Tags: #logbook - Denison
Links:

Logbook_06_220201

A Numeric Project

> **Aims**
>
> - [ ] Create a 2D grid of the applied frequency shift and OC

A.1 Notes

A.2 Results

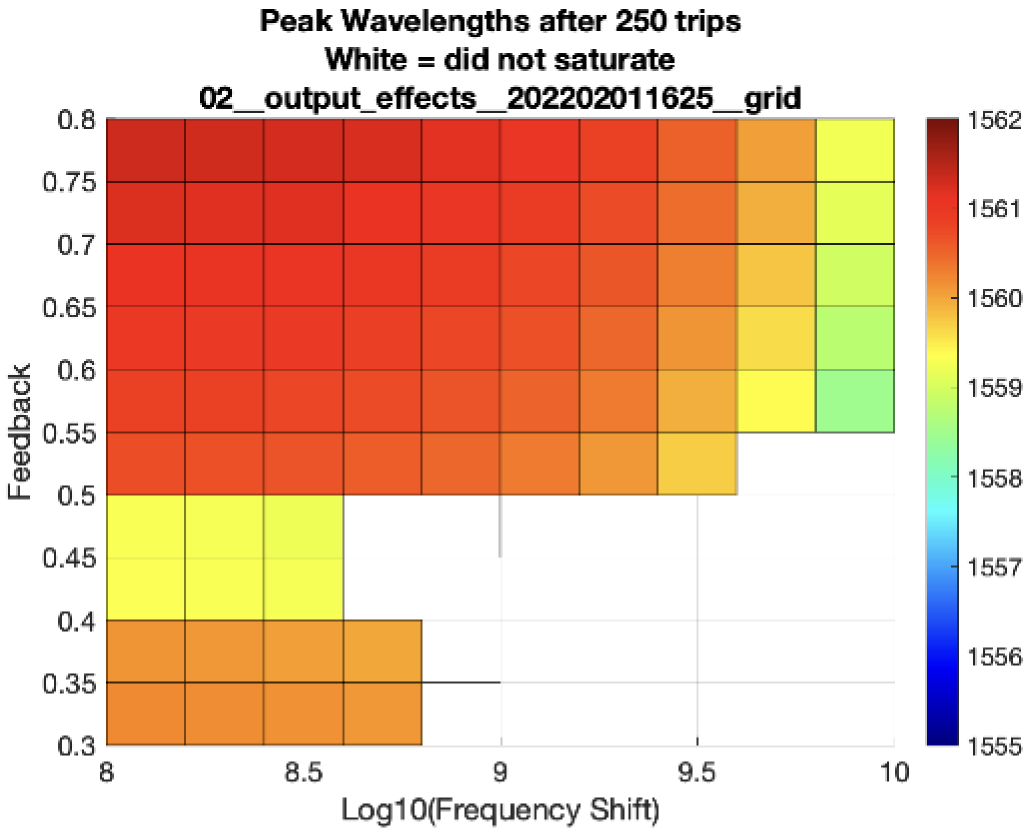
Using `output_effects__220131__grid.m`, swept over the values of applied frequency shifts and feedback to produce a 2D heatmap:

Data: `02__output_effects__202201311758__grid`

> `02__output_effects__202202011610__grid.mp4`

- Anything which goes deep blue, or jumps in colour can be deemed to not be a stable soliton. The gradual changing reds in the top left are actual solitons
- So there appears to be this phase shift around `feedback = 0.5`

> `02__output_effects__202202011625__grid.fig`

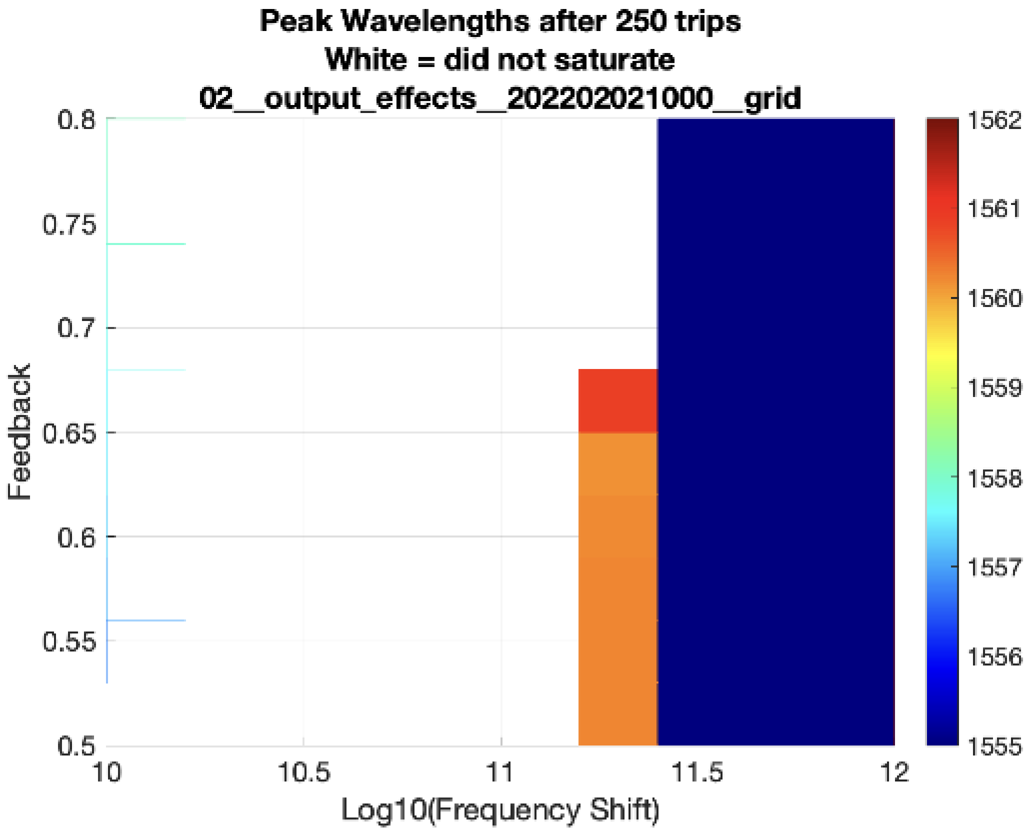


- Below that feedback level, the solitons do not form correctly. This effect is exacerbated at higher input frequency shifts

Moving to higher frequency shifts:

Data: `02__output_effects__202202011634__grid.mat`

> `02__output_effects__20220202100__grid.fig`



So even at high feedbacks, we aren't able to last with such high frequency shifts

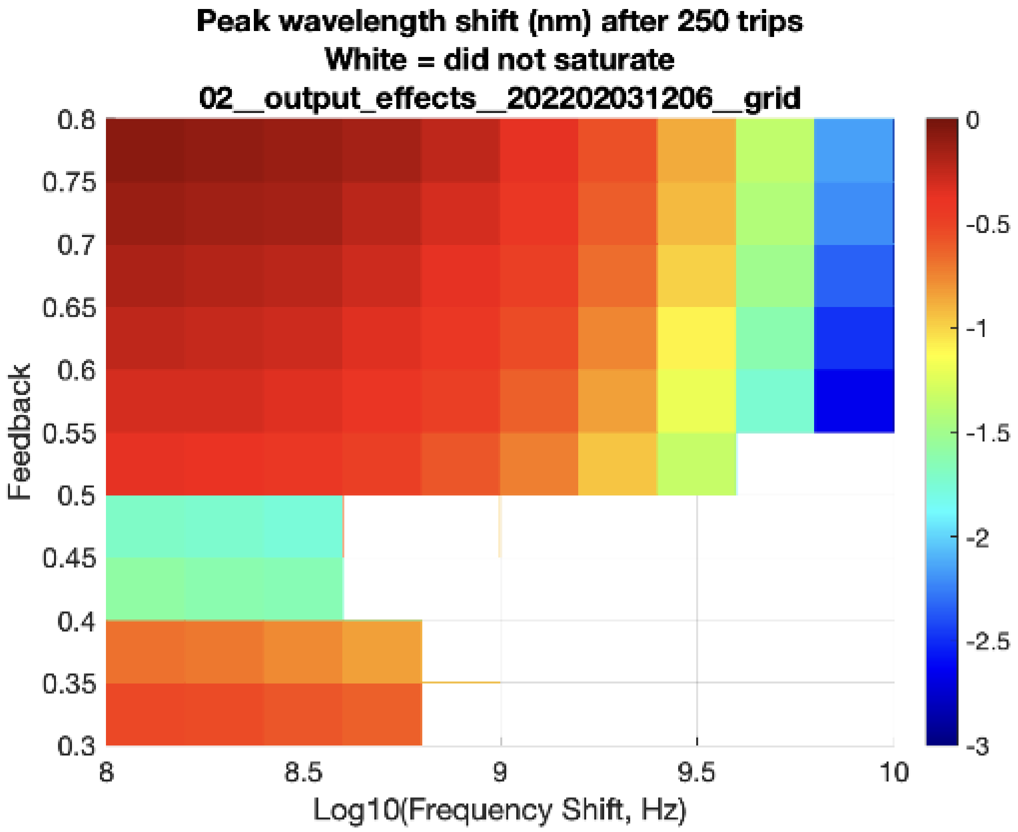
```
02__output_effects__202202021020__grid.mp4
```

- So it probably breaks down around $10^{10.5}$ Hz for `feedback = 0.8`
- (Note that the colorbar in this mp4 is slightly different scale)

A.2.1 Delta wavelength

Ran the simulations to see what the baseline values were for the wavelength `02__output_effects__202202021029__grid.mat` and used these to plot $\Delta\lambda$ instead of just λ.

