The mystery of the quantum cakes

P. G. Kwiat
Physics Division, P-23, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

L. Hardy
Clarendon Laboratory, Department of Physics, University of Oxford, Oxford OX1 3PU, United Kingdom

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In an attempt to make the concept and consequences of quantum mechanical entanglement more accessible to the non-physicist, we present a simple ''real-world'' explanation of the proof of quantum mechanical nonlocality without the use of inequalities.

I. INTRODUCTION

As one of the cornerstones of quantum mechanics (QM), and thus of modern science, it is a bit unfortunate that the concept of *entanglement* is so difficult to grasp, even for physicists, and even more difficult to convey to the non-physicist. Described by Schrödinger as that aspect of quantum mechanics ''that enforces its entire departure from classical lines of thought,''[1] entangled states are most often associated with nonlocality (although there are other ways in which quantum mechanics could be said to be nonlocal[2]). In particular, it was considering the position/momentum entangled state of two particles that led to the contention by Einstein, Podolsky, and Rosen that quantum mechanics might not be a complete description of nature.[3] Although initially the preference between the ''spook-like action at a distance'' of QM and a more intuitive ''hidden-variable'' model (that preserved local realism) was a philosophical one, in 1964 John Bell showed that the two theories gave different predictions for certain gedanken experiments.[4] The well-known Bell inequalities set limits on the *statistical* correlations that could be found between two particles described by any local realistic theory; quantum mechanics was predicted to violate these inequalities. Starting with the work of Clauser *et al.*,[5,6] the ideal inequalities of Bell were modified to relate to real-world experiments. Since then, there have been very many tests of the inequalities,[7] with the vast majority showing good agreement with QM, and, with the inclusion of some supplementary assumptions to account for low detection efficiency and slow or nonrandom switch settings,[8] have disproved local realistic theories.

However, even allowing for the auxillary assumptions necessary to date, the results may still seem somewhat unsatisfying to the non-physicist, for the violation appears only at the statistical level, and then only after a fair amount of logical reasoning. While we in no way dispute the validity of the logic, it has in practice been rather difficult to explain it to the casually interested party, this despite some very nice pedagogical articles published on the topic.[9]

More recently, Greenberger, Horne, and Zeilinger have demonstrated the inconsistency of quantum theory and local realism *without* the need for statistics;[10] however, their arguments require *three* correlated particles, which is also rather complicated to explain in terms of some sort of classical analog. For this reason, the recent work by Hardy,[11] and others,[12,13] in which the contradiction of quantum theory and local realism is shown for only two particles without the need for inequalities,[14] has great utility in trying to bring quantum mechanics to a wider audience. And the arguments themselves that lead to the inconsistency can be made remarkably simple. Nevertheless, there remains a difficulty in conveying them to non-physicists, who in general do not have a firm understanding of polarization, not to mention spin! At least, it has been our experience that it is difficult to explain nonlocal correlations when the listener does not grasp what the correlations are in. (The kind but bewildered listener will often simply feign an understanding; however, this ruse is easy to detect, for they will not display the appropriate amount of consternation at the final result!)

Mermin attempted to circumvent this problem by reducing the discussion to correlations between lights on boxes with switches.[13] While his article was in fact crucial for the final understanding of the phenomenon by one of us (PGK), we have found that the average person may not take too easily to the mysterious boxes. Again, they are rather far removed from everyday experience.

With all this in mind, at a conference three summers ago we undertook to construct a ''real-world'' example that might be useful in explaining to the layperson just how strange the correlations of entangled states actually are. The primary obstacle was coming up with reasonable noncommuting observables, as these are not familiar to most non-physicists. The result, involving pairs of entangled *cakes*, is presented below. Of course, since we will employ everyday articles, like cakes and ovens, such a system could never be realized in practice. However, there is a direct correspondence between our description and, for example, one based on photon polarizations. We will discuss this briefly after the main argument.

II. THE QUANTUM KITCHEN

Consider, then, the situation depicted in Fig. 1. We have a kitchen with two opposing doors, out of which come conveyor belts, and on the belts come pairs of ovens, one to each side. There is an experimenter on each side, call them Lucy (left side) and Ricardo (right), who will make measurements on the ovens; later the two will come together to compare their results. In particular, there are two types of measurements that can be made on a given oven. The tester could wait until the oven reaches the end of the conveyor belt before opening it. Inside, he/she finds a cake, which can then be tested to see whether it tastes Good or Bad. This is one observable, the taste of the cake. Alternatively, the tester can open the oven midway on its journey, to see whether or not the batter has Risen or Not Risen early, the second observable. Assuming we have some sort of soufflé, it is easy to justify why these measurements might be noncommuting—

Fig. 1. Lucy and Ricardo explore nonlocal correlations through quantum mechanically (non-maximally) entangled cakes. Because Ricardo's first cake (far right) rose early, Lucy's cake tastes good.

re-closing the oven in the middle will cause the cake to collapse, and the result will always be a poor cake (perhaps even worse than it would naturally have been). Hence, only *one* of these qualities can be measured on a given cake.

Each experimenter will randomly decide which measurement they will make, and record the results obtained. Comparing the records later on will reveal the strangeness which arises if the cakes are quantum mechanically correlated. There are three main classes to consider, depending on whether Lucy and Ricardo both opened their respective ovens in the middle, one waited until the end to do so, or both did. Below, we describe the results which would be obtained (assuming the cakes are correctly described by a particular quantum mechanical (entangled) state, which we will write down in the Appendix).

#1. In cases where Lucy and Ricardo both checked their ovens midway, they find that 9% of the time, both cakes rose early. The rest of the time, only one or neither did.

In cases where one checked midway and the other waited:

#2. whenever Lucy's cake rose early, Ricardo's tasted good; and

#2′. whenever Ricardo's cake rose early, Lucy's tasted good.

Note that these correlated results lead us to postulate some congruence in the cakes' histories, i.e., perhaps they came from the same batter. Given this, it is then easy to motivate the results #2 and #2′, for it is not unreasonable that cakes which come from an early rising batter will necessarily taste good. (However, a cake may taste good anyway, even if the batter did not rise early.)

Finally, we ask what to expect if both Lucy and Ricardo performed taste-tests on their respective cakes. Consider the 9% of cases where both cakes *would have* been seen to rise early (had Lucy and Ricardo made those measurements instead). Here we are considering what *would have* happened if they had measured one thing even though they actually measure another. Since (in this 9% of cases) Lucy's cake would have risen early, #2 implies that Ricardo's cake will taste good. Likewise, since (again in this 9% of cases) Ricardo's cake would have risen early, #2′ implies that Lucy's cake will taste good. Hence, on the basis of this reasoning, we expect that both cakes will taste good in at least 9% of cases. Somewhat shockingly, the quantum mechanical result (which we are fairly confident is the correct one) is that

#3. *both cakes NEVER taste good*!

That is, at least one of the cakes always tastes Bad. If one starts with #2, #2′, and #3 as the basic conditions, the contradiction between QM and local realism arises as soon as one event from class #1 is observed. In this way one claims to have a nonstatistical violation of the assumptions of local realism.[14]

In explaining the correlations with cakes it must be emphasized that this is only an analog. These correlations could not actually be realized with cakes. Nevertheless, precisely analogous correlations can be seen in measurements on quantum particles. For example, we could take two photons that are (nonmaximally) entangled with respect to their polarization. In the cake example we considered measurements of two properties of the cake—the taste and whether it had risen. In the case of a polarization entangled state, these two measurements would correspond to measurements of polarization in two different bases. Such a measurement could be performed by having a rotatable polarizing beamsplitter with two detectors, one placed in each output port. When this polarizing beamsplitter is oriented at one angle, a click at one detector would correspond to the cake tasting Good and a click at the other detector would correspond to the cake tasting Bad. When the polarizing beamsplitter is rotated to another angle, a click at one detector would correspond to the cake having Risen, and a click at the other detector would correspond to the cake Not having Risen. With these correspondences understood, if we identify, for example, Good and Bad with Horizontal (0°) and Vertical (90°) polarization, and Risen and Not Risen with linear polarizations at $-50.8°$ and 39.2°, the predictions #1, #2, #2′, and #3 will hold exactly, assuming one starts with the nonmaximally entangled state given in the Appendix. Experiments have actually been performed[16] to test the above predictions by employing photon pairs postselected in a nonmaximally entangled state.[17] These experiments successfully demonstrated that quantum mechanics does have the above properties to within some experimental uncertainties.[14] More recently, a method for *direct* production (i.e., without postselection) of the necessary quantum state has been implemented; a very large (122σ) disagreement with the predictions of a local theory was observed.[17]

