

PQS Project

Meeting 2022-01-28 1:00pm

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Project 1

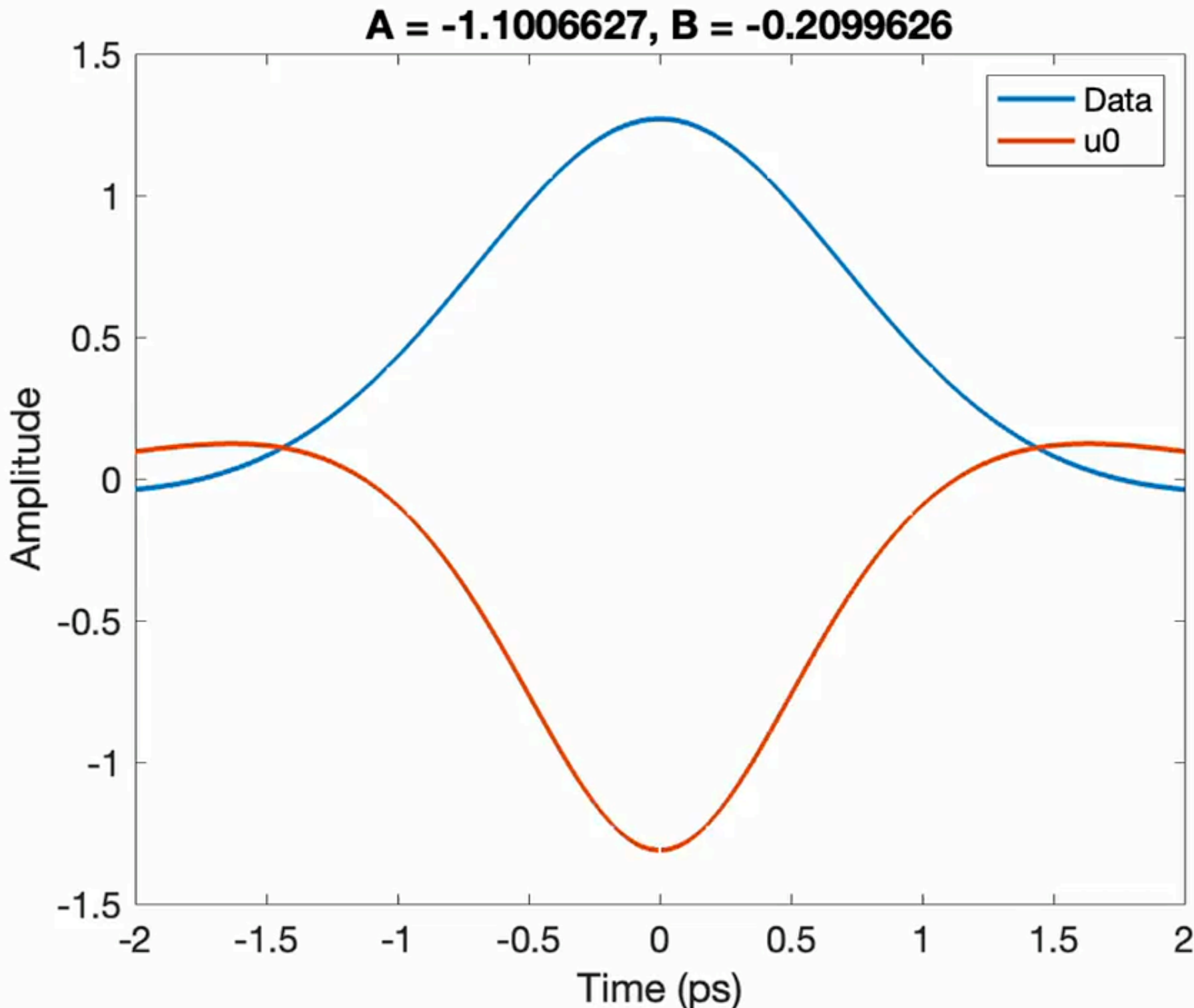
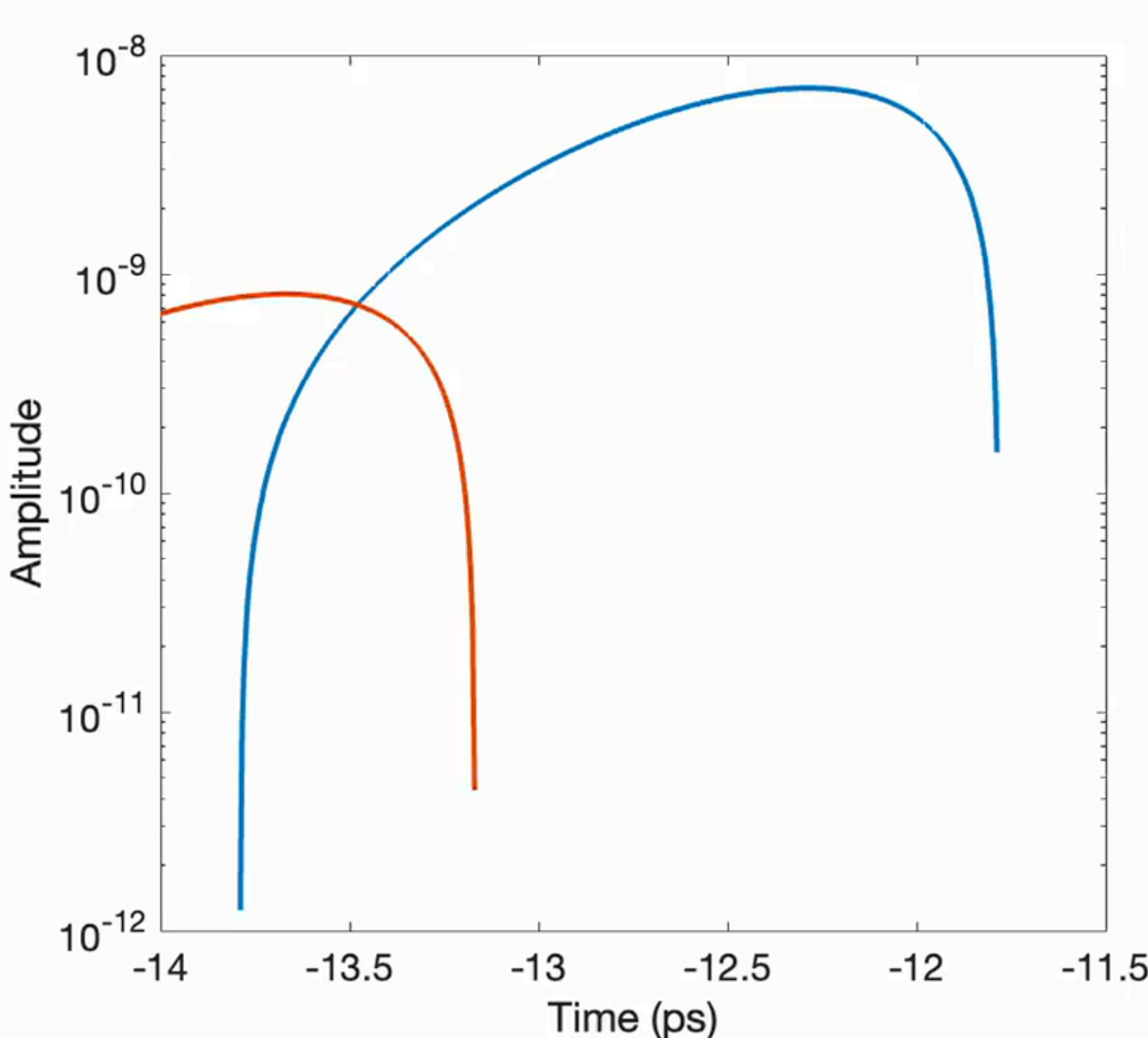
$$\underbrace{u'''' + 4\sigma^4 u}_L + \underbrace{\Gamma u^3}_N = 0, \quad \sigma^4 := \frac{6\mu}{|\beta_4|}, \Gamma := \frac{24\gamma}{|\beta_4|}$$

