Tags: #logbook - Denison
Links:

Logbook_220124

A Numeric Project

Aims

- [ ] Determine how many round trips is required before staturation
- [ ] Re-run the plots to determine output frequency vs applied at saturation

A.1 Notes

The gain of the laser is applied in an inverse parabolic profile, centred about a wavelength $\lambda_0 \sim 1560$ nm. It has a bandwidth of ~ 50 nm. If the soliton moves far from this wavelength, then the gain will have less of an effect.

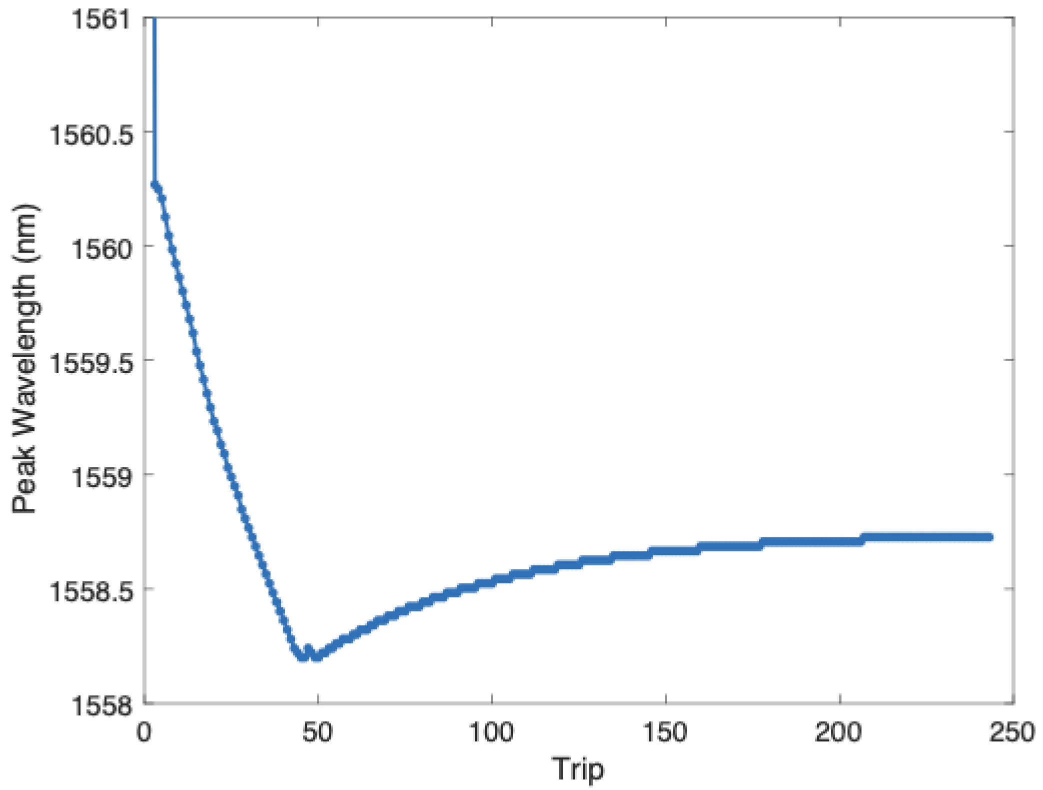
- Therefore, we expect that there will be a **saturation** point where the frequency shifter is pushing the soliton one direction, but it isn't gaining enough energy to be stable beyond that
  - And so the frequency will stabilise at a certain value
- I need to determine that value, and maybe then the plots from Logbook_02_220117 will not be as linear

A.2 Results

A.2.1 Saturation Point

Run the simulation for as long as needed until the peak wavelength no longer changes.

```
02__output_effects__2201251450__saturation.fig
```



- The wavelength increases in steps after reaching the minimum value
    - May be a numerical consequence (e.g. with the FFT)
- Interesting that it goes down then back up again

A.3 Outcomes

- We need around 250 round trips to reach a stable wavelength

A.4 To Do

- [x] ~~Repeat the graphs from the previous logbook except with 250 round trips~~
- [ ] Repeat with quartic dispersion
    - [ ] Use an initial condition that is our solution from the Analytic Project, and check that it is stationary (doesn't change with time)
    - [ ] Then repeat the graphs

