three matrices, and to define σ_0 as the 2×2 identity matrix. Most often, however, we use the notations I, X, Y and Z for $\sigma_0, \sigma_1, \sigma_2$ and σ_3, respectively.

Information theory and probability

As befits good information theorists, logarithms are *always* taken to base two, unless otherwise noted. We use $\log(x)$ to denote logarithms to base 2, and $\ln(x)$ on those rare occasions when we wish to take a natural logarithm. The term *probability distribution* is used to refer to a finite set of real numbers, p_x, such that $p_x \geq 0$ and $\sum_x p_x = 1$. The *relative entropy* of a positive operator A with respect to a positive operator B is defined by $S(A||B) \equiv \mathrm{tr}(A \log A) - \mathrm{tr}(A \log B)$.

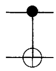
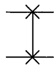
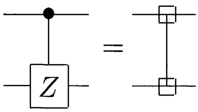
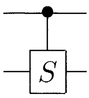
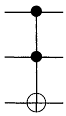
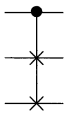
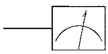
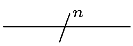
Miscellanea

$\oplus$ denotes modulo two addition. Throughout this book 'z' is pronounced 'zed'.

Frequently used quantum gates and circuit symbols

Certain schematic symbols are often used to denote unitary transforms which are useful in the design of quantum circuits. For the reader's convenience, many of these are gathered together below. The rows and columns of the unitary transforms are labeled from left to right and top to bottom as $00\ldots 0$, $00\ldots 1$ to $11\ldots 1$ with the bottom-most wire being the least significant bit. Note that $e^{i\pi/4}$ is the square root of i, so that the $\pi/8$ gate is the square root of the phase gate, which itself is the square root of the Pauli-Z gate.

Hadamard	$-\boxed{H}-$	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Pauli-X	$-\boxed{X}-$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y	$-\boxed{Y}-$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z	$-\boxed{Z}-$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Phase	$-\boxed{S}-$	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$	$-\boxed{T}-$	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$

controlled-NOT		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
swap		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
controlled-Z		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
controlled-phase		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix}$
Toffoli		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$
Fredkin (controlled-swap)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
measurement		Projection onto $\lvert 0\rangle$ and $\lvert 1\rangle$
qubit		wire carrying a single qubit (time goes left to right)
classical bit		wire carrying a single classical bit
n qubits		wire carrying n qubits