

Thinking Inductively

COS 326

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Administration

- Assignment 1 due at 11:59 tonight!
- Program style guide:
 - <http://www.cs.princeton.edu/~cos326/style.php>
- Read notes:
 - functional basics, type-checking, typed programming
 - thinking inductively (today)
 - Real World OCaml Chapter 2, 3

Options

A value **v** has type **t option** if it is either:

- the value **None**, or
- a value **Some v'**, and **v'** has type **t**

Options can signal there is no useful result to the computation

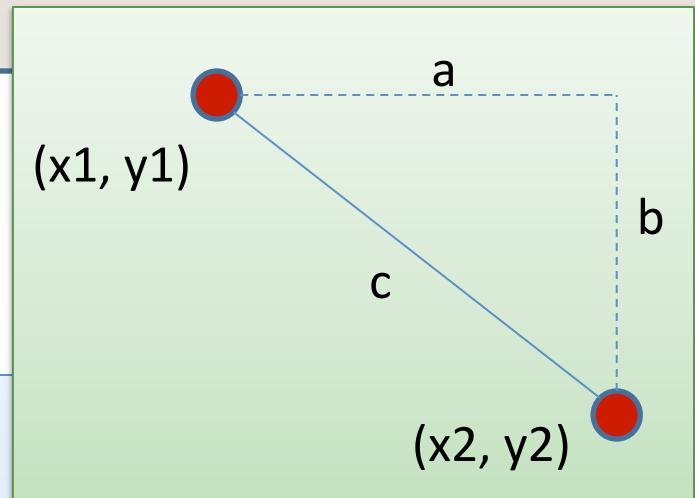
Example: we look up a value in a hash table using a key.

- If the key is present, return **Some v** where **v** is the associated value
- If the key is not present, we return **None**

Slope between two points

```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float =
```

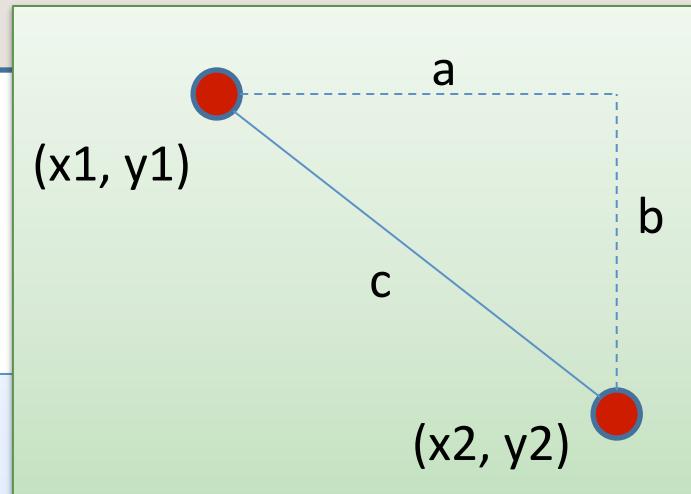


Slope between two points

```
type point = float * float
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```
let slope (p1:point) (p2:point) : float =  
  let (x1,y1) = p1 in  
  let (x2,y2) = p2 in
```

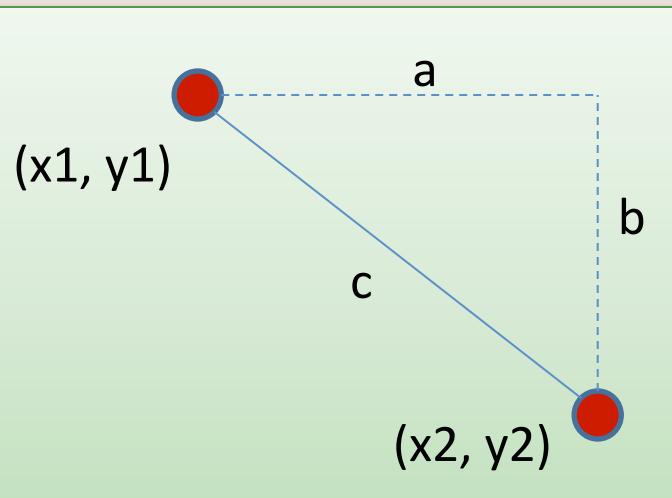
deconstruct tuple



Slope between two points

```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    (y2 -. y1) /. xd
  else
    ???
```