> `02__output_effects__202202031206__grid`



`02__output_effects__202202031206__grid.mp4`

A.3 Outcomes

A.4 To Do

- [x] ~~Examine in more detail how at what ranges this phase shift occurs~~
    - [x] ~~How hard can we push the 0.6 feedback level?~~
- [x] ~~Plot these using $\Delta\lambda$ rather than just λ~~
- [x] ~~Examine quartic dispersion~~

B Analytic Project

Aims

- [ ] Experiment with methods of finding A, B using the DE

B.1 Notes

The previous results showed that taking the Taylor expansion about $x = 0$ don't really give us solutions close to the numerical one. Perhaps we need to consider how the DE behaves over a wider range of values, instead of just at the origin.

We can do this by evaluating the criterion

$$I = \int_{\mathbb{R}} w(x') \left[D\left(x';\ u^{(0)} + u^{(1)},\ (A, B) \right) \right]^2 \mathrm{d}x'$$

where $D(\cdot)$ is the differential equation, and $w(\cdot)$ is an integrable **weight** function

This weight function must:

- Be integrable so that I exists
- Positive
- $w(0) = 0$ so that the numeric integration can evaluate $w \times D$ at $x = 0$, where D may diverge

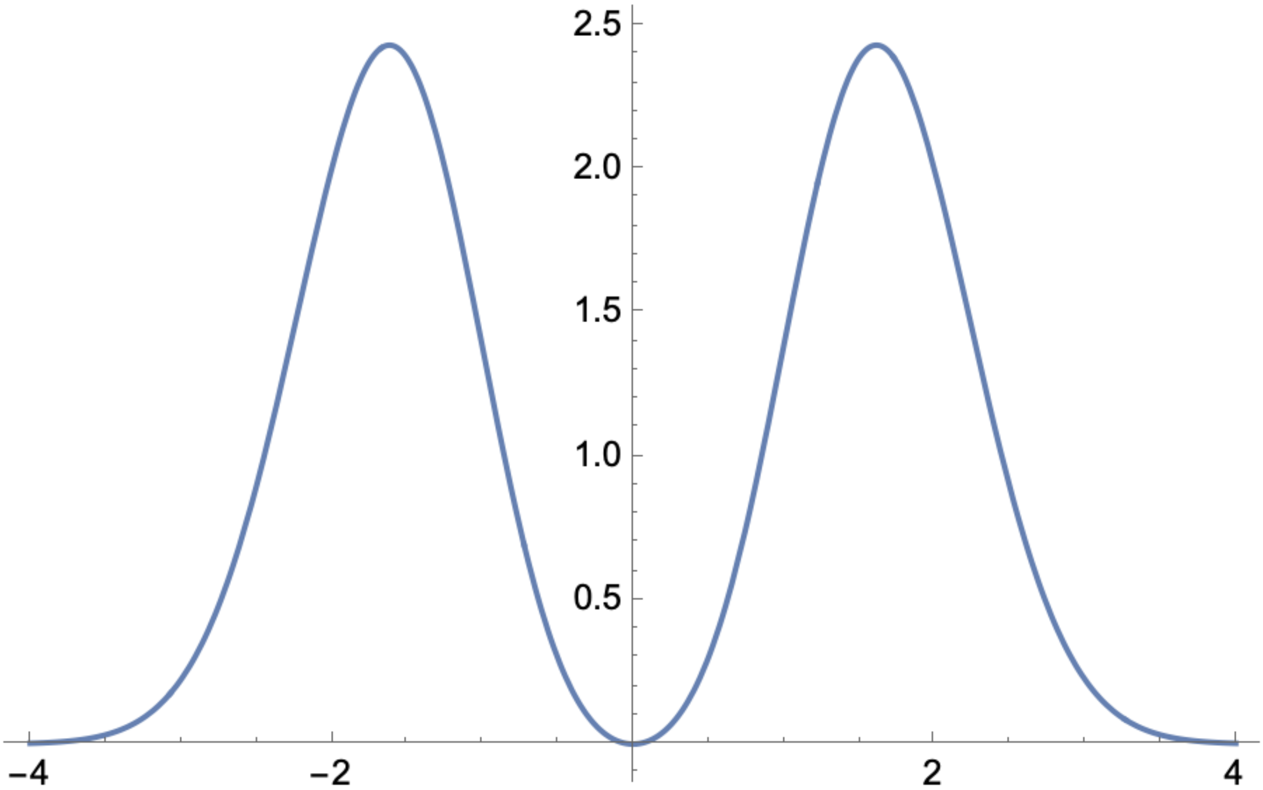
We can also consider the scaled $J := \frac{I}{(A+B)^2}$ so that the value is more invariant to the amplitude of A, B.

