Lecture Notes VII: Elementary Particles

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Quantum mechanics and special relativity were the first two major paradigm shifts of the twentieth century. In the spirit of unification, theoretical physicists soon began to look for a way to combine the two theories.

Experimentalists had begun to discover tiny particles that made up matter as early as 1897, when J.J. Thomson showed that the mysterious "cathode rays" were in fact individual particles (now known as electrons). In 1911 Ernest Rutherford performed his famous α-particle experiment demonstrating that most of an atom was contained in a dense nucleus. He proceeded to break off hydrogen nuclei (which we now call protons) several years later. James Chadwick proved that nuclei also contained uncharged particles (neutrons) in 1932.

1 What is quantum field theory?

Fundamentally, quantum field theory (which we often abbreviate to QFT) is the unification of quantum mechanics and special relativity. The theory of quantum mechanics we discussed at the beginning of the course was invariant under the Galilean transformations, not the Lorentz boosts. Since both special relativity and quantum mechanics were extremely successful at describing a large variety of systems, it's natural to try to unify them into a theory with an even greater scope.

Non-relativistic quantum mechanics, using the formalism of the state vectors in Hilbert space, is also fundamentally incapable of describing processes in which the number or type of particles in the universe changes. This second problem is also a sign that it can't describe a fully relativistic universe, where we know that mass is not conserved (only the four-momentum).

1.1 Fields

We didn't discuss the position basis because of the additional mathematical complexity. Many introductory quantum courses spend a lot of time discussing states in this basis, which are called wavefunctions and represented something like this

$$\psi(x) = \langle x|\psi\rangle \tag{1}$$

Do not interpret this to mean that the physical state is a function of position! The function $\psi(x)$ simply specifies a superposition of position eigenstates. There is a position operator, $\hat{x}$. For technical reasons we can't usually ask for the probability that the particle is at a particular position, but we can ask a similar question: what is the probability that the particle will be found between $x = x_1$ and $x = x_2$[1].

On the other hand, we saw that it's perfectly acceptable to have a quantum state depend on time, which we would write something like $|\psi(t)\rangle$. Time is not an operator, it is a parameter of the theory. Since particles are conserved, asking any sort of question about *when* we could find a particle makes no sense.

The take home message is that non-relativistic quantum mechanics treats time and space differently. That is definitely not going to be consistent with relativity, so something must be done. Either we can promote time to an operator, or we have to demote position to a parameter. The correct prescription turns out to be the latter.

Quantum field theory uses a very different formalism than the non-relativistic theory. As the name implies, objects called fields replace the kets we used before. Although the specifics are far beyond the scope of this course, we can at least get an idea of what fields are and some of the implications of the theory.

As mentioned, the fields are going to be functions of space time. For bosonic fields, we typically use the symbol $\phi(\mathbf{x}, t)$. Fermionic fields are mostly denoted by $\psi(\mathbf{x}, t)$.

A *field* is a collection of things, defined for every value of its argument. The things can vary in nature. A vector field is a collection of vectors, one at every point in space (or spacetime). The "things" that make up quantum fields create particles when they act on the vacuum state. A different amount of probability is made at every point in spacetime, hence the dependence on $\mathbf{x}$ and t. In a quantum theory, the fields obey commutation relations which enforce bosonic or fermionic spin statistics under particle exchange.

1.2 Local symmetries

When I introduced symmetries on the very first day, I implicitly assumed that they would act the same way everywhere in the universe. If I change my coordinate system, I change it in the same way on Earth as on Mars or in a distant galaxy. A new example of this theory is the phase shift invariance. In non-relativistic quantum mechanics, making the substitution

$$|\psi\rangle \to e^{i\theta}\,|\psi\rangle \tag{2}$$

[1]Contrast this with the case of the spin 1/2 system.

everywhere will leave all observable quantities (anything calculated with both a bra and a ket) unchanged. We can do the same thing in quantum field theory with the fields:

$$\phi(\mathbf{x}, t) \rightarrow e^{i\theta}\phi(\mathbf{x}, t) \tag{3}$$

This is a continuous symmetry, so you should be wondering what conservation law it implies. In the Schrödinger theory it implies particle number conservation[2]. In a quantum field theory with no interactions, the conserved quantity is the number of particles minus the number of antiparticles. I am willing to bet that you have heard of antimatter before. But what is antimatter?

