

**EEL 4837**

# Programming for Electrical Engineers II

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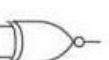
# Excursion 2 – Technology Mapping

Reading:

- Excursion 2 Description

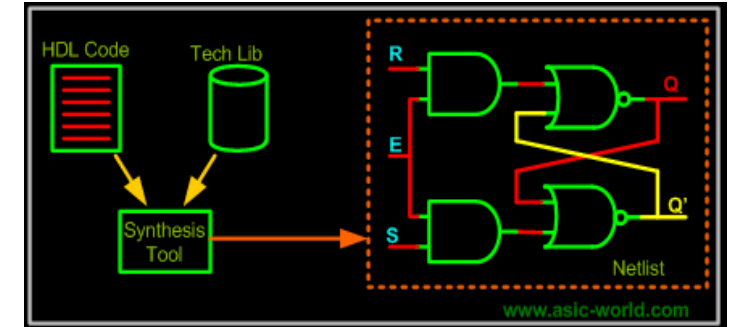
# Logic Gate

- Logic gate
  - A device that acts as a building block for digital circuits.
  - One or more inputs (0/1) and only one output (0/1).
  - Performing a specific logic function.
- Some basic logic gates:

| Name        | NOT  | AND   | NAND   | OR  | NOR   | XOR   | XNOR  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|-------------|--|---|--|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Alg. Expr.  | $\overline{A}$   | $AB$  | $\overline{AB}$  | $A+B$   | $\overline{A+B}$  | $A\oplus B$   | $\overline{A\oplus B}$  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Symbol      |                       |  |  |  |  |  |  |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Truth Table | <table><tr><th>A</th><th>X</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table> | A   | X  | 0   | 1   | 1   | 0   | <table><tr><th>B</th><th>A</th><th>X</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | B | A | X | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | <table><tr><th>B</th><th>A</th><th>X</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table> | B | A | X | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | <table><tr><th>B</th><th>A</th><th>X</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | B | A | X | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | <table><tr><th>B</th><th>A</th><th>X</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table> | B | A | X | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | <table><tr><th>B</th><th>A</th><th>X</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table> | B | A | X | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | <table><tr><th>B</th><th>A</th><th>X</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | B | A | X | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| A           | X  |   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 1  |   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 0  |   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| B           | A  | X   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 0  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 1  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 0  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 1  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| B           | A  | X   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 0  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 1  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 0  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 1  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| B           | A  | X   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 0  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 1  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 0  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 1  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| B           | A  | X   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 0  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 1  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 0  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 1  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| B           | A  | X   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 0  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 1  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 0  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 1  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| B           | A  | X   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 0  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0           | 1  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 0  | 0   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1           | 1  | 1   |  |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

# Logic Synthesis

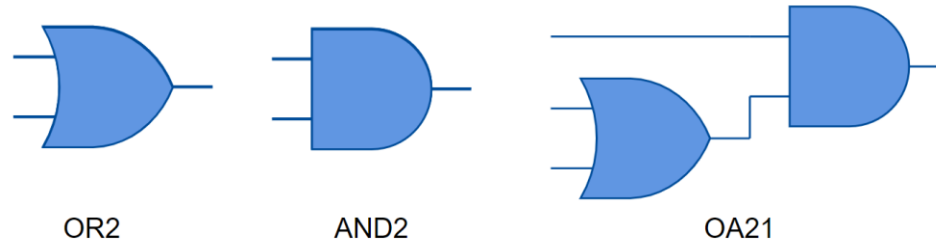
- **Logic synthesis**
  - The process of automatic production of logic components, in particular digital circuits.
- It includes:
  - **Logic optimization**
    - Simplifies logic expressions theoretically.
  - **Technology mapping**
    - Implements logic gates with physical layouts.
    - **The topic of Excursion 2**