B.2 Results

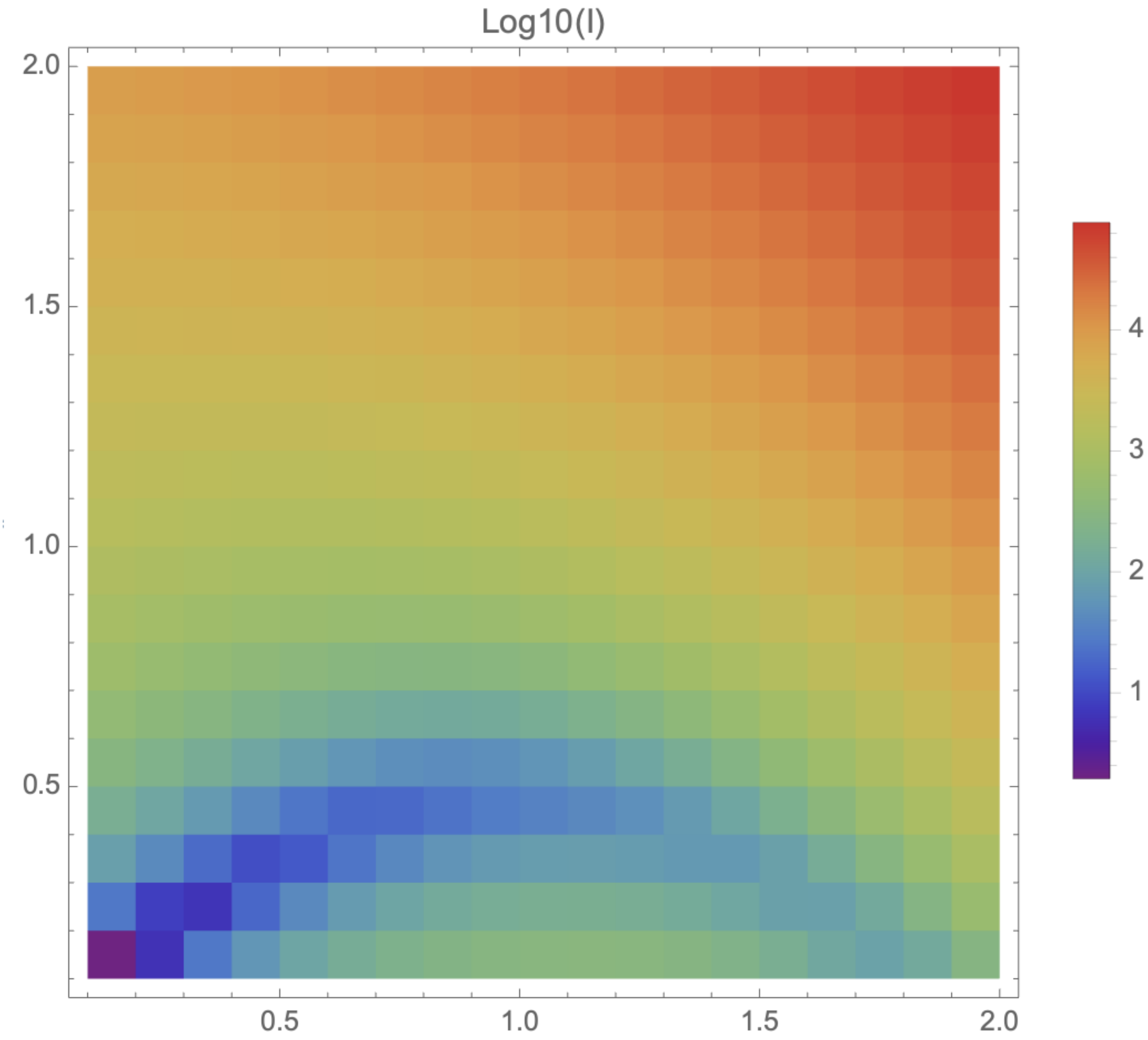
```
05_weighted_220201.nb
```

B.2.1 Weight Function 1

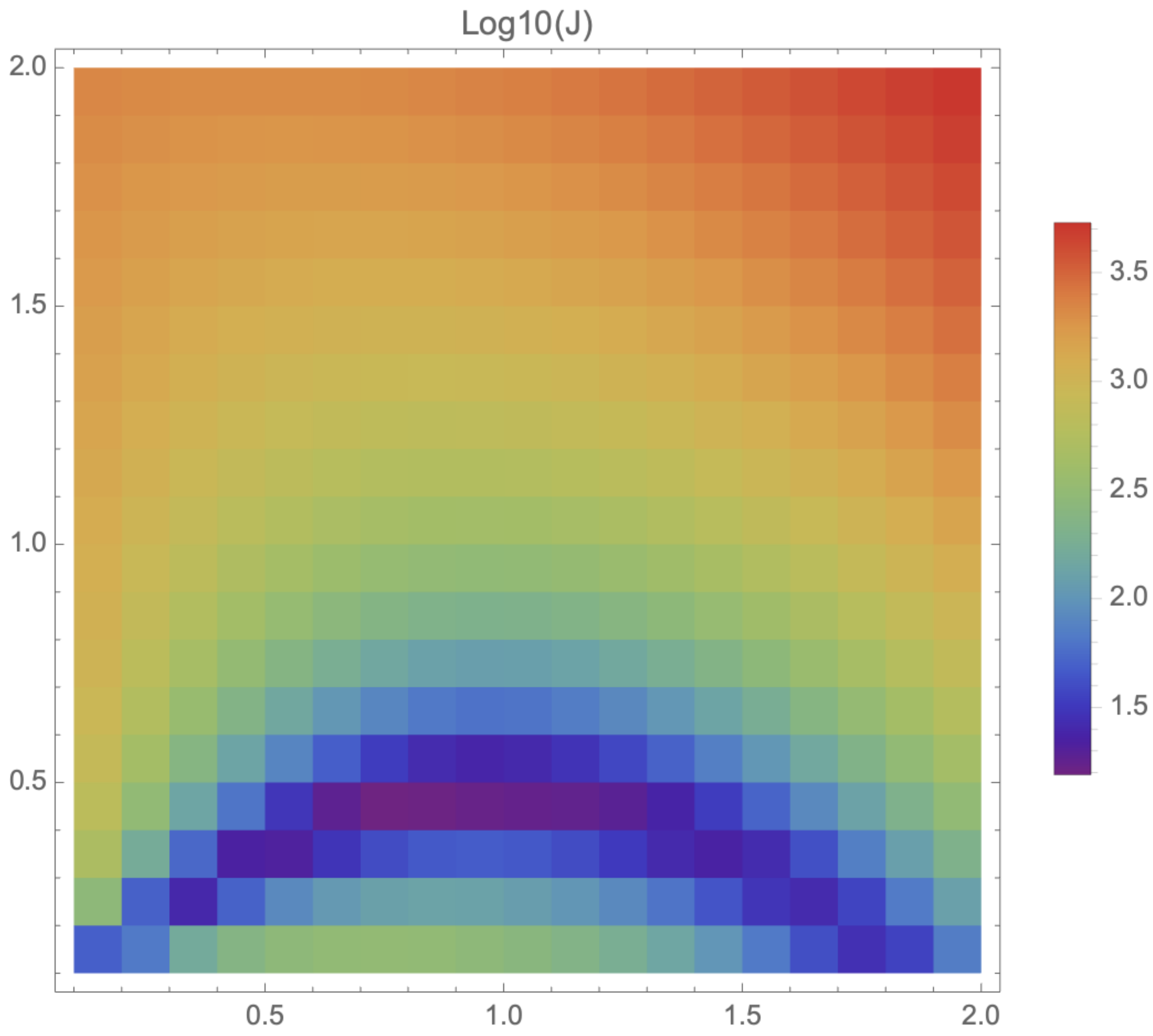
$$w_1(x) = x^2 e^{-x^2} \cosh(2x)$$



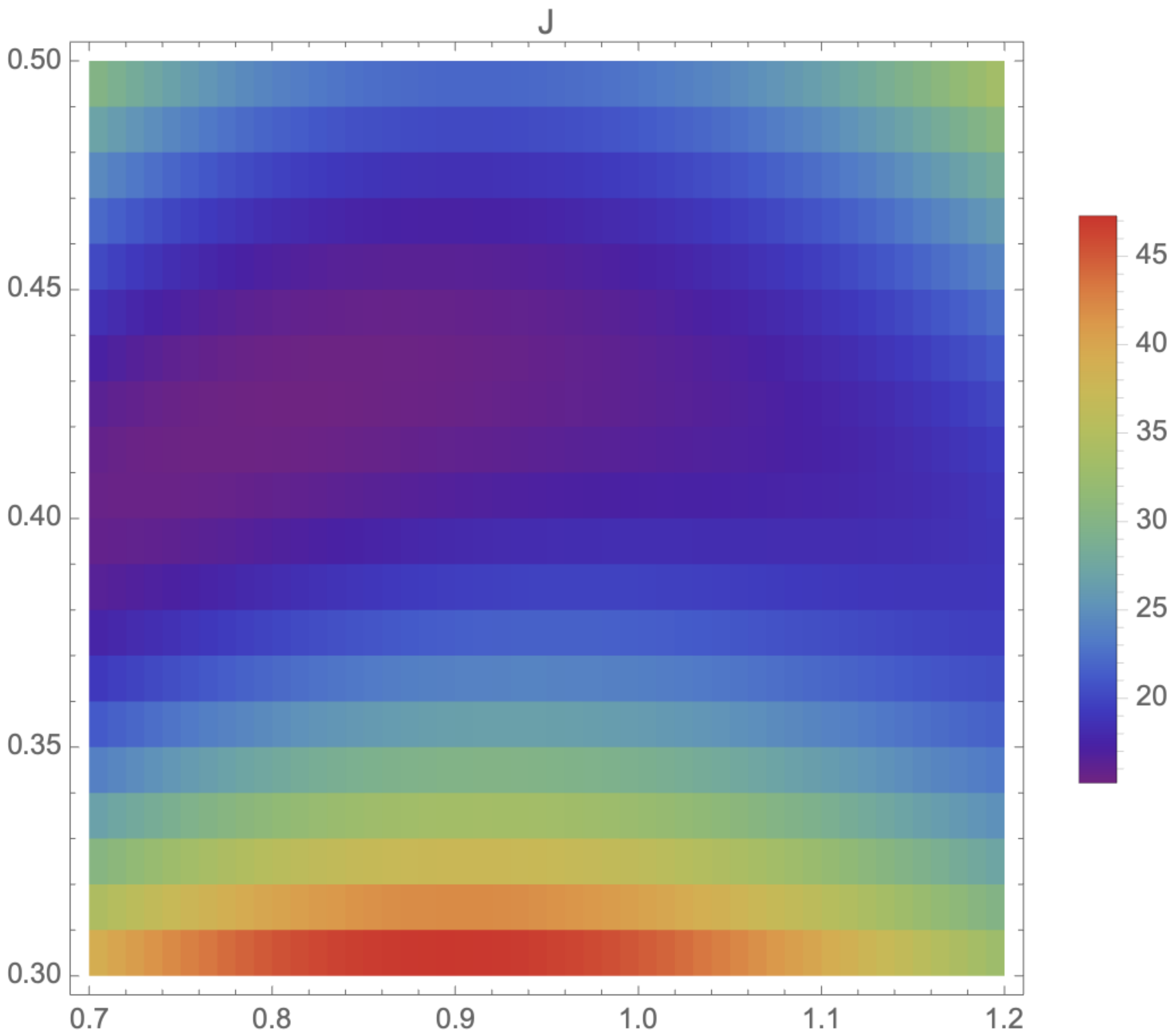
This gives the unscaled:



and scaled:



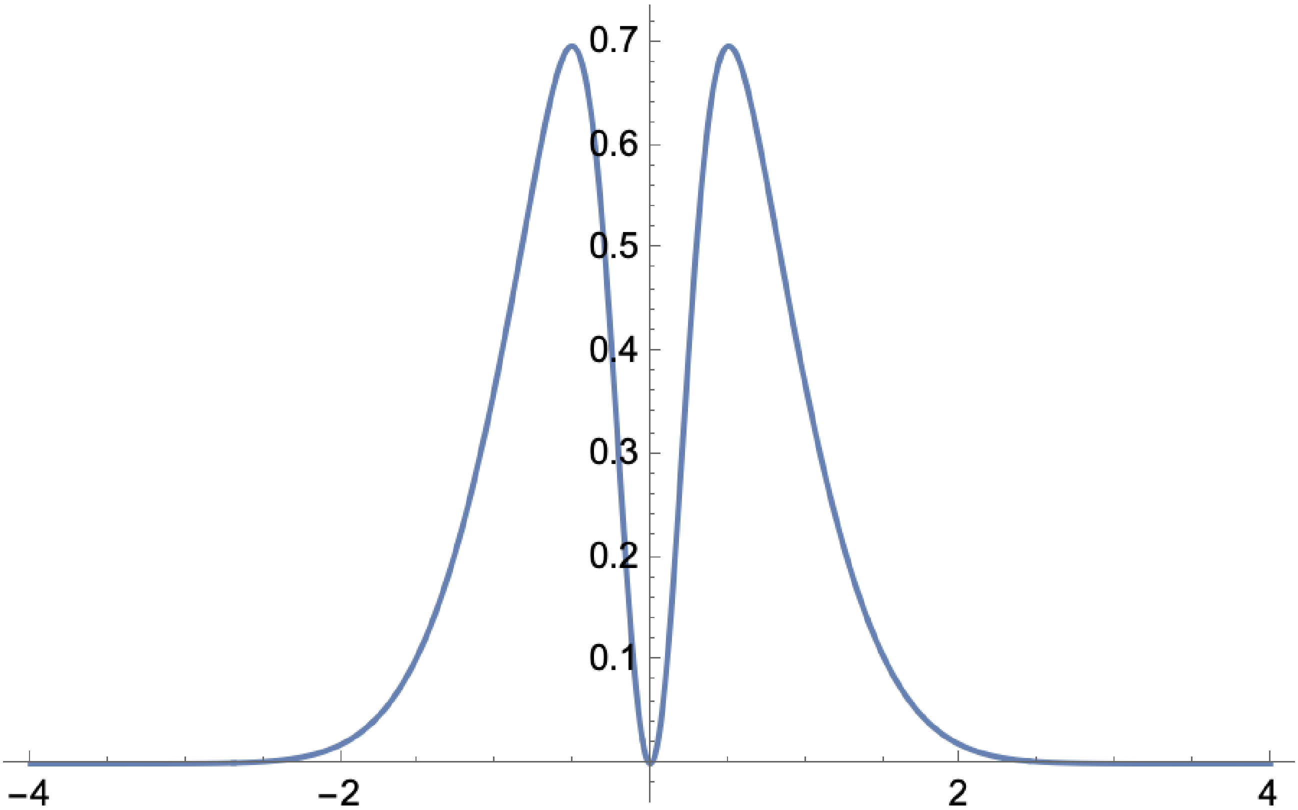
Log10(J)
2.0
1.5
1.0
0.5
0.5
1.0
1.5
2.0
3.5
3.0
2.5
2.0
1.5



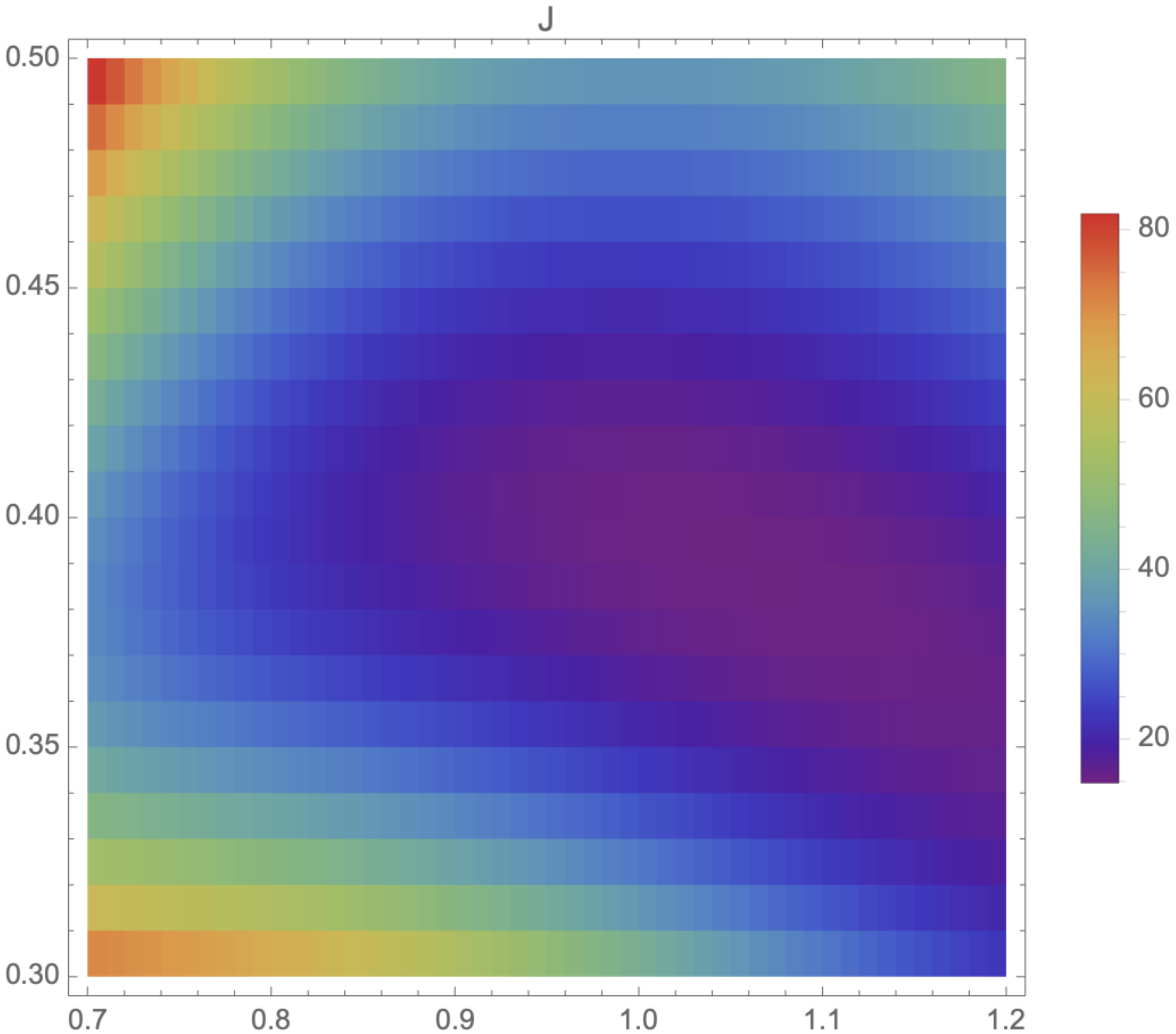
- The minimum values are near to our estimates $(A, B) = (0.45408, 1.02817)$ but not exact

