Chapter 1

Problem Statement and Previous Work

1.1 Adversarial Examples

Adversarial examples were originally defined in the domain of image classification in the form of a constrained optimization problem. The following definition is similar to [1]. We take images to be vectors in $\mathbb{R}$, and S is a finite set of classes, so that a classifier $f : \mathbb{R} \to S$ assigns each image a unique class.

Definition. An adversarial derivation, $x + r \in \mathbb{R}^m$, of an image, $x \in \mathbb{R}^m$ for a classifier, f, is a solution to the following optimization problem:

minimize $||r||_2$ subject to:

1. $f(x + r) \neq f(x)$
2. $x + r \in [0, 1]^m$

We may also denote the adversarial derivation as $x^* = x + r$
In words, an adversarial derivation is a sample with the minimum distance to a true sample such that it is classified as something different. We also require that every element stays in the interval $x_n + r_n \in [0, 1]$ because actual pixels must remain in some constant bounded interval.

Definition. A classifier, f, over a set, D, is a mapping, $f : D \to \{1, 2, \ldots, K\} = C$.

Clearly, this adversarial derivation definition does not translate well to the problem of natural language classification where the domain is not even numeric. We give a more general constrained optimization defintion below, which we will adapt to the problem of creating adversarial derivations for text.

Definition. A distance function, d, over a set S is a mapping, $d : S \times S \to \mathbb{R}$ such that $\forall x, y, z \in S$:

1. $d(x, y) \geq 0$
2. $d(x, y) = 0 \iff x = y$
3. $d(x, y) \leq d(x, z) + d(z, y)$

One important example of a distance metric is the discrete metric, given by

$$\rho(v, v^*) = \begin{cases} 0 & \text{if } v = v^* \\ 1 & \text{if } v \neq v^* \end{cases}$$

It is also true that if S is a normed linear space, $d(x, y) = ||x - y||$ is a distance metric over that space.

Definition. Let d_D be a distance function over D, and d_C be a distance function over C. Then the point x^* is an adversarial derivation (AD) of $x \in D$ with tolerance $\epsilon > 0$ given by

$$\begin{aligned} \min_{x^* \in D(f)} \quad & d_D(x, x^*) \\ \text{subject to} \quad & d_C(f(x), f(x^*)) > \epsilon \end{aligned}$$

Note that image classifier would have the domain $[0, 1]^m$ and codomain $\{1, ...K\}$ where K is the number of classes. So the definition of an adversarial derivation for an image classifier is the same as the general definition if we let $\epsilon = 1$, $d_D(x, x^*) = ||x - x^*||^2$, $d_C(y, y^*) = |y - y^*|$.

As long as the function, f, has a non-singleton codomain, the constraint is satisfiable for for small enough ϵ. In the case that the solution does not exist, $\delta = \inf\{d_D(x, x^*) \text{ s.t. } d_C(f(x), f(x^*)) > \epsilon\}$ exists, in which case we may find $\hat{x}$ that is arbitrarily close to δ. Satisfiability is true from the fact that for any distance metric, $d(y, y^*) > 0$ for $y \neq y^*$. That all being said, the distance between the sample and the derived sample may be so large that they are easily recognized to be different. We therefore define a similar problem with very different properties

Definition. An absolutely adversarial derivation (AAD) of x with similarity δ, difference ϵ, domain metric d_D, and codomain metric d_C is given by any solution to the following two constraints.

$$d_D(x, x^*) < \delta$$

$$d_C(f(x), f(x^*)) > \epsilon$$

In this less relaxed version of the problem, a solution may not exist. In fact, for a given continuous function, f, defined on an open domain, and tolerance, ϵ it is guaranteed that

$$\exists \delta > 0 \text{ s.t. } d_C(f(x), f^*(x)) < \epsilon$$

meaning no solution exists. However, it appeals to a more intuitive concept and allows for the possibility of a model immune to adversarial attacks. It says that the sample and its absolute adversarial derivation must be sufficiently close and the distance between the model outputs must be sufficiently far.