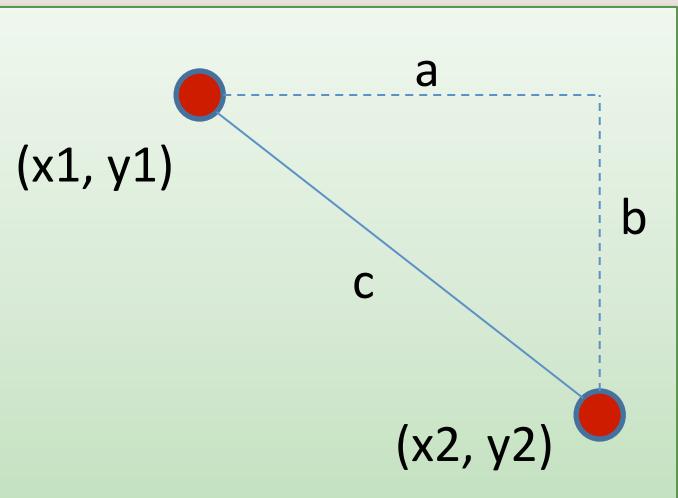
avoid divide by zero

what can we return?

Slope between two points

```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    ???
  else
    ???
```

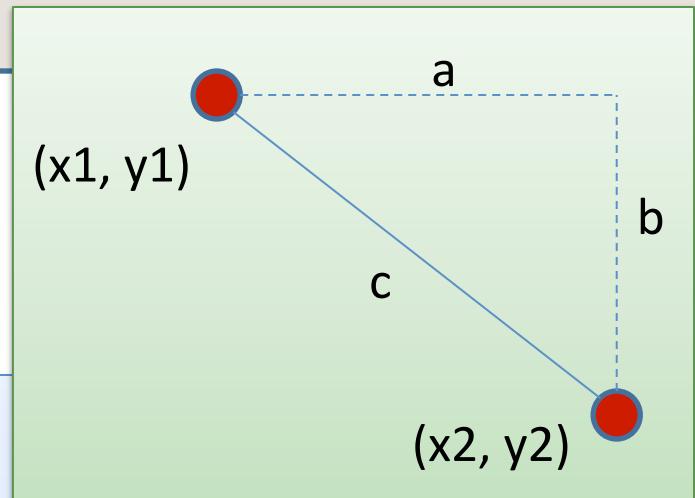


 we need an option type as the result type

Slope between two points

```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    Some ((y2 -. y1) /. xd)
  else
    None
```



Slope between two points

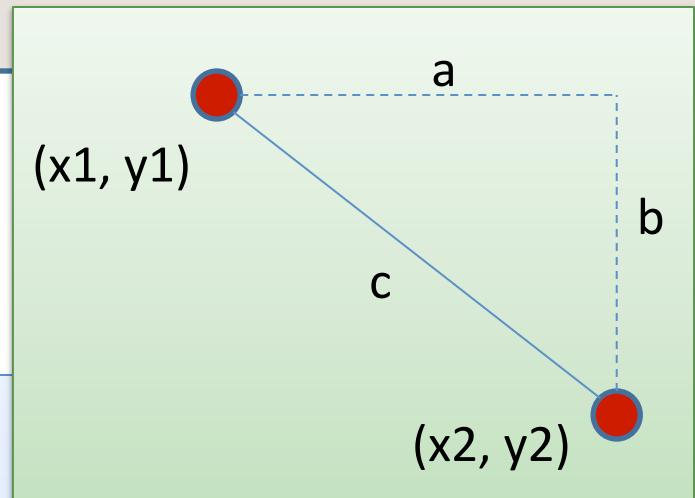
```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
     $(y2 - y1) /. xd$ 
  else
```

None

Has type **float**

Can have type **float option**



Slope between two points

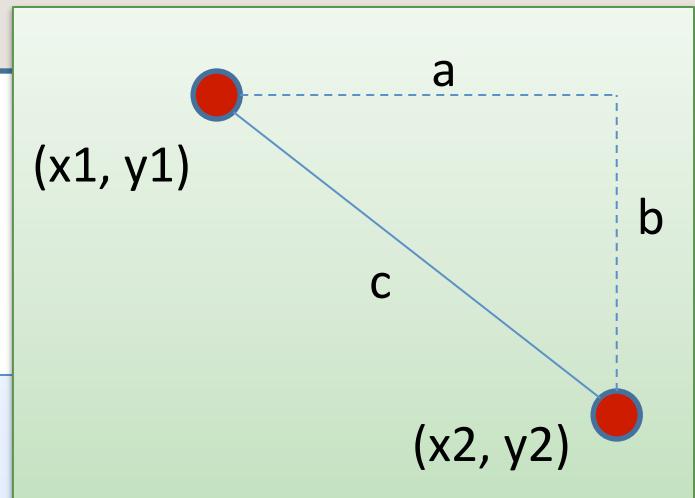
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```

```
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
     $(y2 - y1) /. xd$ 
  else
    None
```