Quick Question: What have you heard about antiparticles?

An antiparticle is nothing special. To preserve causality, quantized field theories must contain an antiparticle for each particle[3]. The antiparticle must have exactly has the same mass as the particle but negative charges (i.e. a particle and it's antiparticle together have no net charge of any kind). An electron has an electric charge of -1 while the antielectron (or positron) has charge +1. Historically we ended up calling the kind of stuff we see the most "matter" and the necessary partner fields "antimatter." It's actually a surprise that we see so much more of one than the other, but if the roles were reversed the universe would look almost the same.

Quick Question: Now that you know more about antiparticles, which of the things from the last quick question do you think are plausible and why?

A lot of popular lore suggests that antimatter is a rare substance which annihilates instantly with any normal matter it touches. There is some truth to this, because when a particle meets its antiparticle they don't violate any conservation laws by annihilating into another particle without any charges. Typically this means conversion of mass into the kinetic energy of a massless gauge boson like a photon. On the other hand, there are plenty of other energetic processes which convert mass into energy.

In field theories, it is also possible to make symmetries which are *local*. A local symmetry acts differently at different points in spacetime (remember we are assuming relativity in QFT). The localized equivalent of (3) promotes the global phase θ to a function of spacetime,

$$\phi(\mathbf{x}, t) \rightarrow e^{i\theta(\mathbf{x},t)}\phi(\mathbf{x}, t). \tag{4}$$

Local symmetries are extremely important because they are present in theories with gauge bosons. Gauge bosons are spin 1 particles which we interpret as force carriers. Photons are the most familiar example. They arise from (4) which is the simplest local symmetry, known to mathematicians from the theory of *Lie groups* as $U(1)$, and they are responsible for the electromagnetic force. We call the quantized theory of electromagnetism *quantum electrodynamics* (QED) We will see some other gauge

[2] Actually there is something called a conserved current as well, but let's ignore that for our purposes.
[3] If a particle is uncharged, it may be its own antiparticle.

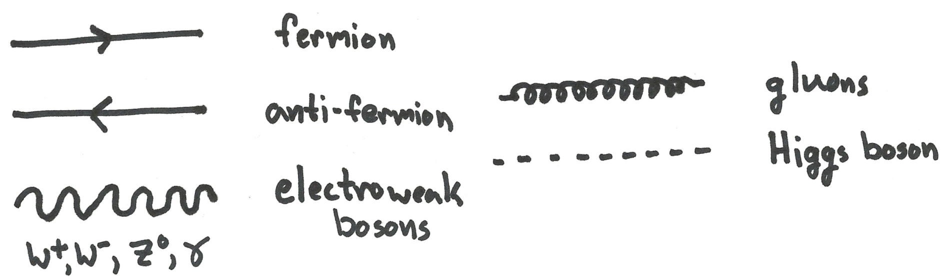

Figure 1: Propagators for particles of the standard model.

bosons shortly, but they arise from more complicated local symmetries that we don't have the tools to discuss in detail.

For interacting relativistic field theories, symmetries like (4) cause the charges of the interaction to be conserved. In QED, our conservation law is the most familiar example of a conserved charge: the electric charge.

1.3 Feynman diagrams for QED

For any physical theory, it's very important to know the appropriate questions the formalism can answer. In QFT, the most common computations calculate the typical time that a particle lives before it decays into other particles or the probability that two particles interact and result in a final state. Richard Feynman developed a very intuitive tool to think about these kinds of calculations, now known as *Feynman diagrams*.

Feynman diagrams are constructed from the pieces shown in Figure 1. I will use the convention that time goes from left to right, so that particles on the left are the initial state and particles on the right are the final state. Many references go from bottom to top instead, so watch out.

Each Feynman diagram must follow certain rules:

1. Each vertex conserves all charges.
2. Each vertex conserves four-momentum. This implies that the final state particles must have the same total four-momentum as the initial state.
3. Each interacting QFT has a set of allowed vertices, only these will appear.

Note that internal legs of the diagram, those particles which only exist as intermediate states, do not have to satisfy the four vector relation $|p^\mu|^2 = -m^2c^2$ like physical particles[4]! This is a consequence of the time-energy uncertainty principle,

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{5}$$

[4]Particles that do satisfy this relation are called *on shell*, short for "on mass shell." Virtual particles which do not are called *off shell*.

