On-Device AI 실습: Quantization for CNN

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Overview

1. **Uniform Quantization**
   – Linear Quantization, Per-channel Quantization, Quantized inference

2. **Non-uniform Quantization**
   – K-Means Quantization, Quantization-aware Training

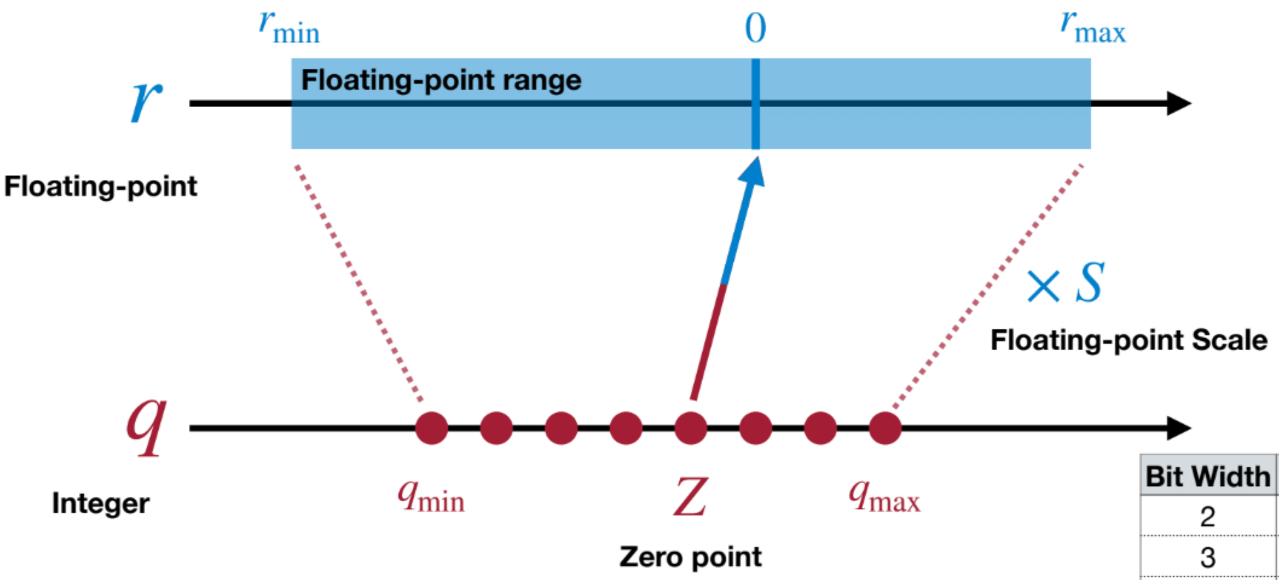
3. **Quantization with PyTorch API**
   – Post-Training Quantization, Quantization Aware Training

Setup

- 실습 자료 "Quantization for CNN.ipnyb"을 colab에서 실행해주세요
- Colab 런타임을 GPU(T4)로 설정해 주세요
- Setup 코드 셀을 실행해 필요한 패키지를 설치해주세요

Linear Quantization

An affine mapping of integers to real numbers $r = S(q - Z)$



Bit Width	q_{min}	q_{max}
2	-2	1
3	-4	3
4	-8	7
N	-2^{N-1}	$2^{N-1}-1$

Linear Quantization

An affine mapping of integers to real numbers $r = S(q - Z)$

weights (32-bit float)	quantized weights (2-bit signed int)	zero point (2-bit signed int)	scale (32-bit float)

$$\begin{bmatrix}2.09 & -0.98 & 1.48 & 0.09\\ 0.05 & -0.14 & -1.08 & 2.12\\ -0.91 & 1.92 & 0 & -1.03\\ 1.87 & 0 & 1.53 & 1.49\end{bmatrix} \Rightarrow \left(\begin{bmatrix}1 & -2 & 0 & -1\\ -1 & -1 & -2 & 1\\ -2 & 1 & -1 & -2\\ 1 & -1 & 0 & 0\end{bmatrix} - -1 \right) \times 1.07$$

$$r = (q - Z) \times S$$

Floating-point | **Integer** | **Integer** | **Floating-point**

Integer (Z):
- quantization parameter
- allow real number $r=0$ be exactly representable by a quantized integer Z

Floating-point (S):
- quantization parameter

[실습1] Linear Quantization 함수 구현

$$q = r/s + z$$

quantized weights (2-bit signed int)		weights (32-bit float)		scale (32-bit float)		zero point (2-bit signed int)

$$\begin{bmatrix} 1 & -2 & 0 & -1 \\ -1 & -1 & -2 & 1 \\ -2 & 1 & -1 & -2 \\ 1 & -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2.09 & -0.98 & 1.48 & 0.09 \\ 0.05 & -0.14 & -1.08 & 2.12 \\ -0.91 & 1.92 & 0 & -1.03 \\ 1.87 & 0 & 1.53 & 1.49 \end{bmatrix} \div 1.07 + -1$$

$$q = r \div S + Z$$

Integer | Floating-point | Floating-point | Integer

[실습1] Linear Quantization 함수 구현

$$q = r/s + z$$

quantized weights (2-bit signed int)		weights (32-bit float)		scale (32-bit float)		zero point (2-bit signed int)
$\begin{bmatrix} 1 & -2 & 0 & -1 \\ -1 & -1 & -2 & 1 \\ -2 & 1 & -1 & -2 \\ 1 & -1 & 0 & 0 \end{bmatrix}$	$=$	$\begin{bmatrix} 2.09 & -0.98 & 1.48 & 0.09 \\ 0.05 & -0.14 & -1.08 & 2.12 \\ -0.91 & 1.92 & 0 & -1.03 \\ 1.87 & 0 & 1.53 & 1.49 \end{bmatrix}$	$\div$	**1.07**	$+$	**-1**
q	$=$	r	$\div$	S	$+$	Z
Integer		Floating-point		Floating-point		Integer