- For the zeroth order ansatz, determined the coefficients that match with data by writing

$$A \frac{\cos}{\cosh} + B \frac{\sin}{\sinh} \equiv \Xi \left(\cos \theta \frac{\cos}{\cosh} + \sin \theta \frac{\sin}{\sinh} \right)$$

- Where Ξ controls the amplitude, θ controls the phase
- Found: $\Xi = 1.1205 \pm 0.0001$, $\theta = 1.1620 \pm 0.0001$ and $A = 0.445408$, $B = 1.02817$

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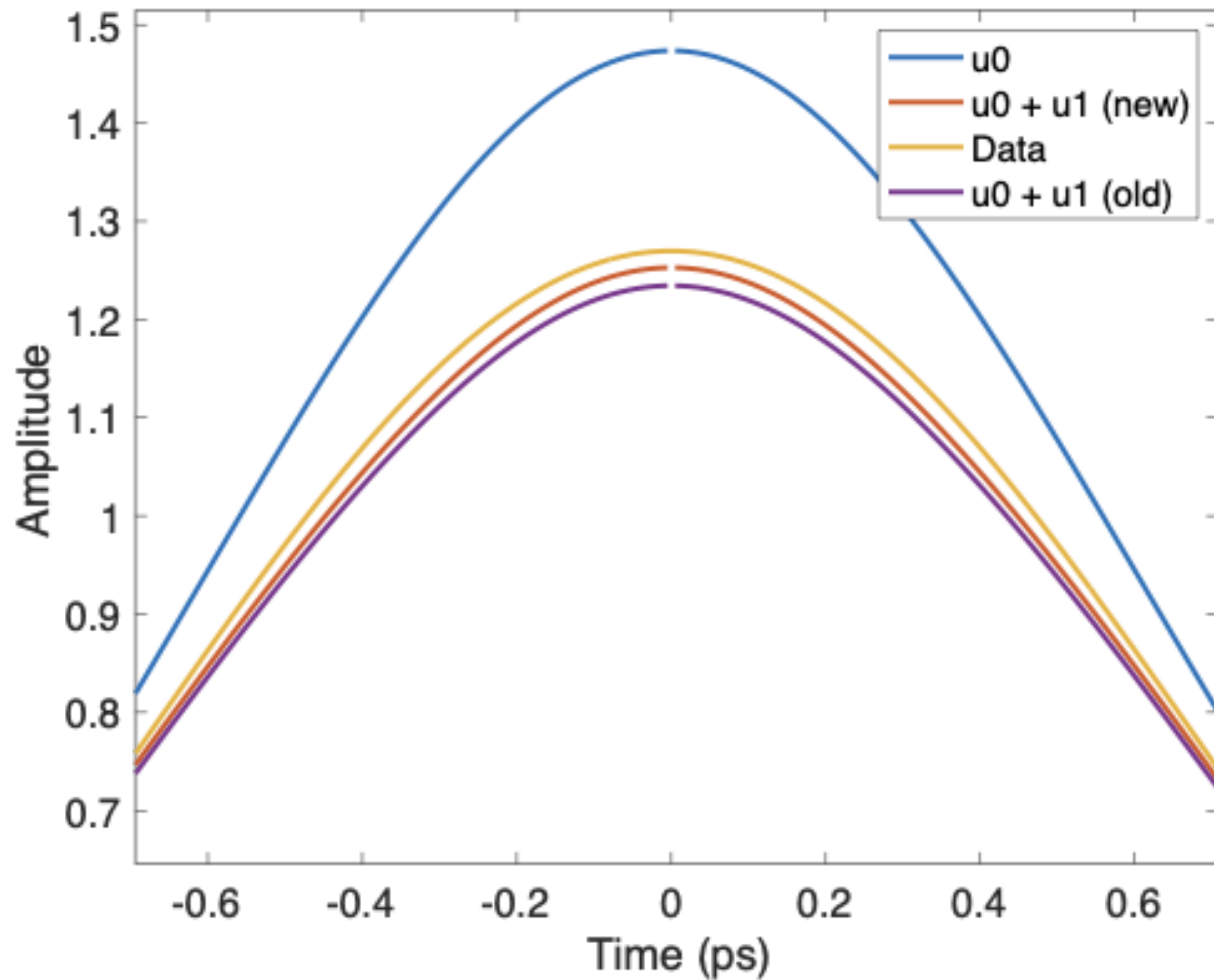
Xi: 1.12051



