Lecture Notes VIII: Gravitation and Cosmology

Cory Schillaci

July 31, 2012

Although the Large Hadron Collider is a spectacular instrument, it can only collide particles with energies at a fraction of the Planck mass where most particle physicists expect truly new physics to emerge. A next generation particle accelerator would probably seek greater precision[1] rather than higher energies as the greater ring sizes required to reach higher energies[2] become more or less absurd.

As particle physicists wrung the last data out of the Tevatron and waited for the LHC to come online, recent developments in astrophysics have enabled a new era of so-called *precision cosmology*. The *cosmic microwave background* is essentially a window to look out at the early universe. By mapping the inhomogeneities, we can deduce very strong constraints on cosmology. All of the data is consistent with a big bang model of cosmology, and therefore we also expect that energies much higher than the Planck mass were reached at very early times.

The cosmos is therefore one of the best laboratories for experimental data about these extreme energies. High energy physics has been able to use cosmological observations to strongly constrain the possible modifications to the standard model. This is one of the most exciting and fruitful interdisciplinary relationships in fundamental physics today.

1 General Relativity

In order to understand cosmology, you first need to know something about the theory which is applicable to the largest scales of the universe: *general relativity* (GR). You are likely familiar with the Newtonian theory of gravity, which is actually an approximation to GR when gravitation is relatively weak.

Quick Question: What does it mean for gravity to be weak? Can you give some examples where gravity is not weak?

---

[1] A linear collider is generally considered the best bet for a followup to the LHC. Such a collider would smash together electrons and positrons rather than protons, thereby eliminating the messiness inherent to QCD processes.

[2] New technologies also help us to increase the energy accessible.

Gravitation requires that we further modify our understanding of space and time. In special relativity, we noted that the length of a four-vector $x^\mu = (ct, x, y, z)$ was non-Euclidean. Instead, Lorentz invariance respects the Minkowski geometry, where

$$ds^2 \equiv |x^\mu|^2 = -t^2 + x^2 + y^2 + z^2. \tag{1}$$

Non-Euclidean geometries further generalize the concept of distance. We could view (1) in the expanded form

$$ds^2 = x \cdot x = (t, x, y, z) \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \tag{2}$$

The matrix is known as the *Minkowski metric*, usually denoted by $\eta^{\mu\nu}$. Clearly, the -1 in the zero-zero entry is what makes the distance non-Euclidean. For a general geometry, the metric is denoted g and is not diagonal (although it is symmetric) and will be a function of location.

Quick Question: What is the metric for our three dimensional Euclidean space?

A thorough study of general relativity requires the mathematical methods of differential geometry, the study of spaces which are locally similar to $\mathbb{R}^n$. However, the idea of a metric is good enough for our purposes. Think of GR as a prescription for finding the metric, given the matter content of the universe.

1.1 The FRW metric

Einstein's equations, which link the metric to the matter content of the universe, are extremely difficult nonlinear differential equations. In very symmetric cases, these equations can be solved exactly.

We expect that the universe, on very large scales,[3] is basically the same everywhere. On the other hand, it seems to be evolving in time. The appropriate symmetries are spatial homogeneity and isotropy, which lead to a solution called the Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - \kappa r^2} + r^2 d\Omega^2 \right] \tag{3}$$

Quick Question: The Minkowski metric is a special case of the FRW metric. Can you see how to recover it?

Let's unpack this a bit. I have separated the radial (r) and angular (abbreviated $d\Omega$) parts. If the observed universe was in $2+1$ dimensions, $r^2 d\Omega^2 = r^2 d\theta^2$ which

[3]This is only true on scales much larger than distances between galaxies. These are VERY large scales.

is the correct way to define distance in polar coordinates. For $3+1$ dimensions we use spherical coordinates and it's a little more complicated to write out the formula, but the idea is fundamentally the same. If you aren't comfortable with curvilinear coordinate systems, feel free to just ignore this term entirely.