[실습1] Linear Quantization 함수 구현

$$q = int(round(r/s)) + z$$

quantized weights (2-bit signed int)

1	-2	0	-1
-1	-1	-2	1
-2	1	-1	-2
1	-1	0	0

=

weights (32-bit float)

2.09	-0.98	1.48	0.09
0.05	-0.14	-1.08	2.12
-0.91	1.92	0	-1.03
1.87	0	1.53	1.49

÷ scale (32-bit float) **1.07**

\+ zero point (2-bit signed int) **-1**

$$q = r \div S + Z$$

Integer = **Floating-point** ÷ **Floating-point** + **Integer**

Integer

Answer


```
################# YOUR CODE STARTS HERE #################
# Step 1: fp_tensor를 scale 하세요.
scaled_tensor = fp_tensor / scale
# Step 2: 부동 소수점 값을 정수 값으로 rounding 하세요.
rounded_tensor = torch.round(scaled_tensor)
################# YOUR CODE ENDS HERE ###################

rounded_tensor = rounded_tensor.to(dtype)

################# YOUR CODE STARTS HERE #################
# Step 3: rounded_tensor를 zero_point 만큼 shift하여 영점을 조정합니다.
shifted_tensor = rounded_tensor + zero_point
################# YOUR CODE ENDS HERE ###################
```

Linear Quantization

An affine mapping of integers to real numbers $r = S(q - Z)$

weights (32-bit float)

2.09	-0.98	1.48	0.09
0.05	-0.14	-1.08	2.12
-0.91	1.92	0	-1.03
1.87	0	1.53	1.49

⇨ (quantized weights (2-bit signed int)

1	-2	0	-1
-1	-1	-2	1
-2	1	-1	-2
1	-1	0	0

− zero point (2-bit signed int) **-1**) × scale (32-bit float) **1.07** =

we will learn how to determine these parameters

reconstructed weights (32-bit float)