B.2.2 Weight Function 2

$$w_2(x) = e^{-x^2} - e^{-10x^2}$$



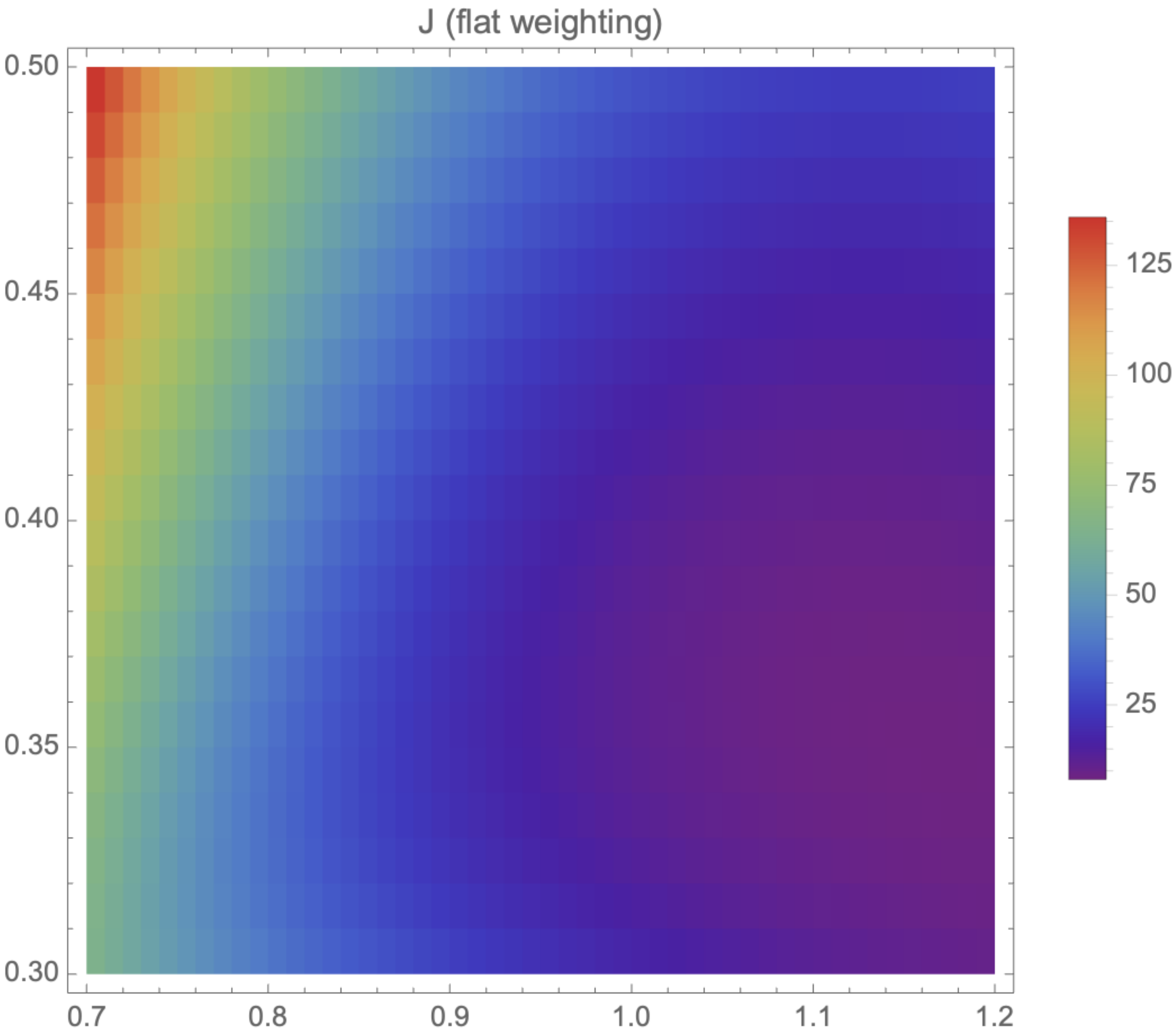
This gives

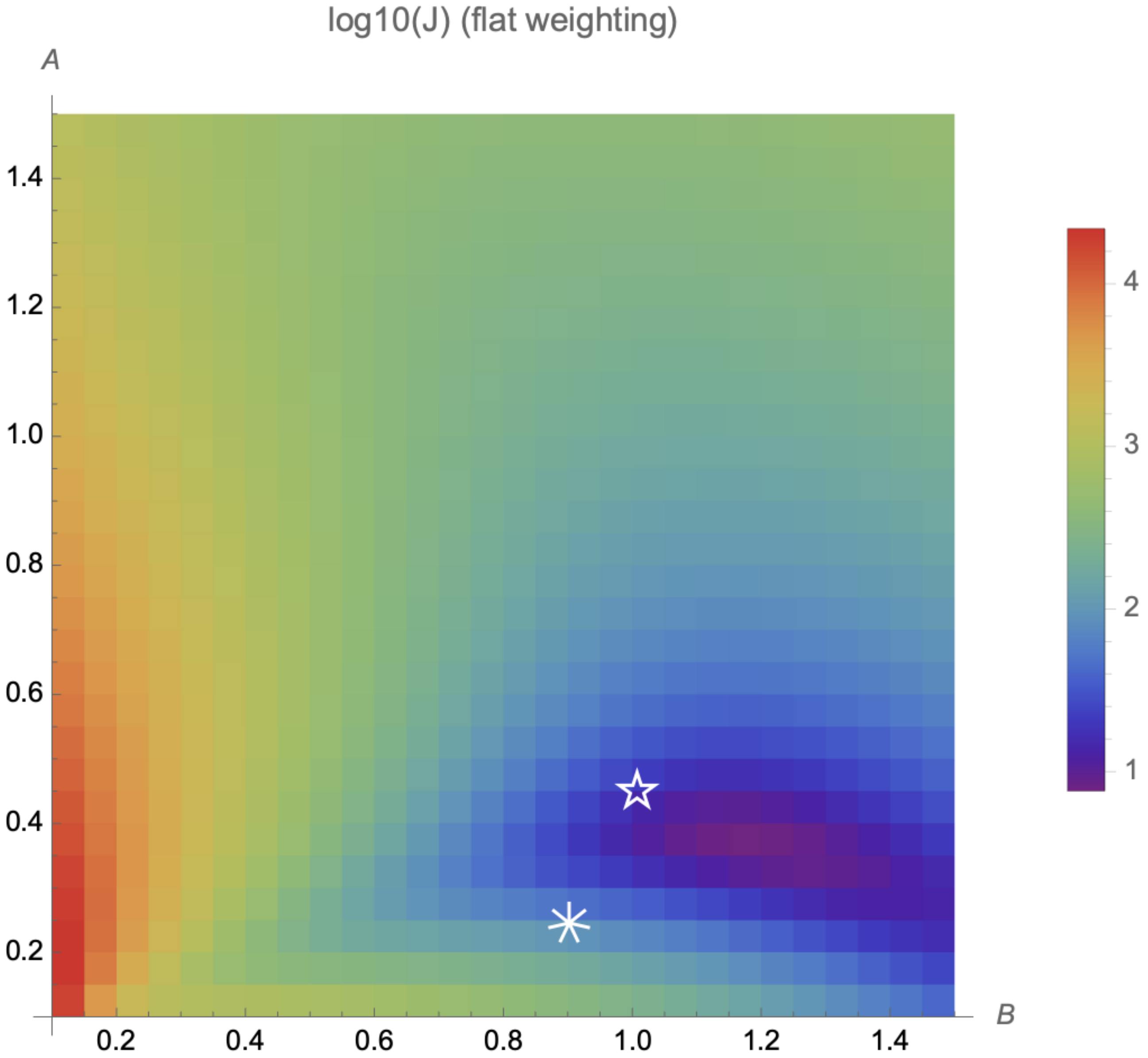


- Again, similar but not exact.
- However, both minimum values are closer than the ones from the Taylor expansion

B.2.3 Flat weight function

$$w_{\text{flat}}(x) = \begin{cases} \frac{1}{4} & 0.01 < |x| < 2 \\ 0 & \text{otherwise} \end{cases}$$