All the interesting stuff happens with the r coordinate anyways. We will discuss the meaning of the *curvature* κ and the details of how $a(t)$ evolves later. For now let's just consider a flat universe with $\kappa = 0$. Then the FRW metric just tells us that distances change with time according to $a(t)$.

In general relativity, things can get farther away without moving. Let's say we have two objects which are distance r apart at time t_1, or equivalently $ds^2 = r^2$. If neither object changes its spatial location, at a later time t_2 the interval between them would be

$$ds^2 = \frac{a^2(t_2)}{a^2(t_1)} r^2 \tag{4}$$

Quick Question: Why didn't I include a t^2 term in the intervals above?

The function $a(t)$ is called the *scale factor*. As it changes, fixed points in space become farther apart or closer together. Let's just consider an expanding universe like our own. If an object is moving away from an observer (its spatial coordinate is changing), there can always be another observer on the other side for whom it's getting closer. In the expanding universe, it doesn't matter where you observe things from. All other points are always getting farther away! This is fundamentally different.

We know that the universe is expanding because that expansion also changes the wavelengths of light. As space stretches, photons experience a *redshift* towards longer wavelengths and lower energies. The redshift z measures this stretching:

$$z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} - 1 \tag{5}$$

From the FRW metric, we see that the stretching of the light is given by

$$z = \frac{a(t_{\text{observed}})}{a(t_{\text{emitted}})} - 1 \tag{6}$$

For small redshift it also makes sense to define a redshift velocity v_r,

$$v_r = cz \tag{7}$$

2 Big Bang Cosmology

If we take GR and the FRW metric seriously, we can run the Friedmann equations backward until the scale factor goes to zero. If we identify this time as $t = 0$, this is the statement that $a(0) = 0$. Nobody knows whether or not this is justified. General

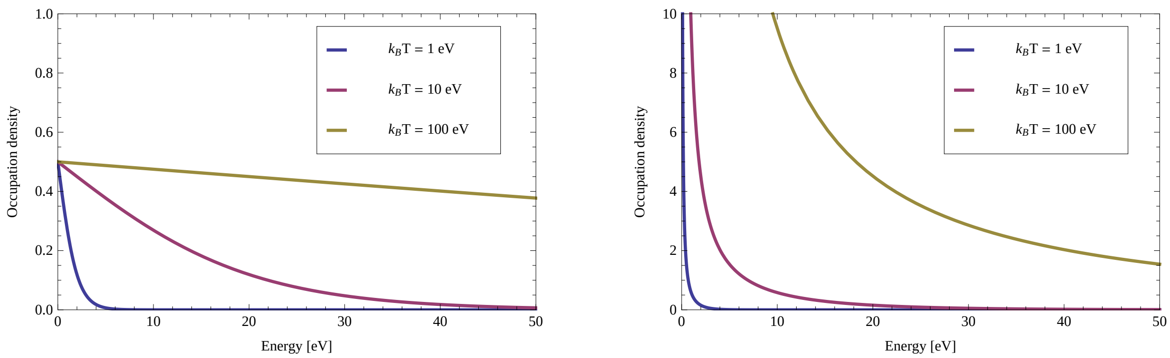

Figure 1: The Fermi-Dirac distribution for fermions (left) and the Bose-Einstein distribution for bosons (right).

relativity is expected to require modification at the quantum level, which may or may not prevent the initial singularity.

In any case, the universe seems to have evolved from a past in which the scale factor was once much smaller than in the present day. The following sections discuss the various phases of the big bang. Here is one place where orders of magnitude are very useful.

It's also useful to have an understanding of what equilibrium and temperature mean. Thermal equilibrium is possible only when constituents of a system interact relatively often. It turns out that the energy distribution of a system with sufficiently strong interactions is basically insensitive to the details of the interactions. Instead, the occupation of energy levels are described by universal distributions. To give you an idea of what this looks like, the number density of particles between energy E and $E + dE$ at temperature T is given by a distribution

$$n(E,T) \propto \frac{1}{e^{E/T} \pm 1} \tag{8}$$

where the plus sign applies to fermions and the minus sign to bosons. These are shown in Figure 1.