B Analytic Project

Aims

- [ ] Verify or find the values of A, B that match with the numerical data (or Ξ, θ)

B.1 Notes

B.1.1 Defining $u^{(0)}$

Instead of writing $A\frac{\cos}{\cosh} + B\frac{\sin}{\sinh}$, we can use a modulus-argument form and write

$$u^{(0)} = \Xi\left(\cos\theta\frac{\cos}{\cosh} + \sin\theta\frac{\sin}{\sinh}\right)$$

- This makes it easier to find these parameters, by aligning the phase of the two terms and also the amplitudes of their peaks

B.1.2 Sign of Γ

There is a different convention used in Martijn's handwritten notes, the actual way we should be writing the DE is

$$u'''' + 4\sigma^4 u + \Gamma u^3 = 0 \qquad \text{(Eq. 1)}$$

where

- $\sigma^4 := \frac{6\mu}{|\beta_4|}$
- $\Gamma := -\frac{24\gamma}{|\beta_4|}$
  Note that this is the **negative** of the way it was defined in Logbook_01_220115 > B 1 Notes.
- This is a bit confusing, since it means that the $u^{(1)}$ will be the negative of what it was before, i.e. moving further away from the data...

- However we can perhaps fix this by checking that we are doing the right thing when defining it in the first place:

B.1.3 Finding $u^{(1)}$

If we define

$$L[u]=\frac{d^4}{dx^4}u+4\sigma^4 u,\quad N[u]=\Gamma u^3$$

then the idea is that we start with

$$L\left[u^{(0)}\right]=0\text{ to 1st order}$$

and then

$$L\left[u^{(1)}+u^{(0)}\right]+N\left[u^{(0)}\right]=0\text{ to 3rd order}\qquad\text{(Eq. 2)}$$

because if it is just $L[u^{(1)}]+N[u^{(0)}]=0$, then the third order terms for $L[u^{(0)}]$ won't be cancelled out.

> **Change**
>
> This is different to the previous work - it used to be just $L\left[u^{(1)}\right]+N\left[u^{(0)}\right]=0\text{ to 3rd order}$

B.2 Results

B.2.1 Revised Approach for $u^{(1)}$

Using the new process (Eq. 2), we get different coefficient expressions for $u^{(1)}$:

$$c_0 = \frac{A}{4} - \left(2A^3 + 9A^2B + 6AB^2 + 9B^3\right)\varphi = 0.15371$$
$$c_1 = -\frac{B}{4} + 3\left(3A^3 + 2A^2B + 3AB^2 - 2B^3\right)\varphi = -0.253315$$
$$c_2 = \frac{A}{4} - 3\left(2A^3 + 3A^2B - 2AB^2 + 3B^3\right)\varphi = 0.137759$$
$$c_3 = -\frac{B}{4} + \left(9A^3 - 6A^2B + 9AB^2 - 2B^3\right)\varphi = -0.259133$$

