Lecture Notes IX: The Universe From Here On Out

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Yesterday we discussed the history of the Big Bang up to structure formation. For now, let's just gloss over the formation of large scale structure. Today we will talk about observations of the present day universe, and possible futures. Then there will be a twist at the end.

For reference purposes, recall that the FRW metric is

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \tag{1}$$

where r is dimensionless and $k \in \{-1, 0, 1\}$.

1 The Present Day Universe

Current evidence suggests that the universe is nearly flat. What does this mean? After all, the curvature can only take discrete values. In addition to the metric (1), we also know how to determine the evolution of the scale factor a using the Friedmann equations. The rate at which the scale factor changes is usually notated as the Hubble parameter H,

$$H \equiv \frac{\dot{a}}{a} \tag{2}$$

where $\dot{a}$ is the rate at which the scale factor changes with respect to time[1].

The Friedmann equations tell us that there is a critical energy density,

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G} \tag{3}$$

If the energy density of the universe is greater than this, the universe must be closed. When $\rho < \rho_{\text{crit}}$, the universe will be open. Of course if the universe has precisely the

[1] If you know calculus, the overdot in physics typically indicates a time derivative. For example, $\dot{x} = \frac{dx}{dt} = v$.

critical density, it should be flat. Recent measurements suggest that the universe is at least very close to flat, i.e. ρ/ρ_{crit} is within .05% of 1.

Note that (3) contains the Hubble parameter, which is changing in time. How can we determine H? Well, I'll tell you:

$$H^2 = \frac{8\pi G}{3}\left(\rho_{\text{matter}} + \rho_{\text{radiation}} + \rho_{\text{vacuum}} - \rho_{\text{curvature}}\right) \tag{4}$$

These ρ's are energy densities due to different types of stuff. Matter means non-relativistic particles, and radiation means relativistic particles (including photons). In our present universe, $\rho_{\text{radiation}} \ll \rho_{\text{matter}}$ so that the second term can be neglected. The other two contributions involve more explanation.

When Einstein formulated general relativity, he noticed that a constant should be added to the equations in order to make a stationary universe. This original motivation was abandoned when Hubble made his famous diagram, but quantum field theory actually predicts that the vacuum is not empty. Instead, it may have some energy density. The vacuum energy density is ρ_{vac}.

Modern observations suggest that not all of the energy density is accounted for by matter, radiation, and curvature. We currently call the excess dark energy, because we don't have a good theory to predict its magnitude or an understanding of its origin. Unfortunately, the prediction from QFT for ρ_{vac} is too big by 120 orders of magnitude! So we might need a new understanding of the vacuum energy's magnitude, or we might be observing something entirely new.

The curvature energy density is not really an energy density at all, just a way of rewriting part of the Friedmann equation to make everything look like an energy density. In the present day this term is negligible, so we can ignore it.

In the present universe, we thus have a roughly comparable vacuum and matter energy density. This conspires to force the curvature to be very small. It is unclear why we are at such a special time in the universe, as $\rho_{\text{mat}} \propto a^{-3}$ and evolves rapidly while ρ_{vac} is constant.

2 The Future of the Universe

Presently, our understanding of dark energy and the shape of the universe is insufficiently precise to distinguish between the possible endings of the universe. The most likely scenarios are probably either the big rip or the big freeze.

2.1 Big Rip

For some kinds of dark energy, the contribution of the dark energy density will actually cause the scale factor to diverge at a finite time. The important property is the

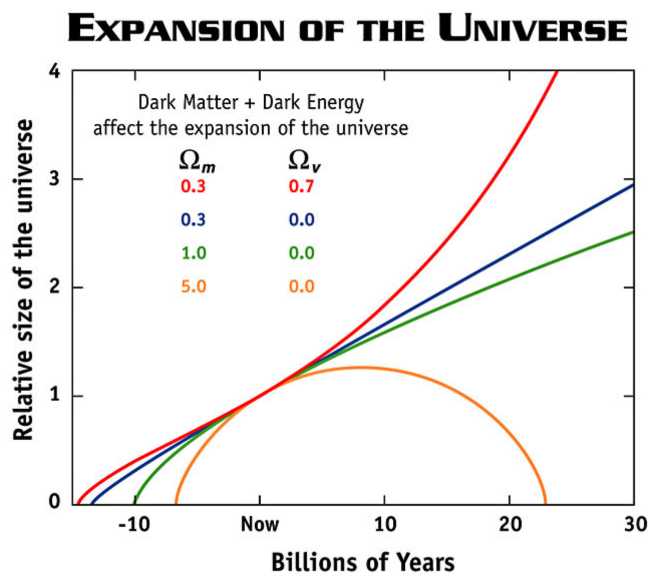

Figure 1: Historically, we thought that the possible outcomes would be either a crunch or eternal inflation. These two possibilities, determined by the balance of the vacuum and matter energy densities, are illustrated here. The Ω's quoted are simply the energy densities divided by the critical density. From http://map.gsfc.nasa.gov/universe/uni_fate.html