theta: -2.95310



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Revised the method that I used to find $u^{(1)}$ etc., and found that the coefficients had an additional linear term:

$$\begin{aligned}c_0 &= \frac{A}{4} - (2A^3 + 9A^2B + 6AB^2 + 9B^3) \varphi = 0.15371 \\c_1 &= -\frac{B}{4} + 3(3A^3 + 2A^2B + 3AB^2 - 2B^3) \varphi = -0.253315 \\c_2 &= \frac{A}{4} - 3(2A^3 + 3A^2B - 2AB^2 + 3B^3) \varphi = 0.137759 \\c_3 &= -\frac{B}{4} + (9A^3 - 6A^2B + 9AB^2 - 2B^3) \varphi = -0.259133\end{aligned}$$

where

$$\varphi := \frac{\Gamma}{1280\sigma^4}$$

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We can find higher order solutions by solving

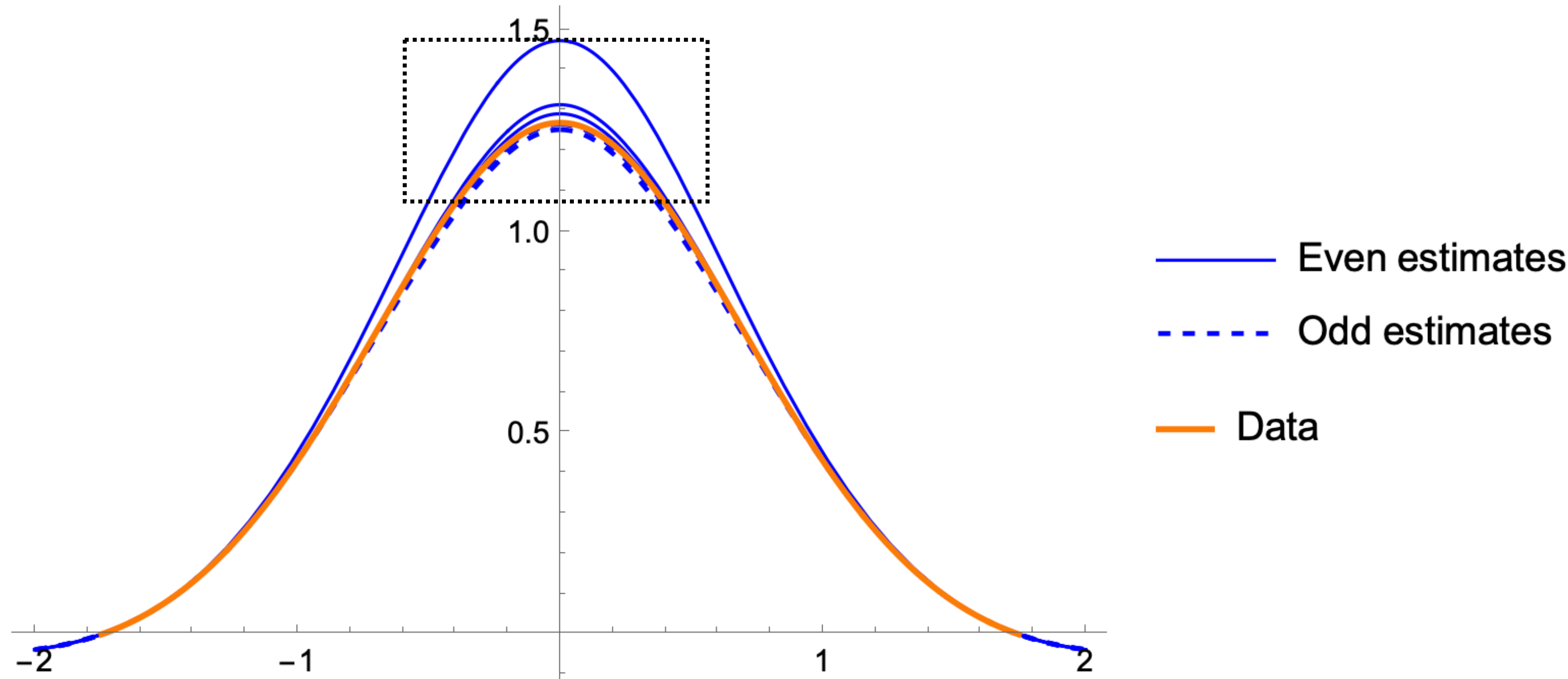
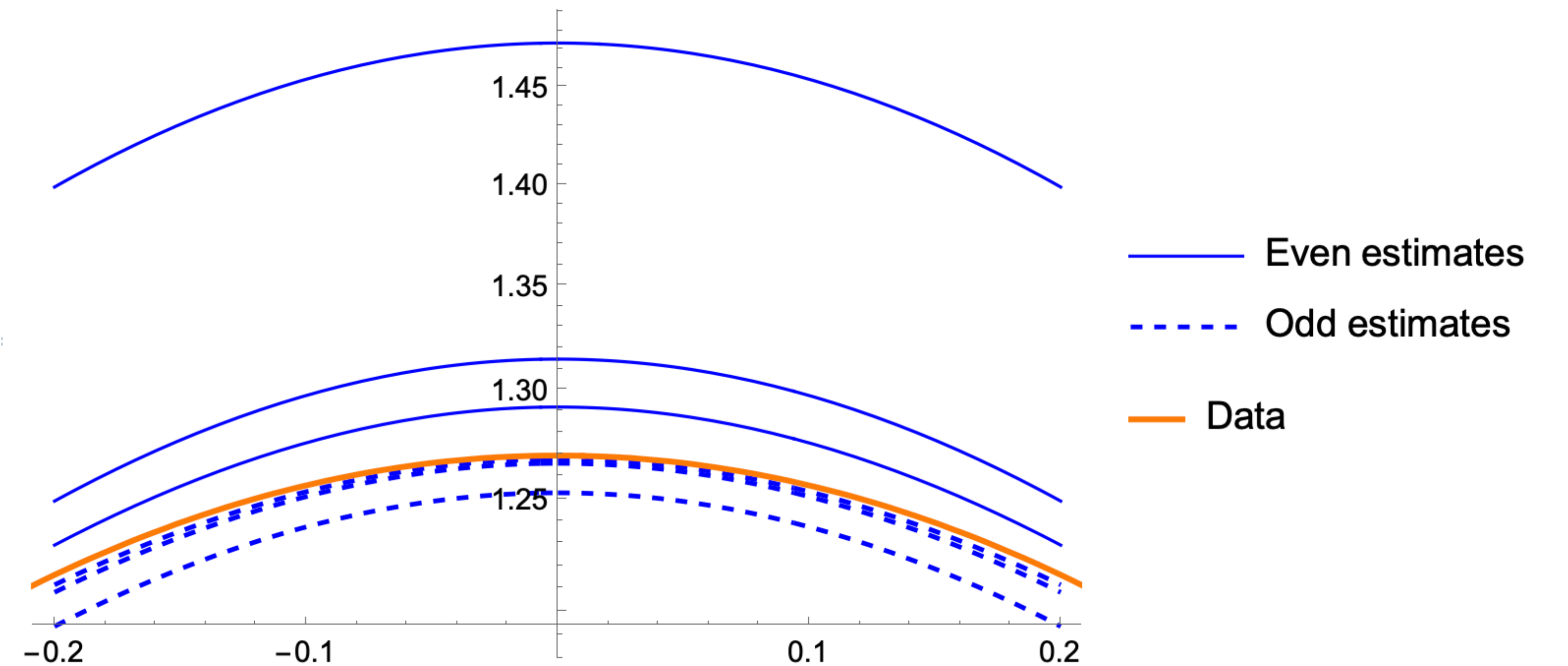
$$L \left[\sum_{j=0}^n u^{(j)} \right] + N \left[\sum_{j=0}^{n-1} u^{(j)} \right] \equiv 0$$

up to $(2n + 1)$ th order.