III. DISCUSSION

The implicit arguments that lead one to predict two good cakes $\geqslant$9% of the time are so straightforward it can be difficult to see where one could have gone astray. We can imagine that the chef in the kitchen is intent on trying to simulate the results using classical cakes, i.e., with a local realistic model. In order to ensure outcome #1, 9% of the time he might specifically use quick-rising batter for both of the cakes. (The other 91% he will never use this batter for both cakes simultaneously.) In order to prevent the occurrence of Good–Good events (condition #3), the chef will need to be a bit creative. For example, if the quick-rising batter necessarily yielded Bad-tasting cakes, then condition #3 could be satisfied. However, we see that if both #2 and #2′ are to hold as well, the final taste of the cake on one side can depend on whether the experimenter on the *other* side checked his/her oven in the middle. In particular, considering only the 9% of cases for which the quick-rising batter was used for both cakes, if one of the experimenters measured in the middle, the other cake would *have* to be Good tasting (to satisfy #2); but if the same experimenter instead tasted his/her cake, then at least one of the cakes will have to come out tasting Bad (to satisfy #3).

In our earlier arguments we implicitly denied such nonlocal influences. That is, we assumed that events at one end are uninfluenced by random choices and events at the other end (e.g., Lucy's cake *would have* tasted the same, regardless of what measurement Ricardo made, and what he observed). But this is precisely where QM entanglement differs from classical correlations—the results on one side of the experiment can depend on the results obtained on the other side, even though the experimental regions may be space-like separated. Nevertheless, we stress that there is still no way to send superluminal *signals* via the cakes because only a fraction of the pairs display the correlations; specifically, one can show that the net probability for Lucy to measure a particular result is independent of what measurement Ricardo makes, and vice versa.[18]

For these results to force nonlocality upon us, it is of course necessary that the conveyor belts be very long and be moving very quickly, so that no measurement made by Lucy could affect the result obtained by Ricardo (and vice versa), unless there were some influence that propagated faster than the speed of light. This feature can be hinted at in the above example, if one postulates that the act of removing one of the cakes from its oven in order to taste it caused a noise or vibration that could propagate through the air or the conveyor belt structure, and thereby cause the other cake to collapse/taste bad. Clearly, to make this impossible, one would need to have the measurement regions separated farther than the ''sound-like'' interval; otherwise, the remarkable lack of Good–Good events would have a perfectly normal explanation, and nonlocality would not be a logical necessity. Similarly, unless the measurements are farther separated than the ''*light*-like'' interval, a chef's accomplice that traveled with each cake could simply watch to see whether the other oven was opened midway or not, and adjust his cake's quality accordingly. Finally, to eliminate the possibility that the peculiar results were somehow *pre*-engineered by the chef, it is necessary that he not know what measurements will be made on a given oven, i.e., that the choice of measurement observable be random.

One can also see how, in a real experiment, it is important to look at as many of the ''cakes'' as possible. In the simplest possible argument, if we were to measure only 91% of the pairs, and still see no Good–Good events, it is possible that the predicted 9% were just those that we did not measure. Obviously, this would require a rather peculiar sampling of the cakes. In real Bell inequality experiments thus far, some sort of ''fair sampling'' assumption was invoked, that the fraction of particles detected was a representative sample of the entire ensemble.[6,8]

Finally, the example gives a venue to discuss ultra-nonlocal theories,[19] in which the occurrence of the case #1 events (both cakes rising early) would be *greater* than 9%, despite condition #'s 2–3 still holding. (It can be shown that, with QM, 9.017% is the upper limit possible.[11,12]) If in some theory there were more than 50% of these case #1 events, one would be forced to accept what has sometimes been referred to as ''strong'' nonlocality: the probability of at least some of the results on at least one side would depend on which sort of measurement was made on the other.[18,20] In this case, one could actually use the nonlocal correlations to send superluminal signals. Curiously, there is a fair amount of freedom in terms of conjecturing ultra-nonlocal theories whose sole constraints are condition #'s 2–3. For instance, even in the limiting case where the cakes are *always* found to rise early (i.e., case #1 happens 100% of the time), it is possible to assign probabilities (consistent with #'s 2, 2′, and 3) so that only one of the parties has a detectable change in his/her measurement probabilities, and then only for one of the types of measurements.[21] The middle ground between the quantum mechanical maximum of ~9% for case #1 events and the 50% maximum to prevent violations of relativistic causality is a very interesting one, deserving of more study.