2.14	-1.07	1.07	0
0	0	-1.07	2.14
-1.07	2.14	0	-1.07
2.14	0	1.07	1.07

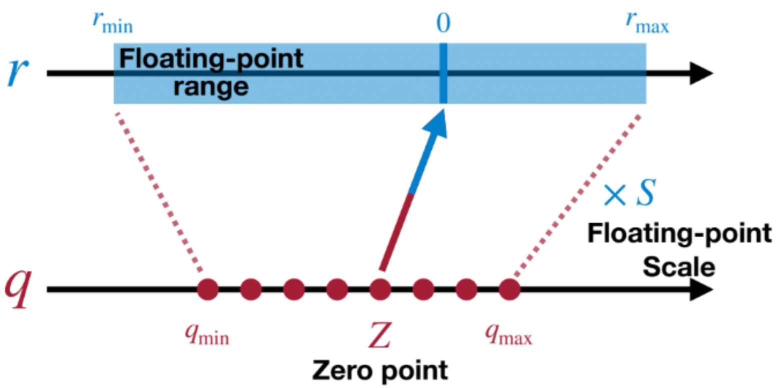
quantization error

-0.05	0.09	0.41	0.09
0.05	-0.14	-0.01	-0.02
0.16	-0.22	0	0.04
-0.27	0	0.46	0.42

Binary	Decimal
01	1
00	0
11	-1
10	-2

Linear Quantization - Scale

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



$$r_{max} = S\left(q_{max} - Z\right)$$

$$r_{min} = S\left(q_{min} - Z\right)$$

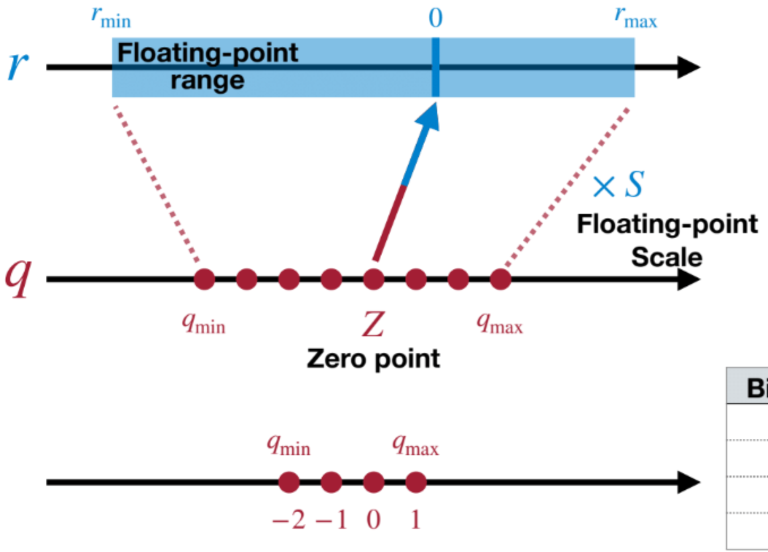
$$\downarrow$$

$$r_{max} - r_{min} = S\left(q_{max} - q_{min}\right)$$

$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

Linear Quantization - Scale

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



2.09	-0.98	1.48	0.09
0.05	-0.14	-1.08	2.12
-0.91	1.92	0	-1.03
1.87	0	1.53	1.49

Min & Max value of fp tensor

Binary	Decimal
01	1
00	0
11	-1
10	-2

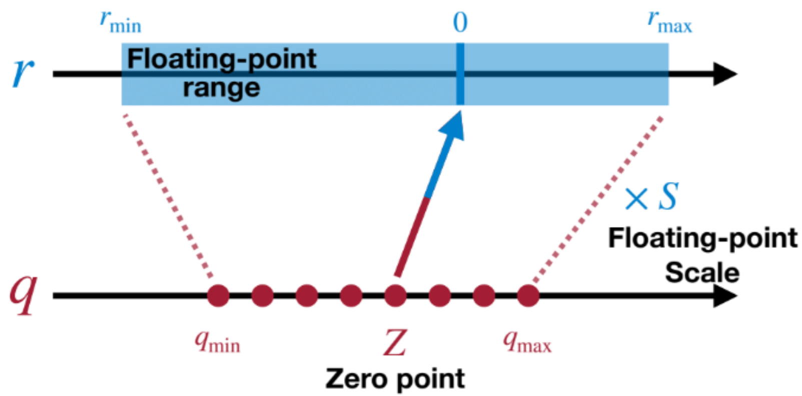
$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

$$= \frac{2.12 - (-1.08)}{1 - (-2)}$$

$$= 1.07$$

Linear Quantization – Zero Point

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



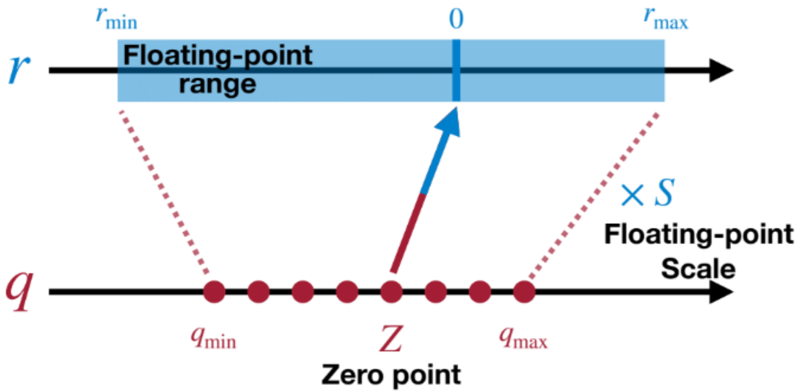
$$r_{\min} = S\left(q_{\min} - Z\right)$$

$$\downarrow$$

$$Z = q_{\min} - \frac{r_{\min}}{S}$$

Linear Quantization – Zero Point

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



$$r_{\min} = S\left(q_{\min} - Z\right)$$

$$\downarrow$$

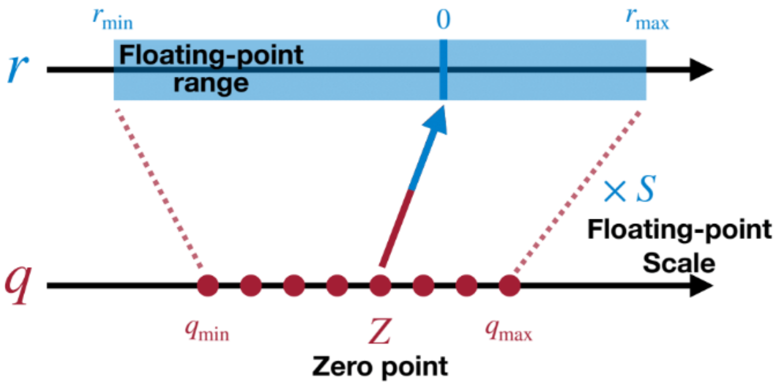
$$\underbrace{Z}_{\text{Integer}} = \boxed{\underbrace{q_{\min}}_{\text{Integer}} - \frac{\overbrace{r_{\min}}^{\text{Floating-point}}}{\underbrace{S}_{\text{Floating-point}}}}$$

$$\downarrow$$

$$Z = \text{round}\left(q_{\min} - \frac{r_{\min}}{S}\right)$$

[실습2] Scale and Zero Point 계산

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

$$Z = \text{round}\left(q_{min} - \frac{r_{min}}{S}\right)$$

Answer

```
################# YOUR CODE STARTS HERE #################
# hint: one line of code for calculating scale
# [VERY IMPORTANT] quantized_max - quantized_min = 2 ** bitwith - 1
scale = (fp_max - fp_min) / (quantized_max - quantized_min)
# hint: one line of code for calculating zero_point
zero_point = round(quantized_min - fp_min / scale)
################# YOUR CODE ENDS HERE ###################
```

Quantized Matrix Multiplication

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following matrix multiplication, when Zw=0.

