EEL 4837
Programming for Electrical Engineers II

Qiangeng Yang
Teaching Assistant
Department of Electrical and Computer Engineering
University of Florida at Gainesville

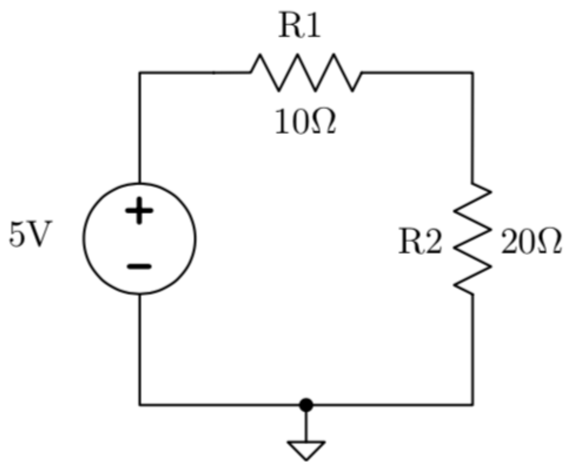
Excursion 1 – Circuit Analysis Tool

Readings:

- Excursion 1 Description

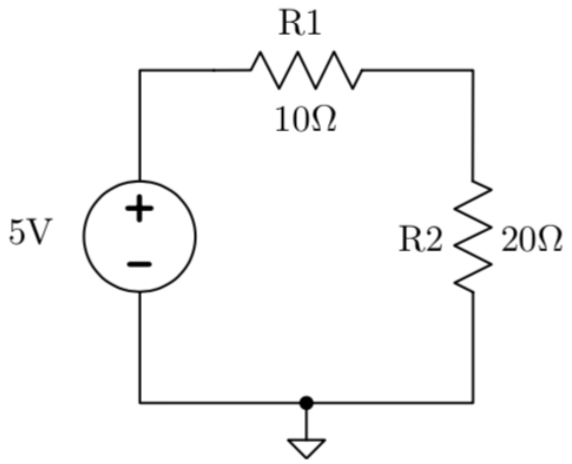
Electronic Circuit

- **Electronic circuit**: a collection of interconnected electronic components.
- A circuit has a well-defined deterministic functionality when running:
  - Voltage sources
  - Current sources
  - Resistors
  - Inductors
  - Capacitors
  - Diodes
  - Transistors
  - …



Electronic Circuit (cont.)

- The functionality of a circuit is fully characterized by:
  - The **current** through each component.
  - The **voltage drop/rise** through each component.
  - The **voltage potential** at each of the interconnections relative to a ground node.