where

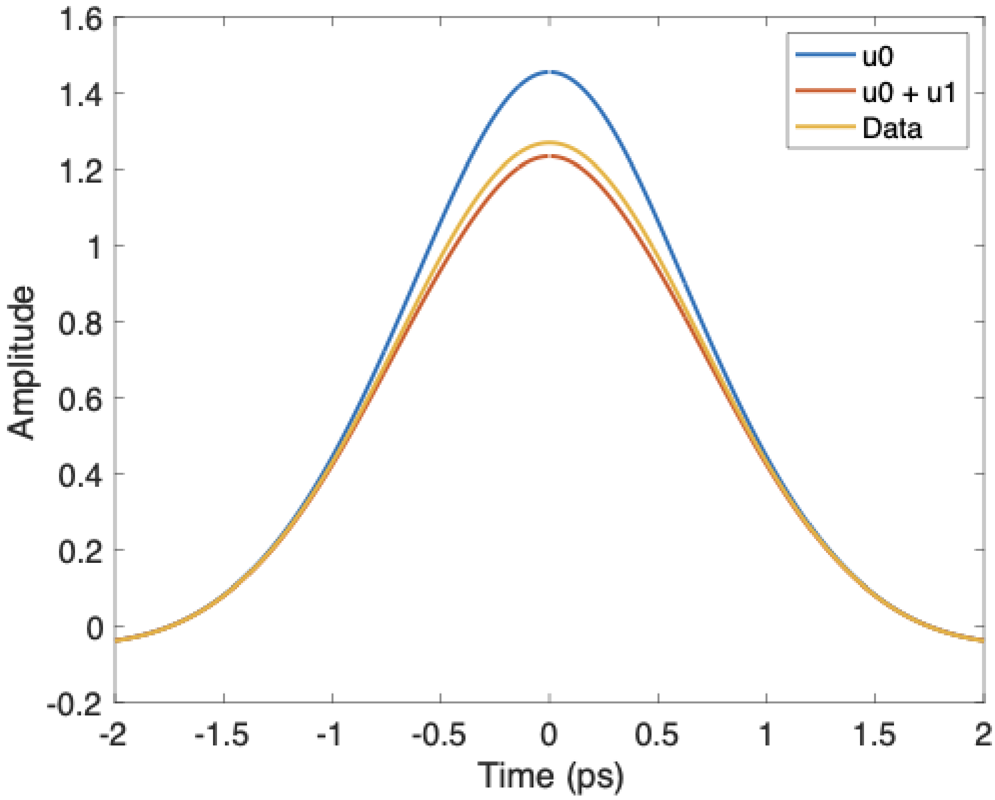
$$\varphi := \frac{\Gamma}{1280\sigma^4}$$

and with the numbers found by substituting in $\Gamma = -24, \sigma^4 = 6$, and $A = 0.4384, B = 1.0170.$

- These are the same expressions as before, except with the $\frac{A}{4}$ and $-\frac{B}{4}$ terms added

This gives:

```
01__compare__202201242100.fig
```



- It results in a much closer fit to the data compared to the incorrect method

- And it works with the new definition of Γ being negative

> **Interesting**
>
> Interestingly, the linear $\frac{A}{4}$, $-\frac{B}{4}$ terms dominate the c expressions. If we see similar results for the fifth order coefficients, then perhaps we can drop the other terms, track the linear terms, and get a closed-form expression?

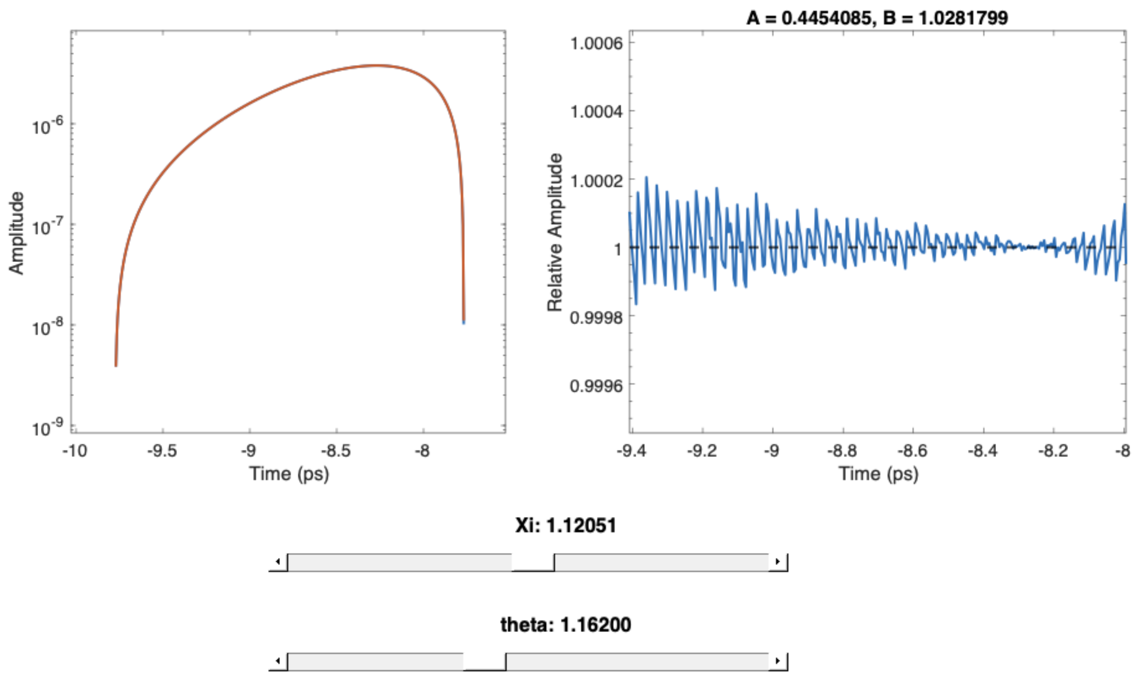
B.2.2 Finding Ξ, θ

To find these values, we want to compare the pattern in the tails (where u^3 is negligible) on a log-linear plot