Can have type **float option**

Has type **float**

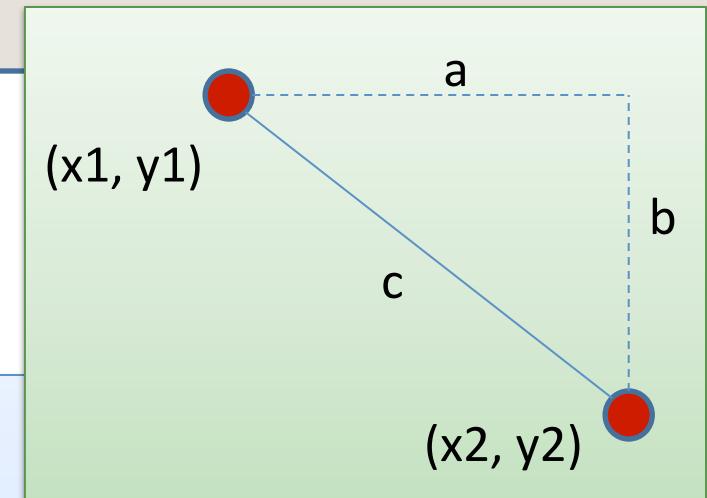
WRONG: Type mismatch



Slope between two points

```
type point = float * float
```

```
let slope (p1:point) (p2:point) : float option =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
     $(y2 - y1) /. xd$ 
  else
    None
```



Has type **float**

doubly WRONG:
result does not
match declared result

Remember the typing rule for if

```
if e1 : bool  
and e2 : t and e3 : t (for some type t)  
then if e1 then e2 else e3 : t
```

Returning an optional value from an if statement:

```
if ... then  
  
    None      : t option  
  
else  
  
    Some ( ... )  : t option
```

How do we use an option?

slope : point -> point -> float option



returns a float option

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
```

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
    slope p1 p2
```



returns a float option;
to print we must discover if it is
None or Some

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
```

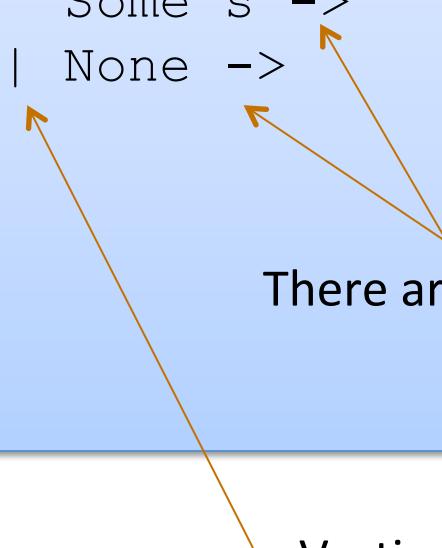
How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =  
  match slope p1 p2 with
```

```
    Some s ->  
  | None ->
```

There are two possibilities



Vertical bar separates possibilities

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
    Some s ->
  | None ->
```

The "Some s" pattern includes the variable s

The object between | and -> is called a pattern

How do we use an option?

```
slope : point -> point -> float option
```

```
let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
    Some s ->
      print_string ("Slope: " ^ string_of_float s)
  | None ->
      print_string "Vertical line.\n"
```

Writing Functions Over Typed Data

- Steps to writing functions over typed data:
 1. Write down the function and argument names
 2. Write down argument and result types
 3. Write down some examples (in a comment)
 4. **Deconstruct** input data structures
 5. **Build** new output values
 6. Clean up by identifying repeated patterns
- For option types:

when the input has type t option , deconstruct with:	when the output has type t option , construct with:
---	--

```
match ... with
| None -> ...
| Some s -> ...
```

```
Some (...)
```

```
None
```

MORE PATTERN MATCHING

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```

(x_2, y_2) is an example of a pattern – a pattern for tuples.

So let declarations can contain patterns just like match statements

The difference is that a match allows you to consider multiple different data shapes

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1,y1) ->
    let (x2,y2) = p2 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```



There is only 1 possibility when matching a pair

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1,y1) ->
    match p2 with
    | (x2,y2) ->
      sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```



We can nest one match expression inside another.

(We can nest any expression inside any other, if the expressions have the right types)

Better Style: Complex Patterns

we built a pair of pairs

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | ((x1, y1), (x2, y2)) ->
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
;
```

Pattern for a pair of pairs: **((variable, variable), (variable, variable))**
All the variable names in the pattern must be different.