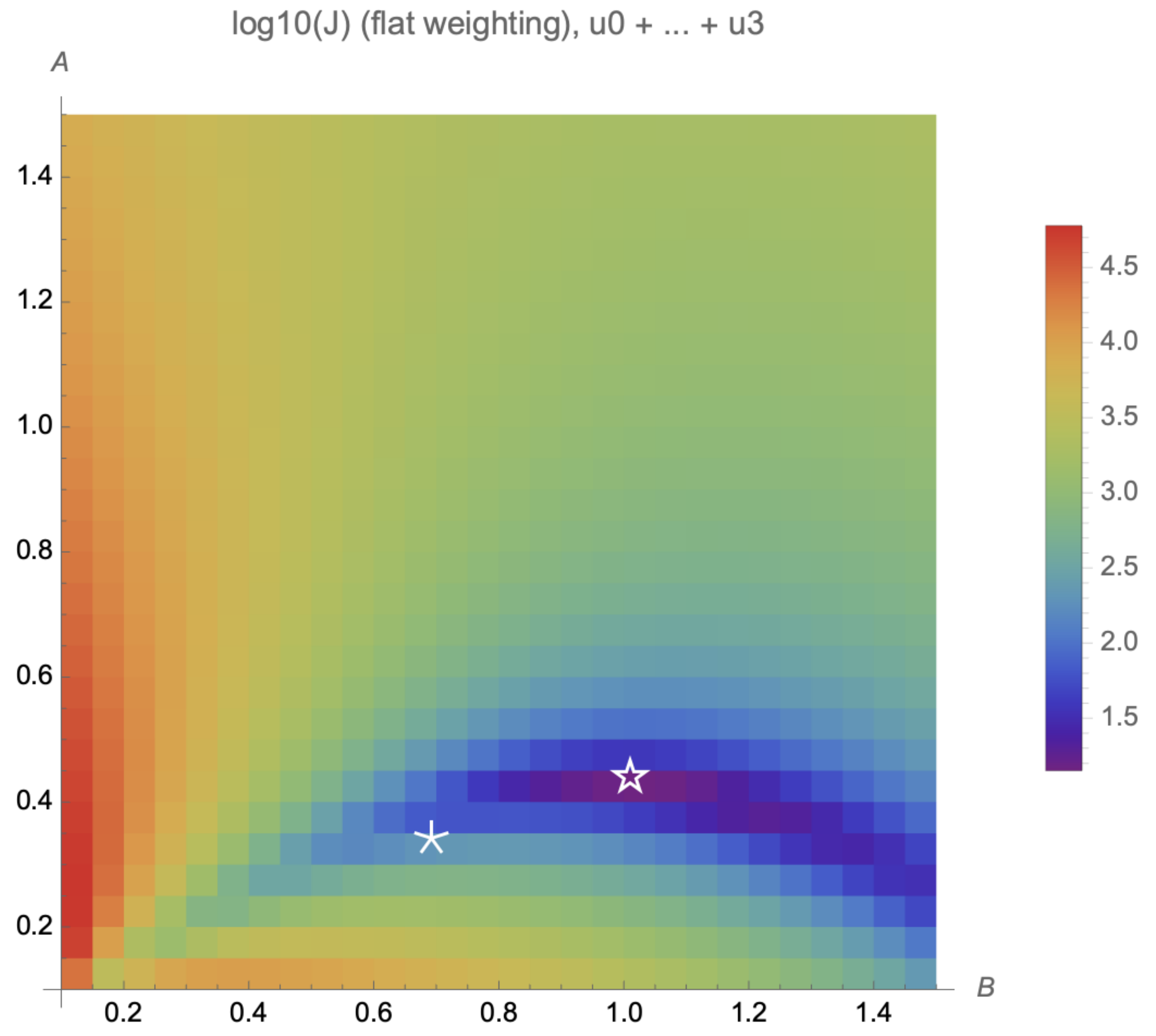

J (flat weighting)
0.50
0.45
0.40
0.35
0.30
0.7
0.8
0.9
1.0
1.1
1.2
125
100
75
50
25



The $*$ is visual estimate, the $\star$ is the Taylor estimate

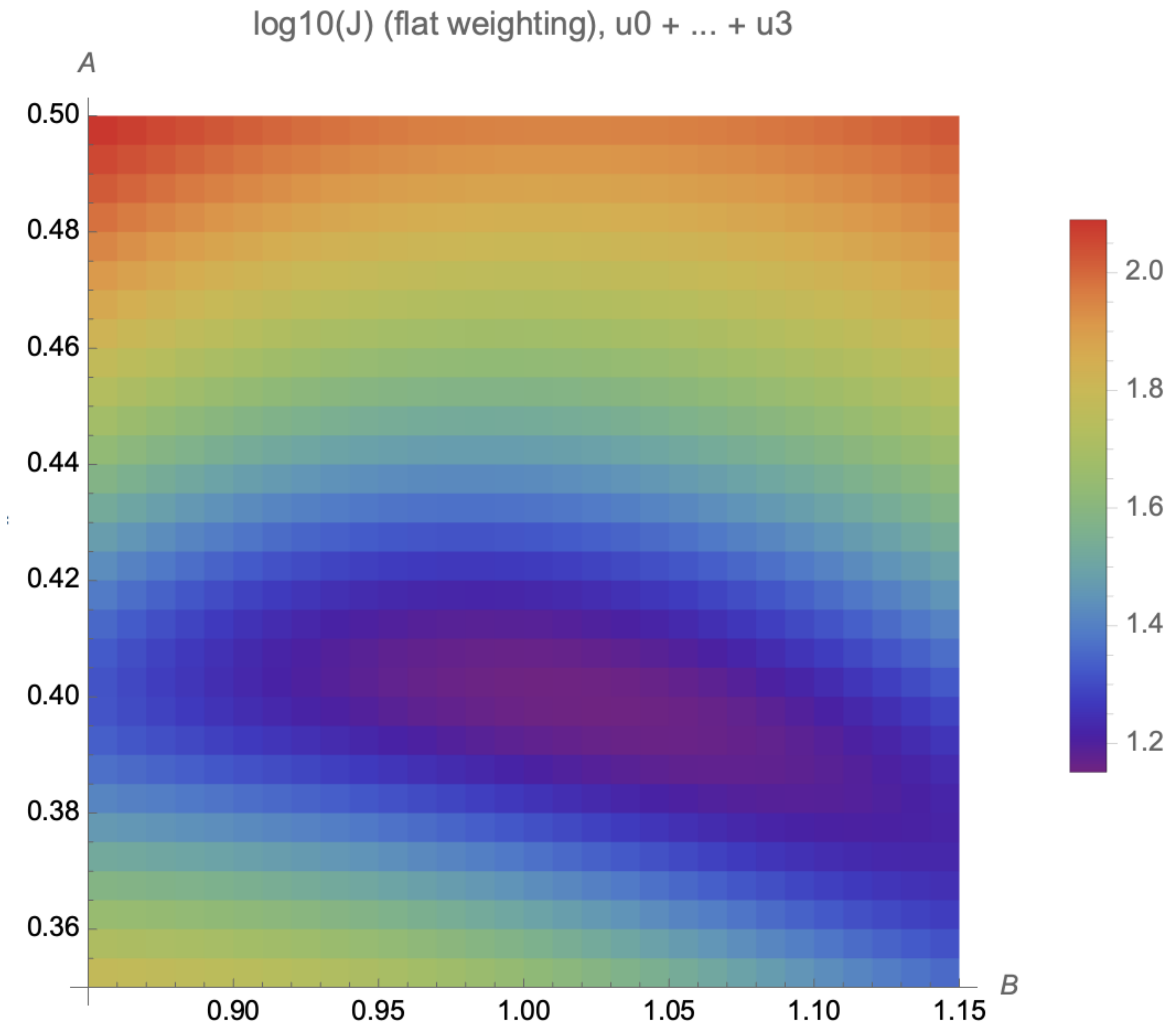
B.2.3.1 Higher Orders

Taking $u^{(0)} + \cdots + u^{(3)}$:



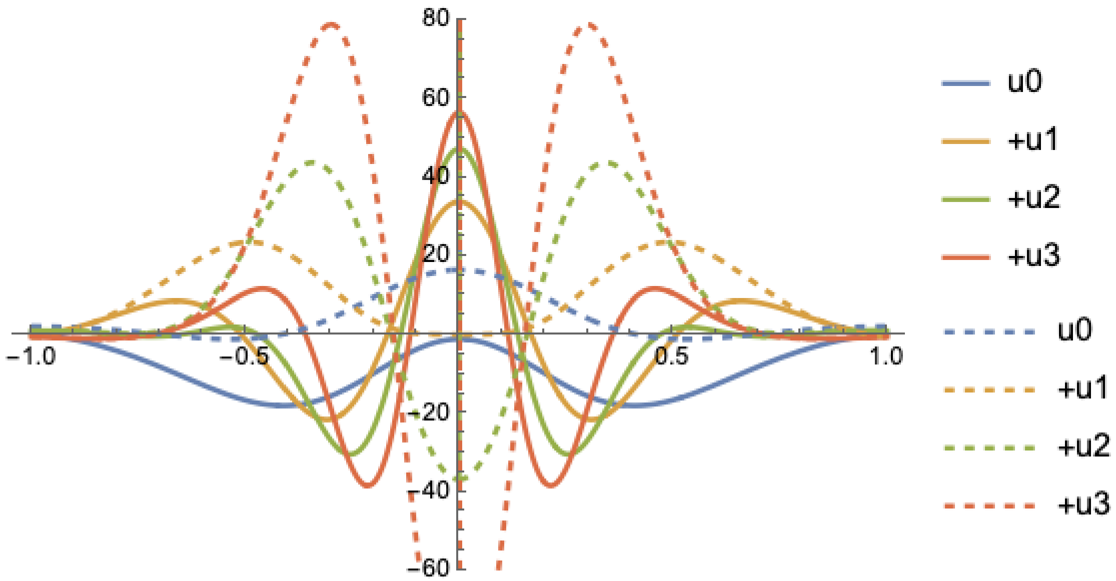
- The $*$ is visual estimate, the $\star$ is the Taylor estimate (for $u^{(3)}$)