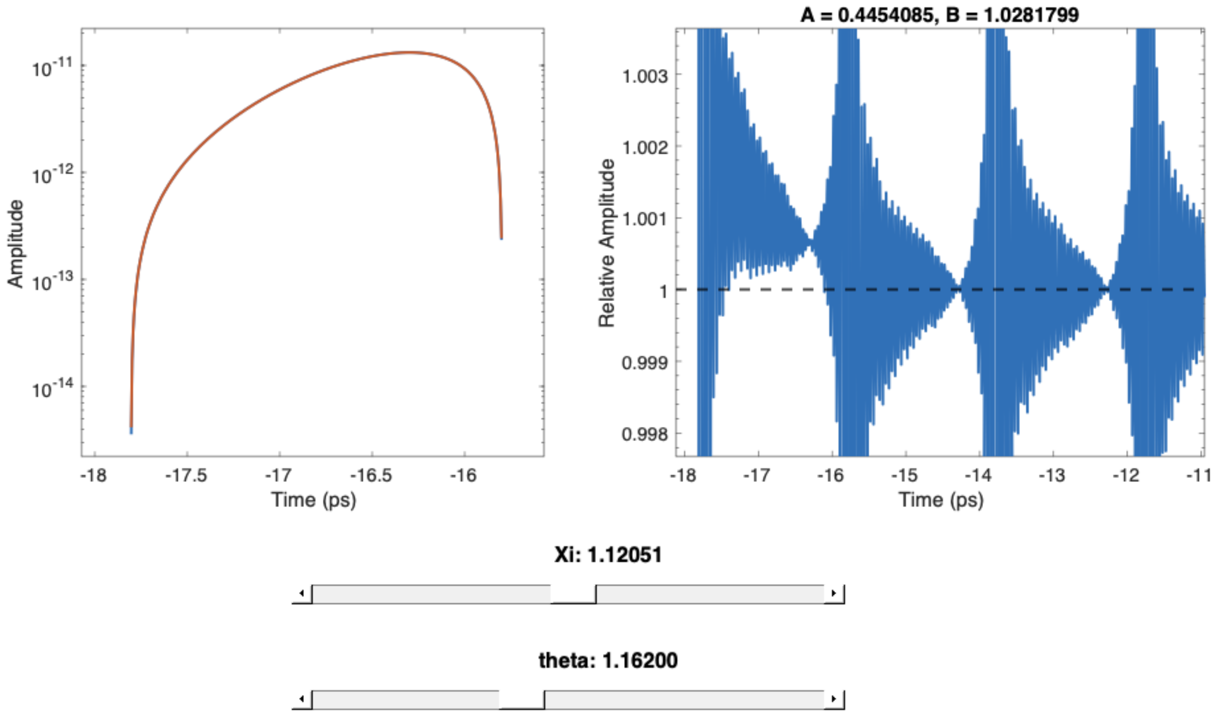
- If we divide the data by the $u^{(0)}$ estimate, we should get a straight line at $y = 1$

