Tags: #logbook - Denison
Links:

Logbook_04_220125

A Numeric Project

> **Aims**
>
> - [ ] Re run the simulations using 250 round trips
> - [ ] Examine quartic dispersion

A.1 Notes

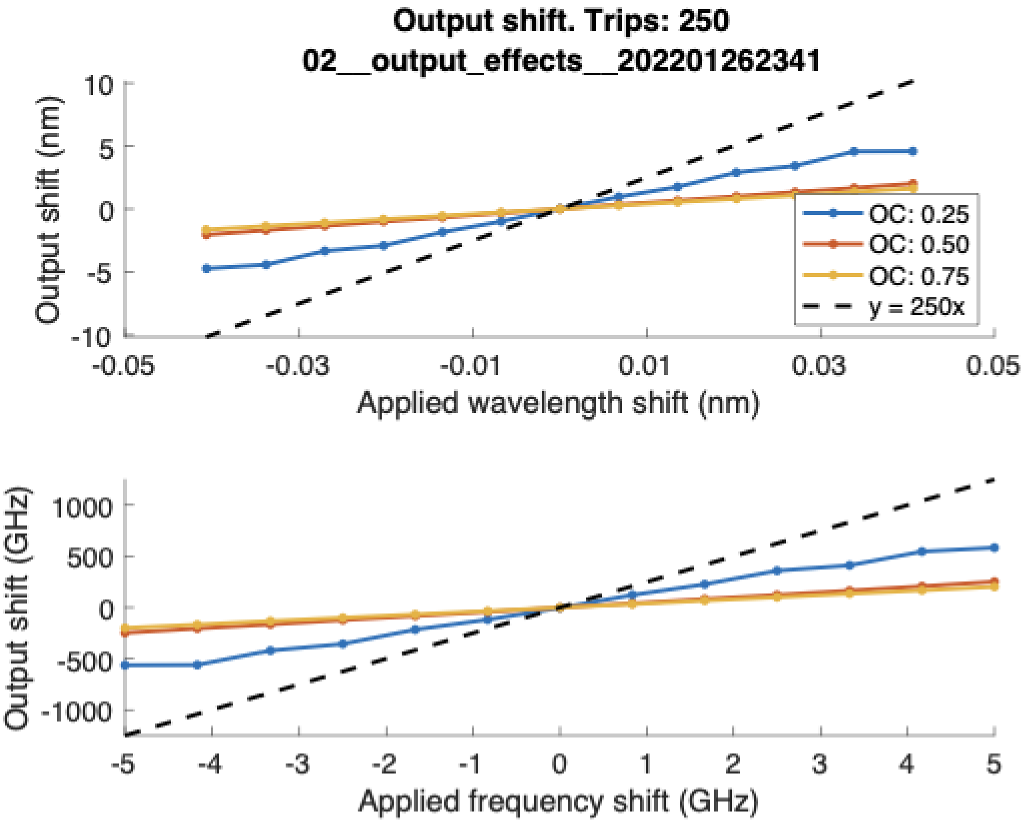
A.2 Results

A.2.1 Applied Shift

Ran `output_effects_220126_freq2.m` with `roundtrips = 250` and only 13 data points (because time)

- Took ~ 5 hours to run

  > `02__output_effects__202201262341.fig`



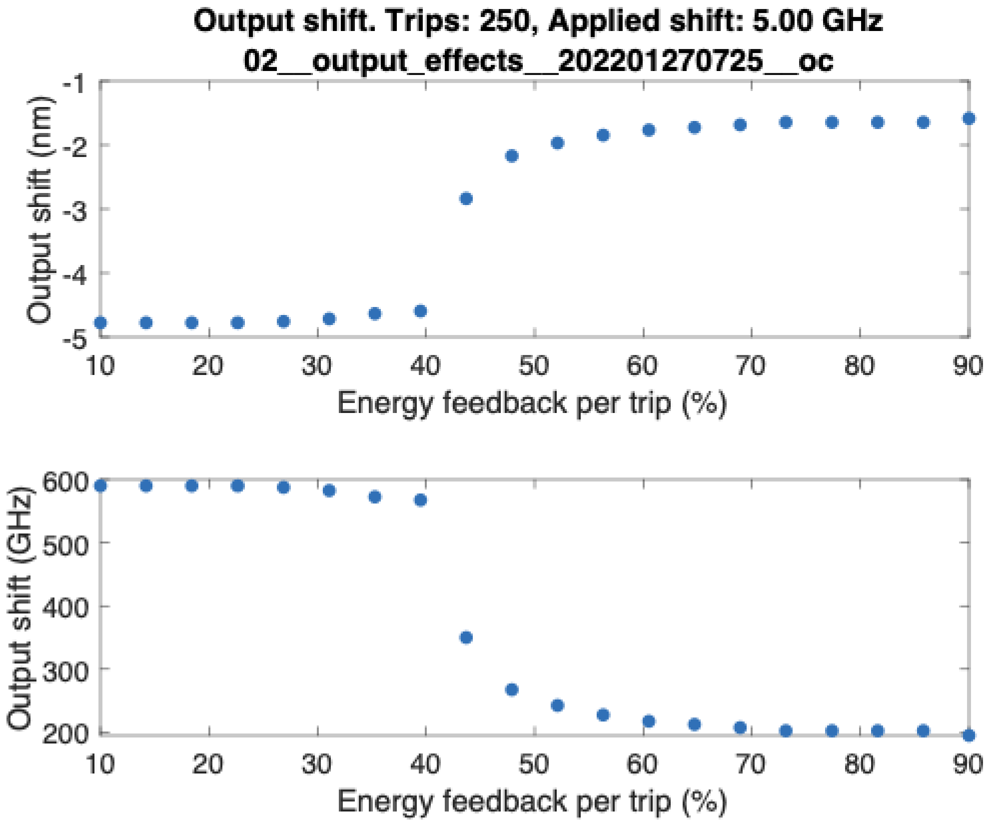
- The biggest difference is with the 0.25 feedback simulations, which appear slightly less linear
- However the other coupling simulations still appear very linear
  - But they are no longer as close to the $y = 250x$ line

A.2.2 Output Coupling

Re-ran `output_effects_220124_oc.m` with `roundtrips = 250`.

- Took ~ 950 seconds per data point, 20 data points means ~ 5 hours to run

```
02__output_effects__202201270725__oc.fig
```



- This is similar to the graph from before, except now there is less noise
  - Because now it has reached a saturation point where it stabilises
- Compared to the previous graph, the low energy feedback points are more extreme
  - Indicating that low feedback means it takes longer to reach saturation?
- The most effective shift appears to be for low energy feedback
  - But the energy in the cavity would be much lower

> **Why**
>
> Why does less energy feedback mean larger shifts?

A.2.2.1 Trip Evolution

Maybe this weird behaviour is due to that the graph has not yet stabilised, so the low energy points are at the dip, but will go back to smaller shifts once they stabilise.