2.1 The Planck Time

Avoiding the question of an initial singularity, we will start our story at the Planck time (10^{-43}s) and energy ($10^{1}9$ GeV). Around this point, the quantum nature of gravity becomes less important. If the gravitational interaction is somehow unified with all of the standard model gauge theories, that symmetry is broken around this time and the graviton acquires its current properties.

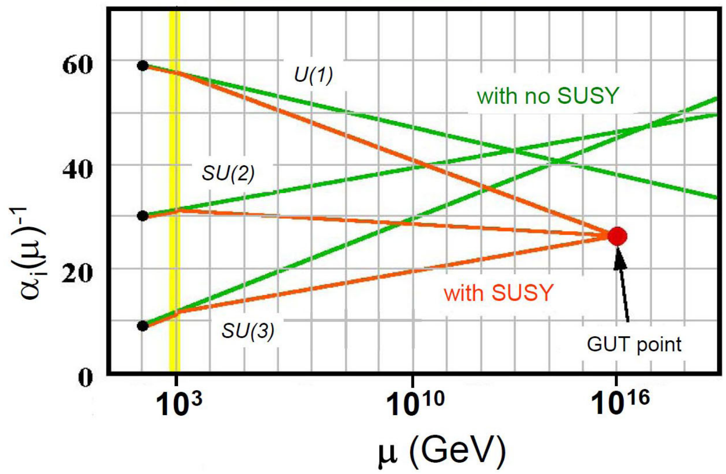

Figure 2: A plot of the running of the coupling constants. Unification appears naturally only in supersymmetric theories. Although there is some model dependence, the unification scale can be taken as approximately 10^{16}GeV. I don't think the vertical scale on this plot is accurate, but the qualitative behavior is correctly demonstrated. From http://atlas.kek.jp.

2.2 GUT Epoch

After the Planck time, we initially expect that all of the gauge interactions are unified. Physicists generally use the term Grand Unified Theory (GUT) to describe a fundamental theory which treats the electroweak and strong interactions in the same way. In supersymmetric theories, the three standard model coupling constants naturally become equal at an energy of about 10^{16} GeV. This corresponds to a time of approximately 10^{-35} seconds, when the GUT epoch comes to an end.

Since all of the SM symmetries are intact, the Higgs has not yet broken the electroweak symmetry and all particles are massless. Energies are so incredibly high that no bound states can form. Particles are so dense that they all interact constantly, remaining in thermal equilibrium.

2.3 Inflation

Around the end of the GUT epoch, the universe is thought to undergo exponential inflation. This mechanism solves the so-called horizon problem, explains the general homogeneity of the universe, and provides quantum mechanical fluctuations which seed the formation of structure in the universe.

During inflation, the size of the universe increase by a factor of approximately 10^{26} very quickly. This is supposedly generated by a field known as the *inflaton*, although we have no known candidate for the inflaton field. The rapid expansion causes cooling (in exact analogy with the cooling of a rapidly expanding gas), but at the end the inflaton decays into energetic standard model particles and reheats the universe. Inflation is expected to end around $t = 10^{-33}$ s, overlapping with the beginning of the electroweak epoch.

Warning: there are many models for inflation which result in a veritable zoo of different outcomes, but this is the general idea.

2.4 Electroweak Epoch

When the grand unified symmetry breaks at 10^{16} GeV, around 10^{-35} seconds into everything, the electroweak and strong interactions become distinct. There was still far too much energy for protons to stay bound, so the universe was a hot soup of massless quarks, leptons, and gauge bosons. The Higgs mass, about 100 GeV, sets the scale for the breaking of electroweak symmetry at about 10^{-11} s.

Somewhere in here *baryogenesis* probably occurs. This is the unknown mechanism by which the universe generates a matter-antimatter asymmetry in the baryons. It is assumed to occur after inflation, which would have dispersed any preciously generated asymmetry, and before the hadrons fall out of equilibrium. Leptogenesis, the analogue for the lepton-antilepton asymmetry, probably occurs at some unknown time before the lepton epoch.

Quick Question: Why does a baryon asymmetry imply a lepton asymmetry?

2.5 Quark Epoch

After the electroweak symmetry breaking, the history of the universe is less speculative. At this point all particles have masses and all the symmetries are broken down to what we observe today. It's still quark soup though, until we get to energies around the hadron mass scale of 1 GeV. This epoch lasts until $t \sim 10^{-}6$ s.

2.6 Hadron Epoch

Until $t \sim 1$s, the hadrons dominate the universe. There are still a lot of anti-hadrons around until later in this epoch. As they fall out of equilibrium, most of the antimatter annihilates and leaves only matter behind.

2.7 Lepton Epoch

Since the baryon asymmetry is not large, very little baryonic matter is left around after annihilation. The lepton epoch gradually sees the heavier leptons falling out of

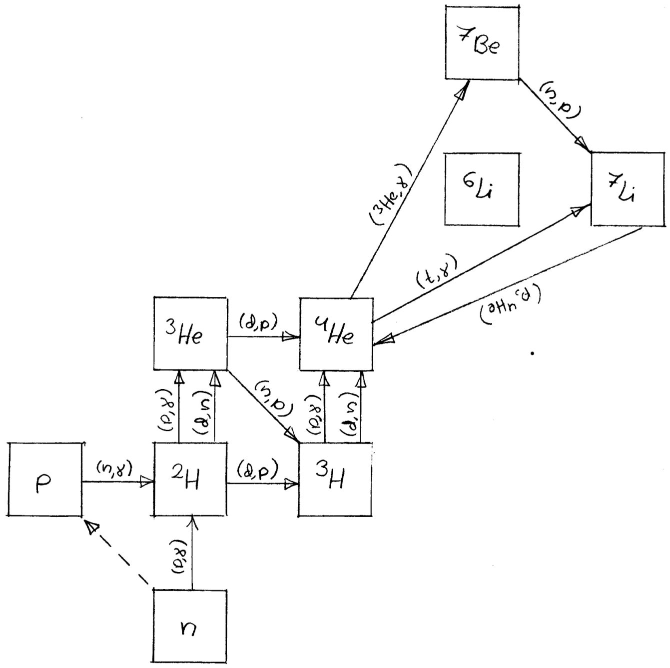

Figure 3: Reactions which produce the light elements during BBN. Hydrogen and helium make up nearly all of the visible baryonic matter in the universe. This figure was created by my advisor, Wick Haxton.

equilibrium as the temperature falls below their masses. By about ten seconds all of the leptons have fallen out of equilibrium and the lepton epoch ends.

Towards the end of the epoch, the lighter leptons also fall out of equilibrium. At about 10^{10} K the neutrinos decouple when the neutral current reactions can no longer keep them in equilibrium, creating the cosmic neutrino background (CνB). After this time, the neutrinos expand freely and cool until the present day, when their temperature is about 1.9 K.

2.8 Big Bang Nucleosynthesis (BBN)

Near the end of the lepton epoch, as the neutrinos decoupled, the temperature of the universe falls below the electron mass. As with the baryons, there is an excess of matter over anti-matter. The positrons all annihilate, leaving the electrons we see today. The BBN era describes the formation of the current hadronic content of the universe, including the formation of the light nuclei. Remember that we are still well above the energies where electrons get stuck to nuclei, so no neutral atoms form at this point and chemistry does not occur.