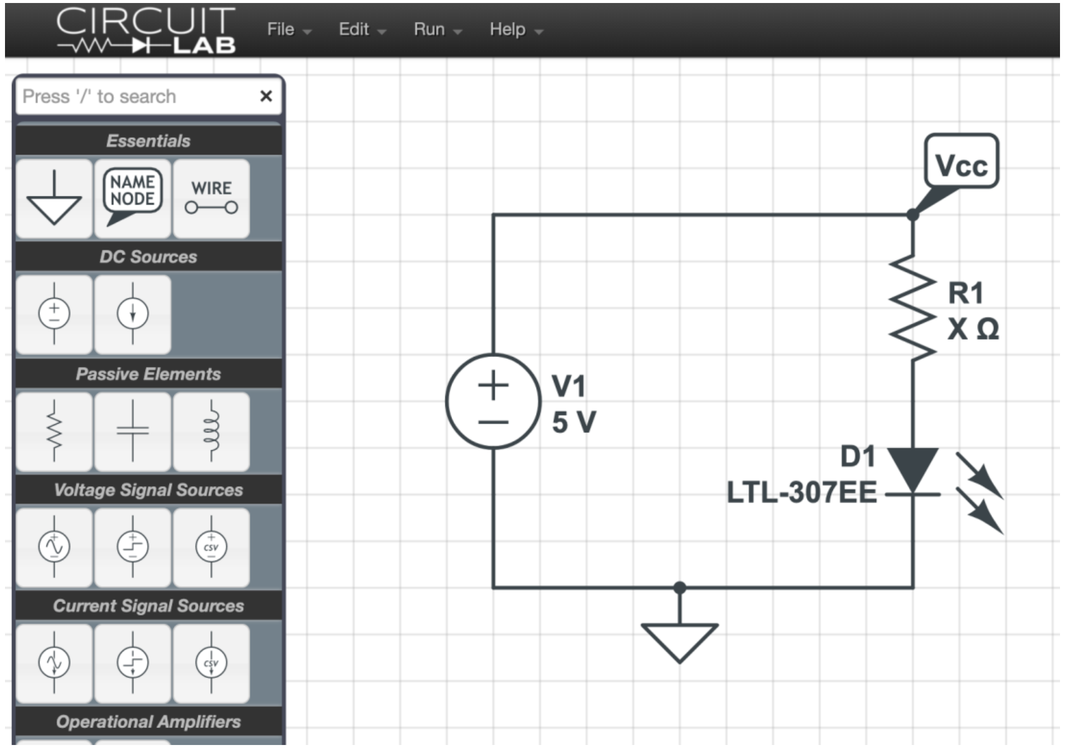


Circuit Analysis

- **Circuit analysis**: a sequence of methods to answer some questions about circuit.
  - e.g., what is the voltage drop/rise across a resistor?
- To do that, we may need:
  - Measuring tools.
    - e.g., Galvanometer, voltmeter, ohmmeter, etc.
  - Theories.
    - e.g., Ohm’s Law, Voltage Law, Current Law, etc.
- Limitations:
  - Sometimes we can only answer questions as needed about the circuit at a time.

Circuit Simulation

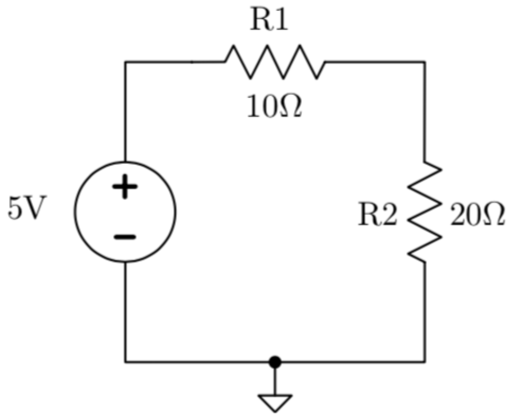
- **Circuit simulation**: a technique to check and verify the design of circuits prior to manufacturing and deployment.
- Almost replaced physical prototype.
- What can we do with simulation?
  - Model a linear circuit with a single matrix algebraic equation.



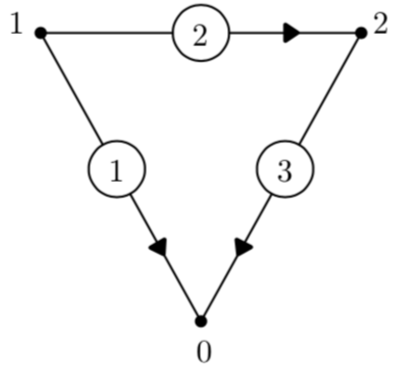
From: CircuitLab

Schematic Diagrams and Digraphs

- **Schematic diagram**: a graphical representation of an electrical circuit.

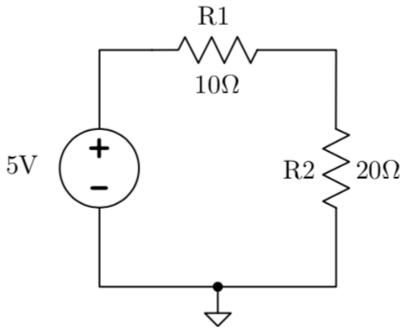


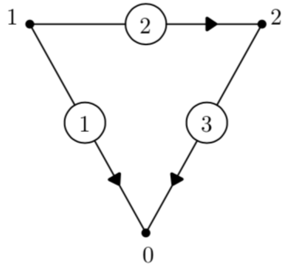
- **Directed graph (digraph):** can represent a circuit more abstractly.
  - From graph theory.



Circuit Netlist

- **Netlist**: describes the connectivity of an electronic circuit.
- It consists of:
  - a list of **electronic components**;
  - a list of **nodes** they are connected to.
- Format:
  - Branch label
    - First character indicates component type.
  - Source node label
  - Destination node label
  - Numeric component value

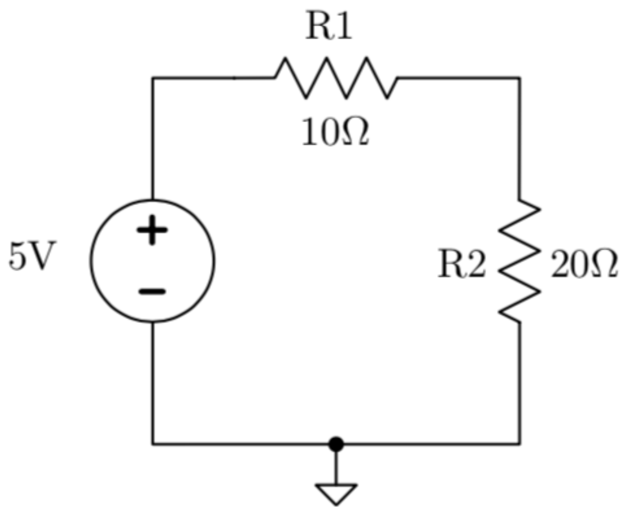





```
V1 1 0 5
R1 1 2 10
R2 2 0 20
```

