Solitons

Meeting 2022-01-17 2pm

Progress

- Analytic:
  - Used Mathematica to evaluate the ODE for the cos/cosh ansatz
  - Checked the working out and then produced expressions for the next order functions when substituted into the ODE
- Numeric:
  - Played around with the code
  - Implemented a frequency shifter
- Other:
  - Still reading the tutorial paper

Analytic

- Calculations for the lowest order match up with the working out given
- For the higher order functions, the output is quite unreadable / the simplifications don't give exactly what we want to determine scaling
  - Initially considered just using Limit[…, x -> Infinity] but that was slow and didn't give conclusive answers when it failed
  - Instead, simplified the expression (`tanh = coth = cos = sin = 1, sinh = cosh`)

```
In[102]:= Collect[ODE[u0] /. Simplification, Sech[σ x]]
```

Out[102]= $\left(A^3\,\Gamma + 3\,A^2\,B\,\Gamma + 3\,A\,B^2\,\Gamma + B^3\,\Gamma - 32\,A\,\sigma^4 - 8\,B\,\sigma^4\right)\,\mathrm{Sech}[x\,\sigma]^3 + \left(5\,A\,\sigma^4 + 5\,B\,\sigma^4\right)\,\mathrm{Sech}[x\,\sigma]^5$

We see that the u0 solution scales with 1/cosh(x)^3

Analytic

- The next order also scales with 1/cosh^3

In[104]:=
```
Collect[ODE[u1] /. Simplification, Sech[σ x]]
```

Out[104]= $\left(352\, c\, \sigma^4 - 192\, d\, \sigma^4 - 96\, e\, \sigma^4 + 64\, f\, \sigma^4\right) \mathrm{Sech}[x\,\sigma]^3 + \left(-696\, c\, \sigma^4 - 96\, d\, \sigma^4 + 8\, e\, \sigma^4 - 96\, f\, \sigma^4\right) \mathrm{Sech}[x\,\sigma]^5 + \left(33\, c\, \sigma^4 + 9\, d\, \sigma^4 + 9\, e\, \sigma^4 + 33\, f\, \sigma^4\right) \mathrm{Sech}[x\,\sigma]^7 + \left(c^3\,\Gamma + 3\, c^2\, d\,\Gamma + 3\, c\, d^2\,\Gamma + d^3\,\Gamma + 3\, c^2\, e\,\Gamma + 6\, c\, d\, e\,\Gamma + 3\, d^2\, e\,\Gamma + 3\, c\, e^2\,\Gamma + 3\, d\, e^2\,\Gamma + e^3\,\Gamma + 3\, c^2\, f\,\Gamma + 6\, c\, d\, f\,\Gamma + 3\, d^2\, f\,\Gamma + 6\, c\, e\, f\,\Gamma + 6\, d\, e\, f\,\Gamma + 3\, e^2\, f\,\Gamma + 3\, c\, f^2\,\Gamma + 3\, d\, f^2\,\Gamma + 3\, e\, f^2\,\Gamma + f^3\,\Gamma\right) \mathrm{Sech}[x\,\sigma]^9$

In[132]:=
```
%104 /. ((# → 1) & /@ {c, d, e, f})
```

Out[132]= $128\,\sigma^4\, \mathrm{Sech}[x\,\sigma]^3 - 880\,\sigma^4\, \mathrm{Sech}[x\,\sigma]^5 + 84\,\sigma^4\, \mathrm{Sech}[x\,\sigma]^7 + 64\,\Gamma\, \mathrm{Sech}[x\,\sigma]^9$

So this also scales with 1/cosh^3

Analytic

- Then u2 scales with 1/cosh^5, u3 with 1/cosh^7 etc.

```
In[195]:= getScalingSimp[2]
```

Out[195]= $4928\,\sigma^4\,\text{Sech}[x\,\sigma]^5 - 4760\,\sigma^4\,\text{Sech}[x\,\sigma]^7 + 270\,\sigma^4\,\text{Sech}[x\,\sigma]^9 + 216\,\Gamma\,\text{Sech}[x\,\sigma]^{15}$

```
In[196]:= getScalingSimp[3]
```