It is easy to see that if an AAD exists for a given sample, then any AD is also an AAD. This means we may solve the more relaxed problem to obtain a valid solution, and so we will work with the more practical objective of creating ADs.

1.2 Word Embeddings

Consider the common scenario of a text classifier which maps plain text files to one of several classes. It is common for the plain text to first be processed into a sequence of tokens, which are then each assigned an integer resulting in a sequence of integers.

Definition. Let s be a sequence of characters. Let $a_n \in \{0, 1, \ldots, V\} \forall n \in \{0, 1, \ldots, N\}$ and $E : s \to \{a_n\}_{n=1}^{N_s}$ then we call E an encoder, V the encoder vocabulary size, and N_s the sample length with respect to E.

In plain words, an encoder maps a string to a sequence of bounded integers. The sequence is some length which depends on both the encoder and the string. We assume a fixed encoder, and therefore vocabulary size, V. Since, after encoding, the distance between one word and another is arbitrary, we further translate into a one-hot encoded vector. That is, the integer n is mapped to a vector where the n^{th} element is 1 and all others are 0. This ensures that all vectors representing words are unit norm and the distance between any two different words is the same. We denote the set of one-hot encoded vectors of size V as 1_V

This simple method of representing words as vectors results in a very high dimension representation of all words in the vocabulary, and thus even a very simple linear model would be very large and be difficult train. Using the word2vec model [2], the dimension of this representation can be significantly reduced, while also encoding information about statistical semantic similarity about each word.

Definition. Let $f : 1_V \to \mathbb{R}^D$. We call f a word embedding and we call D the size of f, or embedding size.

Let $W \in M_{D\times V}(\mathbb{R})$. Then clearly any word embedding, f, of size D may be represented as the matrix multiplication $Wv \; \forall v \in 1_V$. The matrix W is called the embedding matrix.

This numerical representation of words is extremely useful since we can now apply more general and modern techniques to solving the problem of classification. We used gradient descent to train a word2vec model with noise contrastive estimation. The hyperparameters we chose are shown in table 1.1

Learning rate	1.0
Batch size	128
Number of Batches	100,000
Embedding size	128
Number of skip windows	8
Size of skip windows	4
Vocabulary size	10,000

Table 1.1: Word embedding hyperparameters

Our corpus was the concatenation of all preprocessed training samples from the training set in the “Large Movie Review Datasets.” [3] Preprocessing consisted of the steps laid out in table 1.2

Start	The movie isn’t {<br \> }good, I give it a 1
Convert to lower case	the movie isn’t {<br \> }good, i give it a 1
Remove HTML	the movie isn’t good, i give it a 1
Expand contractions	the movie is not good, i give it a 1
Remove punctuation	the movie is not good i give it a 1
Expand numbers	the movie is not good i give it a one
Remove extra whitespace	the movie is not good i give it a one

Table 1.2: Preprocessing algorithm

The processing resulted in a corpus of 5,887,178 words totaling 31,985,233 characters. Since there are 25,000 training samples, each review is on average about 234 words, and 1279 characters. Of all 25,000 reviews training reviews, 27 had more than 1024 words. The word embedding converged to an average noise contrastive loss of about 5.07. Semantic difference between two words is measured by the angular distance between their embeddings, that is,

$$\frac{cos^{-1}\left(\frac{v^T u}{||v||_2||u||_2}\right)}{\pi}$$

The eight nearest neighbors for a few common words are shown in table 1.3. We can see that the first few nearest neighbors are fairly high quality, and would usually make grammatical sense for replacement in a sentence. The quality of replacement falls off quickly after that however.