Zoom in to the region:



B.2.4 Remainders

Continuing on with the remainder analysis from last logbook, worked in `06_remainder_220202.nb` to consider the remainders with higher order u included:



- Solid lines are using A, B from our visual estimate ({A → 0.445408, B → 1.02817})
- Dashed lines are using the A, B by setting the remainder for $u^{(0)} + u^{(1)}$ to be 0 up to x^2 ({A → 0.242423, B → 0.898534})
    - As we can see, since the dashed $+u1$ line is 0 and flat at $x = 0$
- The dashed lines seem to be larger in magnitude than the solid
- Both cases seem to diverge at larger terms, not converge as we might expect of a function that gives a s good fit to the DE

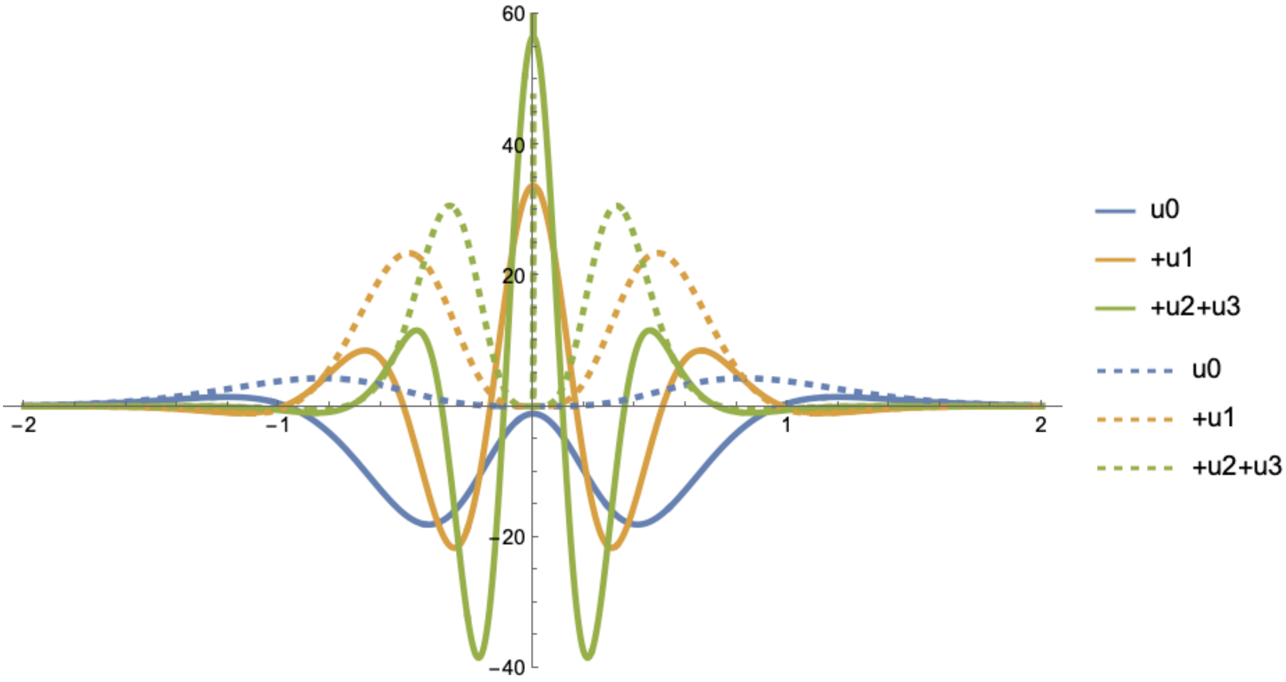
> Maybe this is linked to Gibb's phenomenon

- The region where it doesn't converge gets increasingly narrow

B.2.5 Correct Taylor

(Found the A, B using `04_taylor_220202.nb`)

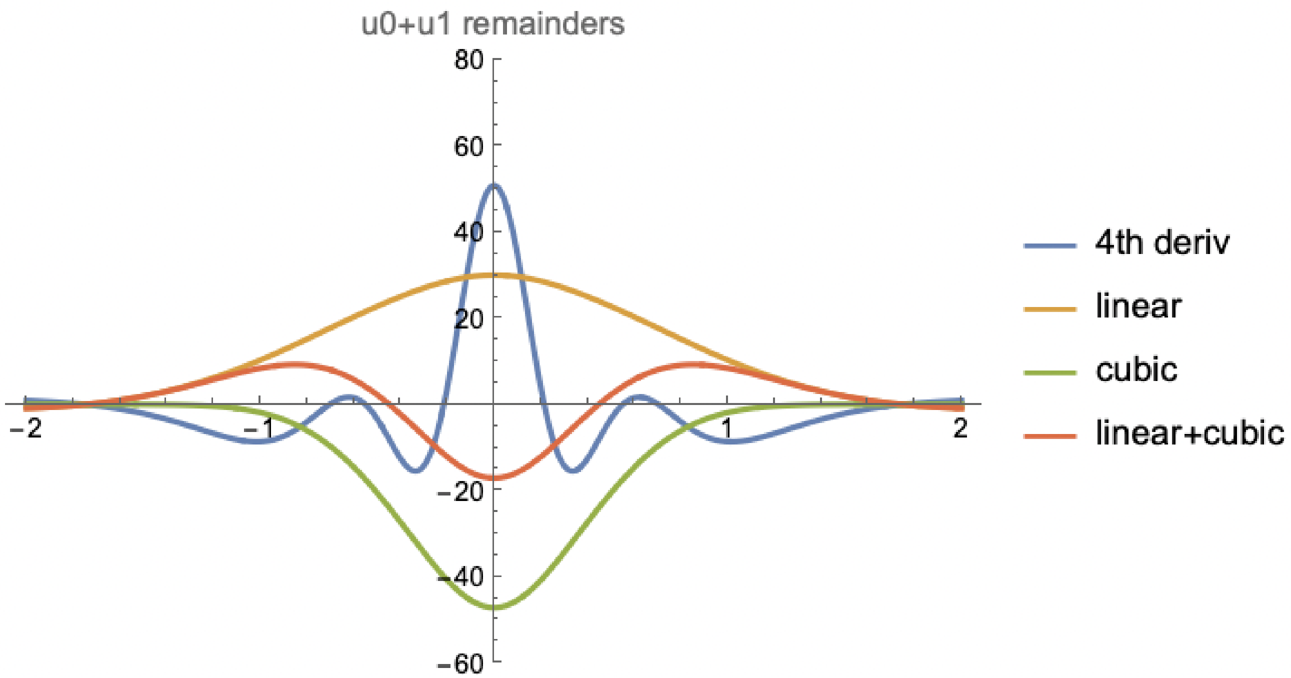
Instead of just using the $u^{(1)}$ Taylor coefficient, each dashed line now has the A, B chosen so that the Taylor expansion is 0 up to x^2 near $x = 0$.