Better Style: Complex Patterns

we built a pair of pairs

```

type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | (p3, p4) ->
    let (x1, y1) = p3 in
    let (x2, y2) = p4 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;

```

A pattern must be **consistent with** the type of the expression
in between **match ... with**
We use (p3, p4) here instead of ((x1, y1), (x2, y2))

Pattern-matching in function parameters

```
type point = float * float

let distance ((x1,y1):point) ((x2,y2):point) : float =
  let square x = x *. x in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
;;
```

Function parameters are patterns too!

What's the best style?

```
let distance (p1:point) (p2:point) : float =  
    let square x = x *. x in  
    let (x1,y1) = p1 in  
    let (x2,y2) = p2 in  
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

```
let distance ((x1,y1):point) ((x2,y2):point) : float =  
    let square x = x *. x in  
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Either of these is reasonably clear and compact.

Code with unnecessary nested matches/lets is particularly ugly to read.

You'll be judged on code style in this class.

What's the best style?

```
let distance (x1,y1) (x2,y2) =  
    let square x = x *. x in  
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

This is how I'd do it ... the types for tuples + the tuple patterns are a little ugly/verbose ... but for now in class, use the explicit type annotations.
We will loosen things up later in the semester.

Combining patterns

```
type point = float * float

(* returns a nearby point in the graph if one exists *)
nearby : graph -> point -> point option

let printer (g:graph) (p:point) : unit =
  match nearby g p with
  | None -> print_string "could not find one\n"
  | Some (x,y) ->
    print_float x;
    print_string ", ";
    print_float y;
    print_newline();
;;
;
```

Other Patterns

- Constant values can be used as patterns

```
let small_prime (n:int) : bool =  
  match n with  
  | 2 -> true  
  | 3 -> true  
  | 5 -> true  
  | _ -> false  
;;
```

```
let iffy (b:bool) : int =  
  match b with  
  | true -> 0  
  | false -> 1  
;;
```

the underscore pattern
matches anything
it is the "don't care" pattern



INDUCTIVE THINKING

Inductive Programming and Proving

An *inductive data type T* is a data type defined by:

- a collection of base cases
 - that don't refer to **T**
- a collection of inductive cases that build new values of type **T** from pre-existing data of type **T**
 - the pre-existing data is guaranteed to be *smaller* than the new values

Programming principle:

- solve programming problem for base cases
- solve programming problem for inductive cases by calling function recursively (inductively) on *smaller* data value

Proving principle:

- prove program satisfies property P for base cases
- prove inductive cases satisfy property P assuming inductive calls on *smaller* data values satisfy property P

LISTS: AN INDUCTIVE DATA TYPE

Lists are Recursive Data

- In OCaml, a list value is:
 - `[]` (the empty list)
 - `v :: vs` (a value `v` followed by a shorter list of values `vs`)



Lists are Inductive Data

- In OCaml, a list value is:
 - `[]` (the empty list)
 - `v :: vs` (a value `v` followed by a shorter list of values `vs`)
- An example:
 - `2 :: 3 :: 5 :: []` has type `int list`
 - is the same as: `2 :: (3 :: (5 :: []))`
 - "`::`" is called "cons"
- An alternative syntax ("syntactic sugar" for lists):
 - `[2; 3; 5]`
 - But this is just a shorthand for `2 :: 3 :: 5 :: []`. If you ever get confused fall back on the 2 basic *constructors*: `::` and `[]`

Typing Lists

- Typing rules for lists:

(1) [] may have any list type $t \text{ list}$

(2) if $e1 : t$ and $e2 : t \text{ list}$
then $(e1 :: e2) : t \text{ list}$

Typing Lists

- Typing rules for lists:

(1) [] may have any list type $t \text{ list}$

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then $(e1 :: e2) : t \text{ list}$

- More examples:

$(1 + 2) :: (3 + 4) :: [] : ??$

$(2 :: []) :: (5 :: 6 :: []) :: [] : ??$

$[[2]; [5; 6]] : ??$

Typing Lists

- Typing rules for lists:

(1) [] may have any list type $t \text{ list}$

(2) if $e1 : t$ and $e2 : t \text{ list}$
then $(e1 :: e2) : t \text{ list}$

- More examples:

$(1 + 2) :: (3 + 4) :: [] : \text{int list}$

$(2 :: []) :: (5 :: 6 :: []) :: [] : \text{int list list}$

$[[2]; [5; 6]] : \text{int list list}$

(Remember that the 3rd example is an abbreviation for the 2nd)

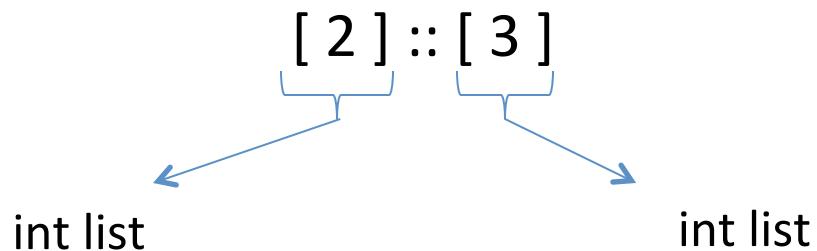
Another Example

- What type does this have?

[2] :: [3]

Another Example

- What type does this have?



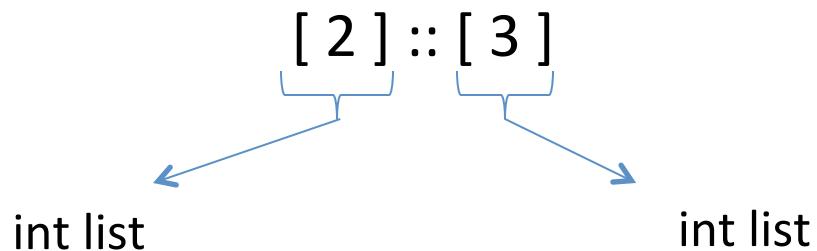
```
# [2] :: [3];;
```

Error: This expression has type int but an
expression was expected of type
int list

