Homework XII: Gravity and Cosmology

Cory Schillaci

Updated July 30, 2013

Set $c = 1$ again in this assignment.

Problem 1 Redshift vs. doppler shift

When a source emitting light is moving away from us, the observed light in our reference frame has a shifted wavelength (similar to the doppler effect for the sound of an ambulance). The redshift due to the relativistic doppler effect is given by

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \tag{1}$$

where β is the speed of the source. Compare this speed to the recession velocity predicted by the Hubble formula for the expansion redshift at $z = 7$.

Numerically check that the two formulas give very similar results for small redshifts.

Actually, neither of these is a satisfying definition of the recession velocity. The most sensible definition of velocity in the FRW metric is somewhere between the two, see Figure 1.

Problem 2 The Cosmological Principle and Lorentz invariance

To derive the FRW metric, we assume that the universe is homogeneous (the same everywhere) and isotropic (all directions are equivalent). Is this a Lorentz invariant property?

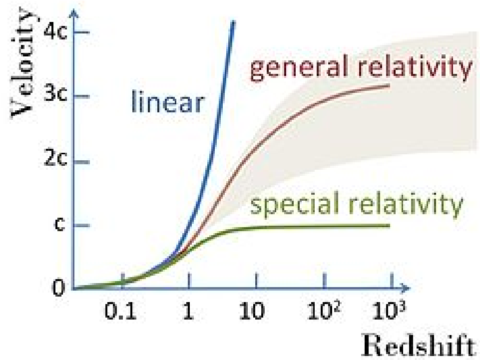

Figure 1: Velocity vs. redshift for different models. The GR result is model dependent, hence the band of color. From Wikipedia.

Problem 3 Looking back in time

Explain why it makes sense to say that cosmic microwave background radiation allows us to look at what happened at a very early time in the universe. When and where did the photons being observed today last interact?

Problem 4 An extra neutrino

At temperatures near 10^{10} K, the relationship between the age and temperature of the universe is given by

$$t = \sqrt{\frac{3}{16\pi G\mathcal{N}a_B}}\frac{1}{T^2} + \text{constant} \tag{2}$$

where a_B is a combination of fundamental constants called the *Stefan-Boltzmann constant* and $\mathcal{N}$ is the number of relativistic particles, counting particles and antiparticles and each spin state separately. Fermions get a funny factor of $\frac{7}{8}$. For example, if we have

- Photons with two spin states.
- Three species each of neutrinos and antineutrinos, but no right handed neutrinos.
- Electrons and positrons with two spin states each.

then $\mathcal{N} = 2 + \frac{7}{8}(6+4) = \frac{43}{4}$.

Suppose there was a fourth neutrino. Qualitatively, how would this effect the rate of cooling? Does this effect the ratio of neutrons to protons? Remember that neutrons are unstable to β-decay until they form nuclei.

Problem 5 Cosmic Neutrino Background (CνB)

Long before the photons decoupled from the charged leptons and baryons, the neutrinos also fell out of equilibrium. The density of the CνB is about 334/cm^3. Sometime later, the electrons annihilate all the positrons in $e^-e^+ \to 2\gamma$ reactions and reheat the photons somewhat without effecting the neutrinos. A fairly simple thermodynamic calculation shows that the temperature ratio is:

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3} \tag{3}$$

Compare the typical neutrino energy $E \sim k_B T$ to the neutrino mass (less than 1 eV). Are the cosmic background neutrinos relativistic? Were they when they decoupled ($T \sim 10^{10} K$)?

Since neutrinos are massive, there must be some reference frame in which the center of mass for the CνB is at rest. In light of Problem 2, does this make sense?

Problem 6 Expansion and Vacuum Energy

Recall that time translation invariance in the laws of physics led to the conservation of energy. One explanation for dark energy is that empty space has an inherent energy, called the *vacuum energy*. Roughly speaking, this is a constant energy density in space. Does this break energy conservation? Can you think of a reason that might be OK?

Problem 7 Reading

These are listed roughly in order of importance, but ultimately you are reading for your own learning so it's up to you

- The article on the website by Lineweaver and Davis, "Misconceptions About the Big Bang," explains some common misconceptions about the expanding universe.
- For some basic ideas about the inspiration for general relativity, read chapters 18-20 of Einstein.
- Oerter pages 203-218 are about the Standard Model, and pages 219-239 discuss about BSM physics.