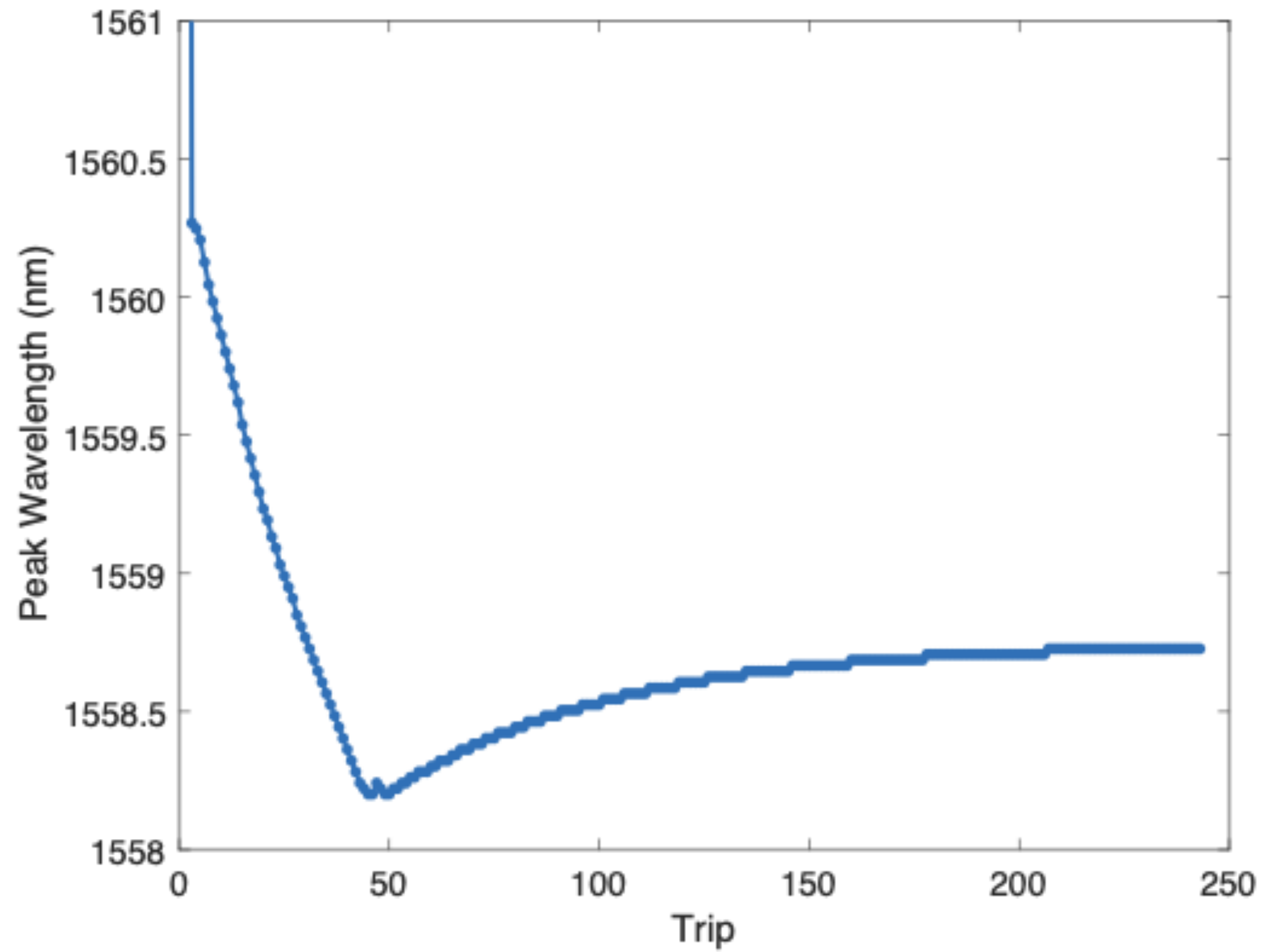
Linear terms appear in most of the coefficients:

Function	$u^{(1)}$	$u^{(2)}$	$u^{(3)}$	$u^{(4)}$	$u^{(5)}$	$u^{(6)}$
Order	3	5	7	9	11	13
Coefficients	A/4	A/8	5A/64	7A/128	21A/512	33A/1024
	-B/4	-B/8	-5B/64	-7B/128	-21B/512	-33B/1024
	A/4	...	-A/64	-A/64	-7A/512	-3A/256
	-B/4	...	B/64	B/64	7B/512	3B/256
		-A/8	-A/64	...	A/256	5A/1024
		B/8	B/64	...	-B/256	-5B/1024
			5A/64	A/64	A/256	...
			-5B/64	-B/64	-B/256	...
				-7A/128	-7A/512	-5A/1024
				7B/128	7B/512	5B/1024
					21A/512	3A/256
					-21B/512	-3B/256
						-33A/1024
						33B/1024