We have presented a model ''real-world'' system with which to describe the nonlocality inherent in quantum mechanical entangled states. While we cannot/do not claim any new physics with this approach, it is our hope that it may make it easier to describe these mind-boggling results to the interested non-physicist.

APPENDIX

One QM state that will yield all of the predictions #1–3 is given by

$$|\psi\rangle = \tfrac{1}{2}|B_L\rangle|B_R\rangle - \sqrt{\tfrac{3}{8}}[|B_L\rangle|G_R\rangle + |G_L\rangle|B_R\rangle], \qquad (1)$$

where B and G are the Good- and Bad-tasting eigenstates, which are related to the R (Risen) and N (Not risen) eigenstates by

$$\begin{aligned} |B\rangle &= \sqrt{\tfrac{2}{5}}|N\rangle + \sqrt{\tfrac{3}{5}}|R\rangle, \\ |G\rangle &= -\sqrt{\tfrac{3}{5}}|N\rangle + \sqrt{\tfrac{2}{5}}|R\rangle. \end{aligned} \qquad (2)$$

Because there is no $|G_L\rangle|G_R\rangle$ term in (1), condition #3 is automatically satisfied. And by substituting the expansions for $|B_L\rangle$ and $|G_L\rangle$ into (1), one can see that the two $|R_L\rangle|B_R\rangle$ terms cancel, implying condition #2; similarly for #2′. Finally, it is simple algebra to verify that the amplitude of the $|R_L\rangle|R_R\rangle$ is -0.3, resulting in 9% of pairs rising.

[1]E. Schrödinger, ''Discussion of probability relations between separated systems,'' Proc. Cambridge Philos. Soc. **31**, 555–563 (1935).

[2]Other examples are the ''collapse'' of the wave function when a photon is detected in one or the other output port of a beamsplitter; and the Aharanov–Bohm effect, in which a particle ''knows'' about enclosed magnetic flux, even though the actual magnetic field can be strictly zero in all regions the particle traverses.

[3]A. Einstein, B. Podolsky, and N. Rosen, ''Can quantum-mechanical description of physical reality be considered complete?'' Phys. Rev. A **47**, 777–780 (1935).

[4]J. S. Bell, ''On the Einstein–Podolsky–Rosen paradox,'' Physics **1**, 195–200 (1964), reprinted in J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge U.P., Cambridge, 1987).

[5]J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, ''Proposed experiment to test local hidden-variable theories,'' Phys. Rev. Lett. **23**, 880–884 (1969); J. F. Clauser and M. A. Horne, ''Experimental consequences of objective local theories,'' Phys. Rev. D **10**, 526–535 (1974).

[6]J. F. Clauser and A. Shimony, ''Bell's theorem: Experimental tests and implications,'' Rep. Prog. Phys. **41**, 1881–1927 (1978).

[7]See, for example, Ref. 6, for a review of experiments through 1977; M. Redhead, *Incompleteness, Nonlocality, and Realism* (Clarendon, Oxford, 1987), pp. 107–113, for experiments through 1987; A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, ''Quantum optical tests of the foundations of physics,'' in the *Atomic, Molecular, & Optical Physics Handbook* (AIP Press, New York, 1996), pp. 907–909, for many of the tests through 1995;

P. G. Kwiat *et al*., ''New high intensity source of polarization-entangled photon pairs,'' Phys. Rev. Lett. **75**, 4337–4341 (1995); D. V. Strekalov *et al*., ''Postselection-free energy-time entanglement,'' Phys. Rev. A **54**, R1–4 (1996); W. Tittel *et al*., ''Violation of Bell inequalities by photons more than 10 km apart,'' Phys. Rev. Lett. **81**, 3563–3566 (1998); and P. G. Kwiat *et al*., ''Ultrabright source of polarization-entangled photons,'' Phys. Rev. A **60**, R773–R776 (1999).

[8]Work is currently in progress at several institutions to perform a so-called ''loophole-free'' test of Bell's inequalities, that would not require the supplementary assumptions present in every experiment thus far. One remarkable exception is the recent experiment by G. Weihs *et al*. [''Violation of Bell's inequality under strict Einstein locality conditions,'' Phys. Rev. Lett. **81**, 5039–5043 (1998)], in which all measurements were performed with truly space-like separations.

[9]For example, N. D. Mermin, ''Is the moon there when nobody looks? Reality and the quantum theory,'' Phys. Today , 38–47 (Apr. 1985); J. S. Bell, ''Bertlmann's socks and the nature of reality,'' reprinted in J. S. Bell, in Ref. 4.