$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

$$\mathbf{q_Y} = \boxed{\frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}}} \boxed{\left(\mathbf{q_W}\mathbf{q_X} - Z_\mathbf{W}\mathbf{q_X} - Z_\mathbf{X}\mathbf{q_W} + Z_\mathbf{W} Z_\mathbf{X}\right)} + \boxed{Z_\mathbf{Y}}$$

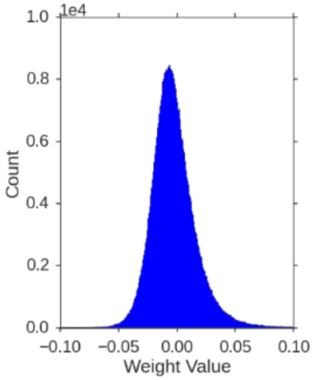
Precompute

N-bit Integer Multiplication
32-bit Integer Addition/Subtraction

N-bit Integer Addition

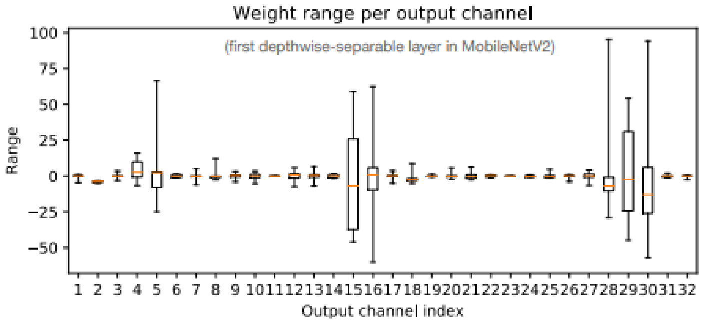
$$Z_\mathbf{W} = 0$$

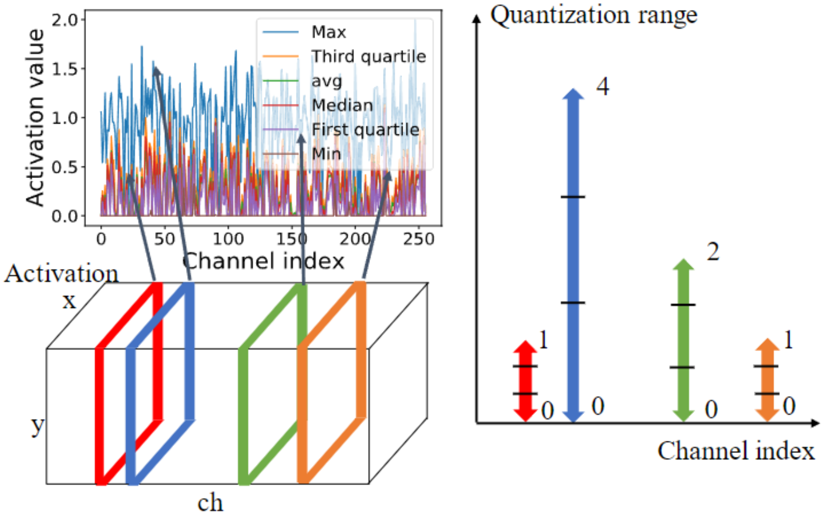
$$\mathbf{q_Y} = \boxed{\frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}}} \boxed{\left(\mathbf{q_W}\mathbf{q_X} - Z_\mathbf{X}\mathbf{q_W}\right)} + \boxed{Z_\mathbf{Y}}$$



Per-Channel Linear Quantization

- Different scaling factors **S** and zero points **Z** for **different output channels** will **perform better**





Quantized Inference

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$S_{\mathbf{Y}}\left(\mathbf{q_Y} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}\left(\mathbf{q_W} - Z_{\mathbf{W}}\right) \cdot S_{\mathbf{X}}\left(\mathbf{q_X} - Z_{\mathbf{X}}\right) + S_{\mathbf{b}}\left(\mathbf{q_b} - Z_{\mathbf{b}}\right)$$

$$\downarrow\ Z_{\mathbf{W}} = 0$$

$$S_{\mathbf{Y}}\left(\mathbf{q_Y} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}S_{\mathbf{X}}\left(\mathbf{q_W}\mathbf{q_X} - Z_{\mathbf{X}}\mathbf{q_W}\right) + S_{\mathbf{b}}\left(\mathbf{q_b} - Z_{\mathbf{b}}\right)$$

$$\downarrow\ Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}}S_{\mathbf{X}}$$

$$S_{\mathbf{Y}}\left(\mathbf{q_Y} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}S_{\mathbf{X}}\left(\mathbf{q_W}\mathbf{q_X} - Z_{\mathbf{X}}\mathbf{q_W} + \mathbf{q_b}\right)$$

Quantized Inference

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0 \downarrow Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}} S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}} + \mathbf{q}_{\mathbf{b}}\right)$$

$$\downarrow$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} + \underbrace{\mathbf{q}_{\mathbf{b}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}}_{\text{Precompute}}\right) + Z_{\mathbf{Y}}$$

$$\downarrow \mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} + \mathbf{q}_{bias}\right) + Z_{\mathbf{Y}}$$

[실습3] Quantized Fully-Connected Layer

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0$$

$$Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

$$\mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - Z_{\mathbf{X}} \mathbf{q}_{\mathbf{W}}$$

$$\mathbf{q}_{\mathbf{Y}} = \boxed{\frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}} \left(\boxed{\mathbf{q}_{\mathbf{W}} \mathbf{q}_{\mathbf{X}} + \mathbf{q}_{bias}} \right) + \boxed{Z_{\mathbf{Y}}}$$

N-bit Int Mult.
32-bit Int Add.