To see this, use `output_effects_220127_oc2.m` and plot the evolution at each round trip:

> `02__output_effects_202201281357.mat`; `oc_animation.mp4`

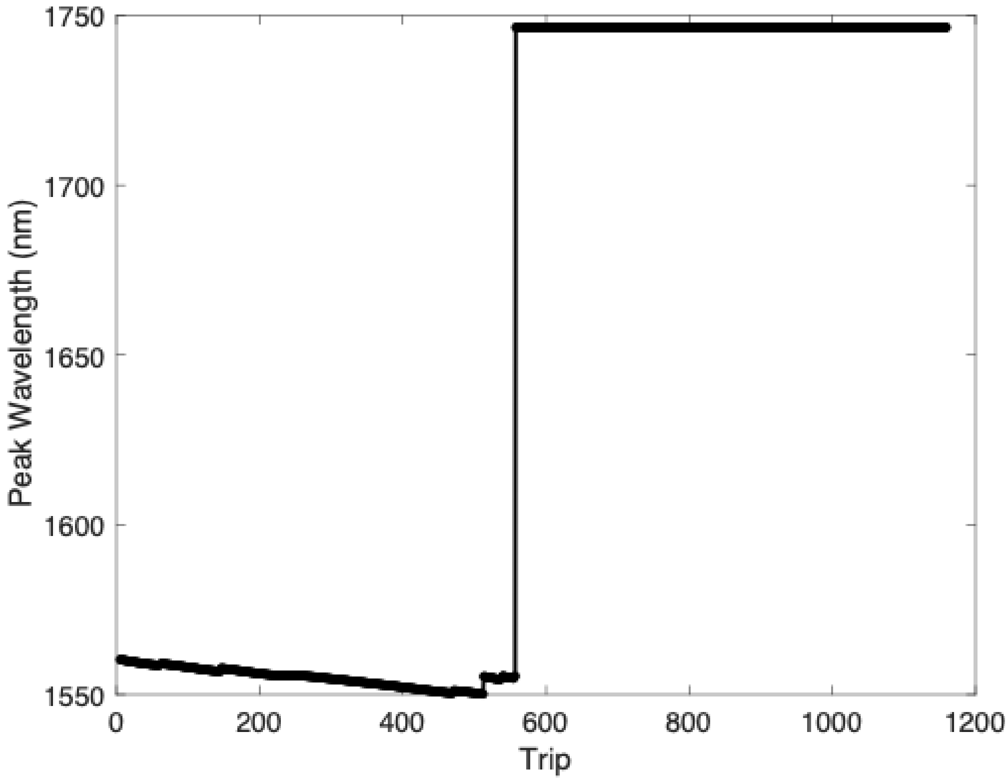
- The points are "on the way back up" when it cuts off, so we do need to perform more than just 250 round trips.

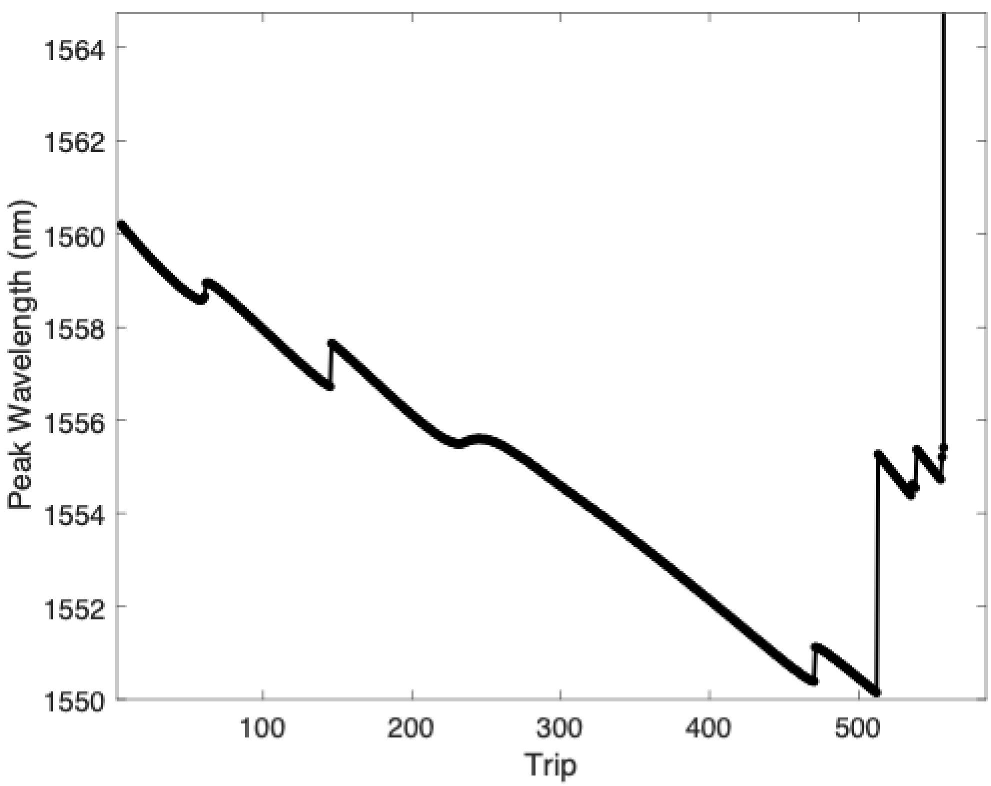
A.2.2.2 Trip Evolution 2

A.2.2.2.1 Feedback = 0.1

- To see this more clearly, re-ran `output_effects_220125_saturation.m` with `feedback = 0.1` and `freqdelta = 5e9` (unchanged) to contrast with the plot from before

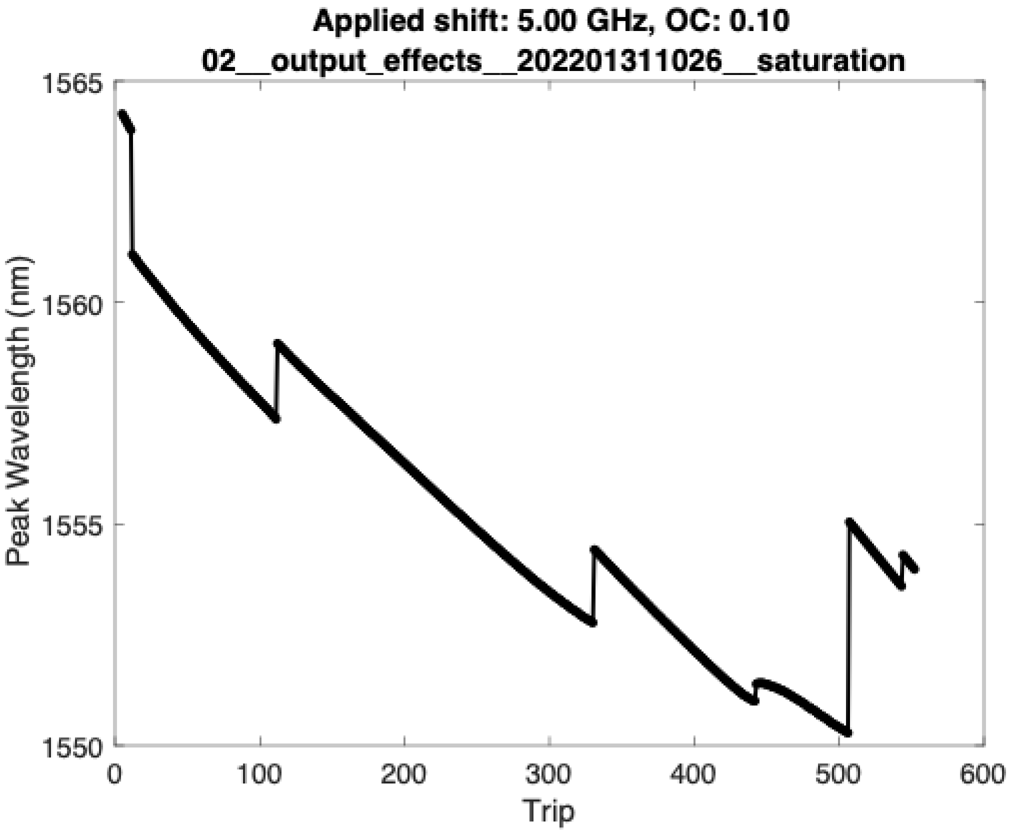
> `02__output_effects__202201281812__saturation.fig`
>
> `feedback = 0.1`



