Solitons

Meeting 2022-01-27 3pm

Progress

- Analytic:
  - Found coefficients for u2 and higher
- Numeric:
  - Reran the simulations with more round trips to achieve saturation

Analytic

$$L\left[\sum_{j=0}^{n} u^{(j)}\right] + N\left[\sum_{j=0}^{n-1} u^{(j)}\right] \equiv 0 \text{ up to } (2n+1)\text{th order}$$

u2 (fifth order):

$$\left\{d[0] \to \frac{A}{8} + \frac{\left(-95\,A^5 + 86\,A^4\,B + 98\,A^3\,B^2 + 352\,A^2\,B^3 + 273\,A\,B^4 + 266\,B^5\right)\Gamma^2}{21\,299\,200\,\sigma^8} - \frac{3\times\left(2\,A^3 + 9\,A^2\,B + 6\,A\,B^2 + 9\,B^3\right)\Gamma}{5120\,\sigma^4},\right.$$

$$d[1] \to -\frac{B}{8} - \frac{\left(86\,A^5 + 671\,A^4\,B + 712\,A^3\,B^2 + 798\,A^2\,B^3 + 626\,A\,B^4 - 273\,B^5\right)\Gamma^2}{21\,299\,200\,\sigma^8} + \frac{3\times\left(3\,A^3 + 2\,A^2\,B + 3\,A\,B^2 - 2\,B^3\right)\Gamma}{5120\,\sigma^4},$$

$$d[2] \to \frac{\Gamma\left(\left(49\,A^5 + 356\,A^4\,B + 50\,A^3\,B^2 + 532\,A^2\,B^3 - 399\,A\,B^4 + 176\,B^5\right)\Gamma - 12\,480\,B\left(3\,A^2 + 4\,A\,B + 3\,B^2\right)\sigma^4\right)}{10\,649\,600\,\sigma^8},$$

$$d[3] \to \frac{\Gamma\left(-\left(\left(176\,A^5 + 399\,A^4\,B + 532\,A^3\,B^2 - 50\,A^2\,B^3 + 356\,A\,B^4 - 49\,B^5\right)\Gamma\right) - 12\,480\,A\left(3\,A^2 - 4\,A\,B + 3\,B^2\right)\sigma^4\right)}{10\,649\,600\,\sigma^8},$$

$$d[4] \to -\frac{A}{8} + \frac{\left(273\,A^5 + 626\,A^4\,B - 798\,A^3\,B^2 + 712\,A^2\,B^3 - 671\,A\,B^4 + 86\,B^5\right)\Gamma^2}{21\,299\,200\,\sigma^8} + \frac{3\times\left(2\,A^3 + 3\,A^2\,B - 2\,A\,B^2 + 3\,B^3\right)\Gamma}{5120\,\sigma^4},$$

$$\left.d[5] \to \frac{B}{8} - \frac{\left(266\,A^5 - 273\,A^4\,B + 352\,A^3\,B^2 - 98\,A^2\,B^3 + 86\,A\,B^4 + 95\,B^5\right)\Gamma^2}{21\,299\,200\,\sigma^8} + \frac{3\times\left(-9\,A^3 + 6\,A^2\,B - 9\,A\,B^2 + 2\,B^3\right)\Gamma}{5120\,\sigma^4}\right\}$$