N-bit Int
Add

Note: both $\mathbf{q}_{\mathbf{b}}$ and $\mathbf{q}_{bias}$ are 32 bits.

Answer


```
################# YOUR CODE STARTS HERE #################
# Step 2: scale the output
output = output*(input_scale * weight_scale / output_scale)


# Step 3: shift output by output_zero_point
output = output + output_zero_point
################# YOUR CODE ENDS HERE ###################
```

[실습4] Quantized Convolution Layer

$$\mathbf{Y} = \text{Conv}\left(\mathbf{W}, \mathbf{X}\right) + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0$$

$$Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

$$\mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - \text{Conv}\left(\mathbf{q}_{\mathbf{W}}, Z_{\mathbf{X}}\right)$$

$$\mathbf{q}_{\mathbf{Y}} = \boxed{\frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}} \left(\boxed{\text{Conv}\left(\mathbf{q}_{\mathbf{W}}, \mathbf{q}_{\mathbf{X}}\right) + \mathbf{q}_{bias}}\right) + \boxed{Z_{\mathbf{Y}}}$$

N-bit Int Mult.
32-bit Int Add.

N-bit Int Add

Note: both $\mathbf{q}_{\mathbf{b}}$ and $\mathbf{q}_{bias}$ are 32 bits.

Answer

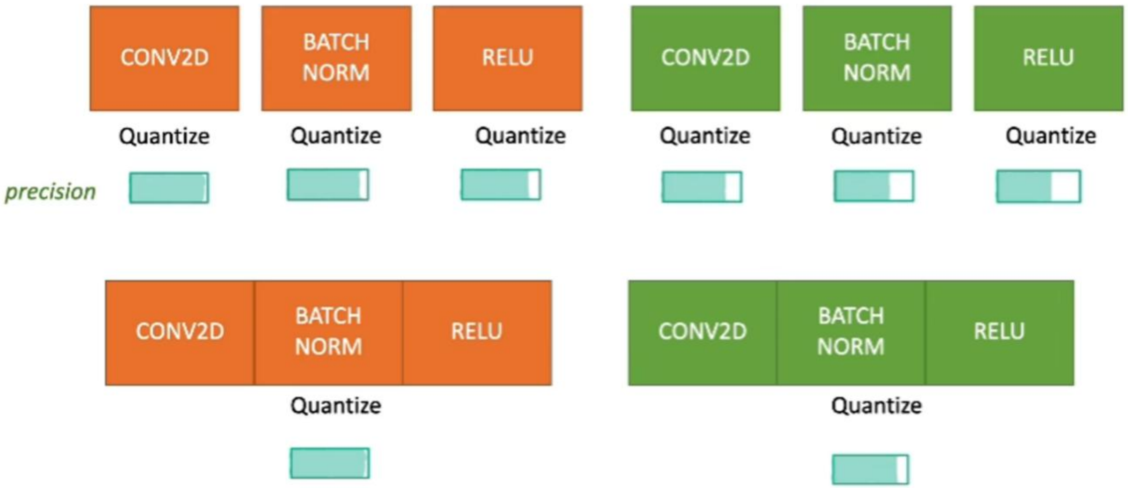

```
################ YOUR CODE STARTS HERE ################
# hint: 이번 코드 블록은 quantized_linear()와 매우 유사합니다.

# Step 2: output 텐서를 scale합니다.
#         hint:
output = output*(input_scale * weight_scale / output_scale)

# Step 3: output을 out_zero_point 만큼 이동하여 영점을 조정합니다.
output = output + output_zero_point
################ YOUR CODE ENDS HERE ##################
```

Model Fusion

- We can **fuse** a **BatchNorm** layer into its previous **convolutional layer**
- Fusing model can reduce the extra multiplication during inference



Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\ldots m}\}$;
Parameters to be learned: γ, β
Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m} (x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

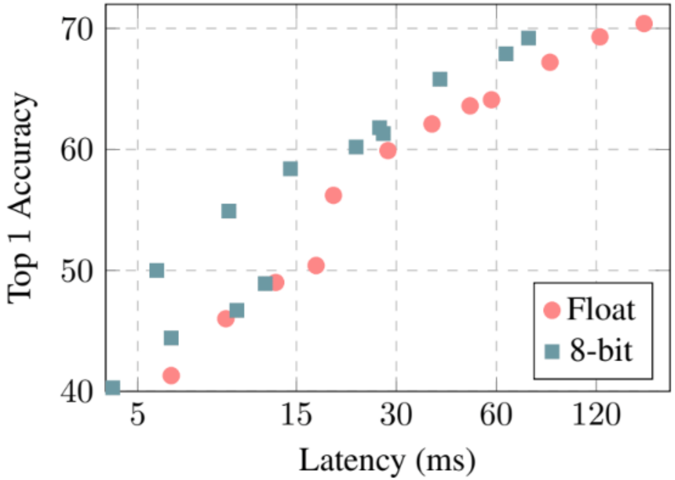
$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad \text{// normalize}$$

$$\boxed{y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i)} \qquad \text{// scale and shift}$$

Algorithm 1: Batch Normalizing Transform, applied to activation x over a mini-batch.