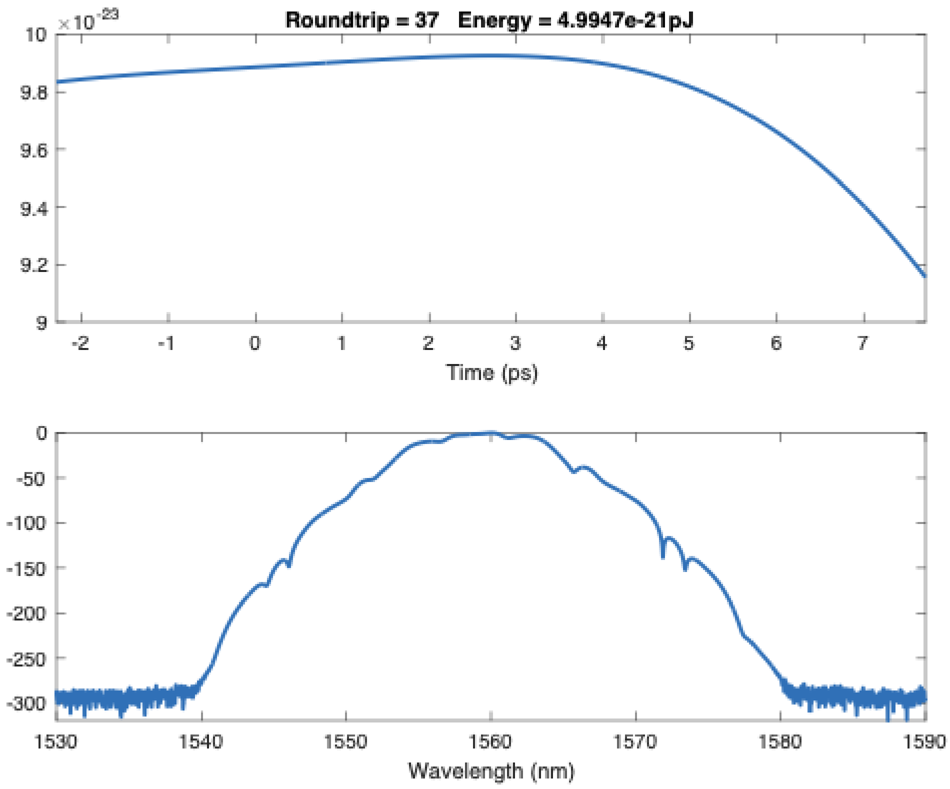


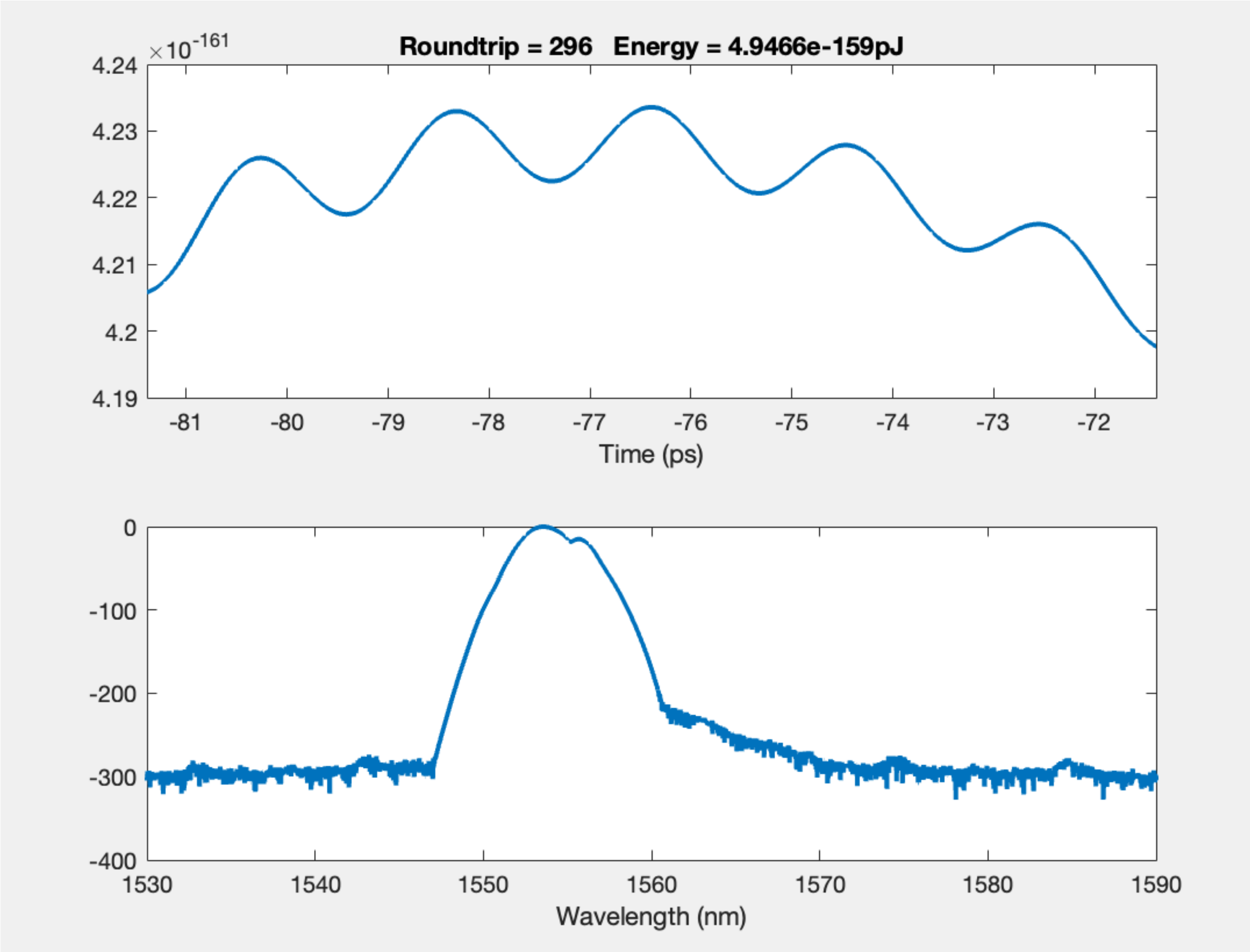
Re-ran with a different seed (`SEED = 222`)

```
02__output_effects__202201311026__saturation.fig
```