all	but	some	and	UNK	just	also	that	so
and	UNK	with	but	also	which	simpler	nerd	just
will	can	would	if	do	could	to	did	you
of	in	from	with	for	UNK	about	which	that
her	his	she	him	he	their	UNK	the	india
she	he	her	him	his	who	UNK	it	that
most	all	best	films	which	other	UNK	some	only
one	three	two	zero	five	only	nine	s	UNK
movie	film	show	story	it	but	really	that	just
film	movie	story	show	which	UNK	it	that	but

Table 1.3: Some examples of nearest neighbors in embedding space

1.3 Adversarial Text Derivation

Now that we have a clear idea of both the domain and numerical representation of words, we may define an adversarial derivation of textual data in the context of a classification model, f. As per the definition of an adversarial derivation, we need only to define the model tolerance, ϵ, as well as the domain metric, d_D and codomain metric, d_C. We will consider primarily two definitions.

Definition. Let $\{v_i\}_{i=1}^N$ be the sequence of vectors obtained from a given word embedding and text sample, then a discrete adversrial derivation is defined has having domain metric, $d_D(v, v^*) = \sum_i^N \rho(v_i, v_i^*)$, codomain metric $d_C(f(v), f(v^*)) = \rho(f(v), f(v^*))$, and tolerance $\epsilon = 1$.

That is, a discrete adversarial derivation $\{v_i^*\}$ of sample $\{v_i\}$ is the sample which changes the least amount of words possible, while changing the classification. This definition is simple, though it may not yield very good results if solved. For example, a positive movie review, "This movie was good" could easily be changed to a negative review by changing just one word resulting in "This movie was bad". These two samples would obviously have different sentiments if read by a human.

Clearly the codomain metric, d_C and difference, ϵ make sense for any defintion in this context, but the domain metric has room for improvement which brings us to the next definition.

Definition. A semantic adversarial derivation is defined as having angular distance as the domain metric and the same difference and codomain metric as a discrete adversarial derivation.

If we minimize this objective, then we would tend to use semantically similar words in substitution. However, this does not necessarily solve the problem of actual sentiment inversion. For instance, in our embedding the semantically closest (measured with the l^2 norm) word to "bad" is "good." This makes sense since they are semantically very similar and would be used in the same contexts, but may result in obvious semantic flips. We will therefore look to other metrics in an attempt to find better results.

Here we will focus on one particular type of model which has proven very effective in text classification and prediction, a recurrent neural network utilizing long short-term memory (LSTM) units. Our base model has one layer of LSTM units where the output of each unit is averaged over time followed by a fully connected layer with two outputs, corresponding to the logits of a positive and negative class. The network's confidence of a given samples class is given by the softmax of both logits.

The model was trained for 50 epochs over the entire training set with a batch size of 1000 and a maximum unfolding length of 1024, meaning that 27 reviews would be clipped. The Adam optimizer with exponential step decay factor of 0.8 every 500 batches was used to minimize the model loss, softmax cross entropy. We used peepholes as well as output dropout for training.

We trained fourteen models, varying the number of hidden units and initial learning rates. The final training and testing accuracies are shown in tables 1.4 and 1.5 respectively. As expected, increasing the model size always increased the training accuracy, as did increasing the learning rate. The testing accuracy was

Number of Hidden Units, Learning Rate = 0.01						
2	4	8	16	32	64	128
0.778	0.784	0.812	0.850	0.878	0.904	0.961
Number of Hidden Units, Learning Rate = 0.1						
2	4	8	16	32	64	128
0.827	0.853	0.895	0.965	0.975	0.997	0.990

Table 1.4: Training Accuracy

Number of Hidden Units, Learning Rate = 0.01						
2	4	8	16	32	64	128
0.753	0.755	0.771	0.809	0.816	0.829	0.834
Number of Hidden Units, Learning Rate = 0.1						
2	4	8	16	32	64	128
0.793	0.816	0.827	0.822	0.815	0.809	0.817

Table 1.5: Testing Accuracy

less predictable. Most accuracies were in the neighborhood of 80%, 30% greater than the baseline of 50% random guessing.