[실습5] Inference with quantized model

Neural Network	ResNet-50	Inception-V3
Floating-point Accuracy	76.4%	78.4%
8-bit Integer-quantized Acurracy	74.9%	75.4%



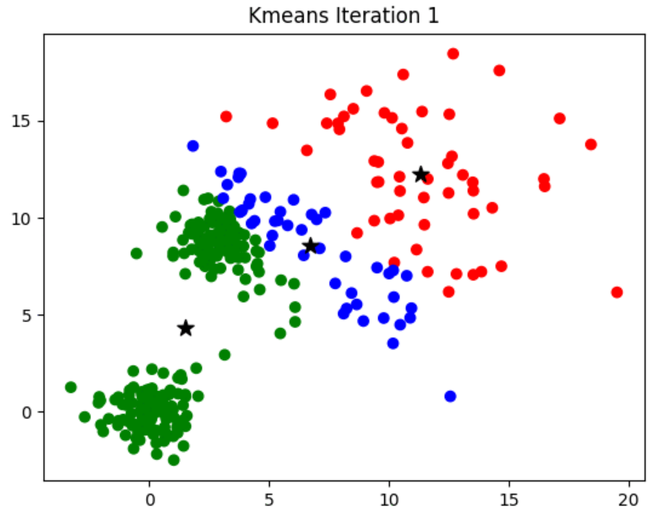
Latency-vs-accuracy tradeoff of float vs. integer-only MobileNets on ImageNet using Snapdragon 835 big cores.

Setup

- “실습 자료 2.2_Nonuniform.ipnyb”를 colab에서 실행해주세요
- Colab 런타임을 GPU(T4)로 설정해 주세요
- Setup 코드 셀을 실행해 필요한 패키지를 설치해주세요

K-Means Clustering

- Popular **unsupervised** learning algorithm to group data into K clusters
- Select K initial centroids and assign each data point into nearest centroids
- Update centroids [illegible]gned points



K-Means-based Weight Quantization

weights
(32-bit float)

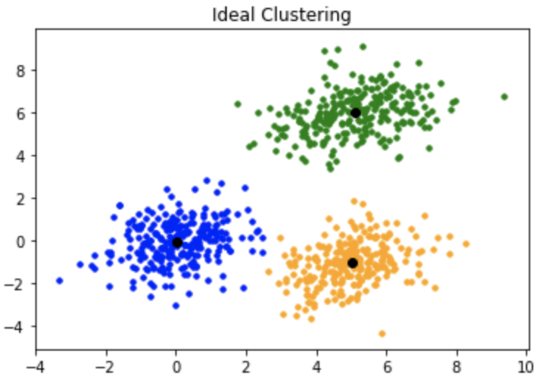
2.09	-0.98	1.48	0.09
0.05	-0.14	-1.08	2.12
-0.91	1.92	0	-1.03
1.87	0	1.53	1.49