```
#
```

Another Example

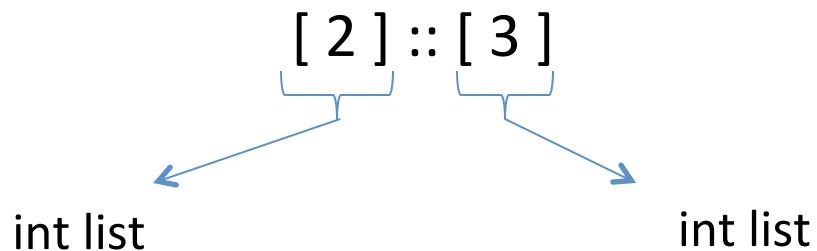
- What type does this have?



- Give me a simple fix that makes the expression type check?

Another Example

- What type does this have?



- Give me a simple fix that makes the expression type check?

Either: $2 :: [3]$: int list

Or: $[2] :: [[3]]$: int list list

Analyzing Lists

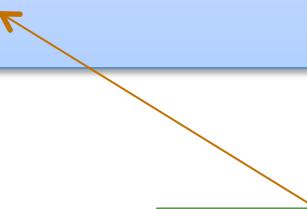
- Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

```
(* return Some v, if v is the first list element;  
   return None, if the list is empty *)  
  
let head (xs : int list) : int option =
```

Analyzing Lists

- Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

```
(* return Some v, if v is the first list element;  
   return None, if the list is empty *)  
  
let head (xs : int list) : int option =  
  match xs with  
  | []  ->  
  | hd :: _ ->
```



we don't care about the contents of the tail of the list so we use the underscore

Analyzing Lists

- Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

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(* return Some v, if v is the first list element;  
   return None, if the list is empty *)  
  
let head (xs : int list) : int option =  
  match xs with  
  | []  -> None  
  | hd :: _ -> Some hd
```

- This function isn't recursive -- we only extracted a small , fixed amount of information from the list -- the first element

A more interesting example

```
(* Given a list of pairs of integers,  
produce the list of products of the pairs  
  
prods [(2,3); (4,7); (5,2)] == [6; 28; 10]  
*)
```

A more interesting example

```
(* Given a list of pairs of integers,  
produce the list of products of the pairs  
  
    prods [(2,3); (4,7); (5,2)] == [6; 28; 10]  
*)  
  
let rec prods (xs : (int * int) list) : int list =
```

A more interesting example

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(* Given a list of pairs of integers,  
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  | [] ->  
  | (x,y) :: tl ->
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```

```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] -> []  
  | (x,y) :: tl -> ?? :: ??
```



the result type is `int list`, so we can speculate
that we should create a list

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(* Given a list of pairs of integers,  
produce the list of products of the pairs  
  
prods [(2,3); (4,7); (5,2)] == [6; 28; 10]  
*)
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```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] -> []  
  | (x,y) :: tl -> (x * y) :: ??
```



the first element is the product

A more interesting example

