Lecture Notes I

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1 Areas of Physics

During the next few weeks we will discuss a variety of themes in physics while ignoring far more. In order to provide some context, it's useful to attempt to categorize the different subjects in physics. Subdividing all of physics neatly and unambiguously is impossible, but we can use the divisions of the American Physical Society (APS) for our purposes. The APS website lists the following divisions of physics[1]:

- Atomic, Molecular and Optical Physics
- Astrophysics
- Biological Physics
- Chemical Physics
- Computational Physics
- Condensed Matter Physics
- Fluid Dynamics
- Laser Science
- Materials Physics
- Nuclear Physics
- Particles and Fields
- Physics of Beams
- Plasma Physics
- Polymer Physics

Now, some of these subfields are far more prominent than others. My bias is certainly at work here, but I will now describe the topics I find to be the most important and influential today, roughly in order of increasing size scale. Don't worry if you don't know all the vocabulary, we should talk about everything I mention by the end of the program!

1.1 Particles and Fields

This field is usually called *particle physics* or *high energy physics*. At the very smallest scales of the universe we find the fundamental particles (which are also fields - we'll get

[1] http://www.aps.org/membership/units/index.cfm

back to this later in the course). A very important topic in physics is to understand how these particles behave, how they interact, what new particles might exist, and how to describe all of this in the simplest way possible. High-energy physicists also work on new conceptual frameworks (like string theory) which might completely alter the way we understand the universe. Experimentalists in this field typically use spectacular instruments like the Large Hadron Collider to study things by smashing them together with as much energy as possible. Exciting topics in this field right now include last year's discovery of the Higgs boson, the search for supersymmetry, development of string theory, and dark matter.

1.2 Nuclear Physics

Nuclear physicists study the next smallest things in nature. When quarks combine to form hadrons[2] or protons and neutrons combine to form atomic nuclei, the nuclear physicists are there. Another important question is the origin of elements, which leads to study of astrophysical objects such as stars and supernova. Because neutrinos are important in nuclear decay and synthesis, nuclear physicists have also made important contributions to understanding the weak force. This is a broad field, and much of its scope is the result of the historical development of physics. It's also my area of study, so feel free to ask questions!

Nuclear experiments, in contrast to high-energy experiments, are typically done at low energies but with very high precision. Many experiments require the expertise of nuclear physicists to properly understand the behavior of complicated atomic nuclei in detectors.

1.3 Atomic, Molecular, and Optical (AMO) Physics

These physicists study how atom sized things interact with one another. The APS gives a pretty good description:

> The Division of Atomic, Molecular and Optical Physics (DAMOP) was founded in 1943, and was the first division of the American Physical Society. Its central focus is fundamental research on atoms, simple molecules, electrons and light, and their interactions. It plays an enabling role underlying many areas of science through the development of methods for the control and manipulation of atoms, molecules, charged particles and light, through precision measurements and calculations of their properties, and through the invention of new ways to generate light with specific properties.

[2] Particles like protons or neutrons which we'll discuss in more detail later

1.4 Condensed Matter Physics

The term condensed matter is roughly short for "condensed phases of matter," typically solids and liquids. These systems are strongly interacting collections of atoms in which the collective behavior is more important than that of any individual constituent. One of the most well-known problems in the field is the search for materials which form new, higher-temperature superconductors and efforts to understand how these so-called high T_C[3] superconductors work at all. Condensed matter is possibly the largest sub-field of physics and overlaps substantially with other physical sciences such as chemistry and materials science.

1.5 Astrophysics (and Cosmology)

Astrophysics is the study of the largest possible scales we can observe: stars, galaxies, even galactic clusters. While we understand many of these things to some extent, there are still many mysteries. How does a core-collapse supernova work? What can the cosmic microwave background tell us about the history of the universe? How do cosmic rays get accelerated to such extreme energies? These are just a few of the questions that astrophysicists study today. Cosmology specifically refers to understanding the early history of the universe, such as the big bang or inflation. Experiments often involve giant radio telescopes, arrays of detectors spread out across 1,200 square miles of Argentina, or satellites.

