# Homework IV: More Quantum Mechanics

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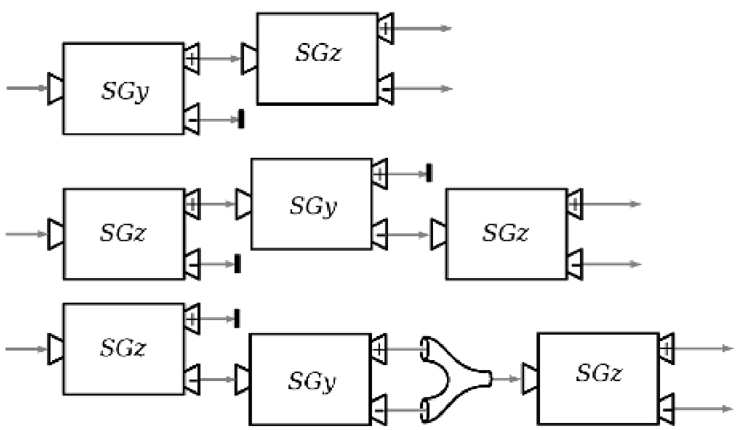
## Problem 1 Looking at the Stern Gerlach experiment

Suppose that an $\mathrm{SG_z}$ device measures the observable related to $\sigma_z$ and that an $\mathrm{SG_y}$ does the same for $\sigma_y$. For each of the sequences of SG machines below, indicate the probability that an unpolarized particle ($|\psi\rangle = (|+\rangle + |-\rangle)/\sqrt{2}$) entering the first machine will be measured in the $|+\rangle$ state by the final $\mathrm{SG_z}$ detector. (That is, what is the probability that the electron comes out the + output of the final machine?) Explain briefly how you know this to be the case.




This problem is adapted from Rob Knop's course at Quest University.

## Problem 2 The Schwarz Inequality

In class we used the relation $|\langle\alpha|\beta\rangle|^2 \leq \langle\alpha|\alpha\rangle\langle\beta|\beta\rangle$. This is called the Schwarz inequality. First, show that it is true in the vector space of arrows we discussed on the second day, i.e. that

$$(\vec{U}\cdot\vec{V})^2 \leq |U|^2|V|^2 \tag{1}$$

Now show that it is true in the vector space of quantum mechanics. Hint: Let $|\gamma\rangle = |\beta\rangle - \frac{\langle\alpha|\beta\rangle}{\langle\alpha|\alpha\rangle}|\alpha\rangle$ and use the requirement from the definition of the inner product that $\langle\gamma|\gamma\rangle \geq 0$.

## Problem 3 Uncertainty Relations for Spin 1 Particles

The two state systems we have focused on represent a quantum degree of freedom called spin, and correspond to a description of particles with a spin of $\frac{1}{2}$. Particles with spin of 1 are a three dimensional system, and the analogues of the $\sigma$ matrices from yesterday are

$$J_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad J_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad J_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Using the uncertainty relation from class, show that $\Delta J_x \Delta J_y \geq \frac{\hbar}{2} \langle J_z \rangle$.

## Problem 4 Origin of spin

Originally, physicists attempted to explain the spin angular momentum we have been talking about using classical ideas. They tried to think of the particles as tiny spinning spheres. Let's see if this could be possible. Newtonian mechanics tells us that $|\vec{L}| = I|\omega|$. For a homogeneous sphere, the inertia is given by $I = 2/5mr^2$. The electron has a mass of $m = 9.1 \times 10^{-31}$kg. The letter $\omega$ (omega) represents the angular velocity. To understand this, we just note that if a sphere has angular velocity $\omega$ then a point a distance $r$ from the axis of rotation is moving at a speed of $v = \omega r$.

Is this model reasonable for the electron? Hint: Nothing can move faster than $c = 3 \times 10^8$m/s.

## Problem 5 The famous uncertainty principle

At some point, you may have heard that "quantum mechanics says you can't measure a particle's position and its speed at the same time." This is the famous Heisenberg uncertainty principle. Unfortunately position and momentum (speed isn't a quantum observable per se, but $\vec{p} = m\vec{v}$ so it is related to momentum) are a very difficult basis to work with because they can't be represented as matrices. You can still understand the meat of this statement though, since now you know what an uncertainty relation is. The relation

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \tag{2}$$

is called the *Heisenberg uncertainty principle.* Discuss what this means.

## Problem 6 Two-Particle States and Statistics

Suppose we have two particles, each of whose state may be either $|+\rangle$ or $|-\rangle$. Ignoring spin degrees of freedom, the two particle states are of the form

$$|\psi\rangle = a\,|{+}{+}\rangle + b\,|{-}{-}\rangle + c\,|{+}{-}\rangle + d\,|{-}{+}\rangle \tag{3}$$

Find the constraints on the complex numbers $a$, $b$, $c$ and $d$ for each of the following cases:

a) The particles are distinguishable.

b) The particles are fermions.

c) The particles are bosons.

Optional: Find the analogue of (3) for three particle states. Again find the constraints on the coefficients for the case of each kind of statistic.

## Problem 7 Measurement of Two-Particle States

For this problem you should use the column vector notation

$$|\uparrow\uparrow\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad |\uparrow\downarrow\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \qquad |\downarrow\uparrow\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \qquad |\downarrow\downarrow\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

- What is the matrix corresponding to SG measurement along the y-axis of just particle two?
- What are the possible outcomes of a measurement of SGz on the first particle for the state
$$|\psi\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$$
- If I perform the measurement in the previous part and find the first particle in $|\uparrow\rangle$, what are the possible outcomes if I immediately measure SGz on the second particle?

## Problem 8 EPR, Entanglement and the Bell Inequalities

There are three articles available for you to read, all of which are related to the experimental evidence that quantum mechanics is a complete description of reality. The first is by Kwiat and Hardy and uses cakes but not inequalities. The second is by Mermin, if you only have time to figure out one article I suggest this one. Bell's article is my favorite, but it's also the longest and most difficult. We will discuss all these concepts on Monday, but I strongly encourage you to struggle with the readings and think about this over the weekend! If you just get it and don't have to struggle, well, you're better at this than I am.