1.4 Stochastic Gradient Analysis

Definition. Let the gradient of a model output, f_i, with respect to an input vector, x be denoted by $g = \nabla f(x)$. The total gradient is given by

$$g_t = \sum_{i=1}^{D} \nabla f(x)_i \tag{1.1}$$

The gradient norm is given by

$$g_n = \sqrt{\sum_{i=1}^{D} |\nabla f(x)_i|^2} = ||\nabla f(x)||_2 \tag{1.2}$$

Both of these measures, along with the gradient itself, are shown for our model as well as a small excerpt of one of the text samples in Figure 1.3. We see that the three words with the largest total gradients are “loved”, “good”, and “bad” which are all sentimental. First, a differentiable function, f, can be locally approximated by

$$f(w_j) \approx f(x_i) + (w_j - x_i)^T \nabla f(x_i) \tag{1.3}$$
$$= f(x_i) + w_j^T \nabla f(x_i) - x_i \nabla f(x_i) \tag{1.4}$$
$$= f(x_i) + w_j^T g - x_i^T g \tag{1.5}$$

Here are considering specificially the affect of replacing the i^{th} input word vector, x_i with the j^{th} embedding word vector, w_j while all other input vectors remain constant.

Definition. With the above defintions of x_i, w_j, and $\nabla f(x_i)$, let

$$X = \Big[x_1, x_2, \ldots, x_N\Big] \in M_{D\times N}(\mathbb{R}) \tag{1.6}$$
$$W = \Big[w_1, w_2, \ldots, w_V\Big] \in M_{D\times V}(\mathbb{R}) \tag{1.7}$$
$$G = \Big[\nabla f(x_1), \nabla f(x_2), \ldots, \nabla f(x_N)\Big] \in M_{D\times N}(\mathbb{R}) \tag{1.8}$$

The approximated perturbation, $f(w_j) - f(x_i)$, of replacing the i^{th} input with the j^{th} embedding word vector is given by

$$D_{i,j} = (G^T W)_{i,j} - (G^T X)_{i,i} \tag{1.9}$$

D is called the approximate delta matrix.

For a perfectly linear function, f, $D_{i,j} = f(w_j) - f(x_i)$. We found that for small output differences, the approximation worked fairly well in that a lesser approximate delta tended to produce a lesser actual deta, as seen in figure **??**. However, the approximation failed almost completely for large differences, as illustrated in figure 1.2. This is not surprising given that the model is highly non-linear but it confirms that using the gradient alone is not a reliable technique for discovering adversarial derivations.

