

Homework VIII

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If you like, you may set $c = 1$ in this assignment.

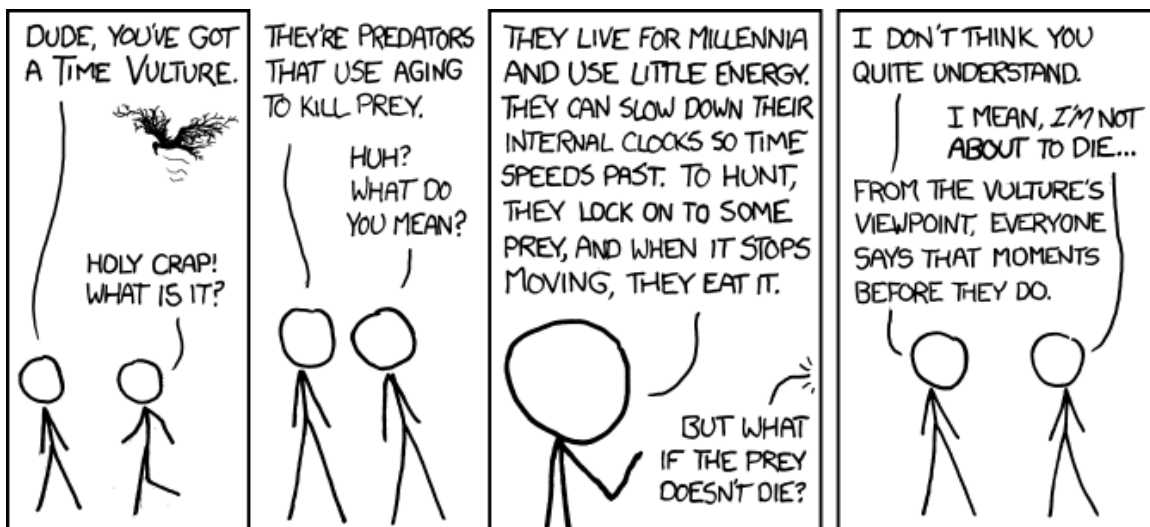
Problem 1 Muon lifetime

Consider the muon, which has a lifetime of $\tau = 2.20 \mu\text{s}$ and a mass of $m = 106 \text{ MeV}/c^2$. Supposing that time dilation did not exist, what would be the maximum typical distance that a muon could travel before decaying? Now, considering the predictions of special relativity, how far does a typical cosmic ray muon of energy $E = 10^{10} \text{ MeV}$ travel? The decay lifetimes of these ultrarelativistic particles provide experimental evidence for time dilation.

Hint: The energy of the muons is given roughly by $E \approx \gamma mc^2$.

Problem 2 Time vultures

Does this xkcd comic make sense based on what you know about special relativity?



Problem 3 Clairvoyance

This problem is taken from *Introduction to Electrodynamics* by David J. Griffiths. The whimsy is all his.

Sophie Zabar, clairvoyante, cried out in pain at precisely the instant her twin brother, 500km away, hit his thumb with a hammer. A skeptical scientist observed both events (brother's accident, Sophie's cry) from an airplane traveling at $\beta = 12/13$ [toward Sophie from her brother's location]. Which event occurred first, according to the scientist? How *much* earlier was it, in seconds?

Problem 4 Lorentz invariant magnitude

Show that the Lorentz transformation

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

Leaves the magnitude $|x^\mu|^2 = -c^2t^2 + |\vec{x}|^2$ unchanged.

Problem 5 Spacetime interval

The spacetime interval between an event A, occurring at $x_A^\mu = (x_A^0, x_A^1, x_A^2, x_A^3)$, and event B at $x_B^\mu = (x_B^0, x_B^1, x_B^2, x_B^3)$ is given by

$$I \equiv |\Delta x^\mu|^2 \quad (2)$$

where

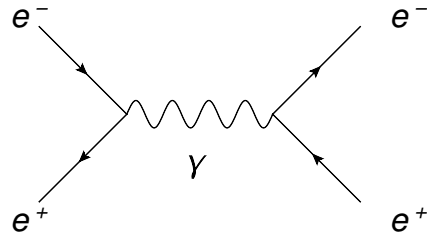
$$\Delta x^\mu \equiv x_A^\mu - x_B^\mu. \quad (3)$$

If $I < 0$, the interval is called *time-like*, if $I = 0$ the interval is called *light-like*, and when $I > 0$ the interval is called *space-like*. Interpret the difference, especially considering the possibility that events A and B influence one another. You might want to start by determining the interval between the two events in Problem 3.

Problem 6 Virtual particles

We learned in class that $\Delta E \Delta t \geq \hbar/2$. This means that particles with finite lifetime have an uncertainty in their mass, and that the virtual particles may not satisfy $|p^\mu|^2 = -m^2c^2$. A particle that does not satisfy the momentum relation is called *off*

shell (short for on mass shell), while one that does is called *on shell*. Show that the invariant mass squared of the photon is nonzero in the following diagram:



Problem 7 Readings from *Relativity: The Special and General Theory*

This is a very nice accessible discussion on relativity from Einstein himself. Read chapters 13, 15 and 17. I recommend that you read the whole book at some point, but it doesn't have to be today.

Problem 8 Readings from *The Theory of Almost Everything*

Read the introduction (pp. 1-12), Chapter 2 (pp. 29-40 only, although the story about Emmy Noether at the beginning of the next section is worth checking out), and Chapter 5.