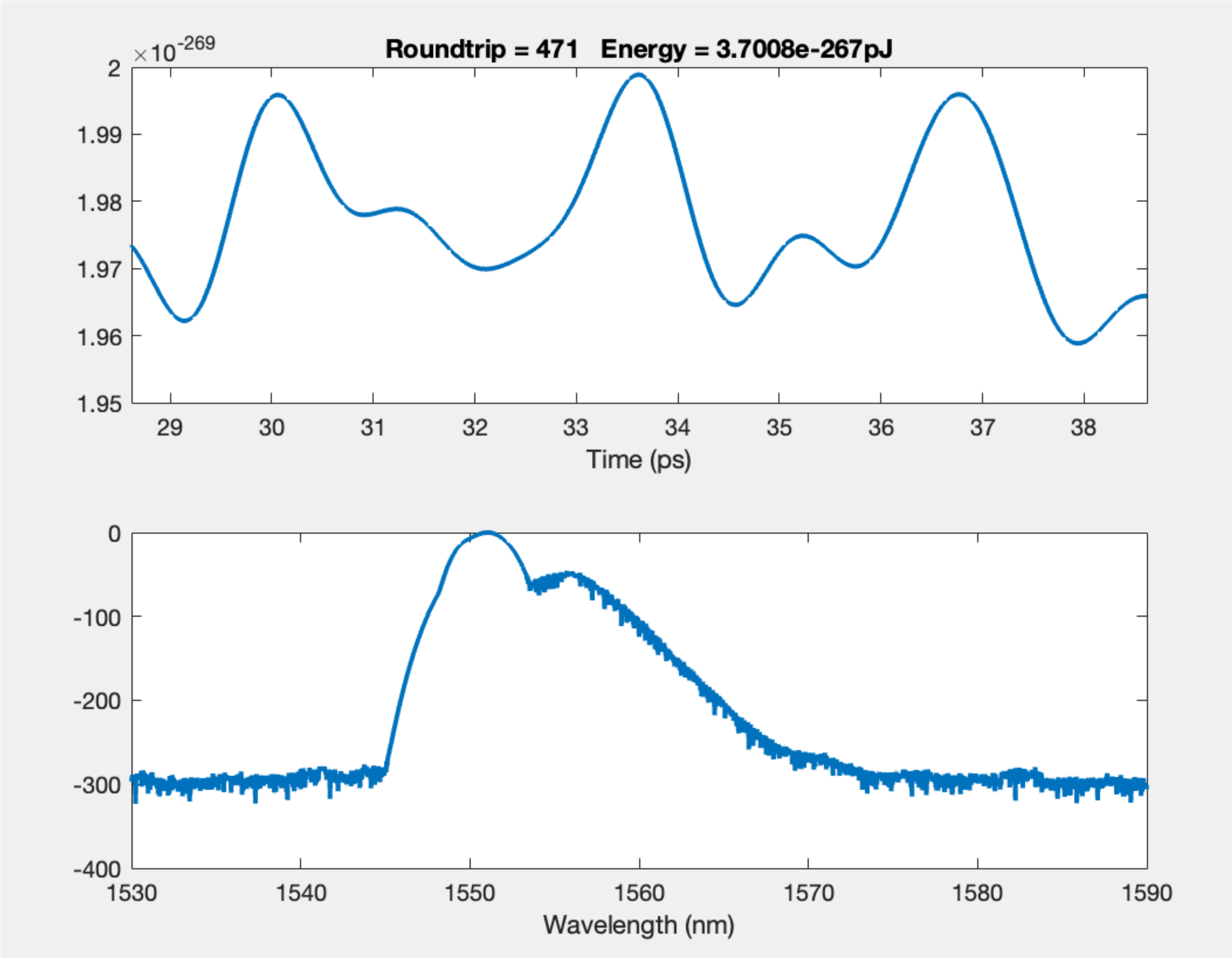


Spectrum and profile:

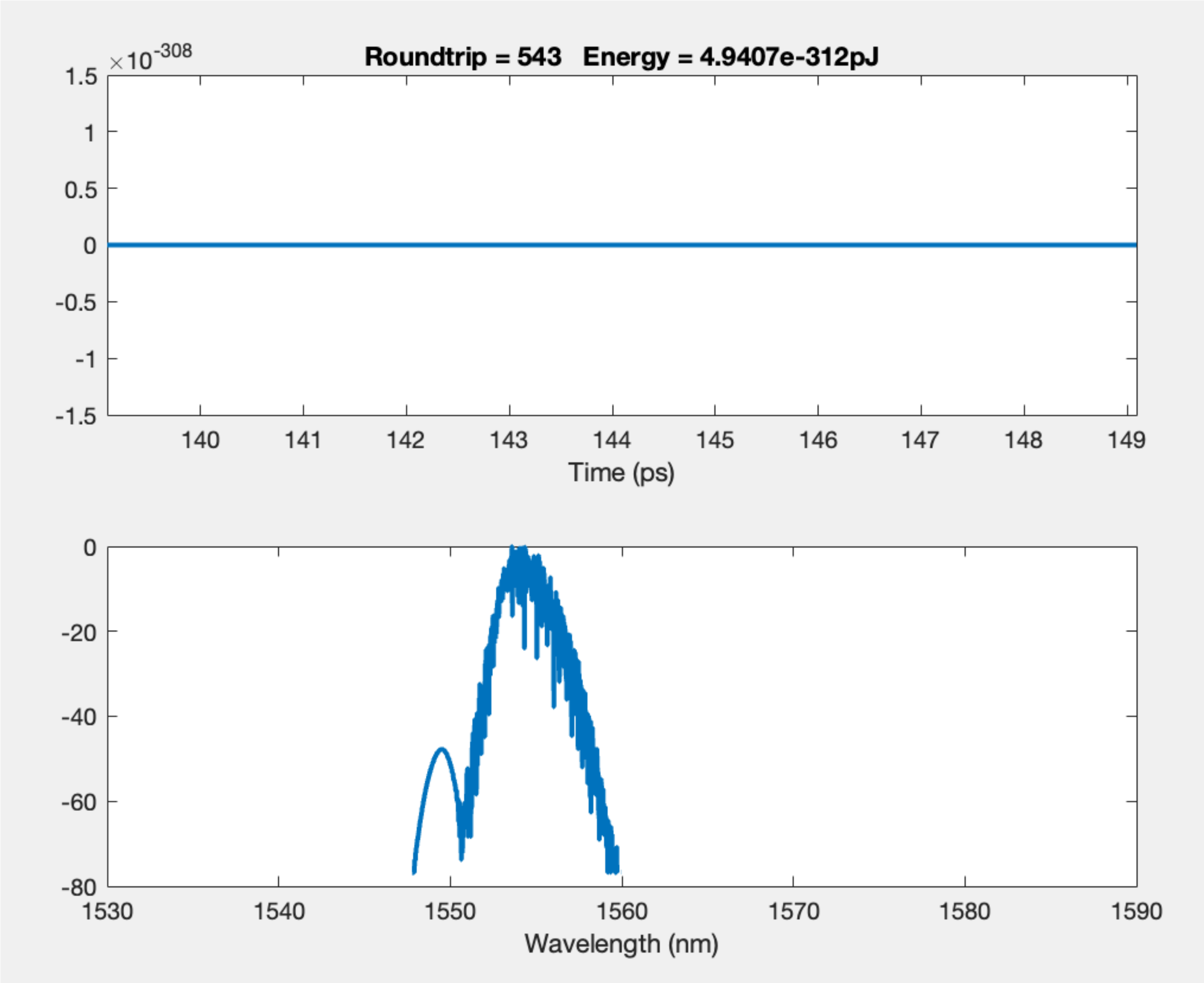




Roundtrip = 296 Energy = 4.9466e-159pJ
×10^-161
4.24
4.23
4.22
4.21
4.2
4.19
-81
-80
-79
-78
-77
-76
-75
-74
-73
-72
Time (ps)
0
-100
-200
-300
-400
1530
1540
1550
1560
1570
1580
1590
Wavelength (nm)



Roundtrip = 471 Energy = 3.7008e-267pJ
×10^-269
2
1.99
1.98
1.97
1.96
1.95
29 30 31 32 33 34 35 36 37 38
Time (ps)
0
-100
-200
-300
-400
1530 1540 1550 1560 1570 1580 1590
Wavelength (nm)