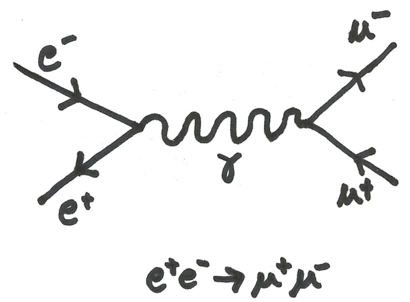

Figure 2: An example of a Feynman diagram. An electron and a positron annihilate to a muon and an antimuon.

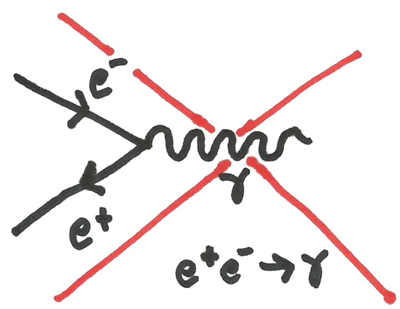

Figure 3: The red lines indicate that this Feynman diagram doesn't exist because it can't conserve momentum.

Physicists interpret this uncertainty relation to mean that energy conservation can be violated for short periods of time in quantum mechanics. The shorter the lifetime of a particle, the less definite its energy will be. Large violations can exist only for very brief periods of time. Another implication of (5) is that particles do not even have a definite energy unless they exist for an infinite amount of time!

These are nice pictures, but don't take them too literally. They are really just a tool that helps people figure out the terms in an expansion called a perturbation series. A perturbation series is a tool to approximate a difficult calculation by a sum of simpler calculations. One example would be the series expansion

$$(1+x)^{1/3} = 1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81} + \dots \tag{6}$$

There are actually an infinite number of terms in this series, and an exact calculation would require that we use all of them. However if x is small, by which I mean $x \ll 1$, the terms get smaller so fast that I can cut off the series and only make a small error. If $x = .1$, the error with only the four terms shown is on the order of 10^-6.

In QED, the expansion parameter is called the *fine structure constant*. The dimensionless quantity formed from the fundamental constants relevant to a quantum theory

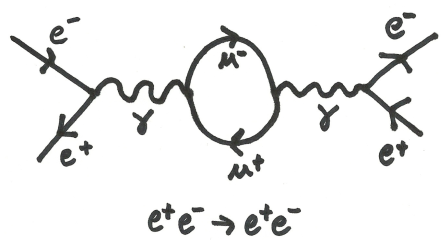

Figure 4: A one loop diagram for the elastic scattering of an electron and a positron.

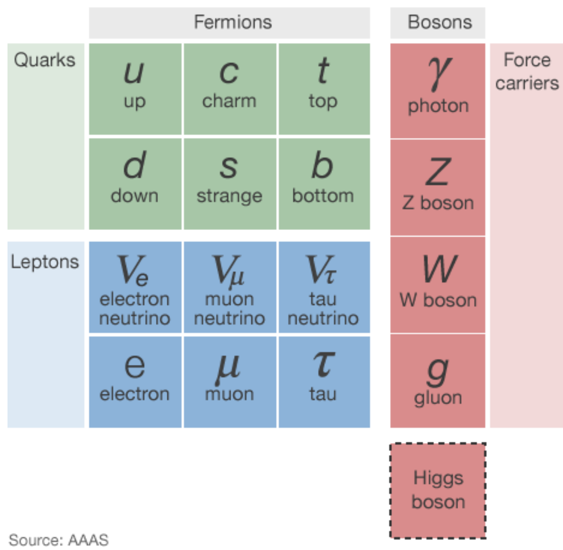

Figure 5: The particles of the standard model.

of electromagnetism is

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.036} \tag{7}$$

where e is the electron charge. This is definitely a small number, so perturbation theory is a useful tool in QED. When we add internal loops to the diagrams, as in Figure 4, these represent higher order terms in the expansion (the math also gets harder very quickly). Fortunately, because α is nice and small, we only need to compute a small number of diagrams in QED. Feynman's diagrams are fundamentally a shorthand for the perturbation expansion, but if you remember this in some corner of your brain it's not too great of a sin to use them intuitively.