Source: <http://www.asic-world.com/verilog/synthesis1.html>

# Technology Mapping

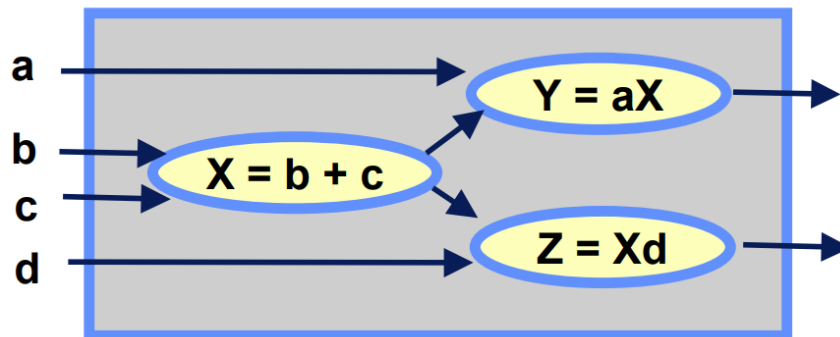
- Suppose we have these gates in our “library”:



OA21: OR-AND, a so-called complex gate.

Number like 2 and 21: the count of direct inputs.

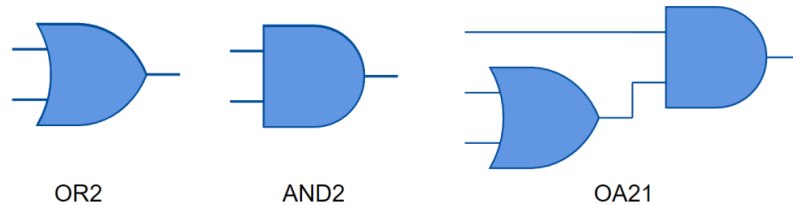
- Suppose we need to build a model as follows:



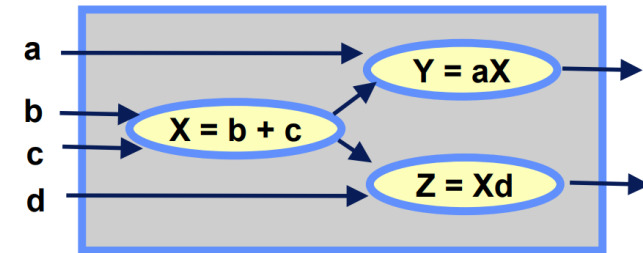
**Q: How can we realize it by using the gates in our library?**

# Technology Mapping

- Library:

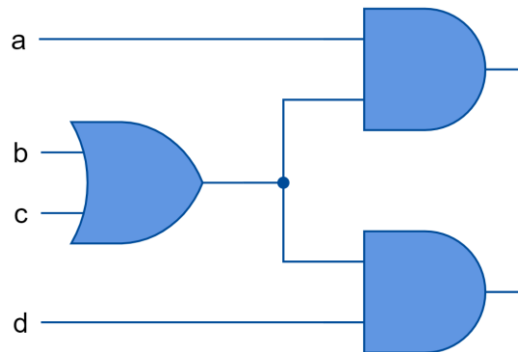


Model:

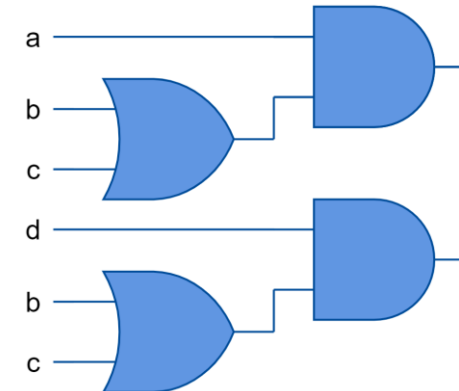


- Solutions:

Solution 1: suppose we have all what we need :)