Project 1

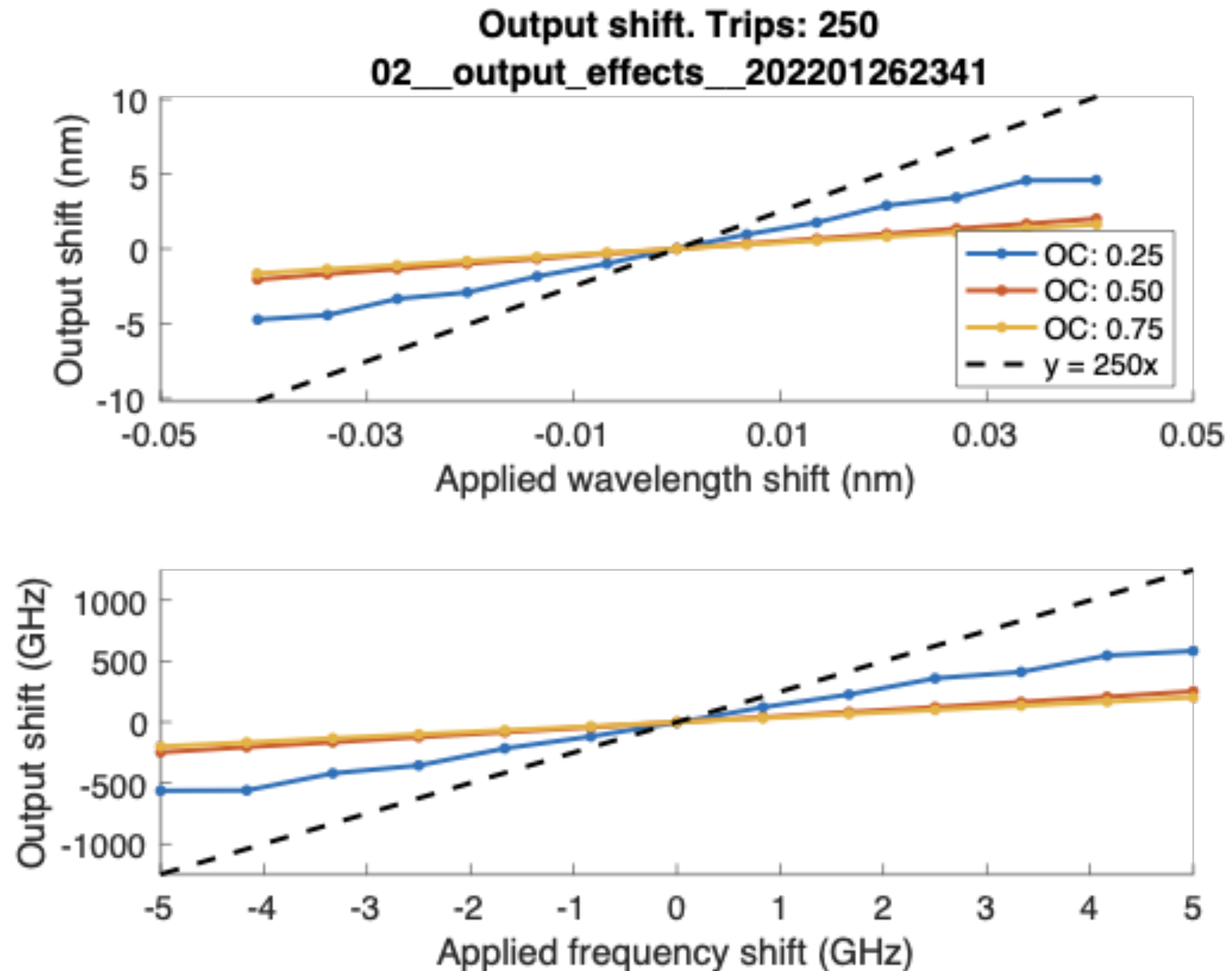


Project 2



Project 2

"OC: 0.3" or "30% energy feedback per trip" means that 30% of the energy returns to the loop after each round trip, 70% exits via the output coupler



Project 2