Circuit Netlist

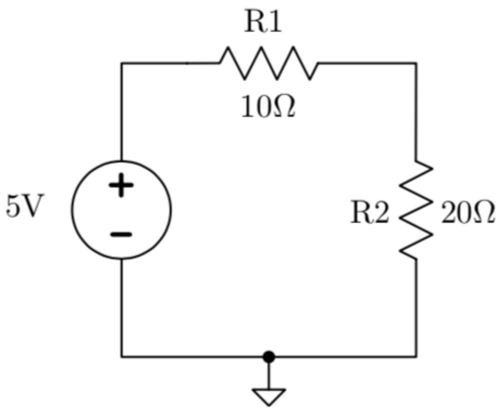
- Some conventions:
  - Ground node is always labeled as 0, while other node can be labeled in any order.
  - Use positive numeric values.
    - Current travels from source to destination.
    - Voltage drops from source to destination.




```
V1 1 0 5
R1 1 2 10
R2 2 0 20
```

Circuit in Computer

- How can we represent a circuit in computer?



Circuit Diagram

```
V1 1 0 5
R1 1 2 10
R2 2 0 20
```

Netlist

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \boldsymbol{A} \\ -\boldsymbol{A}^{\boldsymbol{T}} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{M} & \boldsymbol{N} \end{bmatrix} \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{v} \\ \boldsymbol{i} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{u_s} \end{bmatrix}$$

Matrix

Excursion 1 Goal

- Design an elementary interactive circuit analysis tool to:
  - Read a circuit.
  - Compute the currents and voltages across different components.

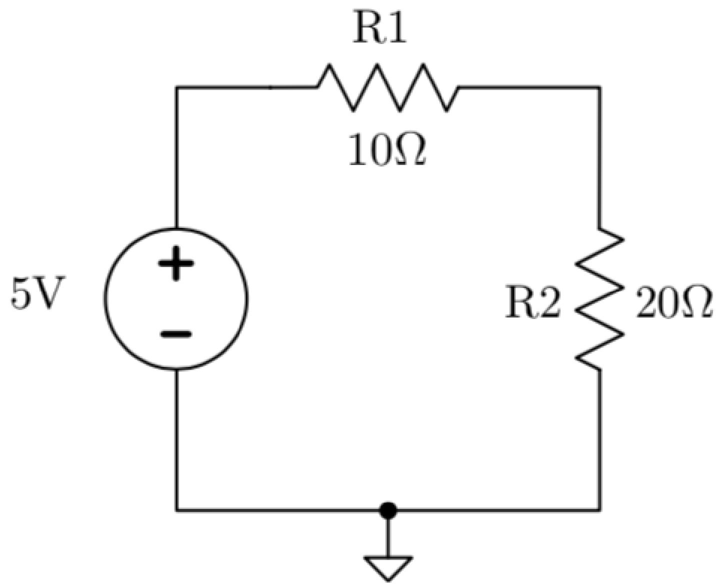
- You can assume that the given circuits only contain:
  - voltage sources
  - resistors

Steps Overview

1. Create a netlist as input to represent a circuit.
2. Use KCL, KVL and Ohm's law to construct three matrix equations.
3. Combine all the matrix equations to solve all the unknown parameters (e, v, i of each part).

Step 1: Create a Netlist

- Based on a circuit diagram, create its corresponding netlist as input.




```
V1 1 0 5
R1 1 2 10
R2 2 0 20
```

Step 2: From Netlist to Matrix

- How can we represent the netlist in computer?

```
V1 1 0 5
R1 1 2 10
R2 2 0 20
```