2 Understanding Units and Scales

I remember that when I was getting started in science classes many of my teachers were very insistent that answers come with units, not just numbers. Cars don't travel at just '60', they go '60 miles per hour.'

Quick Question: List all the different named units that you can. Indicate which are in the SI system (Système international d'unités).

As you learn about science, you also keep learning new units with new concepts. A lot of these are composite units, for example the joule (which is the SI unit of energy):

$$1\,\text{joule} = 1\,\frac{\text{kilogram}\cdot\text{meter}^2}{\text{second}^2} = 1\,\text{newton}\cdot\text{meter} \tag{1}$$

Do we really need all of these? The short answer is no. We could actually make do with a set of just a few units. We are free to choose just what these are, and we should do so to make our lives as simple as possible. For example, a common choice

[3]Pronounced "tee-see," stands for the critical temperature below which the material is superconducting.

in Newtonian mechanics is to use time, distance, and mass as the base units. All other units are then constructed from these.

Quick Question: What would be an example of a bad choice? What makes this choice bad?

2.1 Dimensional Analysis

Because many physical quantities carry units, physicists have developed a couple of ways to use this to our advantage. The first is a consistency check: after going through a calculation, does the answer have the correct units? If not, it can't be right! We can also use dimensional analysis to make educated guesses at answers. Let's start with a very simple example of how this works.

Say you are given a uniform sphere with radius $r = 1$m, mass $m = 45$kg, and a temperature of $T = 300$K. Pretend for a moment that you don't know the formula for the volume of a sphere. How would we construct a formula for the volume out of the possible properties of the sphere? Let's use the notation $[l]$ to indicate some quantity with dimensions of length, $[m]$ for mass, $[t]$ for time, and $[T]$ for temperature. We have to know that volume is something with dimensions of $[l]^3$. How can we combine the properties of the sphere into a quantity with these units? There is only one way:

$$V \propto r^3 \tag{2}$$

This example may seem ridiculous, but it illustrates a few key points. First, we started with all the possible properties of the sphere. We had to make reasonable assessment of what these are. In this problem, I assumed that we could ignore things like the sphere's construction material, its velocity, etc. Making this cut can sometimes be tricky, and can trip you up.

The symbol $\propto$ should be read as "is proportional to," and this leads to the second point. We didn't get the factor of $4\pi/3$ right at all! Constants like these cannot be found by dimensional analysis because they don't carry units. While dimensional analysis can give us an idea of how the volume depends on the radius (here it's a cubic power law), exact answers can be found only by coincidence or by mixing some more 'educated' into the educated guess. However, if you do a lot of dimensional analysis you will begin to notice that the constants are usually not that big. In our example, it was roughly four which is on the order of one (see Section 2.3).

2.2 Natural Units

Now that we've spent a while describing how to use units, I'm going to make almost all of them disappear. High-energy physicists typically work in a system of units

Name	Symbol	SI Value
Dirac Constant	$\hbar$	$1.05 \cdot 10^{-34}$J·s
Boltzmann Constant	k_B	$1.38 \cdot 10^{-23}$J/K
Speed of Light	c	$3.00 \cdot 10^{8}$m/s

Table 1: A list of the fundamental constants which are set to one in natural units.

Unit Type	SI Unit	Natural Unit
Mass	kg	eV
Distance	m	1/eV
Time	s	1/eV
Temperature	K	eV

Table 2: The dimensions of some common quantities in natural units.

that makes their life much easier. Since there is a lot of algebra[4], and some pesky constants show up all over the place, they decided to set all of these constants and their units to be equal to one. For a complete list, see Table 1.

After this is done, the only unit that ever shows up is energy. Since particle physicists study only a small number of particles at a time, joules are not a very relevant energy scale. Instead, everyone talks about electron volts[5]. See Table 2 for a summary of how to give some common quantities in eV when using natural units.