Analytic

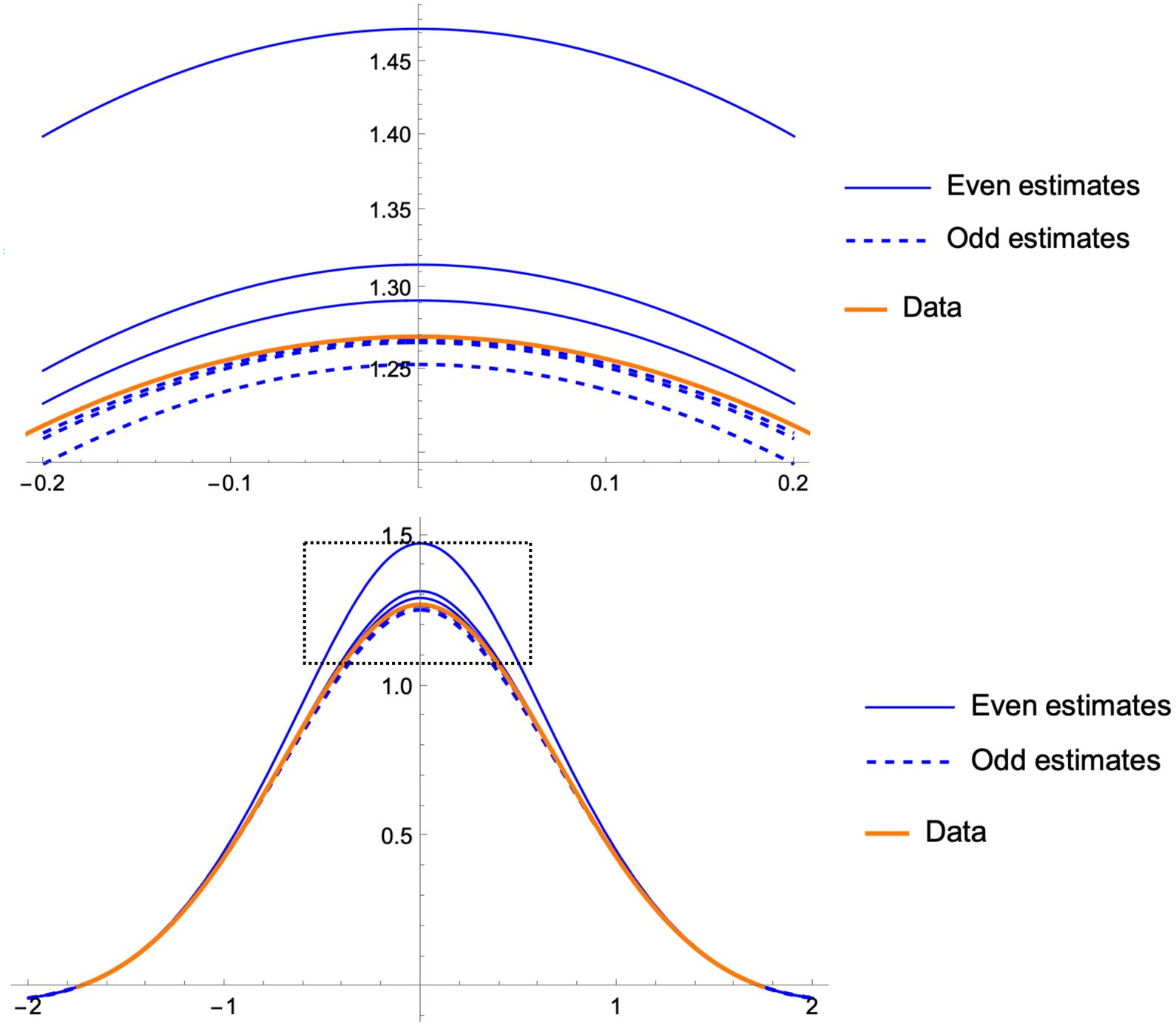
Linear terms:

- The middle coefficients of the even-numbered functions do not have linear terms
- The coefficients are mirrored, with A <—> -B for u1, u3 etc
- And A <—> B for u2, u4 etc
- The signs for a column are repeating (A, -B, -A, B)

- Looking at the actual values of the coefficients, the linear terms are not dominating the other terms