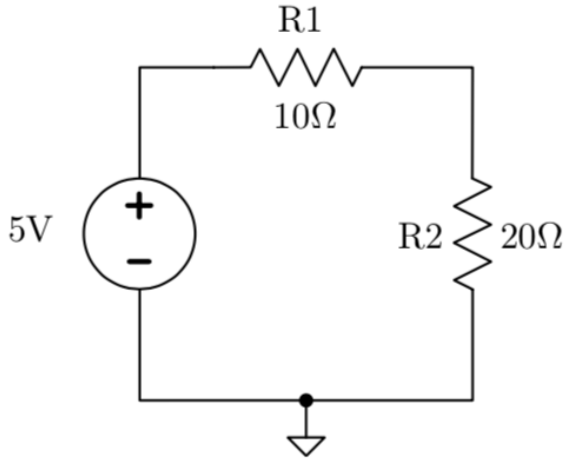
$$A_a = \begin{array}{c} n_0 \\ n_1 \\ n_2 \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} -1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \\ \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \end{array}$$

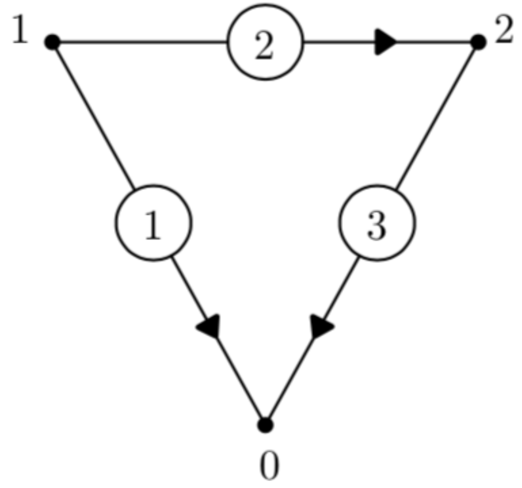
node

branch

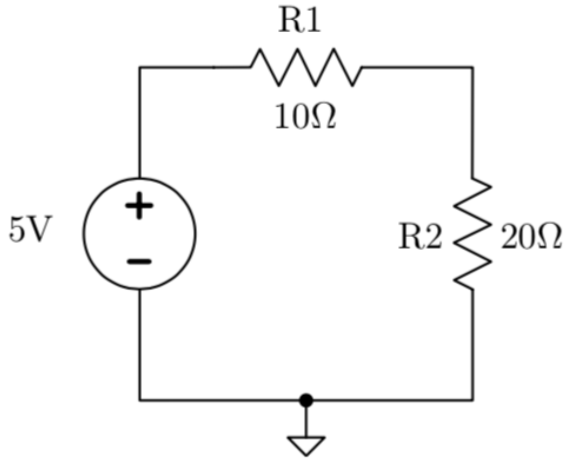
Assume “1” is source and “-1” is destination.

In each column, we put a “1” in the source node row and “-1” in the destination node row.





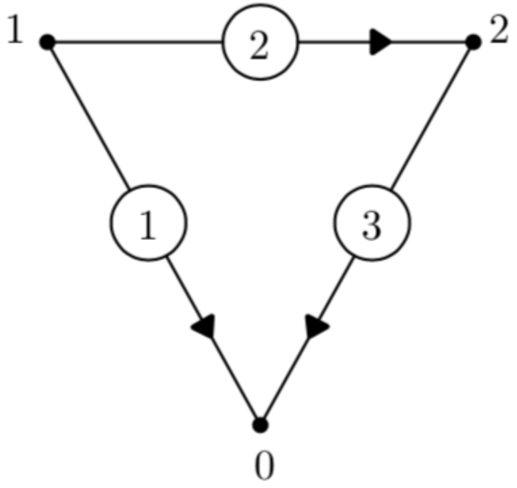
Step 2: From Netlist to Matrix



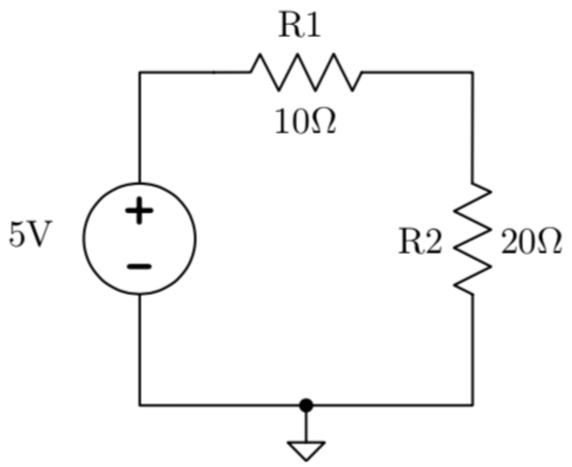
- (Cont.) We can further remove the row for one node (e.g., ground node) to get a reduced incidence matrix.

$$\boldsymbol{A}_a = \begin{array}{cc} \begin{array}{c} \cancel{n_0} \\ n_1 \\ n_2 \end{array} & \begin{bmatrix} \cancel{-1} & \cancel{0} & \cancel{-1} \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \\ & \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \end{array}$$