ratio

$$w = \frac{p}{\rho} \tag{5}$$

of the pressure to energy density. For $w < -1$, a big rip will occur at

$$t - t_0 \approx \frac{2}{3|1-w|H_0\sqrt{1-\rho_{\mathrm{mat}}/\rho_{\mathrm{crit}}}} \tag{6}$$

Using realistic values of the Hubble constant and matter energy density along with the arbitrary choice $w = -1.5$, this formula predicts the big rip in about 22 billion years. Note that a cosmological constant has $w = -1$.

2.2 Big Freeze

If dark energy is due to the cosmological constant then $w = -1$ and the universe will eventually be vacuum energy dominated. When the other terms in (4) become negligible, the scale factor becomes exponential in time:

$$a(t)^2 = e^{Ht} \tag{7}$$

As $t \to \infty$, the universe cools. The CMB and CνB are redshifted to arbitrarily low energies. The density of matter becomes too small to sustain stellar genesis and the stars eventually all burn away. In GUT theories, the protons eventually decay and stellar remnants dissipate. Only those stars which evolved into black holes leave lasting marks as the universe becomes black hole dominated. Even black holes will eventually evaporate as the universe becomes uniform and matter densities become arbitrarily small.

2.3 Big Crunch

In a big crunch scenario, the gravitational attraction of matter causes the universe to eventually collapse back in on itself. The end state of the universe would then be something like the initial state in which $a \to 0$. Current cosmological data suggests that the universe is not just expanding, but that its expansion is accelerating. This makes a big crunch unlikely.

2.4 Big Bounce

The big bounce is a cyclic model of the universe. If the universe eventually experiences a big crunch, but quantum effects prevent a complete collapse back to $a = 0$, a big bang could follow. Then another crunch, another bang, ad infinitum. Since it doesn't look good for the crunch, this scenario is also unlikely.

3 The Cosmological Constant Problem, the Multiverse and the Anthropic Principle

Supposing that some unknown physics corrects the QFT prediction for cosmological constant. In order to reproduce the correct value, the correction has to agree with the QFT vacuum energy up to the 120$^{\text{th}}$ decimal place. This is called fine tuning, and it's fine tuning to a degree far more severe than some other areas of physics (such as the Higgs or neutrino masses).

One of the predictions from inflation is that there should be causally disconnected regions of spacetime, i.e. other universes. The collection of these is called the multiverse. At the same time, string theory has a "problem" in that the vacuum of string theory appears to be non-unique. If we combine these ideas, we can imagine that each universe might have a different set of physical constants covering all of the possibilities. But how likely is it that we find ourselves in the universe we see? Why not in one of the many, many other possibilities?

Many physicists are now beginning to argue that an additional selection principal is at work. The very fact that we are here to learn about physics requires that we are in a special universe that supports intelligent life. Thus we select only from the possible universes in which we could exist. this is called the anthropic principal. It is currently our best explanation for the small value of the cosmological constant, and could also explain other fine tuning problems of physics. Unfortunately, it seems to be ultimately untestable.

4 The Measure Problem

In the last section we asked an innocent question about probabilities. It turns out that probabilities are a bit hard to define in infinite systems.

Quick Question: What do you think the problems are? Can you think of ways around them?

The main problem is that any event with a finite probability per volume will occur an infinite number of times. Infinities are tricky animals. As an example, let's compare the number of positive integers to the number of integers. Naively, it seems like there are about two integers for every positive integer. But I can place the negative and positive integers in one-to-one correspondence[2]. This means that the sets are the same size. Hmmm...

Quick Question: Which is bigger, the set of all integers or the set of all rational numbers?

Of course I could also map two positive integers to each integer, which makes it look like there are more of them. Even very rare events will happen an infinite number of times in the multiverse, so there is no notion of relative probabilities that can be formed by taking ratios. Normally, we can define a probability as

$$P(A) = \frac{\text{number of occurences of } A}{\text{number of chances for } A \text{ to occur}}$$

Of course in the multiverse this gives $\frac{\infty}{\infty}$ which is indeterminate. This is roughly what physicists call the *measure problem*.

[2] In other words, there exists a bijection from $\mathbb{Z}$ to $\mathbb{Z}^+$.