~~2.09, 2.12, 1.92, 1.87~~

↓

2.0

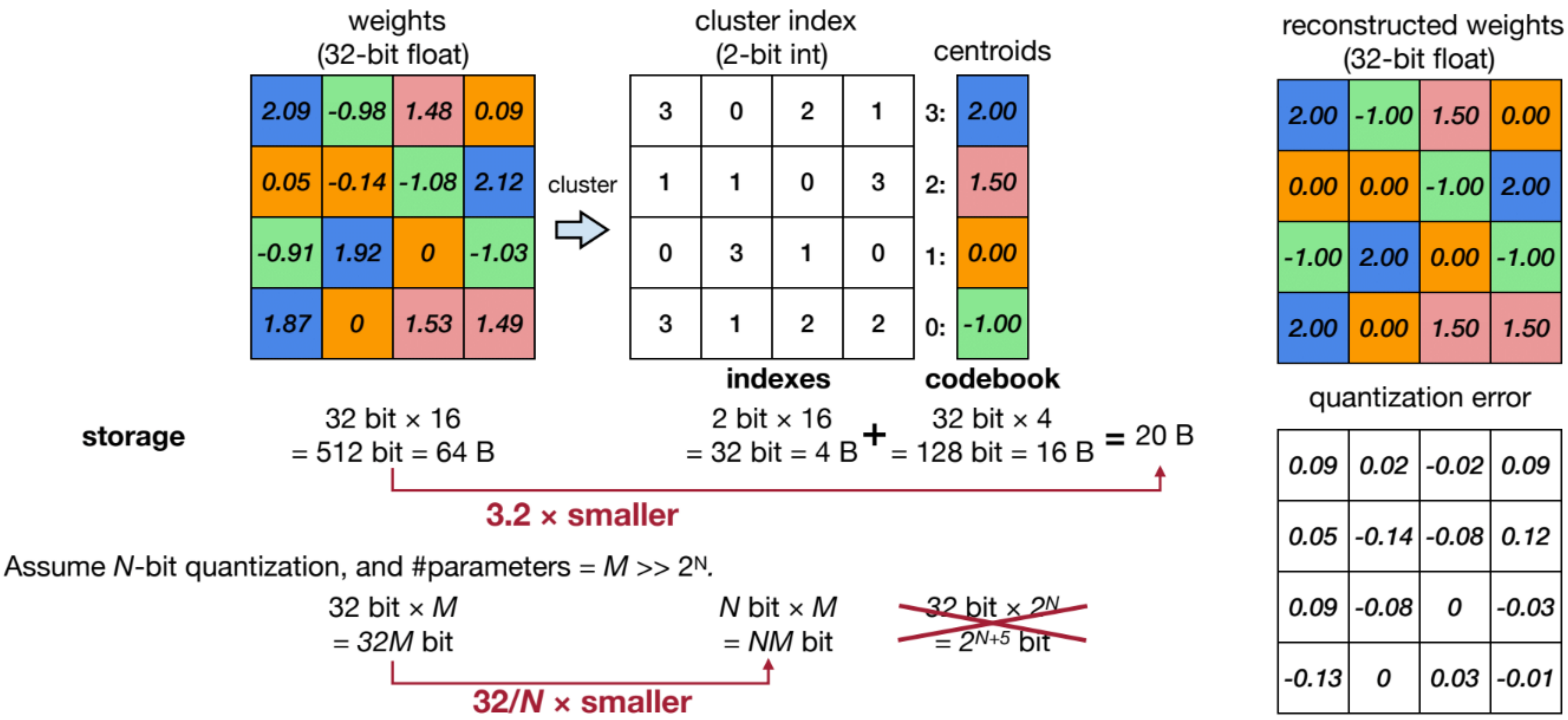
Cluster →



weights
(32-bit float)

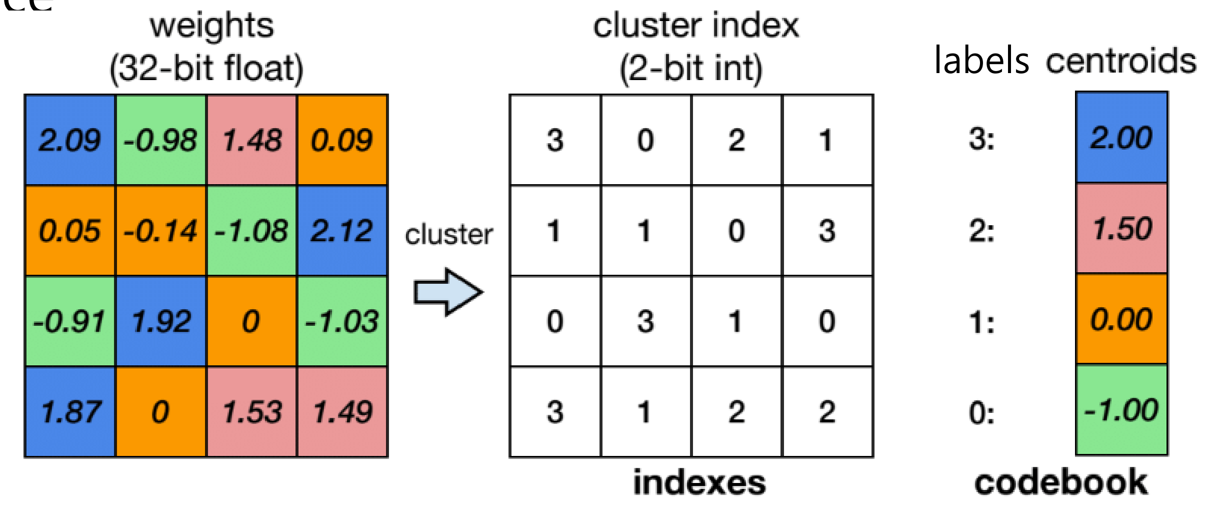
2.09	-0.98	1.48	0.09
0.05	-0.14	-1.08	2.12
-0.91	1.92	0	-1.03
1.87	0	1.53	1.49

K-Means-based Weight Quantization



[실습6] K-Means Quantization 함수 구현

- Get number of clusters (2^n) based on the quantization precision
- **Decode** the codebook into k-means quantized tensor for inference