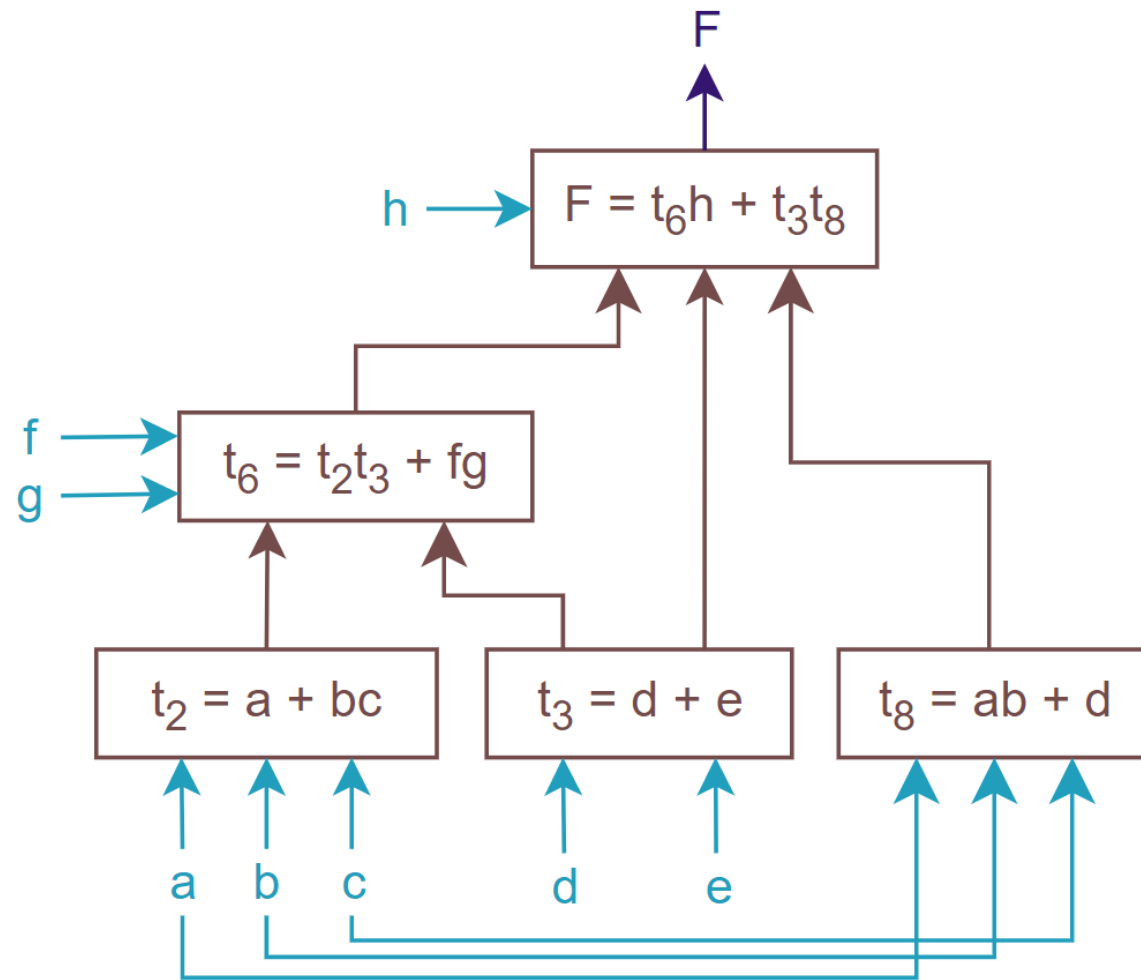
Solution 2: suppose we only have OA21 :(



# Excursion 2

- Goal:
  - Implement a tool for technology mapping.
  - Convert the original logic expressions into NAND and NOT gates.
  - Compute the minimal cost.
- General steps:
  1. Read a Boolean netlist.
  2. Convert the original structure into a NAND-NOT tree.
  3. Recursively compute the minimal cost.

# Step 1: Read a Boolean netlist



Note: some middle steps were combined only for elegance.

## Netlist format:

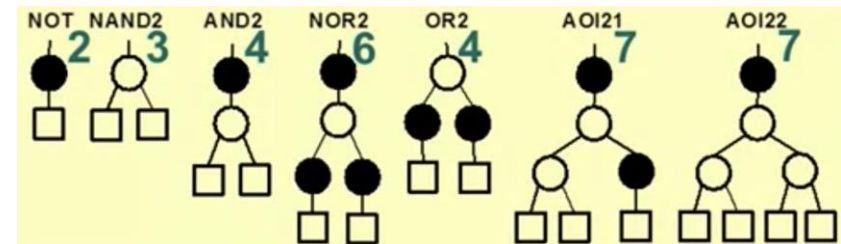
```
a INPUT
b INPUT
c INPUT
d INPUT
e INPUT
f INPUT
g INPUT
h INPUT
F OUTPUT
t1 = AND b c
t2 = OR a t1
t3 = OR d e
t4 = AND f g
t5 = AND t2 t3
t6 = OR t4 t5
t7 = AND a b
t8 = OR t7 d
t9 = AND t3 t8
t10 = AND t6 h
F = OR t9 t10
```



## Step 2: Conversion into a NAND-NOT tree

- Only NAND and NOT gates are available.
- Represent a gate of AND/OR/NOT by NANDs and NOTs only.
  - Transforming equations (De Morgan's laws):

- $\text{NOT } A = \text{NOT } A$
- $A \text{ AND } B = \text{NOT } (A \text{ NAND } B)$
- $A \text{ OR } B = (\text{NOT } A) \text{ NAND } (\text{NOT } B)$

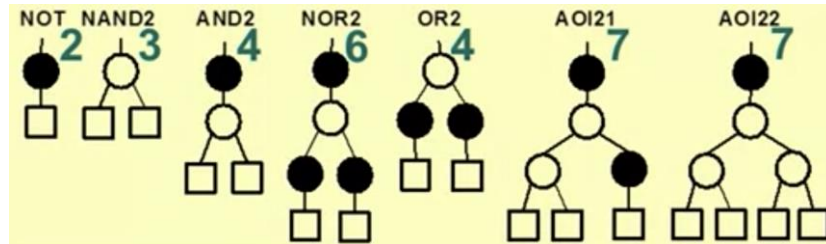


Black Circle = NOT  
White Circle = NAND  
White Square = input

- Best practice: topologically traverse from output to input.

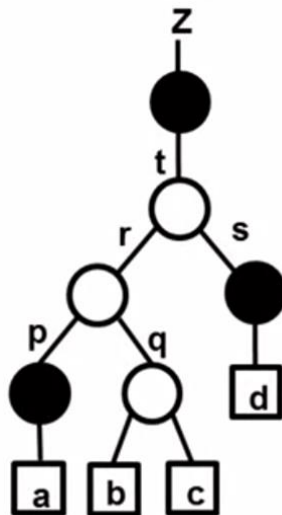
# Step 3: Recursively compute minimal cost

- Library:



Black Circle = NOT  
White Circle = NAND  
White Square = input  
Number: cost

- Consider a tree like this:



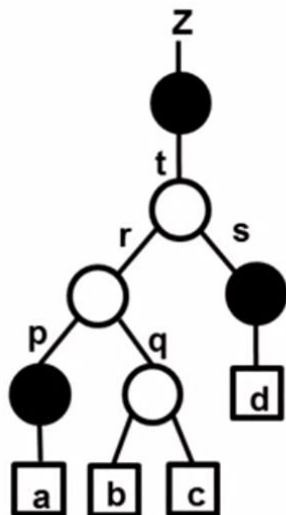
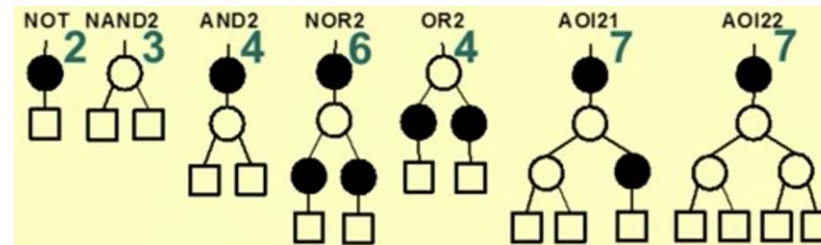
How can we describe the tree?

What components provided in the library were used?

| At node | Can Match | With min cost                               |
|---------|-----------|---|
| Z       | NOT       | $2 + \text{mincost}(t)$                     |
|         | AND2      | $4 + \text{mincost}(r) + \text{mincost}(s)$ |
|         | AOI21     | $7 + \text{mincost}(p) + \text{mincost}(q)$ |
|         |           | } Minimum                                   |

# Step 3: Recursively compute minimal cost

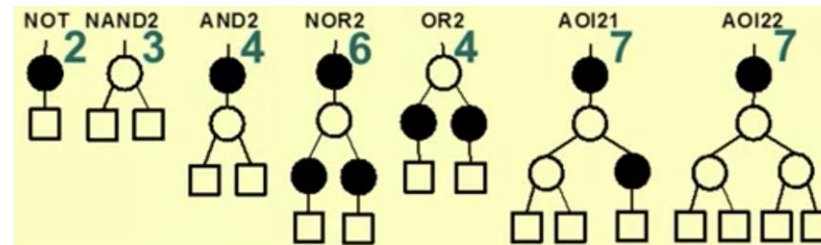
- Continue the recursion to compute the minimal cost at the other nodes.