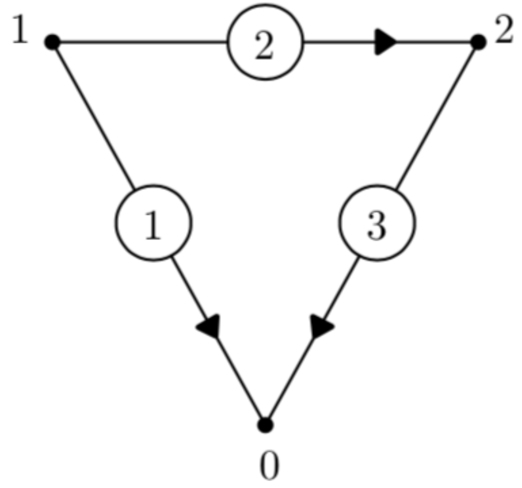
$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$



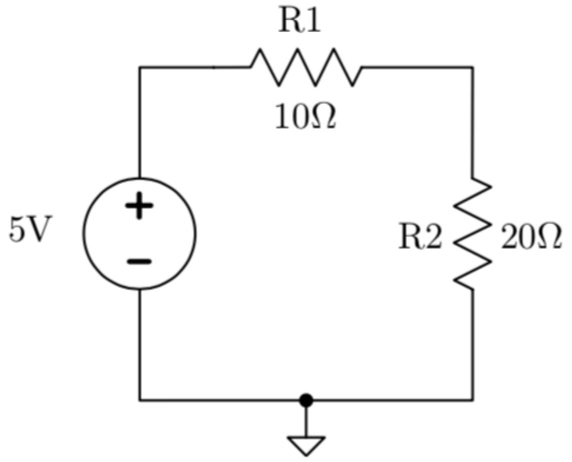
Step 2: From Netlist to Matrix



- **Kirchoff's Current Law (KCL):** the algebraic sum of currents that leave any node is zero.
- KCL equation for each node:
  - Assume leaving is positive and coming is negative:
    - At Node 0: branch currents i_1 and i_3 are coming, so
      $$-i_1 - i_3 = 0$$
    - At Node 1: branch currents i_1 and i_2 are leaving, so
      $$i_1 + i_2 = 0$$
    - At Node 2: branch current i_2 is coming, while i_3 is leaving, so
      $$-i_2 + i_3 = 0$$

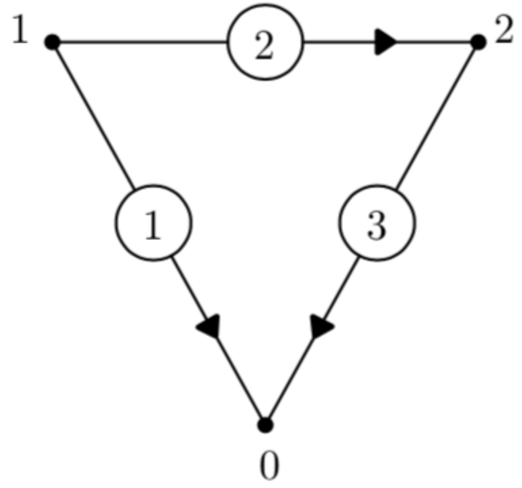


Step 2: From Netlist to Matrix



- (Cont.) Now we have:

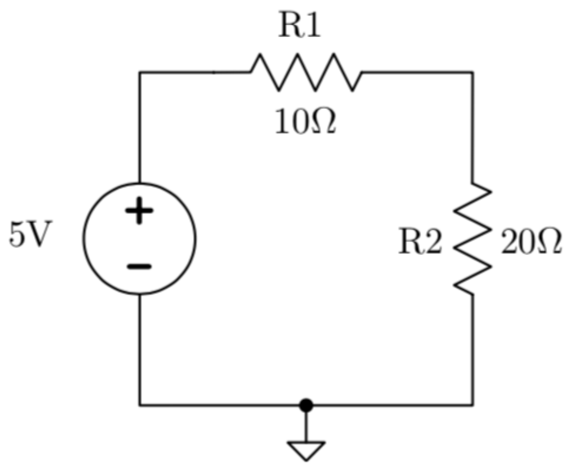
$$\begin{aligned} -i_1 - i_3 &= 0 \\ i_1 + i_2 &= 0 \\ -i_2 + i_3 &= 0 \end{aligned}$$