```
(* Given a list of pairs of integers,  
produce the list of products of the pairs  
  
prods [(2,3); (4,7); (5,2)] == [6; 28; 10]  
*)
```

```
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] -> []  
  | (x,y) :: tl -> (x * y) :: ??
```



to complete the job, we must compute
the products for the rest of the list

A more interesting example

```
(* Given a list of pairs of integers,  
produce the list of products of the pairs  
  
    prods [(2,3); (4,7); (5,2)] == [6; 28; 10]  
*)  
  
let rec prods (xs : (int * int) list) : int list =  
  match xs with  
  | [] -> []  
  | (x,y) :: tl -> (x * y) :: prods tl
```

Three Parts to Constructing a Function

(1) Think about how to *break down* the input in to cases:

```
let rec prods (xs :  
  match xs with  
  | [] -> ...  
  | (x, y) :: tl -> ...  
    ...  
    ...
```

This assumption is called the Induction Hypothesis. You'll use it to prove your program correct.

(2) *Assume* the recursive call on smaller data is correct.

(3) Use the result of the recursive call to *build* correct answer.

```
let rec prods (xs : (int*int) list) : int list =  
  ...  
  | (x, y) :: tl -> ... prods tl ...
```

Another example: zip

(* Given two lists of integers,
return None if the lists are different lengths
otherwise stitch the lists together to create
Some of a list of pairs

```
zip [2; 3] [4; 5] == Some [(2,4); (3,5)]  
zip [5; 3] [4] == None  
zip [4; 5; 6] [8; 9; 10; 11; 12] == None  
*)
```

(Give it a try.)

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
```

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
```

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) ->
  | ([], y :: ys') ->
  | (x :: xs', []) ->
  | (x :: xs', y :: ys') ->
```

Another example: zip

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let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y :: ys') ->
  | (x :: xs', []) ->
  | (x :: xs', y :: ys') ->
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Another example: zip

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let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
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  | (x :: xs', []) -> None
  | (x :: xs', y :: ys') ->
```

Another example: zip

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let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
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  | ([], []) -> Some []
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  | (x :: xs', []) -> None
  | (x :: xs', y :: ys') -> (x, y) :: zip xs' ys'
```



is this ok?

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y :: ys') -> None
  | (x :: xs', []) -> None
  | (x :: xs', y :: ys') -> (x, y) :: zip xs' ys'
```

No! zip returns a list option, not a list!
We need to match it and decide if it is Some or None.



Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y :: ys') -> None
  | (x :: xs', []) -> None
  | (x :: xs', y :: ys') ->
    (match zip xs' ys' with
     | None -> None
     | Some zs -> (x, y) :: zs)
```



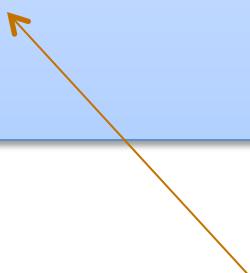
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Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y :: ys') -> None
  | (x :: xs', []) -> None
  | (x :: xs', y :: ys') ->
    (match zip xs' ys' with
     None -> None
     | Some zs -> Some ((x, y) :: zs))
```

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | (x :: xs', y :: ys') ->
    (match zip xs' ys' with
     | None -> None
     | Some zs -> Some ((x, y) :: zs))
  | (_, _) -> None
```



Clean up.
Reorganize the cases.
Pattern matching proceeds in order.

A bad list example

```
let rec sum (xs : int list) : int =  
  match xs with  
  | hd::tl -> hd + sum tl
```

A bad list example

```
let rec sum (xs : int list) : int =
  match xs with
  | hd :: tl -> hd + sum tl
```

```
#      Characters 39-78:
```

```
..match xs with
  hd :: tl -> hd + sum tl..
```

Warning 8: this pattern-matching is not exhaustive.

