probability of A eventually winning. If the two probabilities do not sum to 1, then something is wrong.)

3. Suppose $X$ is a continuous random variable with probability density function (pdf)

$$f(x) = \begin{cases} cx^{-6} & 1 \leq x < \infty \\ 0 & x < 1 \end{cases}$$

   (a) Find $c$.

   (b) Compute $F(x)$, the cumulative distribution function (cdf) of $X$.

   (c) Find the 40$^{\text{th}}$ percentile of $X$, i.e., the number $\gamma$ such that $P(X \leq \gamma) = 0.4$.

   (d) Compute $E[X]$ and $\mathrm{Var}(X)$.

(e) Compute $E[X^5]$.

4. Suppose you and your friend just finished shopping at Wegmans and are checking out separately. You use one of those self-checkout kiosks and know that you will finish checking out in exactly 5 minutes. Your friend is waiting in a traditional checkout lane, and the amount of time until your friend finishes checking out is uniformly distributed between 0 and 15 minutes. The two of you will leave Wegmans together when you both finish checking out. Find $E[T]$, where $T$ is the number of minutes between now and the time you leave.

5. The gain from one share of stock in company $i$ over the coming year is $X_i$, where $i = 1, \ldots, 10$. (Negative values of $X_i$ represent losses.) Suppose that the $X_i$ are independent Normal$(100, 196)$ random variables (i.e., with mean $\mu = 100$ and variance $\sigma^2 = 196$).

   For the follow questions, **you should first derive an expression of the probability in terms of the CDF $\Phi(\cdot)$ of a standard normal r.v., and then give a numerical answer.**

   (a) Compute $P(X_1 \geq 90)$, the probability that company 1's gain is at least 90% of its expectation.

   (b) Compute $P(\sum_{i=1}^{10} X_i \geq 900)$, the probability that the combined gain of all 10 companies is at least 90% of the expectation.

   (c) Compute $P(X_1 - 2X_2 \geq 10)$.

6. Suppose that $X$ is an exponential random variables with parameter $\lambda = 3$.

   (a) What is the cumulative distribution function of $Y = -2X + 2$?

   (b) What is the mean and variance of $Y$?

   (c) What is the 0.9 quantile of $Y$?

   (d) What is the probability density function of $Y$?