2 The Standard Model

The standard model of particle physics, usually just called the standard model, has been frustratingly successful. The standard model starts with basic assumptions about symmetries and describes all of the known fundamental particles (see Figure 5), their interactions via the electroweak and strong forces, and the surprising origin of mass. There are approximately twenty measurable parameters. Since the theory was completed more or less in its present form in the 1970's, experiment after experiment has produced results consistent with standard model predictions. Some of these measurements have been made with dazzling precision. Heroic calculations were made to ensure that theory could make predictions with the same accuracy. As yet, we have not observed any definite violation of any prediction from the standard model[5].

Later we will discuss some reasons that physicists are confident that the standard model is not the full story. That's where the frustration comes in: we know it's not right, but no one can find any exceptions. Many people actually hoped that the LHC wouldn't find a Higgs boson, unfortunately they seem to have continued the trend.

2.1 Electroweak interactions

Quantum electrodynamics was the first area of the standard model worked out by physicists. Eventually, as we continued to explore the nature of fundamental particles and their interactions, we discovered a new interaction called the weak force. But it turned out that we were thinking about this wrong: the electric force and the weak force are not separable. The electroweak interactions are actually unified by the Higgs boson. Since the electroweak sector is the best understood part of the standard model and the Higgs boson has been in the public spotlight recently, let's discuss this part of the standard model first.

2.1.1 Handedness

We have discussed spin 1/2 systems extensively in this class. For a particle with non-zero momentum, we can replace the simple up or down description with an equivalent notion: the *handedness* of a particle. There is an intuitive definition for massless particles, which is safe enough to use for the purposes of intuition even when particles have mass. If a particle's spin is aligned with its momentum, we call it *right-handed*. Otherwise, the particle's spin and and momentum must be anti-aligned and the particle is called *left-handed*.

Quick Question: For a massive particle, do all observers see the same handedness? Hint: Consider the viewpoint an observer moving in the same direction as the particle but faster (all of this is according to some second observer, of course).

Another important property of handedness is the behavior under charge conjugation (the discrete symmetry relating particles and antiparticles). An antiparticle has the

[5]We did modify our understanding of neutrinos, but this was not a fundamental change to the theory. More on this later.

opposite handedness of a particle. In other words, the "anti-(left-handed electron)" is a "right-handed positron."

Every fermion field actually represents four particles. These are the two particles, one of each handedness, and their antiparticles. For most standard model fields, all of these are distinct. A left-handed down quark is not the same thing as a right-handed down quark!

This is not the naive expectation for a model of the universe. We would expect parity, or coordinate inversion, to be a good symmetry of nature. But an experiment performed in the late 1950's demonstrated conclusively that nature distinguished between left and right! We will now see how the electroweak theory predicts the violation of this symmetry.

2.1.2 Breaking symmetries

The electroweak theory is a unification of two symmetries, $U(1) \times SU(2)$, and their corresponding forces. We have already discussed a $U(1)$ symmetry and described the Feynman diagrams associated with it, which we called QED. We call the charge associated with the $U(1)$ symmetry *hypercharge* and denote using the symbol Y. On the other hand, the $SU(2)$ charge is called the *weak isospin* and associated with the symbol T^3.

The best way to understand $SU(2)$ it is to think of all the left handed particles as being in *doublets*:

$$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \ldots \tag{8}$$

The top particles have $T^3 = +1/2$, while the bottom particles have $T^3 = -1/2$. Right handed particles do not get associated in this way, and they all have $T^3 = 0$.

Quick Question: Does the notation of (8) and the possible charges $T^3 = \pm 1/2$ remind you of anything?

Indeed, the $SU(2)$ symmetry mixes the pairs in each doublet using the Pauli matrices. It has no effect on right-handed particles.

If we have four symmetries, we should get four gauge bosons. The $U(1)$ boson is conventionally called the B and the $SU(2)$ bosons are denoted A^1, A^2, A^3. Our B boson couples via hypercharge with strength g, while all of the $SU(2)$ bosons have the same coupling strength g'. Unfortunately, trying to add mass to these gauge bosons spoils the gauge symmetry and leads to lots of infinities. Experiments had shown that there were at least two massive gauge bosons, the $W^{\pm}$, involved in the weak interactions, so something is wrong with this picture.

