OLG

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OLG básico

Demografía

$$N_t = (1+n)N_{t-1}$$

Debe tenerse presente que $l_t^1 \neq 0$ y $l_t^2 = 0$.

Hogares

$$U_t = u(c_t^1) + \beta u(c_{t+1}^2)$$

$$u'(\cdot) > 0, \quad u''(\cdot) < 0, \quad \lim_{c \to 0} u'(c) = \infty$$

$$w_t = c_t^1 + s_t$$
$$c_{t+1}^2 = (1 + r_{t+1}) s_t$$

$$c_t^1 + \frac{c_{t+1}^2}{1 + r_{t+1}} = w_t$$

$$\mathcal{L} = u(c_t^1) + \beta u(c_{t+1}^2) + \lambda \left[w_t - c_t^1 - \frac{c_{t+1}^2}{1 + r_{t+1}} \right]$$

Ecuación de Euler:

$$u'(c_t^1) = \beta(1 + r_{t+1}) u'(c_{t+1}^2)$$

Empresas

$$Y_t = F(K_t, L_t)$$

$$\frac{Y_t}{L_t} = F\left(\frac{K_t}{L_t}, 1\right) \equiv f(k_t)$$

$$\Pi_t = Y_t - w_t L_t - r_t K_t$$

$$w_t = \frac{\partial F(K_t, L_t)}{\partial L_t} = f(k_t) - k_t f'(k_t)$$
$$r_t = \frac{\partial F(K_t, L_t)}{\partial K_t} = f'(k_t)$$

Equilibrio

$$N_t l_t^1 = N_t = L_t$$
$$K_{t+1} - K_t = I_t$$
$$I_t = N_t s_t - K_t$$
$$K_{t+1} = N_t s_t$$

Dividiendo entre N

$$(1+n)k_{t+1} = s_t = s(w_t, r_{t+1})$$

Estado estacionario

$$k_t = k_{t+1} = k$$
$$(1+n)k = s(w(k), r(k))$$

Esta solución puede ser resuelta para k.

Ejemplo numérico

$$U_t = \ln(c_t^1) + \beta \ln(c_{t+1}^2)$$

$$\text{s. a } c_t^1 + \frac{c_{t+1}^2}{1 + r_{t+1}} = w_t$$

Para las empresas

$$Y_t = K_t^\alpha L_t^{1-\alpha}$$
$$\frac{Y_t}{L_t} = \frac{Y_t}{N_t} = y_t = f(k_t) = \left(\frac{K_t}{N_t}\right)^\alpha = k_t^\alpha$$
$$w_t = (1 - alpha) k_t^\alpha$$
$$r_t = \alpha k_t^{\alpha - 1}$$

$$s_t = \frac{\beta}{1+\beta} w_t$$

$$k_{t+1} = \frac{s_t}{1+n} = \frac{\beta}{1+\beta} \frac{1-\alpha}{1+n} k_t^\alpha$$

Estado estacionario:

$$k = \left(\frac{\beta}{1+\beta} \frac{1-\alpha}{1+n}\right)^{\frac{1}{1-\alpha}}$$

OLG Pensiones

Pensiones - Fully funded

$$U_t = u(c_t^1) + \beta u(c_{t+1}^2)$$

$$w_t = c_t^1 + s_t + d_t$$
$$c_{t+1}^2 = (1 + r_{t+1})(d_t + s_t)$$

$$u'(c_t^1) = \beta (1 + r_{t+1}) u'(c_{t+1}^2)$$

$$s_t + d_t = (1+n)k_{t+1}$$

Trabajo elástico

$$U(c_t^1, c_{t+1}^2, l_t) = u^y(c_t^1, l_t) + u^o(c_{t+1}^2)$$

$$(1-\tau)w_t l_t = c_t^1 + s_t$$
$$c_{t+1}^2 = (1+r_{t+1})(d_t + s_t)$$
$$d_t = \tau w_t l_t$$
$$w_t l_t = c_t^1 + \frac{c_{t+1}^2}{1+r_{t+1}}$$

