Homework X: Gravity and Cosmology

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Set $c = 1$ again in this assignment.

Problem 1 Redshift vs. doppler shift

When a source emitting light is moving away from us, the observed light in our reference frame has a shifted wavelength (similar to the doppler effect for the sound of an ambulance). The redshift due to the relativistic doppler effect is given by

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \tag{1}$$

where β is the speed of the source. Compare this speed to the recession velocity predicted by the Hubble formula for the expansion redshift at $z = 7$.

Numerically check that the two formulas give very similar results for small redshifts.

Actually, neither of these is a satisfying definition of the recession velocity. The most sensible definition of velocity in the FRW metric is somewhere between the two, see Figure 1.

Problem 2 The Cosmological Principle and Lorentz invariance

To derive the FRW metric, we assume that the universe is homogeneous (the same everywhere) and isotropic (all directions are equivalent). Is this a Lorentz invariant property?

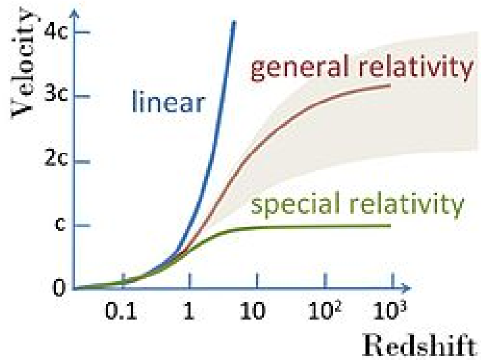

Figure 1: Velocity vs. redshift for different models. The GR result is model dependent, hence the band of color. From Wikipedia.

Problem 3 Cosmic Neutrino Background (CνB)

Long before the photons decoupled from the charged leptons and baryons, the neutrinos also fell out of equilibrium. The density of the CνB is about 334/cm^3. Sometime later, the electrons annihilate all the positrons in $e^-e^+ \to 2\gamma$ reactions and reheat the photons somewhat without effecting the neutrinos. A fairly simple thermodynamic calculation shows that the temperature ratio is:

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3} \tag{2}$$

Compare the typical neutrino energy $E \sim k_B T$ to the neutrino mass (less than 1 eV). Are the cosmic background neutrinos relativistic? Were they when they decoupled ($T \sim 10^{10} K$)?

Since neutrinos are massive, there must be some reference frame in which the center of mass for the CνB is at rest. What are the implications for Einstein's principal of relativity?

Problem 4 Timeline of the Big Bang

Make a timeline of the various stages of the big bang. Give energies, temperatures, and times for each of the major processes. If you like, you may use the internet to fill in more detail than was discussed in class.

Problem 5 Expansion and Vacuum Energy

Recall that time translation invariance in the laws of physics led to the conservation of energy. One explanation for dark energy is that empty space has an inherent energy, called the *vacuum energy*. Roughly speaking, this is a constant energy density in space. Does this break energy conservation? Can you think of a reason that might be OK?

Problem 6 Reading

Read chapters 18-20 of Einstein and the article on the website by Lineweaver and Davis, "Misconceptions About the Big Bang."