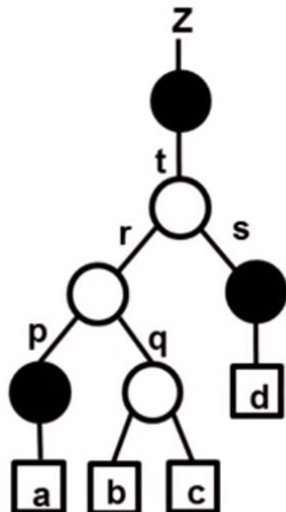
| At node | Can Match            | With min cost   |
|---------|----------------------|---|
| Z       | NOT<br>AND2<br>AOI21 | $\begin{cases} 2 + \text{mincost}(t) \\ 4 + \text{mincost}(r) + \text{mincost}(s) \\ 7 + \text{mincost}(p) + \text{mincost}(q) \end{cases}$ |
| t       | NAND2                | $3 + \text{mincost}(r) + \text{mincost}(s)$   |
| r       | NAND2                | $3 + \text{mincost}(p) + \text{mincost}(q)$   |
| p       | NOT                  | 2   |
| q       | NAND2                | 3   |
| s       | NOT                  | 2   |

# Step 3: Recursively compute minimal cost

- Continue the recursion to compute the minimal cost at the other nodes.



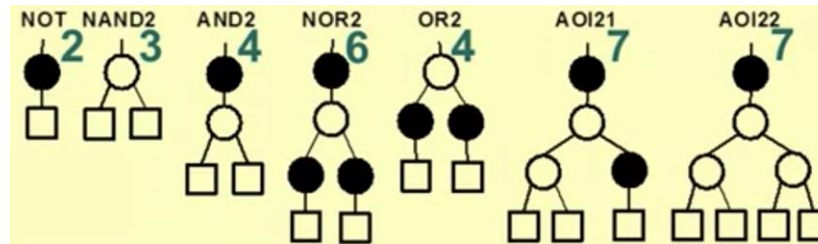
Black Circle = NOT  
White Circle = NAND  
White Square = input  
Number: cost



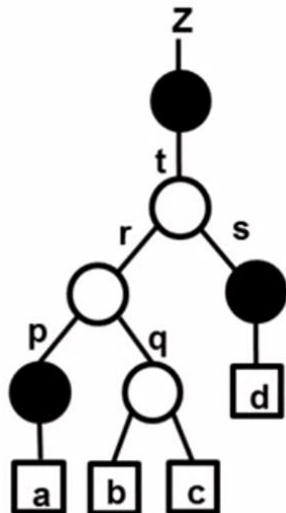
| At node | Can Match | With min cost               |           |
|---------|-----------|-----------------------------|-----------|
| Z       | NOT       | $2 + \text{mincost}(t)$     | } Minimum |
|         | AND2      | $4 + 2 + \text{mincost}(r)$ |           |
|         | AOI21     | $7 + 2 + 3 = 12$            |           |
| t       | NAND2     | $3 + \text{mincost}(r) + 2$ |           |
| r       | NAND2     | $3 + 2 + 3 = 8$             |           |
| p       | NOT       | 2                           |           |
| q       | NAND2     | 3                           |           |
| s       | NOT       | 2                           |           |

# Step 3: Recursively compute minimal cost

- Continue the recursion to compute the minimal cost at the other nodes.