You should note that, in natural units, length scales and energy scales are inverses of each other. Sometimes physicists will say, for example, that a particle accelerator probes small distance scales rather than saying that it probes high energies. These statements are entirely equivalent. There are a few ways to understand this from a fundamental physics standpoint, hopefully we can discuss this topic later in the course.

2.3 Orders of magnitude

Physicists also strongly rely on the concept of the order of magnitude. Basically, this is the exponent on the ten in scientific notation. The numbers $9.5 \cdot 10^{10}$ and $\sqrt{2} \cdot 10^{10}$ are said to be of the same order of magnitude. On the other hand, $5 \cdot 10^{-10}$ is five orders of magnitude larger than $7 \cdot 10^{-15}$. Any number which doesn't need scientific notation at all is a special case, you can say that it is “on the order of one” or “of order unity”.

We can often gain just as much intuition for how something works by making order of magnitude estimate, and save a lot of time by sweeping a few details under the rug. This is often called a ”back of the envelop” calculation, to evoke the idea that

[4]When physicists say algebra, they mean manipulating equations. This often involves calculus or other mathematical methods beyond basic algebra.
[5]Abbreviated eV and often called ‘ee-vee’ for brevity

it can be done quickly and with limited resources. Enrico Fermi, one of the great physicists of the 20th century, is famous for being very good at doing this sort of estimate. Here is his account of how he estimated the yield of the first atomic bomb detonation:

> About 40 seconds after the explosion the air blast reached me. I tried to estimate its strength by dropping from about six feet small pieces of paper before, during, and after the passage of the blast wave. Since, at the time, there was no wind I could observe very distinctly and actually measure the displacement of the pieces of paper that were in the process of falling while the blast was passing. The shift was about 2 1/2 meters, which, at the time, I estimated to correspond to the blast that would be produced by ten thousand tons of T.N.T.

Orders of magnitude are also especially useful when comparing quantities. It's quite intuitive to think of a baseball and a tennis ball as approximately the same size. Actually, the radius of a baseball is 1.14 times larger. If we say that two quantities which have the same order of magnitude are about the same, then this is consistent with our intuition. If the order of magnitude of two quantities is different, they are clearly not the same. For example, the weight of a typical car is only one order of magnitude greater than the weight of any normal adult!

There are many jokes about physicists only doing calculations to an order of magnitude. Of course, sometimes more accuracy does matter! One of the skills a good physicist needs is the ability to make these estimates, but at the same time know whether or not an order of magnitude estimate is enough information for the purpose at hand.

3 Guiding Principles of Physics in the 20$^{\text{th}}$ Century

Two major principles have guided fundamental physics throughout the last century. The first of these is symmetry. Symmetry first reared its head in classical mechanics, but its true power came to be appreciated only with the development by particle physicists of gauge theories in the mid 20$^{\text{th}}$ century. Today we'll learn about the former case, and later we will discuss the gauge symmetries in the standard model of particle of particle physics. Second was reductionism, the belief that the ultimate theory of nature should contain a minimal number of unconstrained parameters and explain everything from the fewest possible fundamental concepts. These are important concepts because they inform the working definition of a good theory. Physicists have had spectacular quantitative successes trying to improve their theories based upon this definition, thereby increasing our confidence in it. Of course, physics has surprised us in the past and might do so again!

3.1 Symmetry

In 1915, Emmy Noether showed that every continuous symmetry corresponds to a conservation law[6]. Let's first try to understand the concept of symmetry in the context of physics.

Quick Question: Can you give a good definition of symmetry?

The circle is an object with obvious symmetry, one which actually has an analog in fundamental physics. No matter how you rotate a circle about it's center in the plane, it never looks any different. That ability to transform the circle without changing anything is what physicists and mathematicians call a symmetry[7].