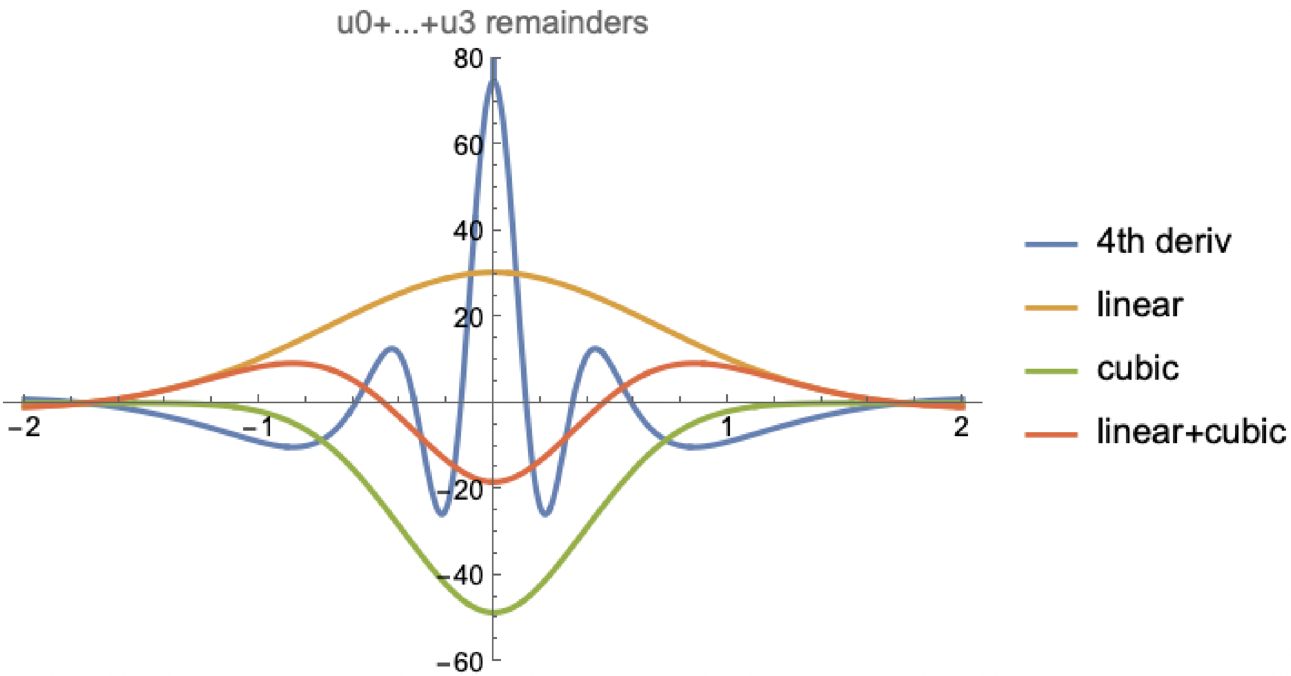


- As expected, the dashed linear are now all flat near the origin
- The $u0$ dashed curve is very close to 0
    - This is somewhat expected, since we know that the plot visually was close to the numeric solution near the peak, but it didn't match in the tails (which only represent a small "area" in this plot)
- With higher orders of the Taylor estimates, the peaks get sharper and move towards 0, so this also links perhaps to the Gibb's phenomenon, where we will always have these bumps, but perhaps higher orders mean we can squish them to being less impactful?

B.2.5.1 Remainder Contributions

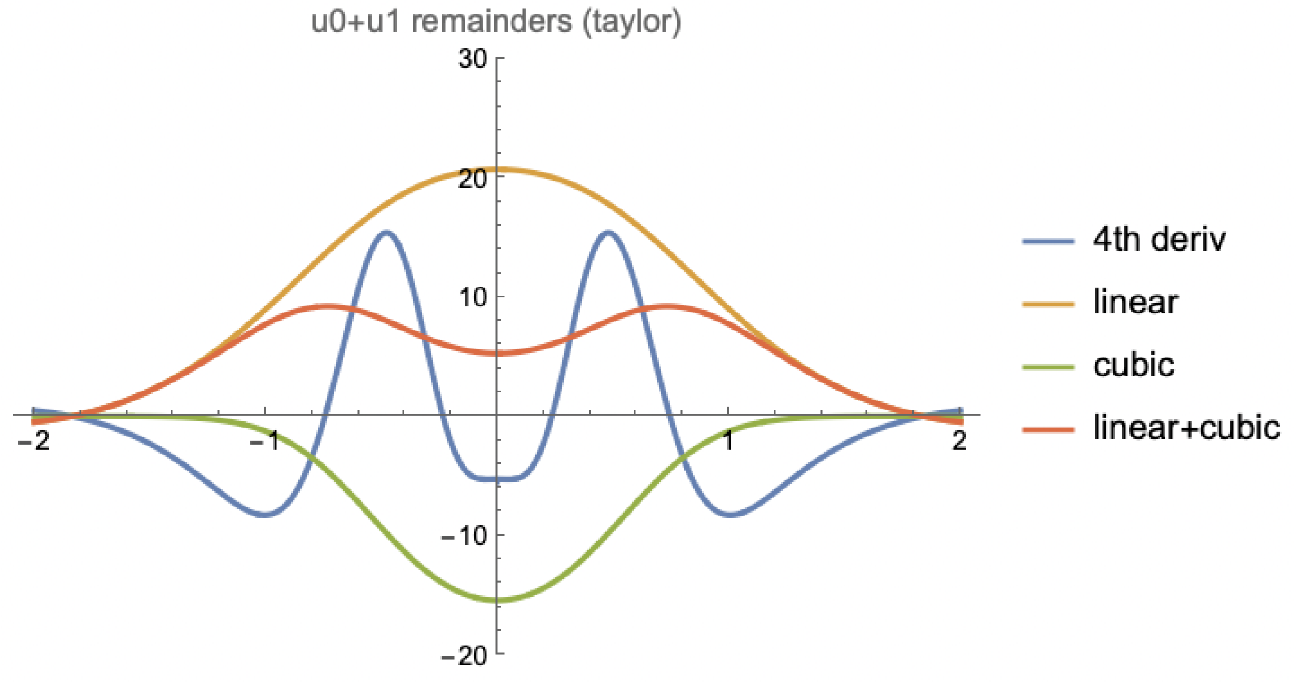
Using our visual estimates of A, B we find:

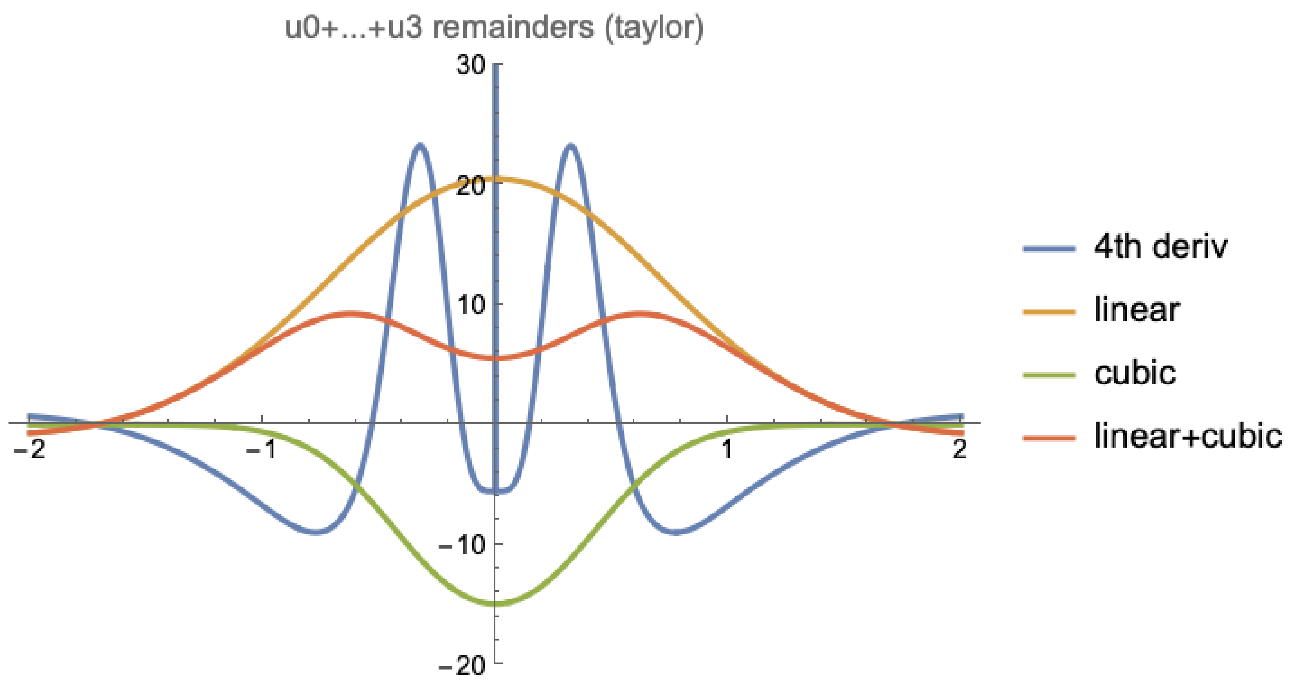




- So the major contribution to the remainder is from the 4th derivative, while the linear and cubic terms roughly cancel to produce only an order of ~ 20, compared to ~ 80 for the 4th derivative
- This makes sense, since we know that our functions quite closely (visually) match that of the numerical answer

Looking at the Taylor estimates:





B.3 Outcomes

- Fundamentally the $u^{(k)}$ ansatz method seems to be flawed - although it does (visually) approach the numerical solution to the zeroth order, the fourth derivative does not
    - There are these wiggles which only get amplified as we take higher $u^{(k)}$ terms
    - These are likely coming from the $\frac{\cos}{\cosh}$ components
    - This could be because we are solving the DE in the tails, yet now examining the fit near the peak

- If this were the correct functional form then it should just approach the solution at the fourth derivative
    - Therefore we may need to think of other functional forms or approaches to satisfy the differential equation

B.4 To Do

- [x] ~~Recalculate the A, B for each of the dashed points (odd u)~~
- [x] ~~!Plot the remainder for just the derivative, the cubic etc~~
- [x] ~~Repeat area calculations for higher us~~

Other

- [x] ~~Find out when Denison ends~~
    - ~~Feb 18th~~