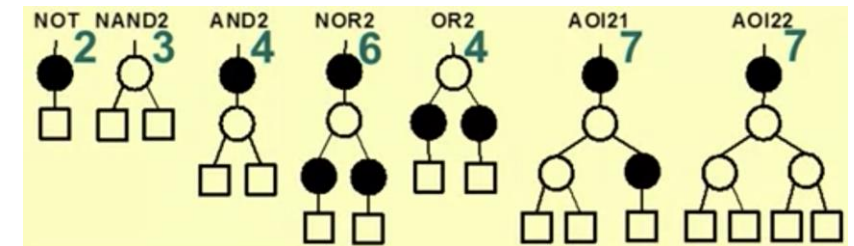
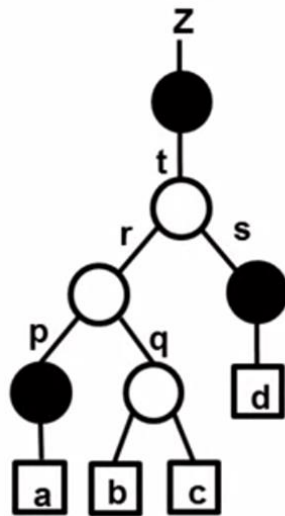
Black Circle = NOT  
White Circle = NAND  
White Square = input  
Number: cost



| At node | Can Match | With min cost    | } Minimum |
|---------|-----------|------------------|-----------|
| Z       | NOT       | $2 + 13 = 15$    |           |
|         | AND2      | $4 + 2 + 8 = 14$ | } Minimum |
|         | AOI21     | $7 + 2 + 3 = 12$ |           |
| t       | NAND2     | $3 + 8 + 2 = 13$ | } Minimum |
| r       | NAND2     | $3 + 2 + 3 = 8$  |           |
| p       | NOT       | 2                | } Minimum |
| q       | NAND2     | 3                |           |
| s       | NOT       | 2                | } Minimum |
|         |           |                  |           |

# Step 3: Recursively compute minimal cost

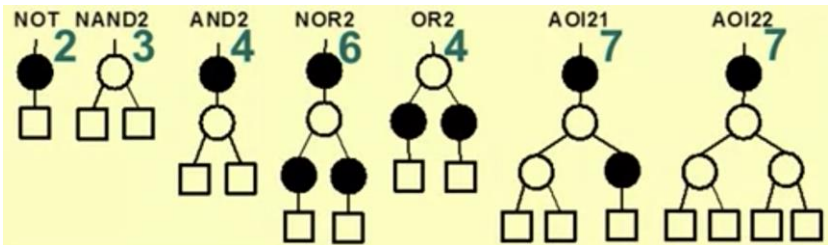
- (Optional, the other ways, **bonus points**)
  - Bottom-up
  - Dynamic programming



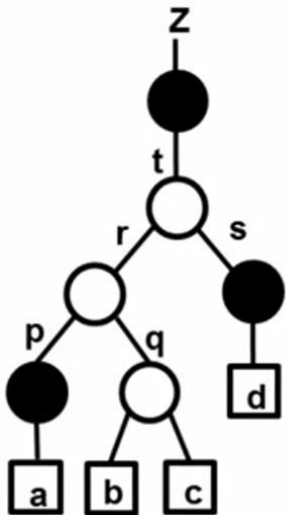
Black Circle = NOT  
White Circle = NAND  
White Square = input  
Number: cost

# Step 3: Recursively compute minimal cost

- Return the calculated optimal cost.



Black Circle = NOT  
 White Circle = NAND  
 White Square = input  
 Number: cost



| At node | Can Match | With min cost    |
|---------|-----------|------------------|
| Z       | NOT       | $2 + 13 = 15$    |
|         | AND2      | $4 + 2 + 8 = 14$ |
|         | AOI21     | $7 + 2 + 3 = 12$ |
| t       | NAND2     | $3 + 8 + 2 = 13$ |
| r       | NAND2     | $3 + 2 + 3 = 8$  |
| p       | NOT       | 2                |
| q       | NAND2     | 3                |
| s       | NOT       | 2                |

} Minimum  **Min cost: 12**

# Quick Review

- Goal:
  - Implement a tool for technology mapping.
  - Convert the original expressions into NAND and NOT gates.
  - Compute the minimal cost.
- General steps:
  1. Read a Boolean netlist.
  2. Convert the original structure into a NAND-NOT tree.
  3. Recursively compute the minimal cost.



# Thank you!

Reading:

- Excursion 2 Description