Here's where the Higgs boson shows up. The Higgs breaks the electroweak symmetry, leaving behind one good symmetry in the process. Although there are four symme-

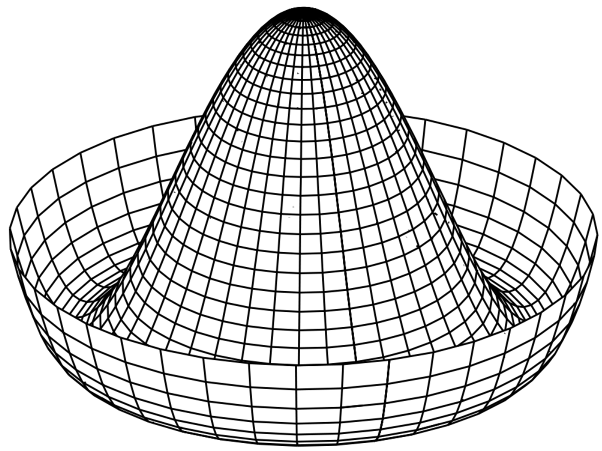

Figure 6: The Mexican hat potential.

tries in the equations describing the standard model, solutions only exhibit one of these!

What kind of equation has a symmetry not exhibited by it's solutions? Consider the following algebraic equation:

$$x^2 + y^2 = 1 \tag{9}$$

Although (9) is symmetric under rotations in the x-y plane, any solution picks out a particular direction[6]! Physical systems whose equations are symmetric but whose solutions are not exhibit the phenomenon known as *spontaneous symmetry breaking*.

Figure 6 shows a potential energy[7] known as the "Mexican hat" potential. The Higgs field has a potential much likes this. Notice that it's rotationally symmetric, but staying at the center is a lot like balancing a pencil on its tip. Perturbing the Higgs at all causes it to roll down into the trough. It turns out this means that there is Higgs field everywhere, since the axes of the potential are essentially measuring "how much Higgs" there is. Since it's lower energy to have some Higgs around, the boson acquires a non-zero value all over the universe. This is known as the *vacuum expectation value* and is denoted by v. This vacuum expectation value picks out a point in the trough of the hat, and breaks the original symmetry.

Since we know that nature contains a massless $U(1)$ gauge boson, it's essential that a $U(1)$ symmetry remains. But it won't be the original hypercharge symmetry! If you look at Table 1 you will see that the Higgs has $Y = +1/2$ and $T^3 = -1/2$. Since it is uncharged under the combination $Q = Y + T^3$, this symmetry will remain as the electric charge. The gauge boson corresponding to this mixture of the original

[6]For example, the solution $(x, y) = (1, 0)$.

[7]Remember that you can think of potential energy like hills and valleys. Going uphill transfers kinetic energy into gravitational potential energy, while going downhill does the opposite.

	T^3	Y	Q
u_L	1/2	1/6	2/3
d_L	-1/2	1/6	-1/3
u_R	0	2/3	2/3
d_R	0	-1/3	-1/3
ν_e	1/2	-1/2	0
e_L	-1/2	-1/2	-1
ν_R	0	0	0
e_R	0	-1/2	-1
h	-1/2	1/2	0

Table 1: The $U(1) \times SU(2)$ or electroweak charges of the standard model.

bosons,

$$A = \frac{1}{\sqrt{g^2 + g'^2}} \left(g' A^3 + gB\right) \tag{10}$$

is known as the photon and remains massless. Photons are essentially oscillations along the trough of the hat potential away from vacuum expectation value. The combinations

$$W^{\pm} = \frac{1}{\sqrt{2}} \left(A^1 \mp iA^2\right) \tag{11}$$

are the massive W bosons and carry electric charge $Q = \pm 1$. These are massive because they correspond to oscillations up and down the walls of the hat, which require energy. They also only couple to left-handed particles! Finally the remaining massive boson is electrically neutral and corresponds to the combination

$$Z^0 = \frac{1}{g^2 + g'^2} \left(gA^3 - g'B\right). \tag{12}$$

In Table 1 you will see the non-conserved Y and T^3 charges of the standard model fermions as well as the Higgs. The conserved electric charge $Q = Y + T^3$ is also given for completeness.

One powerful result of this symmetry breaking is that the masses of the gauge bosons are related! The math shows that

$$m_{W^{\pm}} = m_{Z^0} \cos\theta_w \tag{13}$$

where θ_w is called the weak mixing angle. Experimentally, $\sin^2\theta_w \approx 2.3$.

