Mathematical Analysis Homework 5: Solutions

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1. Let $\{a_n\}_{n=1}^{\infty} \subseteq [0, \infty)$ and assume $\lim_{n\to\infty} a_n = a, a \subseteq \mathbb{R}$. Prove that $a \geq 0$.
   Proof:
   $a_n \geq 0 \implies a - a_n \leq a \implies a_n - a \geq -a$
   Suppose $a < 0 \implies -a > 0 \implies a_n - a > 0$
   Thus $|a_n - a| \geq |a|$
   But since $|a| > 0$, $\lim_{n\to\infty} a_n \neq a$, a contradiction.
   So $a \geq 0$. ■

2. Assume $\lim_{n\to\infty} b_n = b$ exists and $b \neq 0$. Prove that $b_n \neq 0$ for $n \geq N$ for some $N \in \mathbb{N}$.
   Proof:
   We give a proof by contrapositive:
   If $b_n = 0$ for some $n \geq N$ for all $N \in \mathbb{N}$
   Then $|b_n - b| = |b|$ for some $n > N$, $\forall N \in \mathbb{N}$
   Either $b = 0$ or
   If $|b| > 0$ we have $\lim_{n\to\infty} b_n \neq b$ which is the negation of the hypothesis.
   ■