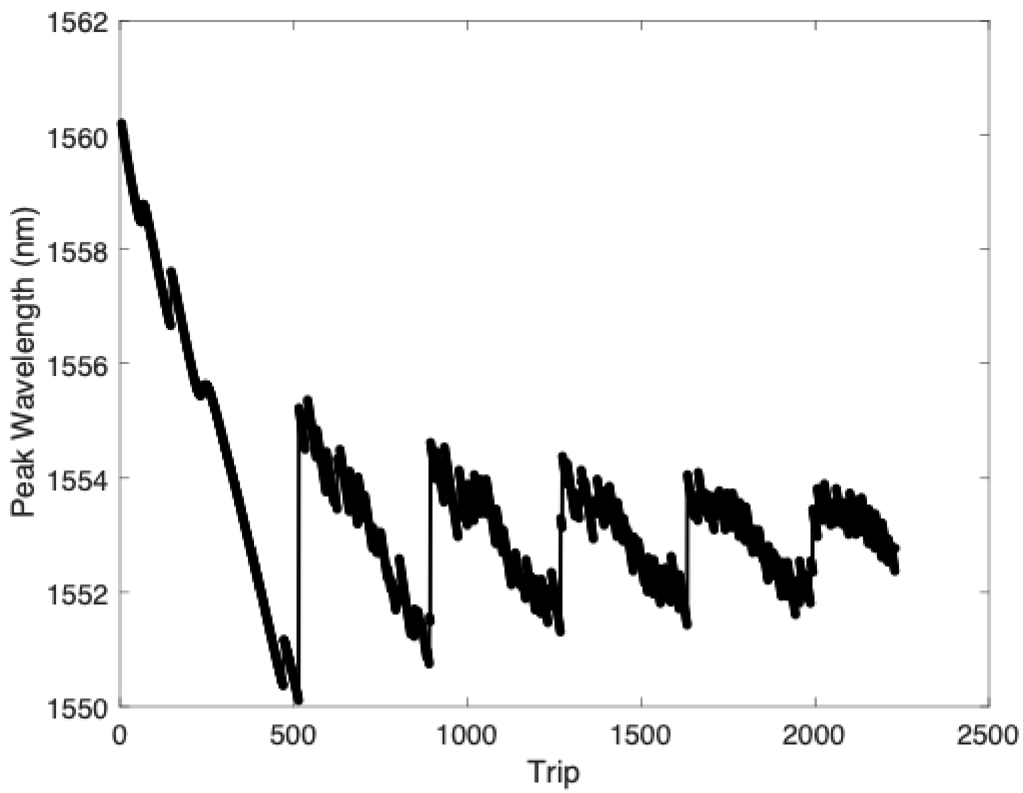


> **Low Feedback**
>
> So if the `feedback` is too low then it just doesn't form a soliton

- The "jumps" in the peak wavelength are most likely due to the peak not being a single value, and so multiple peaks can "overtake" each other, causing it to change values discontinuously

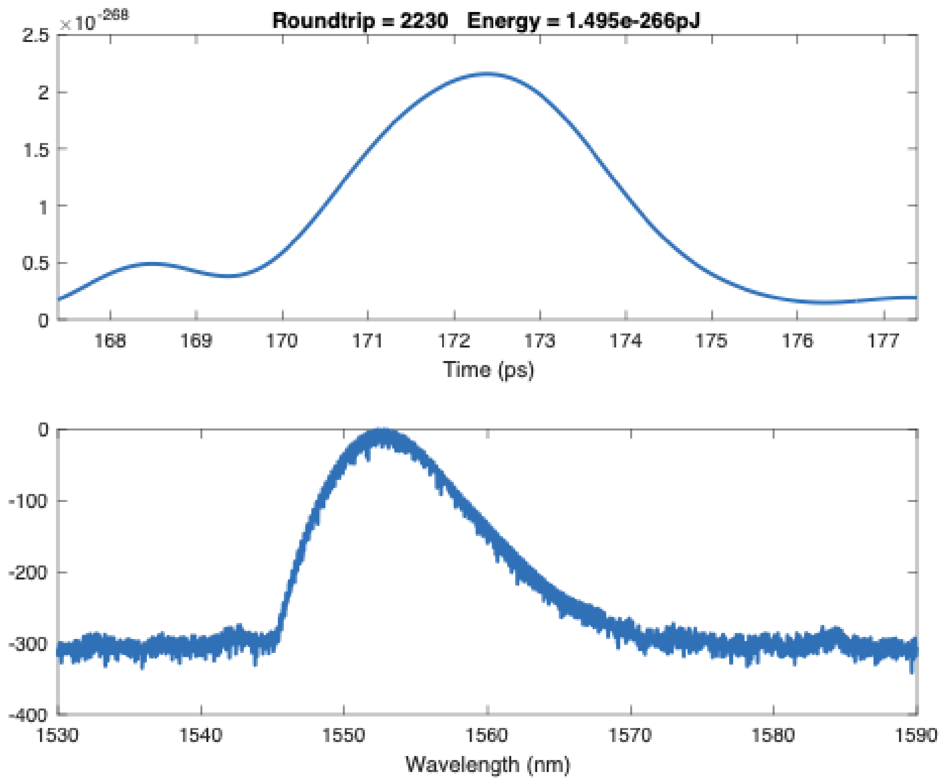
A.2.2.2.2 Feedback = 0.3

```
02__output_efects_202201282136__saturation_01.fig
OC = 0.3
```