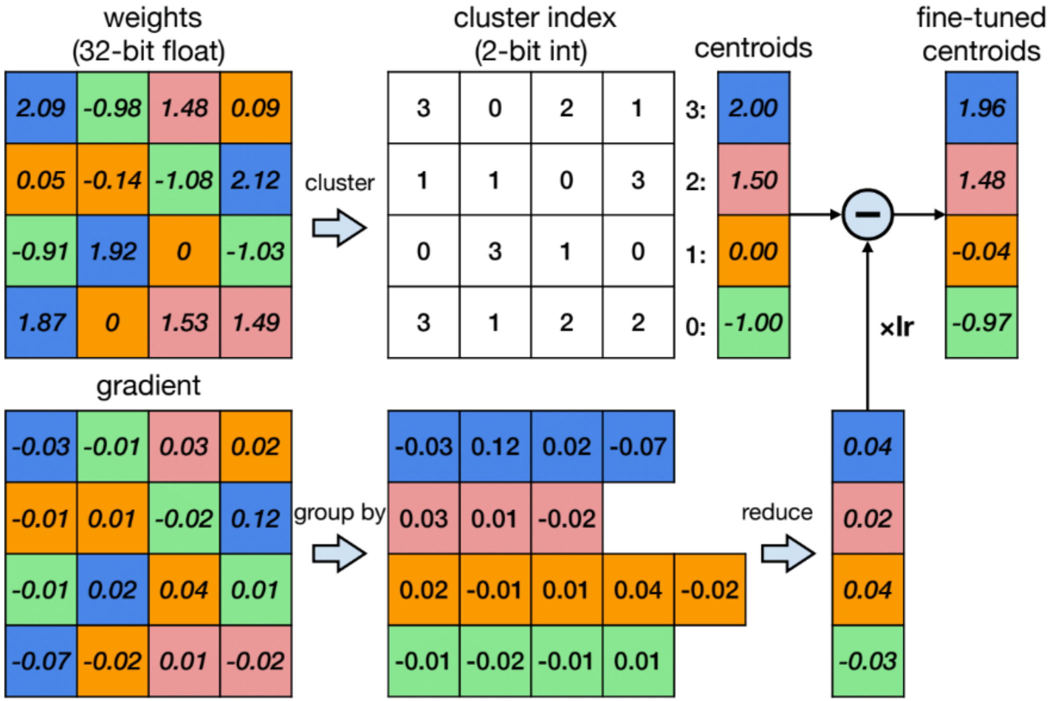


Answer


```python
    ################# YOUR CODE STARTS HERE #################
    # bitwidth에 따라 클러스터 수를 설정하세요.
    n_clusters = 2**bitwidth
    ################# YOUR CODE ENDS HERE ###################
    # k-means를 사용하여 quantization centroid를 얻습니다.
    kmeans = KMeans(n_clusters=n_clusters, mode='euclidean', verbose=0)
    labels = kmeans.fit_predict(fp32_tensor.view(-1, 1)).to(torch.long)
    centroids = kmeans.centroids.to(torch.float).view(-1)
    codebook = Codebook(centroids, labels)
################# YOUR CODE STARTS HERE #################
# 추론에서 사용할 quantized tensor를 구하기 위해 코드북을 디코딩하세요.
quantized_tensor = codebook.centroids[codebook.labels]
################# YOUR CODE ENDS HERE ###################
```

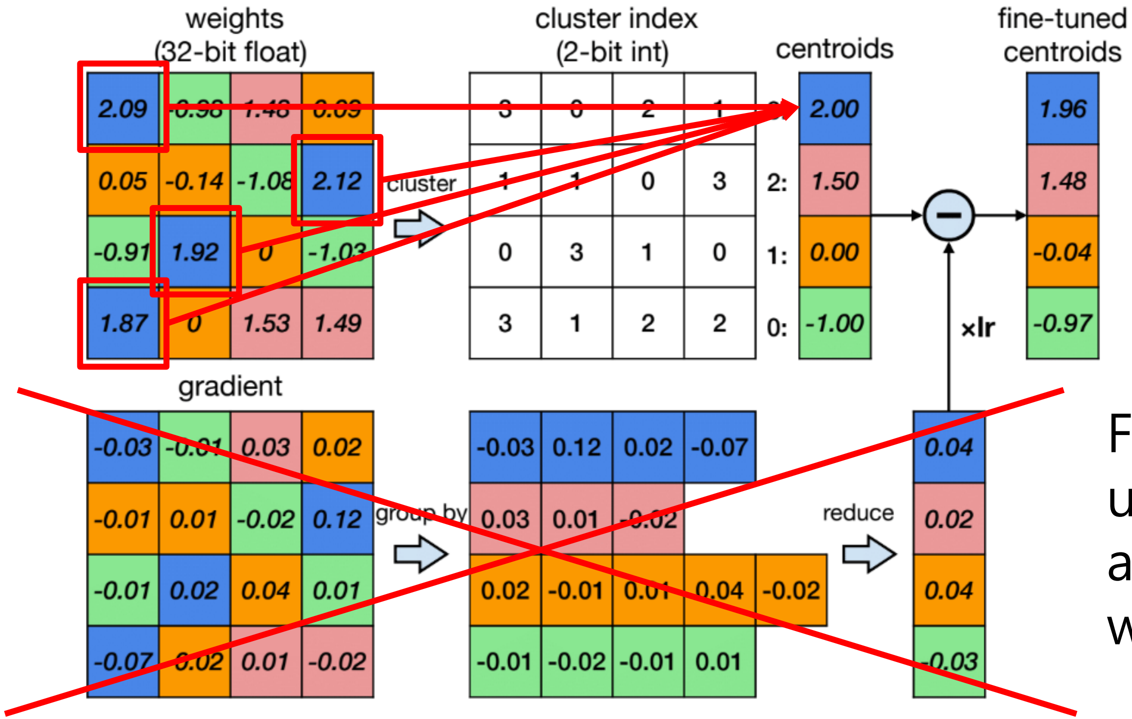
QAT with K-means quantization

Fine-tuning Quantized Weights



[실습7] Fine-tuning K-means quantization

Fine-tuning Quantized Weights



For simplicity, **directly** update the centroids according to the latest weights

Answer


```
################## YOUR CODE STARTS HERE #################
    codebook.centroids[k] = torch.mean(fp32_tensor[codebook.labels==k])
################## YOUR CODE ENDS HERE ###################
```

Quantization with PyTorch API

Machine Learning Libraries

Quantization with PyTorch API

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