Fermions can interact with the vacuum expectation value of the Higgs boson. In fact, the Higgs also gives all of the leptons and quarks their masses. We can think of particle masses in terms of Feynman diagrams like that in Figure **??**, where particles interact with the vacuum expectation value and switch handedness. More massive particles couple more strongly to the Higgs, so they experience this interaction more often. Switching from right-handed to left-handed or vice versa is acceptable for a massive particle, since we know that handedness is not the same to different observers and so isn't a well defined particle property.

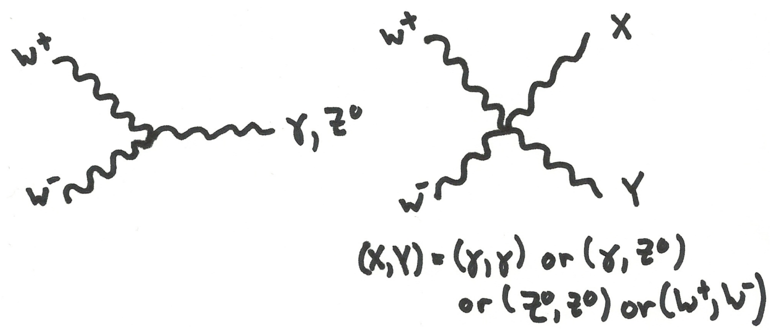

Figure 7: The new vertices coupling only gauge bosons in the electroweak theory.

2.1.3 New vertices

Our complete electroweak theory has a much richer palate of vertices than QED. Clearly, there must be vertices involving all of the new particles. You can find the new vertices coupling fermions and bosons in Figure ?????. Remember that the W bosons only couple to the left-handed particles or right-handed antiparticles. The Z^0 couples to all particles, but with different strengths.

Additionally, $SU(2)$ is different than $U(1)$ because it has multiple *generators* (the three Pauli matrices) which do not commute. A gauge theory of this type is called *non-Abelian* and the gauge bosons can couple directly to one another, subject to all of the usual conservation requirements. These new couplings are shown in Figure (7).

2.2 Strong Interactions

Quantum chromodynamics (QCD) is the theory of the strong interactions. This force is responsible for binding quarks into mesons and hadrons and for confining protons and neutrons to a small dense atomic nucleus despite the repulsive electric charge.

QCD's gauge symmetry is known as $SU(3)$. Each quark can be charged under SU(3) as red, green, or blue. The $SU(3)$ symmetry mixes up the colors in analogy with the up-down mixing of $SU(2)$. The generators are eight 3×3 matrices, known as the Gell-Mann matrices, rather than the three $2x2$ Pauli matrices. We have no use for writing these things out, Google them if you are interested in knowing what they look like. The gluons each carry two charges, one color and one anticolor. Although

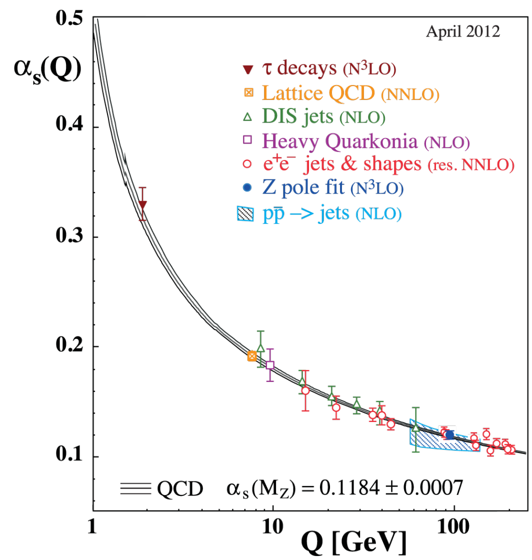

Figure 8: Theoretical prediction and experimental data showing the running of α_s. The horizontal axis is essentially the energy transfer, don't worry about the details. Taken from the Review of Particle Physics.