Out[196]= $34\,304\,\sigma^4\,\text{Sech}[x\,\sigma]^7 - 15\,456\,\sigma^4\,\text{Sech}[x\,\sigma]^9 + 616\,\sigma^4\,\text{Sech}[x\,\sigma]^{11} + 512\,\Gamma\,\text{Sech}[x\,\sigma]^{21}$

Numeric

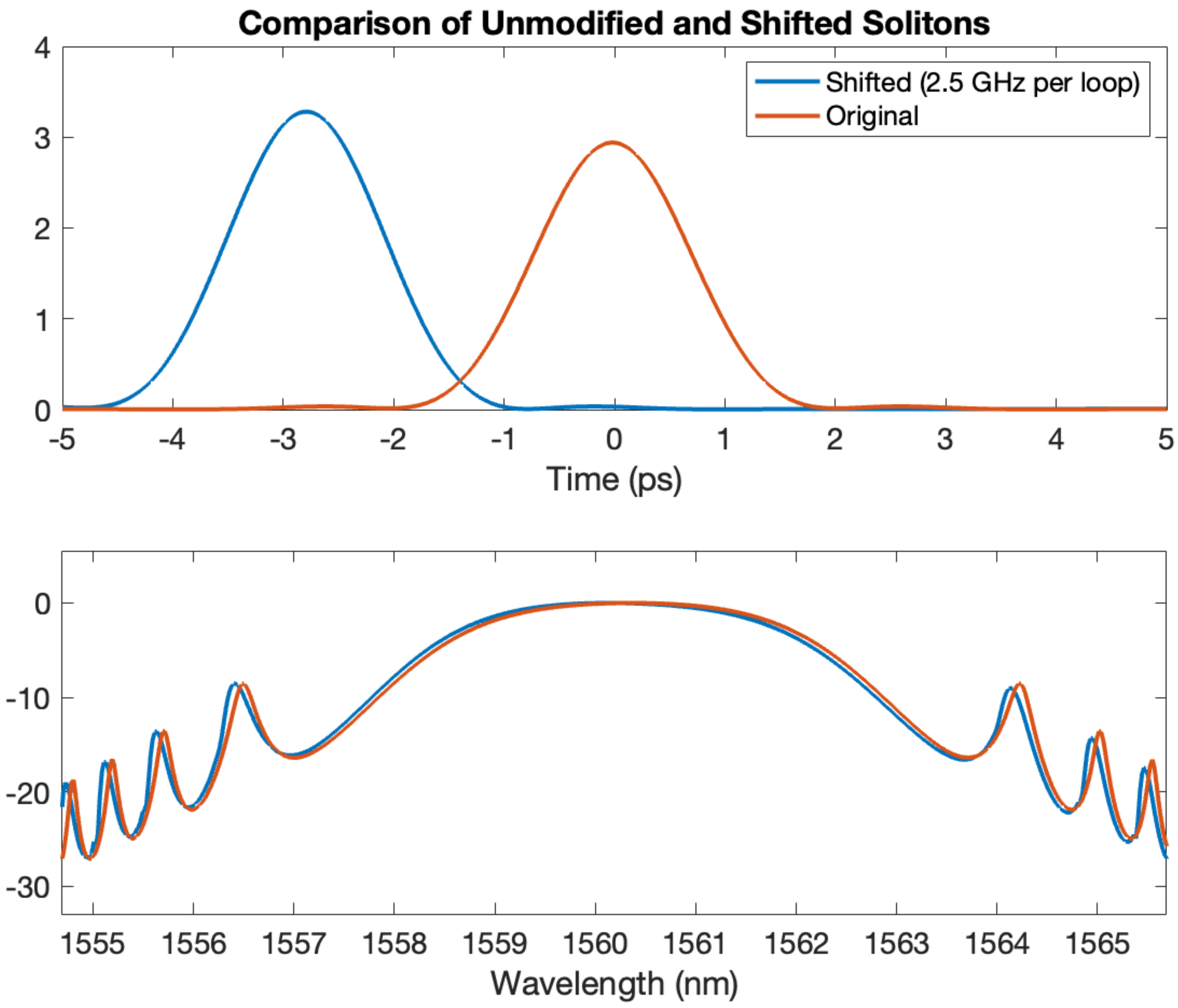
- My implementation of the frequency shifter is:

1. Compute the Fourier transform of the field
2. Shift each element forwards by a certain amount and pad the start with 0s
3. Compute the inverse Fourier transform to return the field

- The resolution of the Fourier transform is df = 2.5 GHz, so that is the minimum shift we can do each round trip
- Placed the shift component after *WaveShaper* in the loop

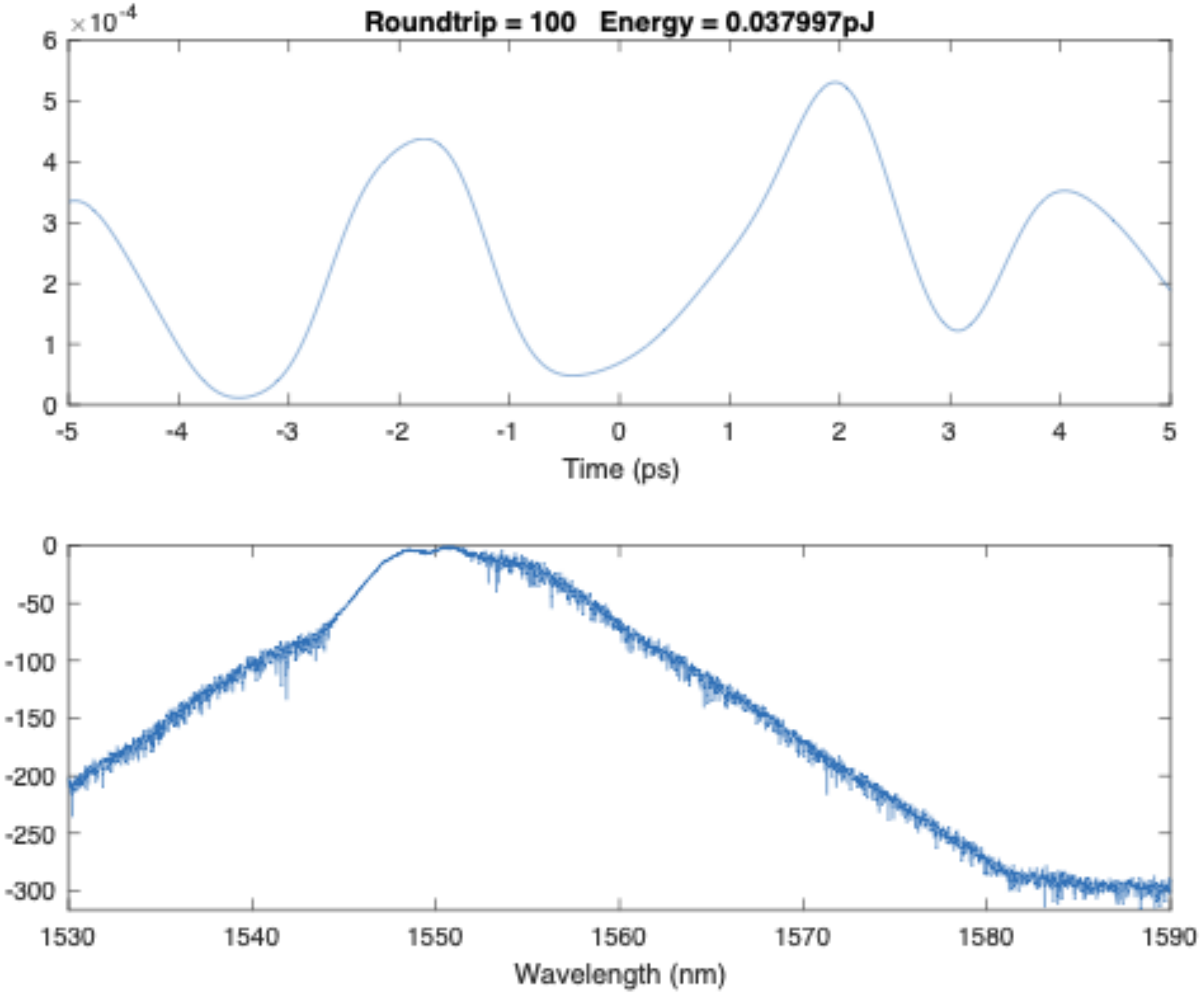
Numeric

- Using a shift of 1df each loop:



Numeric

- Using a shift of 10df each loop:



- No shift: