Adaptive Filters Homework 1

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1. Let $\{u_i\}_{i=1}^r$ be r linearly independent $M \times 1$ vectors, and $\{v_i\}_{i=1}^r$ be r linearly independent $N \times 1$ vectors. Show that the $M \times N$ matrix A given by:

$$A = \sum_{i=1}^{r} u_i v_i^H$$

Proof:
Let $y \in \mathcal{R}_A$, then for some $x \in M^{N \times 1}$
$y = Ax = \left(\sum_{i=1}^r u_i v_i^H\right) x$. Since matrix multiplication is distributive,
$y = Ax = \sum_{i=1}^r u_i v_i^H x$. Since matrix multiplication is associative,
$y = Ax = \sum_{i=1}^r u_i (v_i^H x)$. Since $v_i^H x$ is scalar,
$y = Ax = \sum_{i=1}^r (v_i^H x) u_i$
Thus since $\{u_i\}$ is linearly independent, it is a basis for $\mathcal{R}_A$ and
$rank\ A = dim\ \mathcal{R} = r$ ■