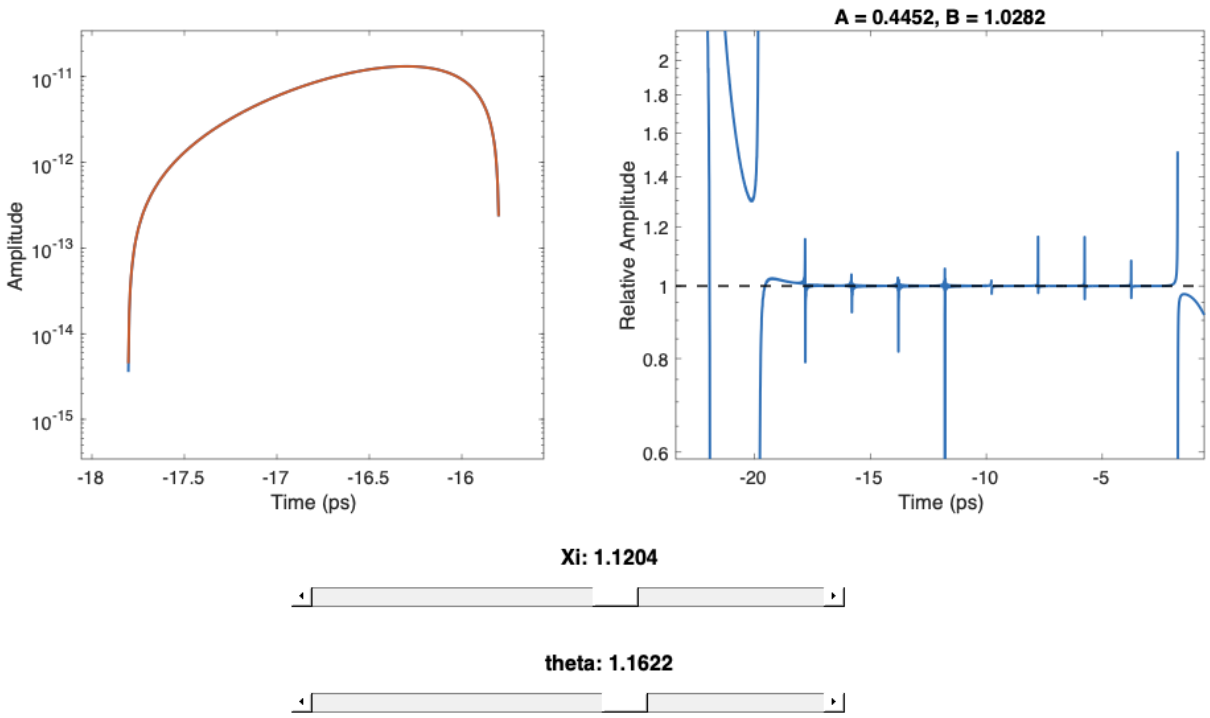
I made an interactive figure `f02_220124_xitheta.m`, `02__interactive_xi_theta__202201241730.fig`

- The left plot shows the data (blue) and the fitted function (orange)
- The right plot shows $\mathrm{data/fitted}$
- By varying Ξ, θ, we can find a "good fit" in the tails of the function



The fit gets bad after around 20 picoseconds:





- This could just be due to precision issues with the numerical solution
- And the grid can cause oscillations, where the numerics goes above and below at different values

However, the peaks before then all match closely

- The values I get are:

$$\Xi = 1.12051,\ \theta = 1.16200; \qquad A = 0.445408,\ B = 1.02817$$

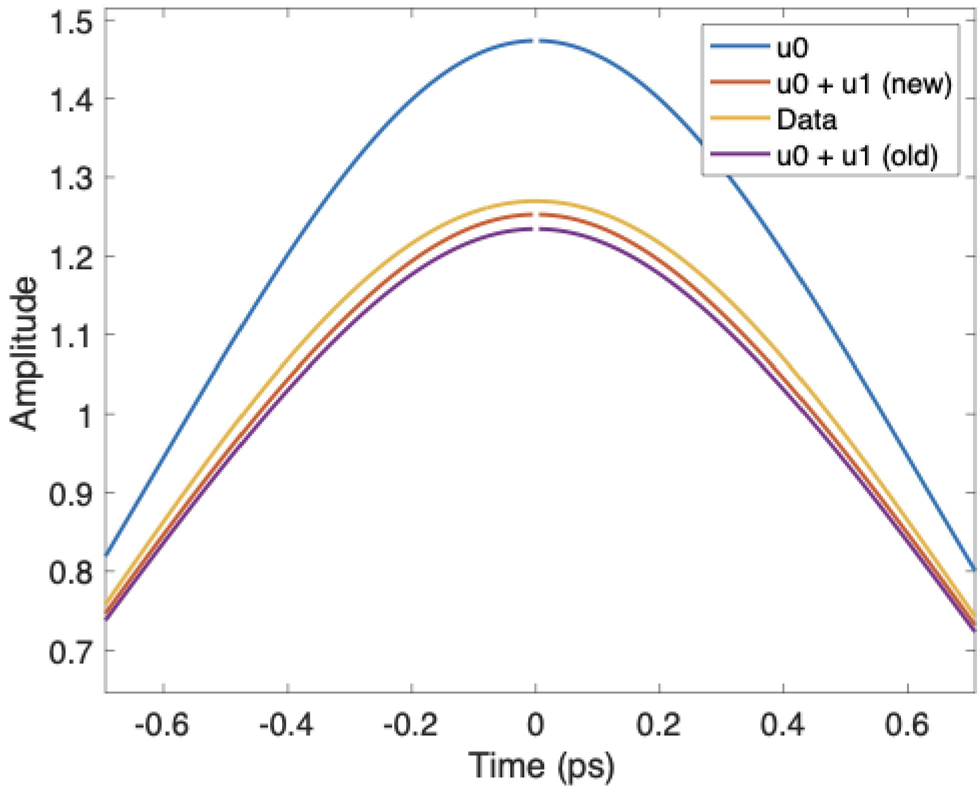
```
which are similar but not exact to the one's from
[[Logbook_01_220115|Long]]
```

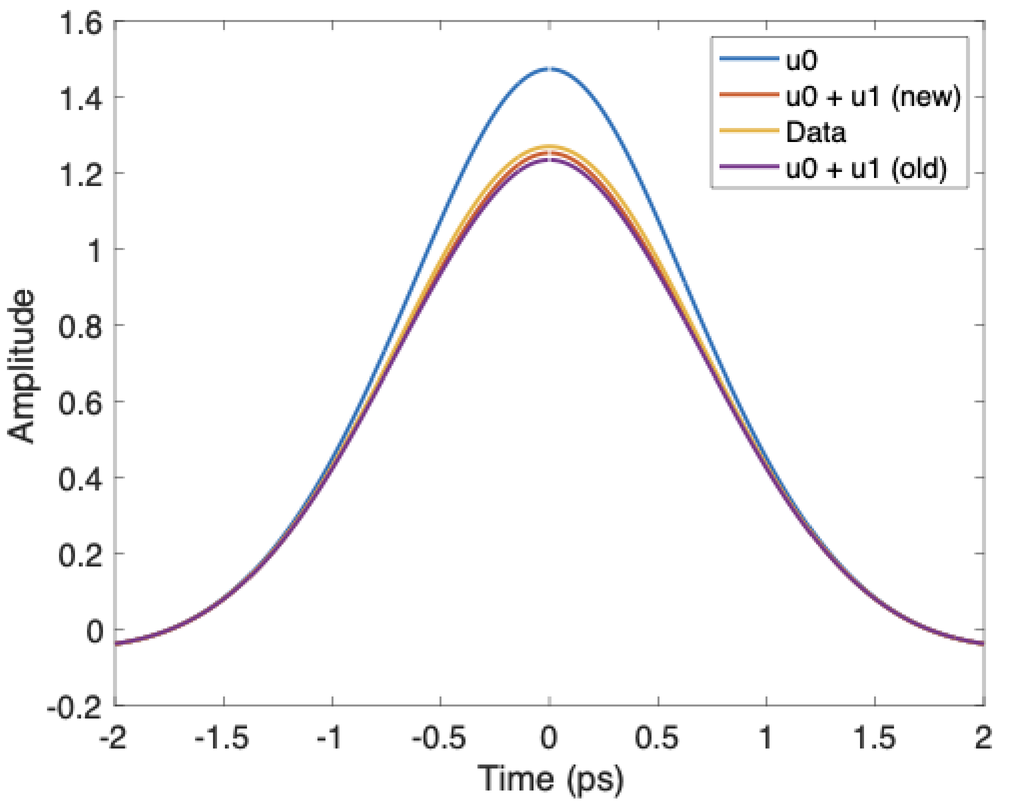
- Resolution was ± 1 in the last digit of Ξ and ± 5 in the last digit of θ