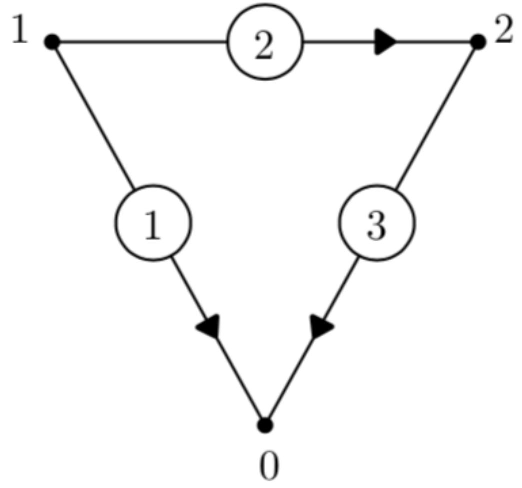
- Represent these equations with matrices:

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \boldsymbol{Ai} = \mathbf{0}$$

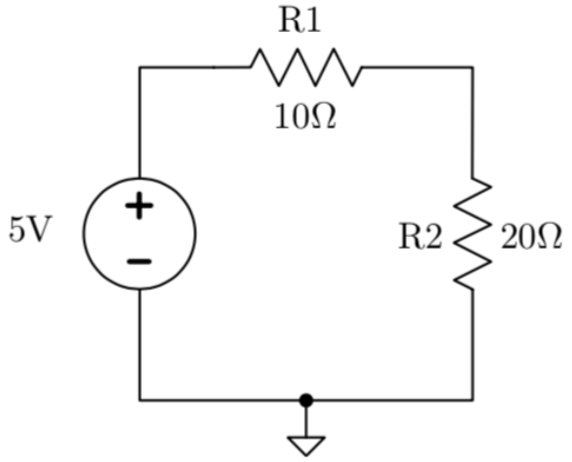
Step 2: From Netlist to Matrix



- **Kirchhoff's Voltage Law (KVL)**: the voltage drop between any two nodes is equal to the difference of the two node voltages.
- KVL equation for each pair of nodes:
  - Voltage drop between n_0 and n_1 (branch 1):
    $$v_1 = e_1 - 0$$
  - Voltage drop between n_1 and n_2 (branch 2):
    $$v_2 = e_1 - e_2$$
  - Voltage drop between n_0 and n_2 (branch 3):
    $$v_3 = e_2 - 0$$

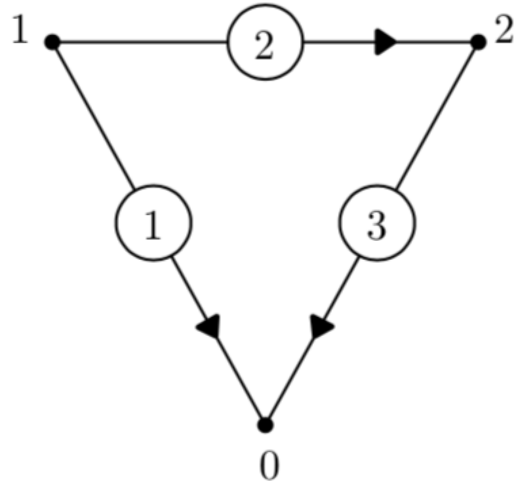


Step 2: From Netlist to Matrix



- (Cont.) Now we have:

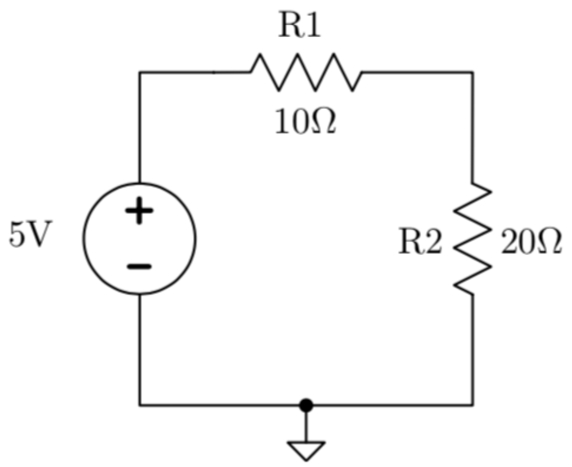
$$
\begin{aligned}
v_1 &= e_1 - 0 \\
v_2 &= e_1 - e_2 \\
v_3 &= e_2 - 0
\end{aligned}
$$



- Represent these equations with matrices:

$$
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \Rightarrow \boldsymbol{v} = \boldsymbol{A}^T \boldsymbol{e}
$$

