Homework VII

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Problem 1 Muon lifetime

Consider the muon, which has a lifetime of $\tau = 2.20\,\mu$s and a mass of $m = 106\,\mathrm{MeV/c^2}$. Supposing that time dilation did not exist, what would be the maximum typical distance that a muon could travel before decaying? Now, considering the predictions of special relativity, how far does a typical cosmic ray muon of energy $E = 10^{10}\,\mathrm{MeV}$ travel? The decay lifetimes of these ultrarelativistic particles provide experimental evidence for time dilation.

Problem 2 Clairvoyance

This problem is taken from *Introduction to Electrodynamics* by David J. Griffiths. The whimsy is all his.

Sophie Zabar, clairvoyante, cried out in pain at precisely the instant her twin brother, 500km way, hit his thumb with a hammer. A skeptical scientist observed both events (brother's accident, Sophie's cry) from an airplane traveling at $\beta = 12/13$ [toward Sophie from her brother's location]. Which event occurred first, according to the scientist? How *much* earlier was it, in seconds?

Problem 3 Lorentz invariant magnitude

Show that the Lorentz transformation

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Leaves the magnitude $|x^\mu|^2 = -c^2t^2 + |\vec{x}|^2$ unchanged.

Problem 4 Spacetime interval

The spacetime interval between an event A, occurring at $x^\mu_A = (x^0_A, x^1_A, x^2_A, x^3_A)$, and event B at $x^\mu_B = (x^0_B, x^1_B, x^2_B, x^3_B)$ is given by

$$I \equiv |\Delta x^\mu|^2 \tag{2}$$

where

$$\Delta x^\mu \equiv x^\mu_A - x^\mu_B. \tag{3}$$

If $I < 0$, the interval is called *time-like*, if $I = 0$ the interval is called *light-like*, and when $I > 0$ the interval is called *space-like*. Interpret the difference, especially considering the possibility that events A and B influence one another. You might want to start by determining the interval between the two events in Problem 2.

Problem 5 Readings from *The Theory of Almost Everything*

Read the introduction (pp. 1-12), Chapter 2 (pp. 29-40 only, although the story about Emmy Noether at the beginning of the next section is worth checking out), and Chapter 5.