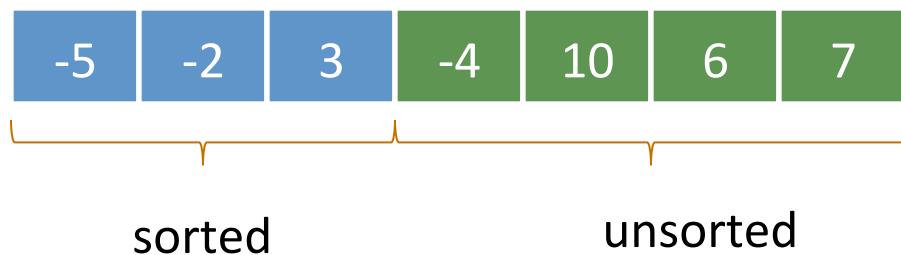
Here is an example of a value that is not matched: []

```
val sum : int list -> int = <fun>
```

INSERTION SORT

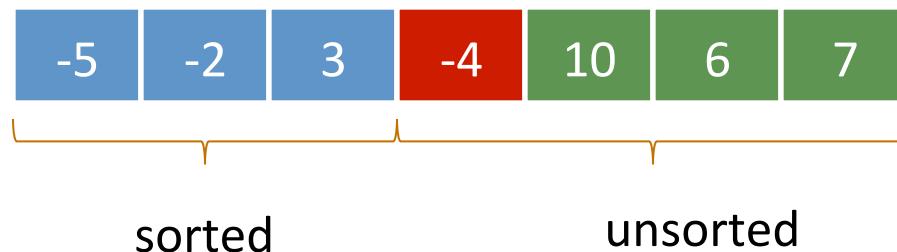
Recall Insertion Sort

- At any point during the insertion sort:
 - some initial segment of the array will be sorted
 - the rest of the array will be in the same (unsorted) order as it was originally

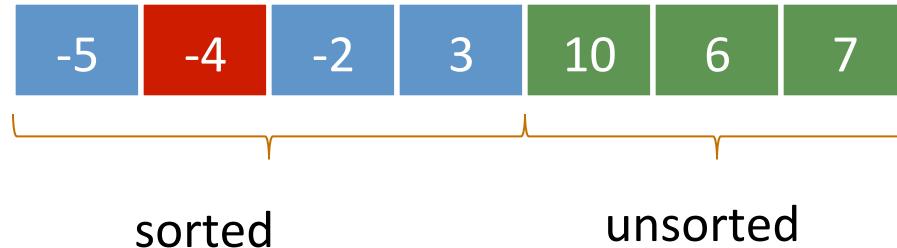


Recall Insertion Sort

- At any point during the insertion sort:
 - some initial segment of the array will be sorted
 - the rest of the array will be in the same (unsorted) order as it was originally



- At each step, take the next item in the array and insert it in order into the sorted portion of the list



Insertion Sort With Lists

- The algorithm is similar, except instead of *one array*, we will maintain *two lists*, a sorted list and an unsorted list

list 1:



sorted

list 2:



unsorted

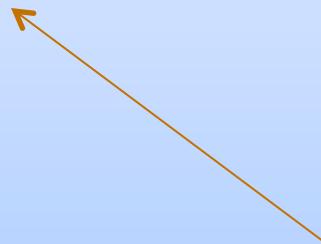
- We'll factor the algorithm:
 - a function to insert into a sorted list
 - a sorting function that repeatedly inserts

Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =
```

Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
  | [] ->  
  | hd :: tl ->
```



a familiar pattern:
analyze the list by cases

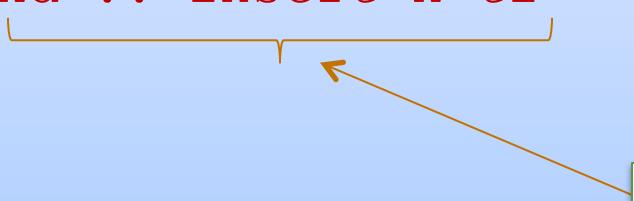
Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
  | [] -> [x]  
  | hd :: tl ->
```

insert x into the
empty list

Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
  | [] -> [x]  
  | hd :: tl ->  
    if hd < x then  
      hd :: insert x tl
```

- 
- build a new list with:
- hd at the beginning
 - the result of inserting x in to the tail of the list afterwards

Insert

```
(* insert x in to sorted list xs *)  
  
let rec insert (x : int) (xs : int list) : int list =  
  match xs with  
  | [] -> [x]  
  | hd :: tl ->  
    if hd < x then  
      hd :: insert x tl  
    else  
      x :: xs
```



put x on the front of the list,
the rest of the list follows

Insertion Sort

```
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =
```

Insertion Sort

```
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =
  let rec aux (sorted : il) (unsorted : il) : il =
    in
```

Insertion Sort

```
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =
  let rec aux (sorted : il) (unsorted : il) : il =
    in
    aux [] xs
```

Insertion Sort

```
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =
  let rec aux (sorted : il) (unsorted : il) : il =
    match unsorted with
    | [] ->
    | hd :: tl ->
in
aux [] xs
```

Insertion Sort

```
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =
  let rec aux (sorted : il) (unsorted : il) : il =
    match unsorted with
    | [] -> sorted
    | hd :: tl -> aux (insert hd sorted) tl
  in
  aux [] xs
```

A SHORT JAVA RANT

Definition and Use of Java Pairs

```
public class Pair {  
  
    public int x;  
    public int y;  
  
    public Pair (int a, int b) {  
        x = a;  
        y = b;  
    }  
}
```

```
public class User {  
  
    public Pair swap (Pair p1) {  
        Pair p2 =  
            new Pair(p1.y, p1.x);  
  
        return p2;  
    }  
}
```

What could go wrong?