Step 2: From Netlist to Matrix



- Comparison between KCL and KVL:

KCL

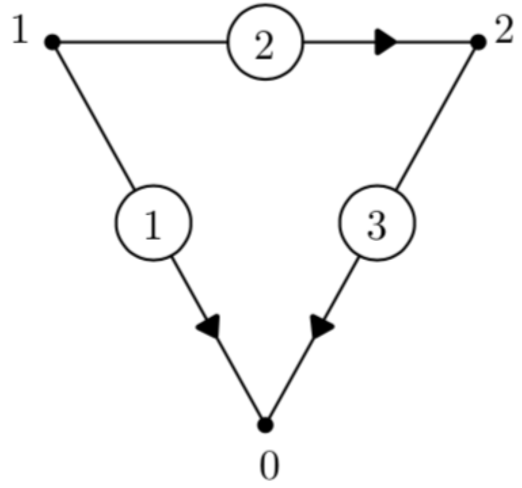
$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{Ai} = \boldsymbol{0}$$

KVL

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

$$\boldsymbol{v} = \boldsymbol{A}^T\boldsymbol{e}$$

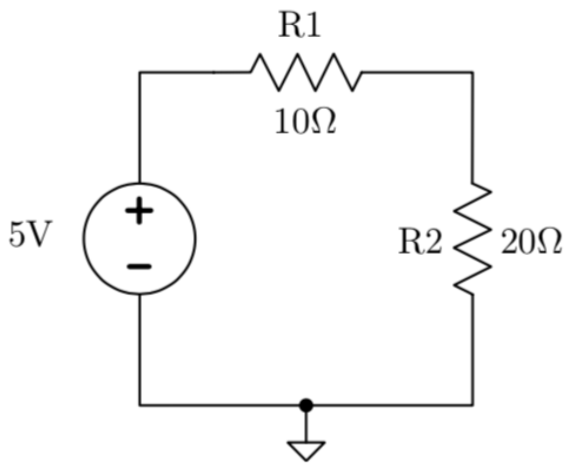


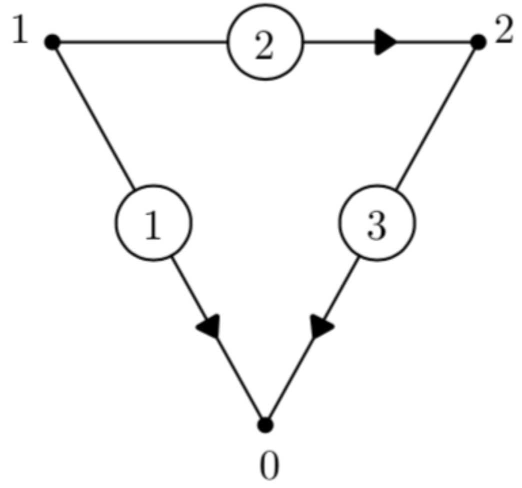
Step 2: From Netlist to Matrix

- Combine the KCL and KVL equations:

$$\boldsymbol{Ai} = \boldsymbol{0} \qquad\qquad \boldsymbol{v} = \boldsymbol{A}^T \boldsymbol{e}$$

$$\boxed{\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A} \\ -\boldsymbol{A}^T & \boldsymbol{1} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{v} \\ \boldsymbol{i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}}$$





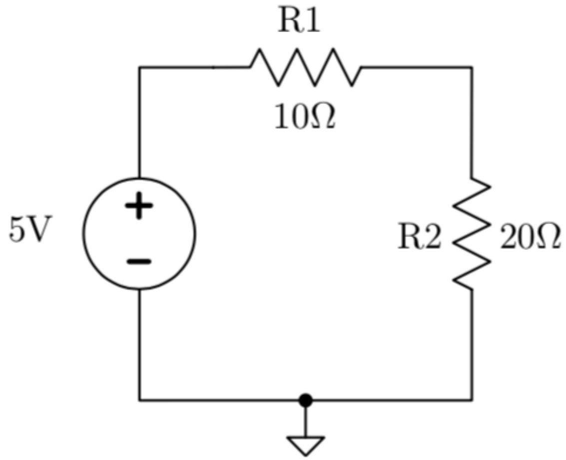
Step 2: From Netlist to Matrix

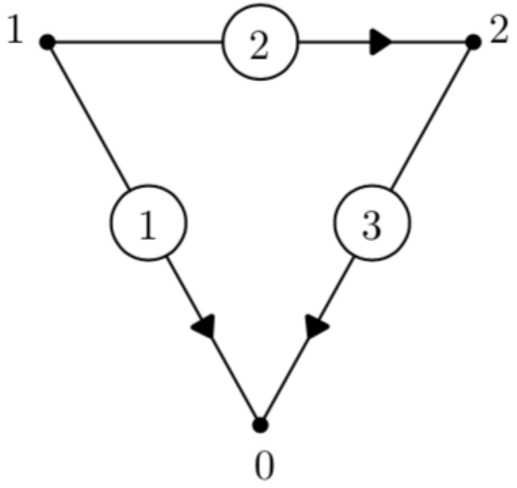
- **Ohm's law**: the current through a conductor between two points is directly proportional to the voltage across the two points.
  - $I = \frac{V}{R}$
- Ohm's law equation for each component:

$$v_1 = 5$$

$$v_2 = 10i_2$$

$$v_3 = 20i_3$$





Step 2: From Netlist to Matrix

- (Continue) Represent these equations with matrices:

$$
\begin{aligned}
v_1 &= 5 \\
v_2 &= 10i_2 \\
v_3 &= 20i_3
\end{aligned}
$$

$$
\begin{aligned}
v_1 &+ 0i_1 = 5 \\
v_2 &- 10i_2 = 0 \\
v_3 &- 20i_3 = 0
\end{aligned}
$$

voltages currents constants

Format: $av_b + bi_b = u_b$

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}
+
\begin{bmatrix} 0 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -20 \end{bmatrix}
\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}
=
\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}
$$

$$\boxed{\boldsymbol{Mv} + \boldsymbol{Ni} = \boldsymbol{u_s}}$$

Step 3: Calculate Unknown Parameters

- Now, what do we have?
  - KCL + KVL:

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \boldsymbol{A} \\ -\boldsymbol{A}^T & \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{v} \\ \boldsymbol{i} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

Ohm's Law:

$$\boldsymbol{Mv} + \boldsymbol{Ni} = \boldsymbol{u_s}$$

- Combine all the equations to create the sparse matrix representation:

$$\boxed{\begin{bmatrix} \mathbf{0} & \mathbf{0} & \boldsymbol{A} \\ -\boldsymbol{A}^T & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{M} & \boldsymbol{N} \end{bmatrix} \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{v} \\ \boldsymbol{i} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{u_s} \end{bmatrix}} \Rightarrow \boxed{\boldsymbol{Tw} = \boldsymbol{u}}$$

Step 3: Calculate Unknown Parameters

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \boldsymbol{A} \\ -\boldsymbol{A}^T & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{M} & \boldsymbol{N} \end{bmatrix} \begin{bmatrix} \boldsymbol{e} \\ \boldsymbol{v} \\ \boldsymbol{i} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{u_s} \end{bmatrix}$$

Put everything we know into the matrices

$$\left[\begin{array}{cc|ccc|ccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ \hline -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -20 \end{array}\right] \left[\begin{array}{c} e_1 \\ e_2 \\ \hline v_1 \\ v_2 \\ v_3 \\ \hline i_1 \\ i_2 \\ i_3 \end{array}\right] = \left[\begin{array}{c} 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 5 \\ 0 \\ 0 \end{array}\right]$$

Final goal: calculate e, v, and i!

Steps Review

1. Create a netlist as input to represent a circuit.
2. Use KCL, KVL and Ohm's law to construct three matrix equations.
    - Construct $\boldsymbol{A}$, $\boldsymbol{M}$, $\boldsymbol{N}$, and $\boldsymbol{u_s}$ to represent the netlist.
3. Combine all the matrix equations to solve all the unknown parameters (e, v, i of each part).
    - Use Gauss Elimination, LU Factorization, etc.

Example 1

- Input:
  - V1 1 0 6
  - R1 1 2 6
  - R2 2 0 3.33
  - R3 2 3 3
  - R4 3 0 11
  - R5 3 4 4
  - R6 4 0 7
  - R7 4 5 7
  - R8 5 0 7

Example 1

- $A = \begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{matrix}$

- $M = \begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}$

Input:
V1 1 0 6
R1 1 2 6
R2 2 0 3.33
R3 2 3 3
R4 3 0 11
R5 3 4 4
R6 4 0 7
R7 4 5 7
R8 5 0 7

Example 1

- $$N = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.33 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -11 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 \end{matrix} \qquad u_s = \begin{matrix} 6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

Input:
V1 1 0 6
R1 1 2 6
R2 2 0 3.33
R3 2 3 3
R4 3 0 11
R5 3 4 4
R6 4 0 7
R7 4 5 7
R8 5 0 7

Thank you!

Readings:

- Excursion 1 Description