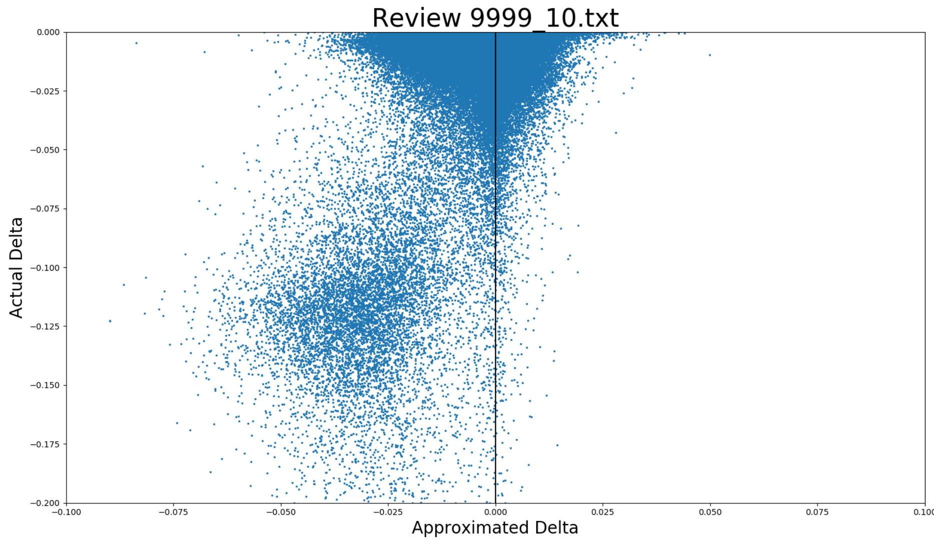
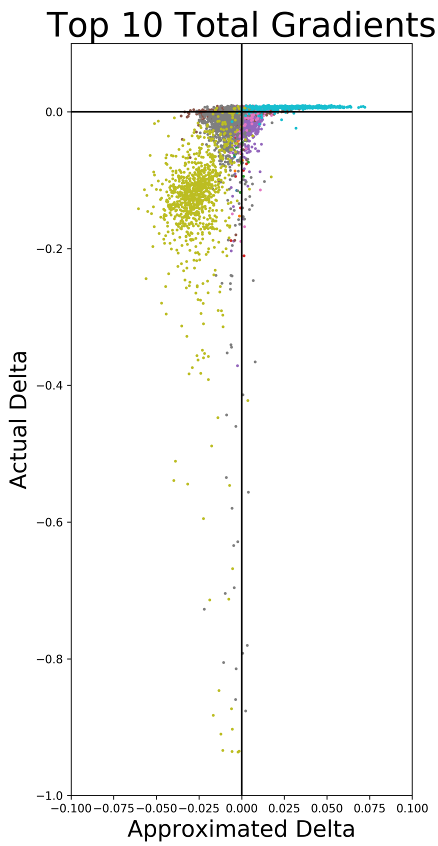
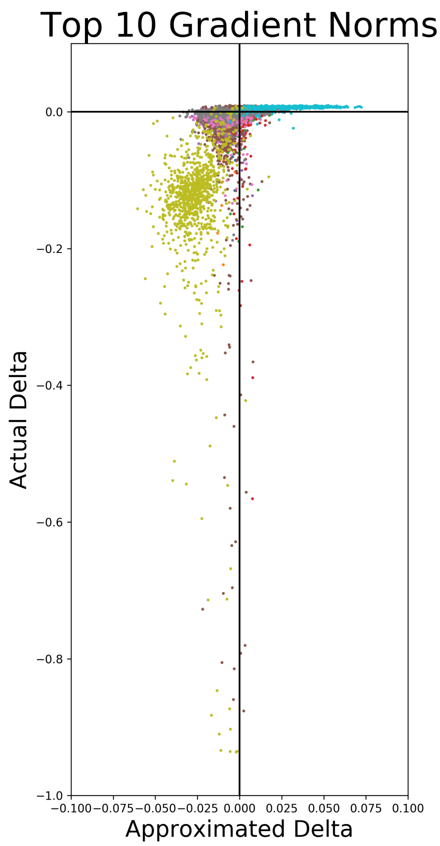


Figure 1.1: Predicted difference in prediction confidence vs. actual difference for sample file 9999_10.txt



(a) The words with the top ten largest absolute total gradeints are chosen



(b) The words with the top ten largest gradient norms are chosen

Figure 1.2: The predicted change in classifier confidence vs. the actual change. Only the most common 1,000 words are considered in replacement for this example.

We see that the approximated delta is not a good choice for determining which specific words to interchange, but that our measures of gradient may be useful in determining which words are susceptible to attack. We give motivation with some simplified probabilistic analysis. Suppose that each of the embedding dimensions is distributed independently and identically across words, with some mean, μ and variance, σ^2. Let $g = \nabla f(u)$ for a given word vector, u and suppose we replace it with a random word vector, v. Recall equation 1.5, we are interested in estimating the term $v^T g$ probabilstically since the term $u \nabla f(u)$ is fixed for a given word.