A Paucity of Types

```
public class Pair {  
  
    public int x;  
    public int y;  
  
    public Pair (int a, int b) {  
        x = a;  
        y = b;  
    }  
}
```

```
public class User {  
  
    public Pair swap (Pair p1) {  
        Pair p2 =  
            new Pair(p1.y, p1.x);  
  
        return p2;  
    }  
}
```

The input **p1** to swap may be **null** and we forgot to check.

Java has no way to define a pair data structure that is *just a pair*.

How many students in the class have seen an accidental null pointer exception thrown in their Java code?

From Java Pairs to OCaml Pairs

In OCaml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

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And if you write code like this:

```
let swap_java_pair (p:java_pair) : java_pair =
  let (x,y) = p in
  (y,x)
```

From Java Pairs to OCaml Pairs

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And if you write code like this:

```
let swap_java_pair (p:java_pair) : java_pair =
  let (x,y) = p in
  (y,x)
```

You get a *helpful* error message like this:

```
# ... Characters 91-92:
let (x,y) = p in (y,x);;
          ^
```

```
Error: This expression has type java_pair = (int * int) option
      but an expression was expected of type 'a * 'b
```

From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

```
let swap_java_pair (p:java_pair) : java_pair =
  match p with
  | Some (x,y) -> Some (y,x)
```

From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
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And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

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let swap_java_pair (p:java_pair) : java_pair =
  match p with
  | Some (x,y) -> Some (y,x)
```

OCaml to the rescue!

```
..match p with
  | Some (x,y) -> Some (y,x)
```

Warning 8: this **pattern-matching is not exhaustive**.
Here is an example of a value that is not matched:
None

From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

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let swap_java_pair (p:java_pair) : java_pair =
  match p with
  | Some (x,y) -> Some (y,x)
```



```
let swap_java_pair (p:java_pair) : java_pair =
  match p with
  | None -> None
  | Some (x,y) -> Some (y,x)
```

From Java Pairs to OCaml Pairs

Moreover, your pairs are probably almost never null!

Defensive programming & always checking for null is
AnNOyinG

From Java Pairs to OCaml Pairs

There just isn't always some "good thing" for a function to do when it receives a bad input, like a null pointer

In OCaml, all these issues disappear when you use the proper type for a pair and that type contains no "extra junk"

```
type pair = int * int
```

Once you know OCaml, it is *hard* to write swap incorrectly
Your *bullet-proof* code is much simpler than in Java.

```
let swap (p:pair) : pair =
  let (x,y) = p in (y,x)
```

Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe just the pairs
- There is no type to describe just the triples
- There is no type to describe the pairs of pairs
- There is no type ...

OCaml has many more types

- use option when things may be null
- do not use option when things are not null
- OCaml types describe data structures more precisely
 - programmers have fewer cases to worry about
 - entire classes of errors just go away
 - type checking and pattern analysis help prevent programmers from ever forgetting about a case

Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe just the pairs
- There is no type to describe pairs +
- There is no type to describe pairs -
- There is no type to describe pairs ×

OCaml

- uses types
- does type inference

SCORE: OCAML 1, JAVA 0

- type checker catches bugs at compile time
- type analysis help prevent programmers from ever forgetting about a case

Example problems to practice

- Write a function to sum the elements of a list
 - $\text{sum } [1; 2; 3] ==> 6$
- Write a function to append two lists
 - $\text{append } [1;2;3] [4;5;6] ==> [1;2;3;4;5;6]$
- Write a function to reverse a list
 - $\text{rev } [1;2;3] ==> [3;2;1]$
- Write a function to turn a list of pairs into a pair of lists
 - $\text{split } [(1,2); (3,4); (5,6)] ==> ([1;3;5], [2;4;6])$
- Write a function that returns all prefixes of a list
 - $\text{prefixes } [1;2;3] ==> [[]; [1]; [1;2]; [1;2;3]]$
- suffixes...