Homework V

Cory Schillaci

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Problem 1   EPRB Experiment

Derive the formulas

$$P(\uparrow_A, \uparrow_B; \alpha, \beta) = P(\downarrow_A, \downarrow_B; \alpha, \beta) = \frac{1}{2} \sin^2 \frac{\alpha - \beta}{2} \tag{1}$$

$$P(\uparrow_A, \downarrow_B; \alpha, \beta) = P(\downarrow_A, \uparrow_B; \alpha, \beta) = \frac{1}{2} \cos^2 \frac{\alpha - \beta}{2} \tag{2}$$

describing the outcomes of the EPRB experiment.

Problem 2   The Greenberger-Horne-Zeilinger (GHZ) experiment

The GHZ experiment is less well known than Bell's inequality but still very interesting.

Suppose that three people are presented with the following game. They may confer initially, but are then separated and brought to distant locations. There, each is asked one of two questions:

1. What is x?

2. What is y?

The players must answer either '+1' or '-1'. They are guaranteed that either all players are asked the first question, or that exactly two are asked the second question. In the former case, they win if the product of their answers is -1. In the latter, the product must be +1. Can this game be played with classical information in such a way that the team always wins? Prove your answer.

There is certainly a quantum mechanical way to win this game. The team prepares the state

$$|\psi\rangle = \frac{|\uparrow_z\rangle_A |\uparrow_z\rangle_B |\uparrow_z\rangle_C - |\downarrow_z\rangle_A |\downarrow_z\rangle_B |\downarrow_z\rangle_C}{\sqrt{2}}$$

Each player then takes one of the particles A, B, or C with them. After being asked the question, they measure with the corresponding SG device and reply with the outcome of their measurement. Verify that this strategy works.

The quantum mechanical strategy has been shown to work experimentally. Does this imply anything about hidden variables?

Problem 3   Entropy in language

Entropy is also meaningful outside of physics. This problem is inspired by xkcd http://xkcd.com/936/, which is worth a read.

Pretend we used a language with only five letters: a, b, c, and d. If a website required you to form a password of four characters with no further restrictions. What is the entropy of a password? Calculate this with two different assumptions:

- Assume first that all characters appear with equal probability.

- Now assume that at least one of the letters is always a, because it's the only vowel.

The strength of a password against attacks by random guesses is characterized by the entropy. If the web admins notice that users always include the letter a, can they make the passwords stronger by forbidding its use?

Problem 4   Deriving quantum mechanics

Read the viewpoint by Brukner at http://physics.aps.org/articles/v4/55 or linked from our class webpage. These are questions that people are asking and thinking about right now!

Problem 5   Can a quantum description of reality be considered complete?

Einstein didn't have the full picture when he answered the title question of this problem in the negative. Now that you know more than Einstein (did you think that was going to happen in this class?), what do you think? While there is a wrong answer, we're not certain which it is yet. My feeling is that quantum mechanics is complete, but there are very intelligent physicists on both sides of the issue.

Problem 6   BECs and Cooling

The JILA website <http://www.colorado.edu/physics/2000/bec/> has an excellent introduction to Bose-Einstein condensation and how innovative techniques helped to finally achieve the incredibly low temperatures necessary to realize them. Read it!