there is no unique way to define the gluons, one possibility is

$$
\begin{aligned}
g_1 &= \frac{1}{\sqrt{2}}\left(\mathrm{r\bar{b}} + \mathrm{b\bar{r}}\right) & g_2 &= \frac{i}{\sqrt{2}}\left(\mathrm{r\bar{b}} - \mathrm{b\bar{r}}\right) \\
g_3 &= \frac{1}{\sqrt{2}}\left(\mathrm{r\bar{r}} - \mathrm{b\bar{b}}\right) & g_4 &= \frac{1}{\sqrt{2}}\left(\mathrm{b\bar{g}} + \mathrm{g\bar{b}}\right) \\
g_5 &= \frac{i}{\sqrt{2}}\left(\mathrm{b\bar{g}} - \mathrm{g\bar{b}}\right) & g_6 &= \frac{1}{\sqrt{2}}\left(\mathrm{g\bar{r}} + \mathrm{r\bar{g}}\right) \\
g_7 &= \frac{i}{\sqrt{2}}\left(\mathrm{g\bar{r}} - \mathrm{r\bar{g}}\right) & g_8 &= \frac{1}{\sqrt{6}}\left(\mathrm{r\bar{r}} + \mathrm{b\bar{b}} - 2\mathrm{g\bar{g}}\right)
\end{aligned}
$$

which is just a choice for the eigenvectors of the Gell-Mann matrices.

Why are these interactions called strong? There are a couple of reasons. First, let's discuss confinement. Gluons are massless, so they can have infinite range just like the photon. We are familiar with how electromagnetic forces work over long distances: they get weaker the farther apart two particles are. But the strong interaction actually gets stronger as I pull the quarks apart! It gets so strong so quickly that if I ever had two color charged objects far apart, they would immediately have an incredibly energetic exchange of particles to fix their problem.

This phenomenon is called *confinement*. We never see an individual quark, we can only see combinations of quarks (called hadrons) which are either *colorless* or *white*. These states are known as color singlets for reasons I won't explain.

Mesons are the colorless combinations of quarks. Each meson is made of one quark and one antiquark. The lightest meson, for example, is the uncharged pion (π^0).

Each π^0 contains an up quark and an anti-up quark (abbreviated as $u\bar{u}$). If we have a green up quark and an anti-green anti-up, the charges cancel and we say that the combination is colorless. Because $SU(3)$ is an unbroken symmetry, there is no physically observable difference between different colors. If the quarks were instead red and anti-red, the properties would be absolutely identical. Actually, mesons are all made of the same superposition which forms the only color singlet for two particles. This singlet is the superposition

$$\frac{\mathrm{r\bar{r}} + \mathrm{g\bar{g}} + \mathrm{b\bar{b}}}{\sqrt{3}} \tag{14}$$

Baryons are combinations of three quarks, one of each color. Labeling the QCD charges as red, green and blue suggests the designation white for such a color (just like your computer screen produces white from a combination of red, blue and green). The proton (uud) and neutron (udd) are the most familiar example of baryons. Again, all hadrons actually have the same color wavefunction. For hadrons, the three quarks are always in the color singlet superposition

$$\frac{1}{\sqrt{6}}\left(\mathrm{rgb} - \mathrm{grb} + \mathrm{gbr} - \mathrm{bgr} + \mathrm{brg} - \mathrm{rbg}\right) \tag{15}$$

Quick Question: How could you make colorless combinations of four or five quarks? These hadrons are called tetraquarks or pentaquarks, but there is no convincing experimental evidence for their existence.

Quick Question: What would be special about a gluon in a color singlet state, like the hadrons? If the symmetry of QCD was $U(3)$ instead of $SU(3)$, this gluon would exist. It is not observed experimentally.

Although we have only definitely observed bound states of quarks in color singlets, the theory also predicts that gluons (which are nonabelian and thus interact directly through the three- or four- boson vertices) also form bound states called *glueballs*. Lattice QCD predicts a mass of about 1.8 GeV for these particles. Bound states of quarks and gluons could also exist. Some composite particles have been identified

The other reason we call the strong interaction strong is that the coupling constant, α_S, is not small which messes up our ability to use perturbation theory. If there is no small parameter, no expansion can be found which converges. Actually, I've been covering up a bit of a dirty secret. Coupling constants in QFT are not really constant, they're just poorly named. For the electroweak interactions, the strength increases at higher energies. QCD is a special kind of theory which exhibits a phenomenon known as *asymptotic freedom*.

At high energies, perturbation theory works fine. Physically, this means that with high energy probes we can actually resolve the quarks inside of the hadrons. Feynman diagrams can be drawn assuming that the quarks are independent, as in Figure ?????, as a first approximation.