[10]D. M. Greenberger, M. Horne, and A. Zeilinger, ''Going beyond Bell's theorem,'' in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe* (Kluwer, Dordrecht, The Netherlands, 1989), pp. 73–76; D. M. Greenberger, M. Horne, A. Shimony, and A. Zeilinger, ''Bell's theorem without inequalities,'' Am. J. Phys. **58**, 1131–1143 (1990).

[11]L. Hardy, ''Nonlocality for two particles without inequalities for almost all entangled states,'' Phys. Rev. Lett. **71**, 1665–1668 (1993).

[12]T. F. Jordan, ''Testing Einstein–Podolsky–Rosen assumptions without inequalities with 2 photons or particles with spin 1/2,'' Phys. Rev. A **50**, 62–66 (1994).

[13]N. D. Mermin, ''Quantum mysteries refined,'' Am. J. Phys. **62**, 880–887 (1994).

[14]Of course, as has been pointed out by many authors (Refs. 13 and 15), any physical test (Refs. 16 and 17) of the Hardy logic will inevitably lead to an inequality (perhaps not surprisingly, it is a version of the original Bell inequality). For example, even in a perfect, noiseless system, one can never prove the *non*existence of a certain class of events to better than $\sim 1/N$, where N is the number of times one looked for the event.

[15]A. Garuccio, ''Hardy's approach, Eberhard's inequality, and supplementary assumptions,'' Phys. Rev. A **52**, 2535–2537 (1995).

[16]J. R. Torgerson, D. Branning, C. H. Monken, and L. Mandel, ''Experimental demonstration of the violation of local realism without Bell inequalities,'' Phys. Lett. A **204**, 323–328 (1995); G. Digiuseppe, F. De Martini, and D. Boschi, ''Experimental test of the violation of local realism in quantum-mechanics without Bell inequalities,'' Phys. Rev. A **56**, 176–181 (1997); D. Boschi, S. Branca, F. De Martini, and L. Hardy, ''Ladder proof of nonlocality without inequalities: Theoretical and experimental results,'' Phys. Rev. Lett. **79**, 2755–2758 (1997).

[17]A. G. White *et al*., ''Non-maximally entangled states: production, characterization, and utilization,'' Phys. Rev. Lett. (to appear, 1999); see also the last article in Ref. 7.

[18]N. D. Mermin, ''The best version of Bell's Theorem,'' in *Fundamental Problems in Quantum Theory* [Ann. (N.Y.) Acad. Sci. **755**, 616–623 (1995)].

[19]S. Popescu and D. Rohrlich, ''Quantum nonlocality as an axiom,'' Found. Phys. **24**, 379–385 (1994).

[20]The probabilities of the various results at the 50% threshold are $P(R,R) = P(N,N) = P(G,B) = P(B,G) = P(R,G) = P(N,B) = P(G,R) = P(B,N) = 1/2$, with all other possibilities zero (''R'', ''N'', ''G'', and ''B'' stand for Risen, Not Risen, Good, and Bad, respectively); clearly, with these probabilities no measurements performed on one side can reveal the type of measurement performed by the other experimenter.

[21]One example would have $P(R,R) = P(R,G) = P(G,R) = P(G,B) = 1$, and all other probabilities zero. Here, Lucy's cakes always Rise early and always taste Good, independent of Ricardo's measurement, and Ricardo only has Risen cakes; but the taste of his cakes is completely determined by Lucy's type of measurement. For the case $P(R,R) = P(R,G) = P(G,R) = 1$; $P(G,B) = P(B,G) = 1/2$; and all other probabilities zero, the nonlocality shows up in the tastes of *both* parties' cakes, but not deterministically, i.e., tasting a single cake would not necessarily reveal whether the ''sender'' had measured in the middle or the end.

OPEN-MINDEDNESS

When he finished his pacing, Bell sat down and said, ''Perhaps I did something to rekindle interest in these questions. People who are younger than me now tend to agree that there are problems to be solved. Of course, most of them don't tackle these problems. They rather work on lines in elementary-particle physics like string theory. But they are generally more open to the idea that there are problems with the foundations of the quantum theory than their teachers were.''

Jeremy Bernstein , ''John Stewart Bell: Quantum Engineer,'' in *Quantum Profiles* (Princeton University Press, Princeton, 1991), p. 86.