B.2.2.1 Data Comparison

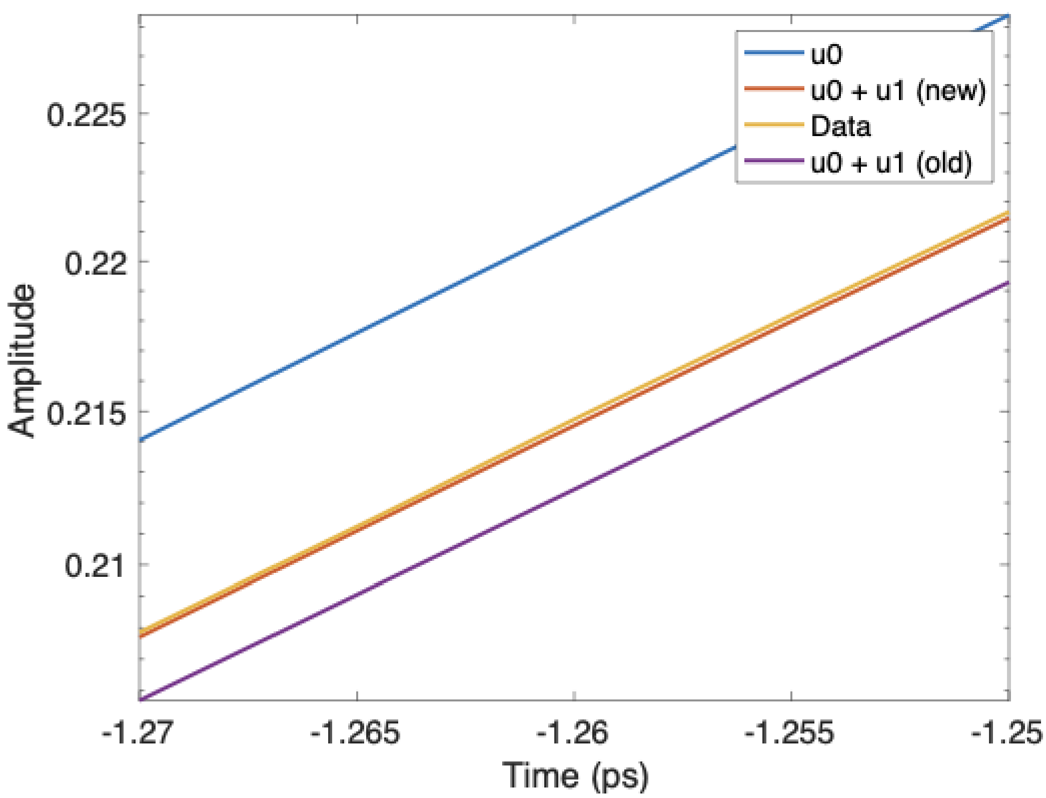
Comparing this fit with the data, we get an even better fit near the peak and in the tails:

> `01__compare__202201242301.fig`






```
01__compare__202201242301.fig
```



B.2.3 How θ affects the function

- When $\theta = 0$, so only $\cos$, it is too thin and the tails are too emphasised
    - When $\theta = \pi/2$ so only $\sin$, it is too wide and flat
- There are θs when the concavity changes (as expected, since $\theta = -\pi$ would just be the negative of $\theta = 0$)

- θ is independent of Γ and σ, because it is just the constant coming out of a differential equation
    - θ is a universal parameter

B.3 Outcomes

- Our best estimates are

$$\Xi = 1.1205 \pm 0.0001, \; \theta = 1.1620 \pm 0.0001$$

$$A = 0.445408, \; B = 1.02817$$

B.4 To Do

- [x] ~~Repeat the "finding Xi, theta" approach using $u^{(1)}$ as well $u^{(0)}$~~

- [x] ~~Check whether or not the new~~ $L + N = 0$ ~~reasoning is correct~~
    - ~~Yes it is~~
- [x] ~~Examine the fifth order solutions~~

B.5 Question

- [x] ~~How does this negative~~ Γ ~~sign impact the accuracy of~~ $u^{(1)}$ ~~etc.?~~
    - ~~Why does it go in the wrong direction?~~
    - ~~It fixes itself when we resolve the error with our old approach to finding~~ $u^{(1)}$~~, now it goes in the correct direction~~