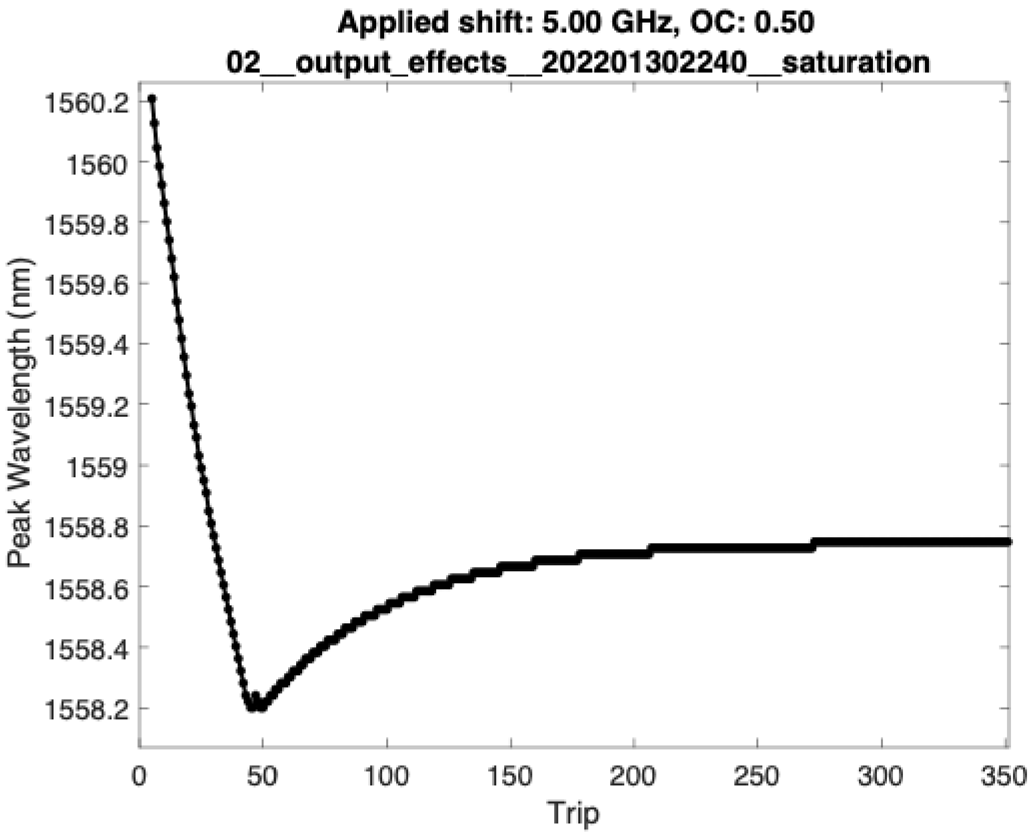
- By the end, the spectrum looks like

`02__output_effects__202201282135__saturation_02.fig`

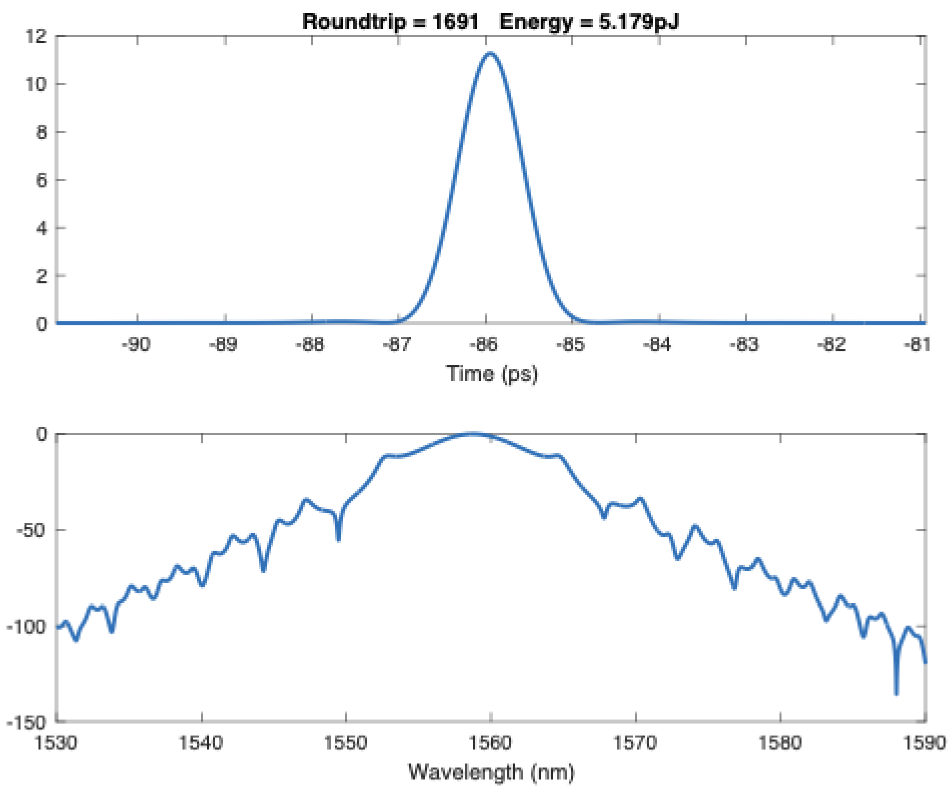


A.2.2.2.3 Feedback = 0.5

```
02__output_effects__202201302240__saturation.fig
```

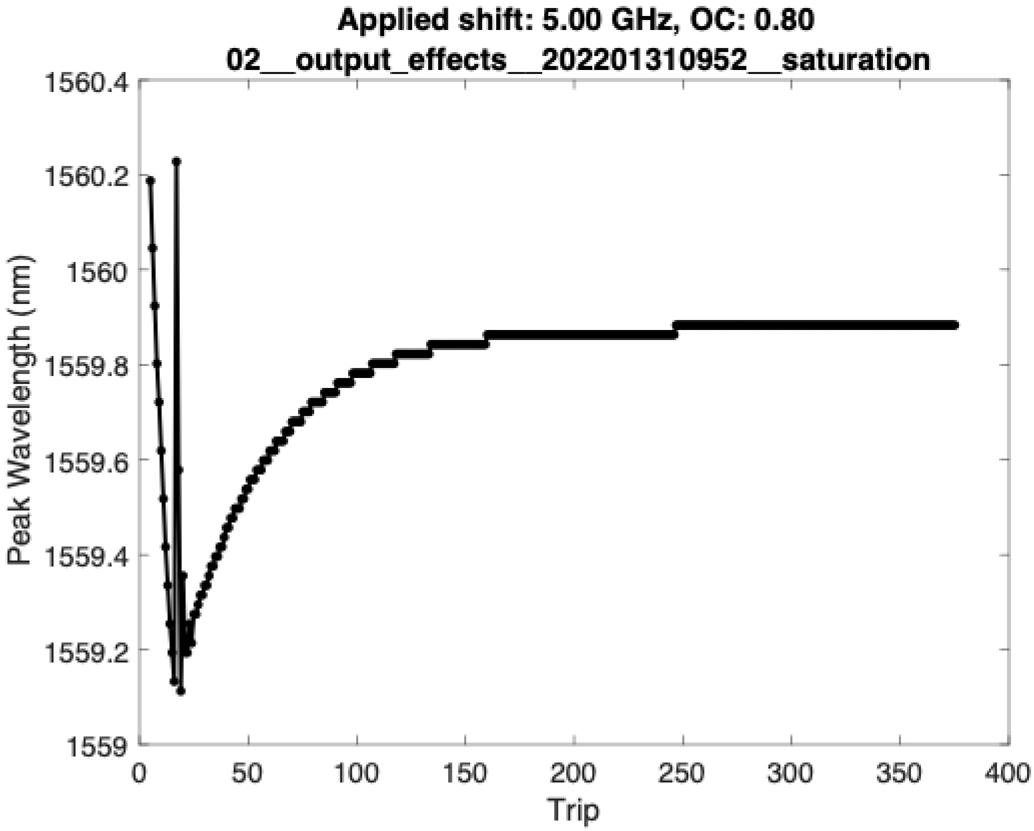



```
02__output_effects__202201302240__saturation_02.fig
```

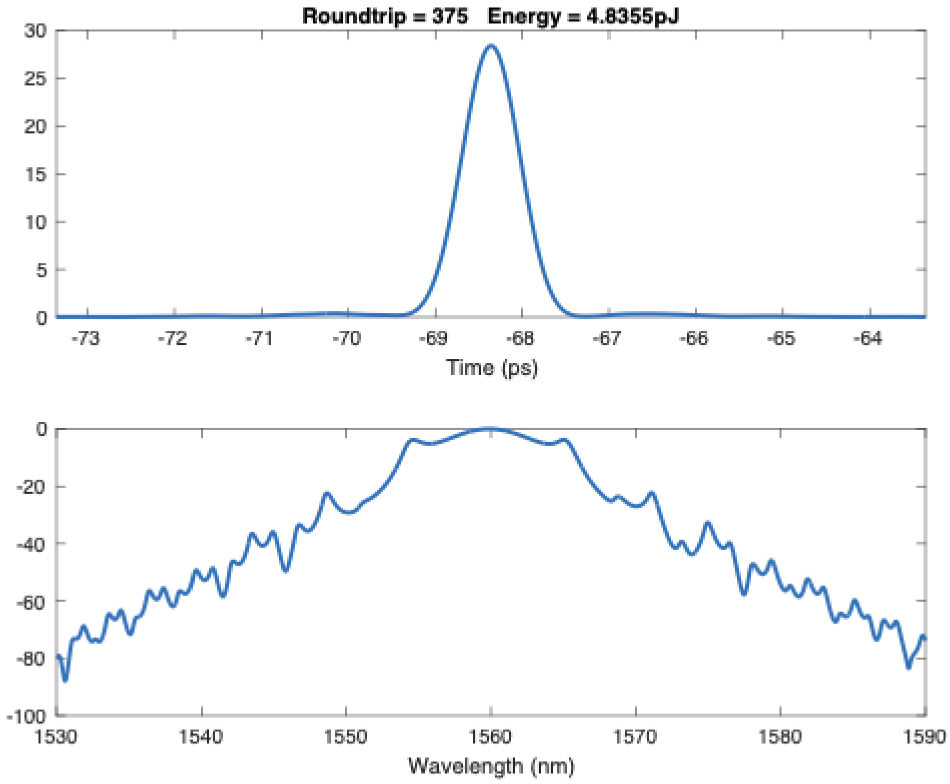


A.2.2.2.4 Feedback = 0.8

```
02__output_effects__202201310952__saturation.fig
```




```
02__output_effects__202201310952__saturation.fig
```



A.3 Outcomes

So there appears to be a critical point where solitons do not form correctly below a certain OC, around 0.45