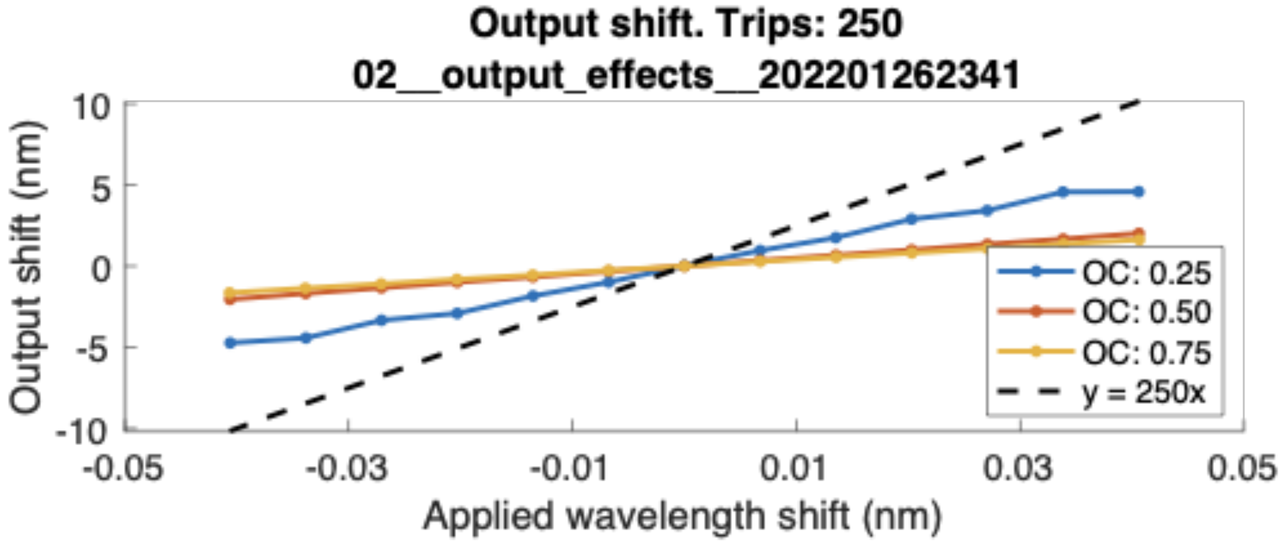
Function	$u^{(1)}$	$u^{(2)}$	$u^{(3)}$	$u^{(4)}$	$u^{(5)}$	$u^{(6)}$
Order	**3**	**5**	**7**	**9**	**11**	**13**
Coefficients	A/4	A/8	5A/64	7A/128	21A/512	33A/1024
	-B/4	-B/8	-5B/64	-7B/128	-21B/512	-33B/1024
	A/4	...	-A/64	-A/64	-7A/512	-3A/256
	-B/4	...	B/64	B/64	7B/512	3B/256
		-A/8	-A/64	...	A/256	5A/1024
		B/8	B/64	...	-B/256	-5B/1024
			5A/64	A/64	A/256	...
			-5B/64	-B/64	-B/256	...
				-7A/128	-7A/512	-5A/1024
				7B/128	7B/512	5B/1024
					21A/512	3A/256
					-21B/512	-3B/256
						-33A/1024
						33B/1024

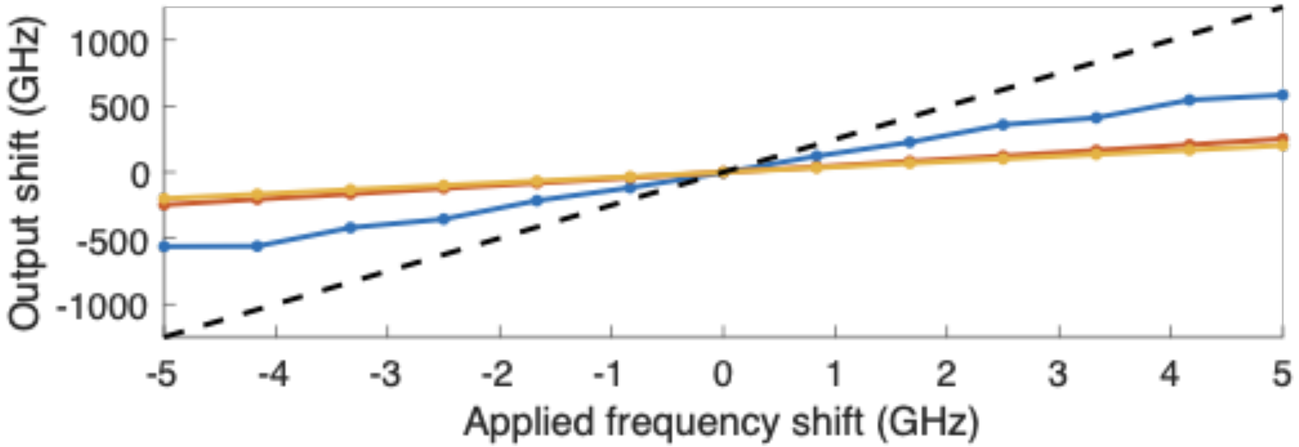
- Ellipsis indicates that there is no linear term for those coefficients
- The first row is the pure $\cos$ / $\cosh$ term (e.g. c_0)


1.45
1.40
1.35
1.30
1.25
−0.2
−0.1
0.1
0.2
Even estimates
Odd estimates
Data
1.5
1.0
0.5
−2
−1
1
2
Even estimates
Odd estimates
Data