At the beginning of this era, we are mostly concerned with neutron-proton balance. Because neutrons are heavier by about 1.3 MeV, the ratio of neutrons to protons will decrease as the temperature drops. The processes which cause interconversion require the neutrinos, so we are still talking about the end of the so-called lepton-epoch. These three processes convert neutrons into protons, and are all reversible.

$$n + \nu \Leftrightarrow p + e^-$$
$$n + e^+ \Leftrightarrow p + \overline{\nu}$$
$$n \Leftrightarrow p + e^- + \overline{\nu}$$

After the neutrinos decouple below 10^{10} K, the electrons and positrons also quickly fall out of equilibrium (the electron mass is 0.5 MeV). The first two processes above become extremely rare by the time T drops to 10^9 K, but free neutrons are unstable and continue to β-decay with a lifetime of about 10^3 seconds.

Around 100 seconds into the life of the universe, temperatures drop below the deuteron binding energy of 2.2 MeV and the remaining neutrons start to bind with protons. At this time, the ratio of neutrons to protons is about 1/7 and stops changing.

Deuteron formation is slowed down somewhat by the very large number of photons in the universe. Studies of the CMB indicate that the ratio of baryons to photons is

$$\eta = \frac{N_b}{N_\gamma} \approx 10^-8 \tag{9}$$

Basically, there are so many photons that they tend to simply blast the deuterons apart until the universe has cooled to the point where the binding energy is significant. This creates the *deuterium bottleneck*, as deuterons are the first step in synthesis of all the light nuclei. A reaction network for BBN is shown in Figure 3. As the universe expands, the nuclei become too diffuse to fuse often and the abundances *freeze out*.

2.9 Recombination and the CMB

Also known as the *time of last scattering*, recombination occurs when the temperature drops low enough that electrons are bound to the nuclei. Naively one expects this to be about the binding energy of the electron in the hydrogen atom, 13.6 eV. However the small baryon to photon ratio suppresses recombination until $kT \approx 10^{-1}$ eV. This occurs somewhere around 300,000 years after the big bang.

Once the electrons are all bound up into neutral atoms, the universe suddenly becomes mostly transparent to photons. The photons which were in thermal equilibrium then enter free expansion and form the cosmic microwave background, which we have measured to very high precision. The CMB is not perfectly uniform, and the anisotropies tell us a great deal about the early universe. CMB observations are one of the main tools of modern precision cosmology.

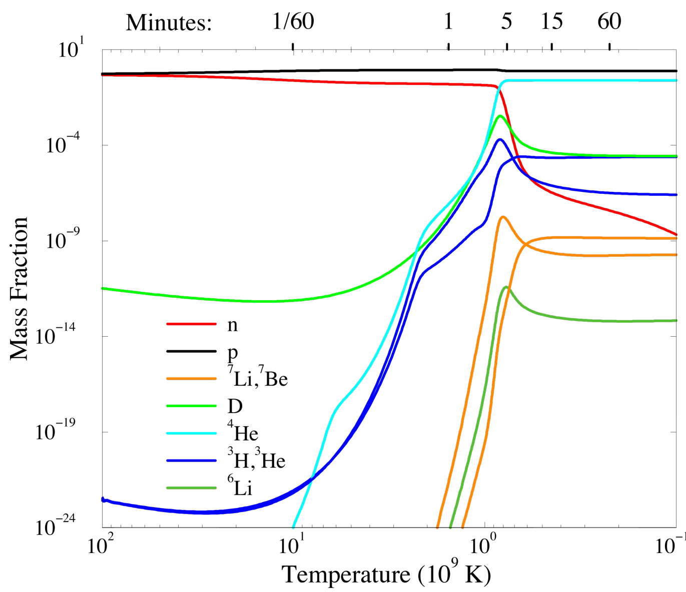

Figure 4: Predictions for the abundances of light elements during BBN. Note especially the deuteron bottleneck until $t \approx 5$ minutes, after which everything starts to really get going. The neutron abundance does not freeze out because free neutrons continue to decay. From a paper by Burles, Nollett, and Turner http://arxiv.org/abs/astro-ph/9903300.