- Because if the feedback is too low, then we are pushing it too hard and it can't re-form
- We next want to find this critical point and how it varies with the applied shift

A.4 To Do

- [x] ~~Why does less energy feedback mean larger shifts?~~
  - [x] ~~Repeat the energy feedback graph except animate what it looks like at each round trip.~~
    - ~~Because it's not actually a soliton~~
- [x] ~~Repeat graphs with different initial conditions (e.g. the one that find saturation from Logbook_03_220124)~~
- [ ] Experiment with quartic dispersion

- Starting from an initial condition which is my analytic solution

- [ ] Do a 2D plot of applied shift (y) and feedback (x) with colour representing frequency (or white if it is an unstable solution)

  - Log scale frequency shift 100 MHz to 5 GHz
  - Don't need too fine a scale, we want to just see "how hard we can push it"
  - With quadratic dispersion for now

B Analytic Project

Aims

- [ ] Examine the higher order solutions and their coefficients

B.1 Notes

We can start finding higher order solutions, by taking

$$L\left[\sum_{j=0}^{n} u^{(j)}\right] + N\left[\sum_{j=0}^{n-1} u^{(j)}\right] \equiv 0 \text{ up to } (2n+1)\text{th order}$$

B.2 Results

Using Mathematica notebook `03_fifthorder_220125.nb`

B.2.1 Fifth Order Coefficients

In[330]:= `answer2 = AnalyticSolve[eq1 /. answer1, 2, x, σ, d] // FullSimplify`

Out[330]=

$$\left\{d[0] \to \frac{A}{8} + \frac{\left(-95 A^5 + 86 A^4 B + 98 A^3 B^2 + 352 A^2 B^3 + 273 A B^4 + 266 B^5\right)\Gamma^2}{21299200\,\sigma^8} - \frac{3 \times \left(2 A^3 + 9 A^2 B + 6 A B^2 + 9 B^3\right)\Gamma}{5120\,\sigma^4},\right.$$

$$d[1] \to -\frac{B}{8} - \frac{\left(86 A^5 + 671 A^4 B + 712 A^3 B^2 + 798 A^2 B^3 + 626 A B^4 - 273 B^5\right)\Gamma^2}{21299200\,\sigma^8} + \frac{3 \times \left(3 A^3 + 2 A^2 B + 3 A B^2 - 2 B^3\right)\Gamma}{5120\,\sigma^4},$$

$$d[2] \to \frac{1}{10649600\,\sigma^8}\Gamma\left(\left(49 A^5 + 356 A^4 B + 50 A^3 B^2 + 532 A^2 B^3 - 399 A B^4 + 176 B^5\right)\Gamma - 12480 B\left(3 A^2 + 4 A B + 3 B^2\right)\sigma^4\right),$$

$$d[3] \to \frac{1}{10649600\,\sigma^8}\Gamma\left(-\left(\left(176 A^5 + 399 A^4 B + 532 A^3 B^2 - 50 A^2 B^3 + 356 A B^4 - 49 B^5\right)\Gamma\right) - 12480 A\left(3 A^2 - 4 A B + 3 B^2\right)\sigma^4\right),$$

$$d[4] \to -\frac{A}{8} + \frac{\left(273 A^5 + 626 A^4 B - 798 A^3 B^2 + 712 A^2 B^3 - 671 A B^4 + 86 B^5\right)\Gamma^2}{21299200\,\sigma^8} + \frac{3 \times \left(2 A^3 + 3 A^2 B - 2 A B^2 + 3 B^3\right)\Gamma}{5120\,\sigma^4},$$

$$\left.d[5] \to \frac{B}{8} - \frac{\left(266 A^5 - 273 A^4 B + 352 A^3 B^2 - 98 A^2 B^3 + 86 A B^4 + 95 B^5\right)\Gamma^2}{21299200\,\sigma^8} + \frac{3 \times \left(-9 A^3 + 6 A^2 B - 9 A B^2 + 2 B^3\right)\Gamma}{5120\,\sigma^4}\right\}$$

In[323]:= `answer2 //. allvalues`

Out[323]= `{d[0] → 0.088257, d[1] → -0.127076, d[2] → 0.0262384, d[3] → 0.00364435, d[4] → -0.061929, d[5] → 0.130676}`

We see that there are linear terms in all coefficients apart from d_2, d_3.

B.2.2 Linear Terms in Coefficients

Tracking the linear terms for higher orders, we get:

Function	$u^{(1)}$	$u^{(2)}$	$u^{(3)}$	$u^{(4)}$	$u^{(5)}$	$u^{(6)}$
Order	**3**	**5**	**7**	**9**	**11**	**13**
Coefficients	A/4	A/8	5A/64	7A/128	21A/512	33A/1024
	-B/4	-B/8	-5B/64	-7B/128	-21B/512	-33B/1024
	A/4	...	-A/64	-A/64	-7A/512	-3A/256
	-B/4	...	B/64	B/64	7B/512	3B/256
		-A/8	-A/64	...	A/256	5A/1024
		B/8	B/64	...	-B/256	-5B/1024
			5A/64	A/64	A/256	...
			-5B/64	-B/64	-B/256	...
				-7A/128	-7A/512	-5A/1024
				7B/128	7B/512	5B/1024
					21A/512	3A/256
					-21B/512	-3B/256
						-33A/1024
						33B/1024

- Ellipsis indicates that there is no linear term for those coefficients
- The first row is the pure $\cos / \cosh$ term (e.g. c_0)

B.2.2.1 Patterns

1. The "no linear term" occurs every 2nd function, in the middle 2 entries
2. For u1, u3, u5, ..., the coefficients are mirrored, with $A \mapsto -B$ and $B \mapsto -A$
3. For u2, u4, ..., the coefficients are mirrored, but with $A \mapsto B$ and $B \mapsto A$
4. Going down a column, the signs are $A, -B, -A, B$ repeating (terminating when reaching the halfway point)

Is there a pattern for the first coefficient?

> **Catalan Numbers**
>
> If we take the denominators to be increasing as 4^k, then the numerators become $1, 2, 5, 14, 42, 132$: These are the Catalan numbers
>
> $$C(n) = \frac{(2n)!}{n!(n+1)!}$$

Similarly, for the third coefficients (so $1/64, 1/64, 7/512$ etc.), these *could* be the **Fourth convolution of Catalan numbers** if we take denominator to again be $4, 4^2, 4^3$ etc.

- But there isn't enough data points to see this conclusively
- It takes too long to compute $u^{(7)}$ at the moment, need to speed up the code somehow

