Homework V - Problem 1 Solution

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Problem 1 EPRB Experiment

Derive the formulas

$$P(\uparrow_A, \uparrow_B; \alpha, \beta) = P(\downarrow_A, \downarrow_B; \alpha, \beta) = \frac{1}{2} \sin^2 \frac{\alpha - \beta}{2} \tag{1}$$

$$P(\uparrow_A, \downarrow_B; \alpha, \beta) = P(\downarrow_A, \uparrow_B; \alpha, \beta) = \frac{1}{2} \cos^2 \frac{\alpha - \beta}{2} \tag{2}$$

describing the outcomes of the EPRB experiment.

We only need to do one calculation. The equalities $P(\uparrow_A, \uparrow_B; \alpha, \beta) = P(\downarrow_A, \downarrow_B; \alpha, \beta)$ and $P(\uparrow_A, \downarrow_B; \alpha, \beta) = P(\downarrow_A, \uparrow_B; \alpha, \beta)$ follow from rotational invariance (the meaning of up and down is arbitrary). Since these are the only possibilities, they must add up to one.

I will show the first relation, $P(\uparrow_A, \uparrow_B; \alpha, \beta) = \frac{1}{2} \sin^2 \frac{\alpha-\beta}{2}$. It's very important to understand what we are calculating. By definition, $P(\uparrow_A, \uparrow_B; \alpha, \beta)$ is the probability of measuring particle A to be up with an SG device oriented in the α direction, then measuring particle B as down along the β direction with another SG apparatus.

The matrix corresponding to SG measurement in the direction $\hat{v} = \sin\alpha \hat{x} + \cos\alpha \hat{z}$ is (see Friday's notes if you don't know why)

$$S_\alpha = \frac{\hbar}{2} \begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{pmatrix} \tag{3}$$

You can verify that an orthonormal choice for the eigenvectors of this matrix are

$$|\uparrow_\alpha\rangle = \begin{pmatrix} \cos\frac{\alpha}{2} \\ \sin\frac{\alpha}{2} \end{pmatrix} \qquad |\downarrow_\alpha\rangle = \begin{pmatrix} \sin\frac{\alpha}{2} \\ -\cos\frac{\alpha}{2} \end{pmatrix} \tag{4}$$

You should now use symmetry to note that $P(\uparrow_A, \uparrow_B; \alpha, \beta) = P(\uparrow_A, \uparrow_B; 0, \beta - \alpha)$. If you are reading and thinking carefully, you might wonder why I'm not concerned that the initial state was defined in the SG$_z$ basis. Since this pair of particles is assumed to be from the decay of a spin zero particle, the sum of the spins must be zero along any axis we choose. Let's talk sometime if you don't find that convincing. You could also do the calculation explicitly for $P(\uparrow_A, \uparrow_B; \alpha, \beta)$.

From the fourth postulate,

$$P(\uparrow_A, \uparrow_B; 0, \beta-\alpha) = \left| \langle \uparrow_z \downarrow_{\beta-\alpha} | \frac{1}{\sqrt{2}} \left(|\uparrow_z \downarrow_z\rangle - |\downarrow_z \uparrow_z\rangle \right) \right|^2 \tag{5}$$

Multiplying out the the parts corresponding to particle A, we are left with (this is just for particle B)

$$P(\uparrow_A, \uparrow_B; 0, \beta-\alpha) = \frac{1}{2} \left| \langle \downarrow_{\beta-\alpha} \mid \downarrow_z \rangle \right|^2 \tag{6}$$

The inner product is found using (**??**) but substituting $\alpha \to \beta - \alpha$,

$$P(\uparrow_A, \uparrow_B; 0, \beta-\alpha) = \frac{1}{2} \cos^2 \frac{\beta-\alpha}{2} \tag{7}$$

This is the desired result. The other three probabilities follow from symmetry as discussed above.