Now consider a triangle. We can rotate the triangle 120°or 240°without seeing anything change. In contrast to the circle, for which we could make any rotation, the triangle only has a few discrete rotations which are symmetries. We say that the circle has a continuous symmetry, because we can rotate it by any angle of any size. The triangle has a discrete symmetry, because we have to use multiples of 120°. Noether's theorem does not apply to discrete symmetries!

Quick Question: List some other symmetric shapes. Give both two- and three-dimensional examples!

In physics, we expect that rotating the universe (similar to rotating the the circle, but we can rotate in two ways because space is three dimensional) would not change anything about the way things work. This is a continuous symmetry, so we get a conservation law: the conservation of angular momentum. Most of you probably aren't very familiar with angular momentum, so let's consider another example.

In order for experiments to be reproducible, it must be true that performing them again a day or two later would give the same results[8]. To make this more rigorous, this means that the laws of physics can't be changing in time. Time invariance leads to the most familiar conservation law: the conservation of energy.

No physically allowed process will ever produce or destroy energy. This is an incredibly strong constraint. If we were to just randomly write down algebraic expressions of time and assign them as particle trajectories, we would probably never write down a set which satisfied the law of conservation of energy. Conservation laws are very beautiful, their relationship with symmetries is unexpected and yet fundamental, and their existence is a very basic hint that the laws of physics have a very specific and special mathematical structure. Luckily, that structure also makes life easier!

Quick Question: List some more symmetries of physics and some conservation laws. Can we match them up? Hint: What else must be true to make experiments

[6]This is easy to see in the beautiful Lagrangian formulation of classical mechanics. One of the greatest tragedies of introductory physics education is the extensive reliance on Newton's practical but structurally opaque formalism of mechanics.

[7]The mathematical study of symmetries, known as group theory, is very important in physics.

[8]At least if the experiment is performed in exactly the same way.

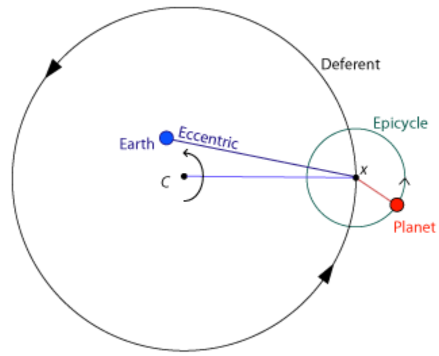

Figure 1: Ptolemy's model of the solar system using epicycles, deferants, and equants.

reproducible?

3.2 Unification

Our readings from *Dreams of a Final Theory* will do a much better job of explaining and advocating the reductionist philosophy than I could. Still, let's provide a quick introduction to the main idea using a historical example. Ptolemy was a Roman in the 2$^{\text{nd}}$ century CE. He developed a model of the solar system based on circles, which were considered to be "natural" shapes at the time. Roughly speaking, he allowed every solar object to move on a circle (called an *epicycle*), the center of which rotated on another circle (called a *deferant*) about the Earth. The center of the deferant for each planet was slightly displaced from earth, and the angular speed with which the epicycles moved was required to appear constant from a point called the *equant* (see Figure 1). This theory was computationally successful and widely accepted until Kepler came along in 1609[9]. From the modern viewpoint, Ptolemy's theory was successful for so long because it contained sufficient free parameters (four for each planet) to reproduce the still rough experimental data.

Newton's theory of gravity reduced the necessary observational data to one constant, G. It further related the motion of the planets to the way objects fall on Earth[10]. That such a wide variety of phenomena can be described by a single idea seems implausible at best, but as experimental data was refined it continued to support Newton's theory until the slight deviations due to Einstein's theory of general relativity were noticed in the 1920s. Reductionism is the belief that all phenomena will eventually be described, at least in principle, by a single theory.

Quick Question: What are some other phenomena that can be predicted from Newtonian gravity?

[9]The third law was published in 1619.

[10]It's interesting to note that in Ptolemy's era, the geocentric model had been argued for because all objects fell towards its center.