$$\mathrm{E}\left[g^T v\right] = \mathrm{E}\left[\sum_{i=1}^{D} g_i v_i\right] = \sum_{i=1}^{D} g_i \, \mathrm{E}\left[v_i\right] = \mu \sum_{i=1}^{D} g_i = \mu g_t$$

If we are interested in purposefully altering classification, however, we might be more interested in the expected maximum value of $g^T v$, that is,

$$Z(g) = \mathrm{E}\left[\max_{0 \leq n \leq V} g^T v_n\right]$$

Unfortunately, there is no simple expression which captures this value. However if we assume that $v_{n,i}$ is distributed normally, we have

$$\mathrm{E}\left[\max_{0 \leq n \leq V} g^T v_n\right] = \mathrm{E}\left[\max_{0 \leq n \leq V} \sum_{i=1}^{D} g_i v_{n,i}\right] = \mathrm{E}\left[\max_{0 \leq n \leq V} p_n\right]$$

where $p_n \sim \mathcal{N}(\mu g_t, \sigma^2 g_n^2)$ There is still no closed form expression for this value, but there is a known upper bound:

$$Z(g) \leq \mu g_t + \sigma g_n \sqrt{2 \log V}$$

This inequality is intuitively satisfying. It says that the expected maximum perturbation grows with both the vocabulary size and the norm of the gradient. The total gradient also plays a role here, increasing or decreasing the expected maximum depending on the sign. Empirical study of our embedding and classifier show that the term including standard deviation is usually much larger. It should

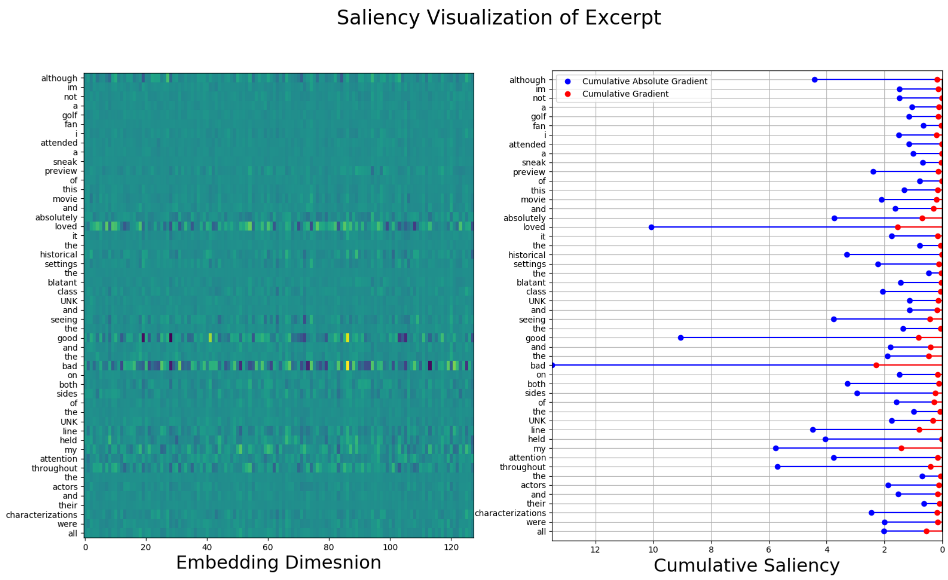


Figure 1.3: Different measures of word sentiment/importance

be noted that the lower bound on the expected minimum, $Y(g)$, is simply given by a sign reversal of the second term:

$$Y(g) \leq \mu g_t - \sigma g_n \sqrt{2 \log V}$$

In our word embedding, the average value of μ was -0.017, but there was significant variation across embedding dimension. The average standard deviation was fairly constant over embedding dimension, with a value of 0.55. Since our vocabulary size is 10,000 we have the empircal formula for the upper bound on expected maximum perturbation:

$$-0.017 g_t + 0.55 g_n \sqrt{2 \log(10,000)} = -0.017 g_t + 2.36 g_n$$

Bibliography

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