$$\frac{\partial u^y(c_t^1, l_t)}{\partial c_t^1} = \beta(1+r_{t+1})\frac{\partial u^0(c_{t+1}^2)}{\partial c_{t+1}^2}$$

$$-\frac{\partial u^y(c_t^1, l_t)}{\partial l_t} = w_t$$

$$w_t = \frac{\partial F(K_t, L_t)}{\partial L_t} = f(k_t) - k_t f'(k_t)$$

$$r_t = \frac{\partial F(K_t, L_t)}{\partial K_t} = f'(k_t)$$

$$(1+n)k_{t+1} = s_t + d_t = (1-\tau)w_t l_t - c_t^1 + \tau w_t l_t = w_t l_t - c_t^1$$

$$L_t \equiv N_t l_t$$

Pensiones - PAYG

$$N_t d_t = N_{t-1} pen_t$$
$$N_t = (1+n) N_{t-1}$$
$$pen_t = (1+n) d_t$$

Para contribuciones constantes $d_t = d$

$$pen_t = (1+n) d$$

La restricción es

$$w_t = c_t^1 + s_t + d_t$$

$$c_{t+1}^2 = (1+r_{t+1}) s_t + pen_{t+1}$$

$$c_t^1 + \frac{c_t^2}{1+r_{t+1}} = w_t - d_t + \frac{pen_{t+1}}{1+r_{t+1}}$$

$$c_t^1 + \frac{c_t^2}{1+r_{t+1}} = w_t + \frac{n-r_{t+1}}{1+r_{t+1}} d$$

$$\mathcal{L} = u(c_t^1) + \beta u(c_{t+1}^2) + \lambda \left[w_t - dt + \frac{pen_{t+1}}{1+r_{t+1}} - c_t^1 - \frac{c_{t+1}^2}{1+r_{t+1}} \right]$$

$$(1+n) k_{t+1} = s[w_t(k_t), r_{t+1}(k_{t+1}), d_t]$$

Ejemplo: trabajo inelástico

$$U_t = \ln c_t^1 + \beta \ln c_{t+1}^2 - \nu_0 \frac{l_t^{1+\frac{1}{\nu_1}}}{1+\frac{1}{\nu_1}}$$

$$c_t^1 + \frac{c_t^2}{1+r_{t+1}} = w_t + \frac{n - r_{t+1}}{1+r_{t+1}} d$$

FOC:

$$\frac{1}{c_t^1} = \lambda_t$$

$$\frac{\beta}{c_{t+1}^2} = \frac{\lambda_t}{1+r_{t+1}}$$

$$c_{t+1}^2 = \beta c_t^1 (1 + r_{t+1})$$

Sustituyendo en la restricción intertemporal

$$c_t^1 = \frac{1}{1+\beta}\left[w_t \bar{l} + \frac{n - r_{t+1}}{1+r_{t+1}} d\right]$$

$$s_t = w_t \bar{l} - c_t^1 - d_t = \frac{\beta}{1+\beta} w_t \bar{l} - \frac{1+\beta+\beta r_{t+1} + n}{(1+\beta)(1+r_{t+1})} d$$

$$(1+n)k_{t+1} = w_t \bar{l} - c_t^1 - d_t = \frac{\beta}{1+\beta} w_t \bar{l} - \frac{1+\beta+\beta r_{t+1} + n}{(1+\beta)(1+r_{t+1})} d$$

$$Y_t = K_t^\alpha (N_t l_t)^{1-\alpha}$$

$$w_t = (1-\alpha) K_t^\alpha (N_t l_t)^{-\alpha} = (1-\alpha) k_t^\alpha l_t^{-\alpha}$$
$$r_t = \alpha K_t^{\alpha-1} (N_t l_t)^{1-\alpha} = \alpha k_t^{\alpha-1} l_t^{1-\alpha}$$