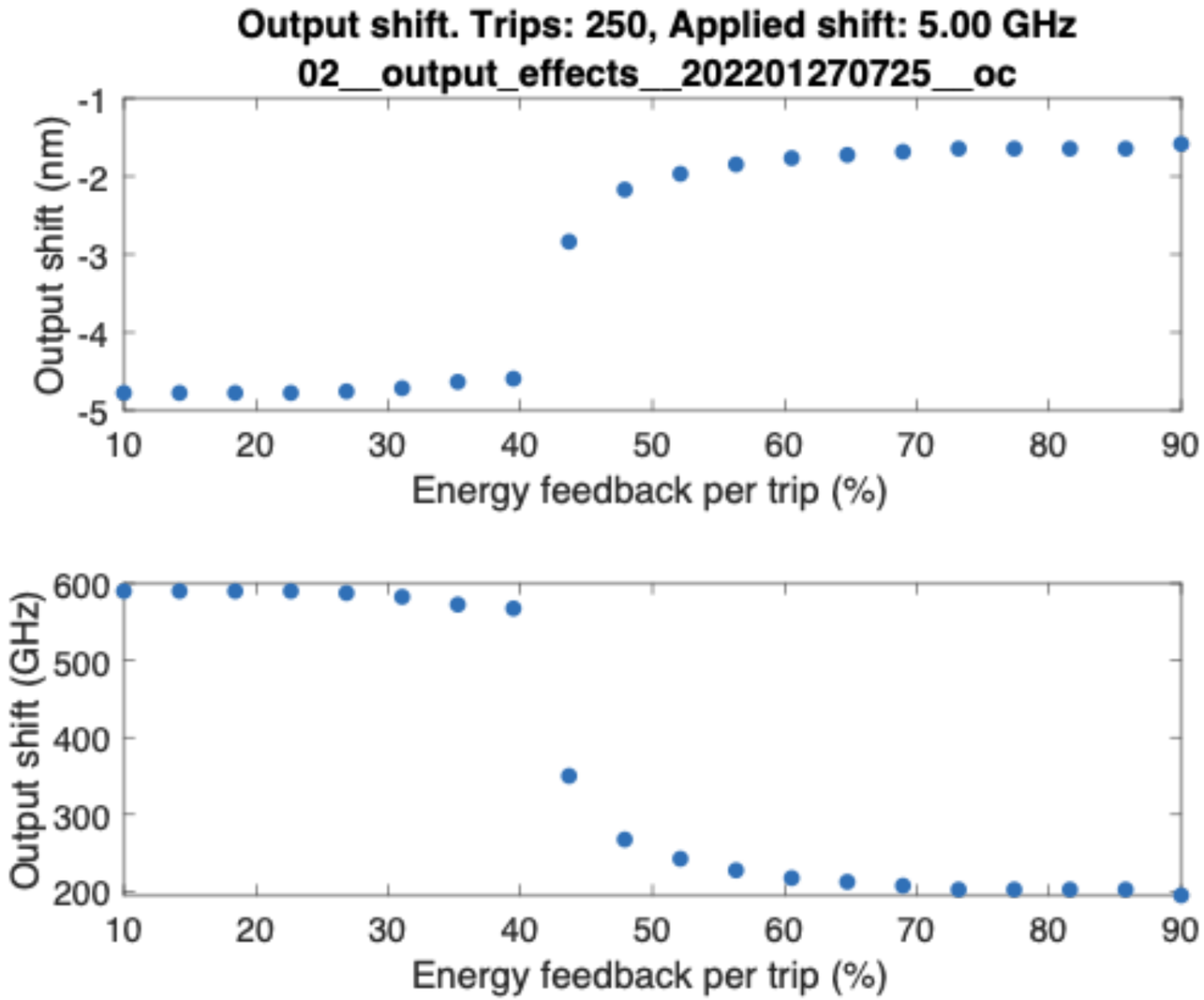
Numeric

"OC: 0.3” or “30% energy feedback per trip” means that 30% of the energy returns to the loop after each round trip, 70% exits via the output coupler





Numeric


Output shift. Trips: 250, Applied shift: 5.00 GHz
02__output_effects__202201270725__oc
Output shift (nm)
-1
-2
-3
-4
-5
10
20
30
40
50
60
70
80
90
Energy feedback per trip (%)
Output shift (GHz)
600
500
400
300
200
10
20
30
40
50
60
70
80
90
Energy feedback per trip (%)

Numeric

"OC: 0.3" or "30% energy feedback per trip" means that 30% of the energy returns to the loop after each round trip, 70% exits via the output coupler