B.2.2.2 Values

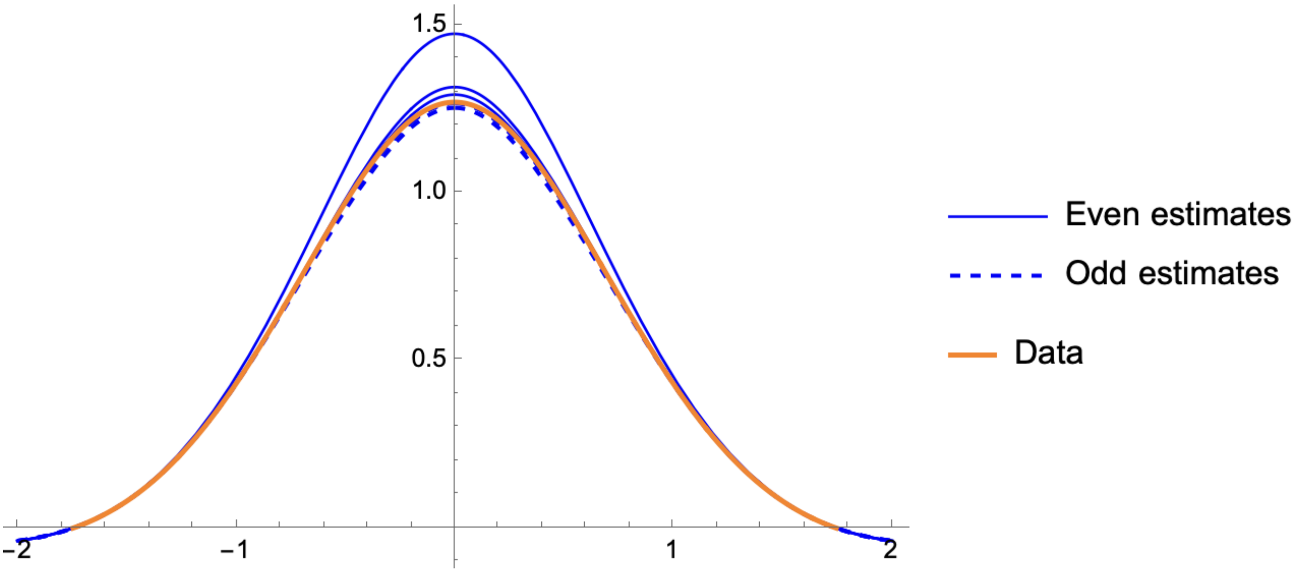
c[0] → 0.157039	d[0] → 0.0903383	e[0] → 0.0609978	f[0] → 0.0448517	g[0] → 0.0348072	h[0] → 0.0280441
c[1] → -0.256215	d[1] → -0.128514	e[1] → -0.0804295	f[1] → -0.0563353	g[1] → -0.0422631	h[1] → -0.0332103
c[2] → 0.140487	d[2] → 0.0271894	e[2] → 0.00526498	f[2] → -0.00152474	g[2] → -0.00390647	h[2] → -0.00469792
c[3] → -0.262153	d[3] → 0.00377046	e[3] → 0.0168306	f[3] → 0.0162891	g[3] → 0.0141419	h[3] → 0.0120898
_	d[4] → -0.0630543	e[4] → -0.0190443	f[4] → -0.00858966	g[4] → -0.00390552	h[4] → -0.0014936
_	d[5] → 0.132236	e[5] → 0.0142077	f[5] → -0.000842331	g[5] → -0.00440549	h[5] → -0.00521731
_	_	e[6] → 0.0366884	f[6] → 0.0133825	g[6] → 0.00733371	h[6] → 0.00430593
_	_	e[7] → -0.0830518	f[7] → -0.015152	g[7] → -0.00337559	h[7] → 0.000378755
_	_	_	f[8] → -0.0244042	g[8] → -0.00985882	h[8] → -0.00593467
_	_	_	f[9] → 0.0583147	g[9] → 0.0136075	h[9] → 0.00456695
_	_	_	_	g[10] → 0.0176156	h[10] → 0.00757355
_	_	_	_	g[11] → -0.0438274	h[11] → -0.0118415
_	_	_	_	_	h[12] → -0.0134343
_	_	_	_	_	h[13] → 0.0344878

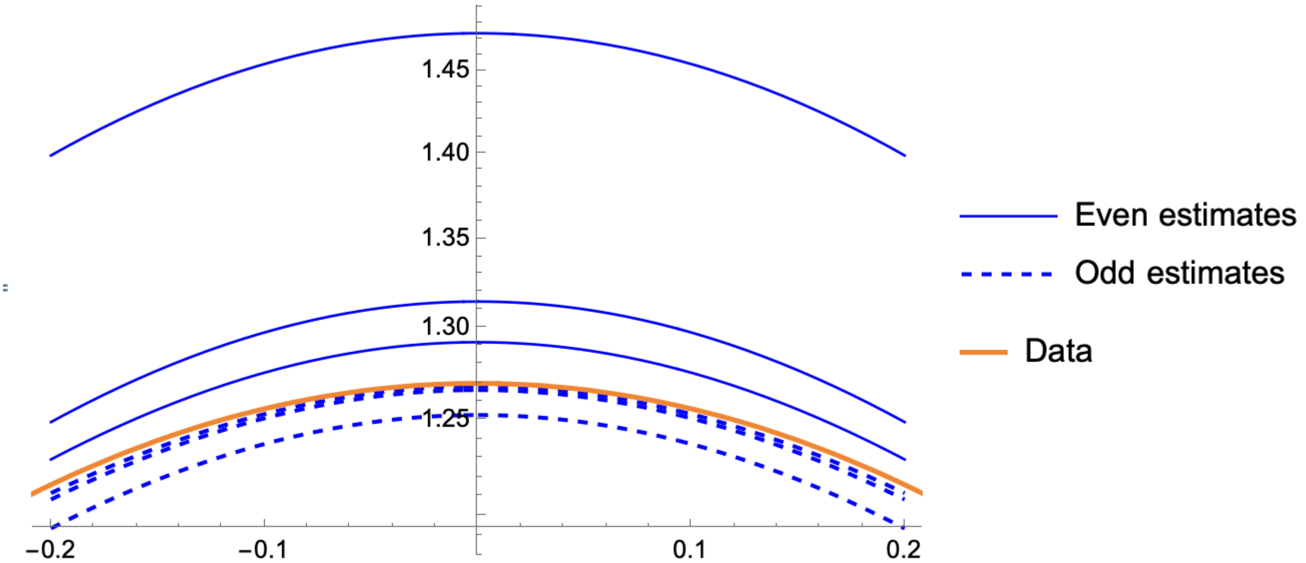
- The magnitude of the values are not decreasing significantly at each new order

Just the linear terms:

0.111352	0.055676	0.0347975	0.0243583	0.0182687	0.014354
-0.257043	-0.128521	-0.0803258	-0.056228	-0.042171	-0.0331344
0.111352	0	-0.0069595	-0.0069595	-0.00608956	-0.00521963
-0.257043	0	0.0160652	0.0160652	0.014057	0.0120489
_	-0.055676	-0.0069595	0	0.00173988	0.00217484
_	0.128521	0.0160652	0	-0.00401629	-0.00502036
_	_	0.0347975	0.0069595	0.00173988	0
_	_	-0.0803258	-0.0160652	-0.00401629	0
_	_	_	-0.0243583	-0.00608956	-0.00217484
_	_	_	0.056228	0.014057	0.00502036
_	_	_	_	0.0182687	0.00521963
_	_	_	_	-0.042171	-0.0120489
_	_	_	_	_	-0.014354
_	_	_	_	_	0.0331344

- If we look at terms with the same linear component, they still have (sometimes significantly) different numerical values
    - So perhaps only considering the linear components is not a viable strategy
- However the function is still converging:





Estimates

The even estimates tend to overshoot, the odd tend to undershoot (but are closer to the actual data)

- "Even estimate" means $u^{(k)}$ where k is even

B.3 Outcomes

- There are linear terms that appear in the coefficients
  - There is a link to the Catalan numbers in these

- But only considering the linear terms wouldn't give us good estimates of the full coefficients because they don't dominate the other terms
- The even estimates overshoot, the odd undershoot but are closer to the numerical solution

B.4 To Do

- [ ] Find patterns in the linear terms
- [ ] Find patterns in the other terms, perhaps we can write them as functions of preceding coefficients + linear term
- [x] ~~Search this series and see if anything comes up on Google~~
    - ~~Catalan numbers~~
    - [ ] ~~Search the other linear terms~~

There should be a condition on A/B, because other solutions will go to ∞ on the sides

- [ ] Look at the zeroes of the polynomials

- [x] ~~Look at Taylor series~~

- [ ] Use $\cos^2$ instead of $\cos$ so that the amplitude isn't affected by θ

- Is there a better set of basis functions we can use?