Problem Set 3

Question 1

Lifetime of a light bulb T is given by $P(T > t) = e^{-t/3}$ for all $t \geq 0$. The bulb has lasted x years, so the conditional probability that it will last at most $x+2$ years is given by the conditional probability equation.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = \int_{x}^{x+2} e^{-t/3}dt = -\frac{1}{3}e^{-t/3}\Big|_{x}^{x+2} = \frac{1}{3}e^{-x/3} - \frac{1}{3}e^{-(x+2)/3}$$

$$P(B) = \int_{0}^{x} e^{-t/3}dt = -\frac{1}{3}e^{-t/3}\Big|_{0}^{x} = -\frac{1}{3}e^{-x/3} + \frac{1}{3}e^{-0/3} = \frac{1}{3} - \frac{1}{3}e^{-x/3}$$

$$P(A|B) = \frac{\frac{1}{3}e^{-x/3} - \frac{1}{3}e^{-(x+2)/3}}{\frac{1}{3} - \frac{1}{3}e^{-x/3}} = \frac{e^{-x/3} - e^{-x/3}e^{-2/3}}{1 - e^{-x/3}}$$

Question 2

Part A

The probability that both children are female, assuming the first is female, is given by the conditional probability equation $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.25}{.5} = .5$$

There is a probability of .5 that given the first child is female that the second will also be female.

Part B

Here the conditional probability equation is used again, but the order of children no longer matters.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.25}{.75} = .333$$

There is a probability of .333 that given as least one child is female that the other will also be female.

Part C

$$P(\text{ff} \mid \geq 1 \text{ Katie}) = \frac{P(\text{ff} \cap \geq 1 \text{ Katie})}{P(\geq 1 \text{ Katie})} = \frac{.25}{.75}$$

Question 3

Part A

A = bag is identified as dangerous
B = bag contains explosives

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{}{10/4000000}$$

Part B

Part C

Question 4

Part A

The set $\mathcal{X}$ of possible values for X is given by $S = \{1, 2, 4, 8, 16, 32, ...\}$ (or by 2^x where x can take all integers starting at 0).

Part B

The PMF of X is given by $p_X(x) = P(X = x)$.

Part C

Part D

Part E

Question 5

Part A

$$\text{f(x)} = \begin{cases} P(X = -2) & = .2 \\ P(X = -1) & = .1 \\ P(X = 0) & = .4 \\ P(X = 1) & = .1 \\ P(X = 2) & = .2 \end{cases}$$

Part B

$$E[X] = \sum_{x \in \mathcal{X}} x \cdot P_X(X = x) = (-2 \cdot .2) + (-1 \cdot .1) + (0 \cdot .4) + (1 \cdot .1) + (2 \cdot .2) = -.4 - .1 + 0 + .1 + .4 = 0$$

Part C

Question 6

Part A

Part B

Part C

Part D

Part E