

## Question 6

$$m(x) = \frac{1}{N} \sum_{i=1}^N x_i \quad \text{cov}(x, y) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(y))$$

$$s^2 = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2 \quad Y = a + bX$$

6.1  $m(a+bX) = \frac{1}{N} \sum_{i=1}^N (a + bx_i)$

$$= \frac{1}{N} \left( \sum_{i=1}^N a + \sum_{i=1}^N bx_i \right)$$

$$= \frac{1}{N} (Na) + b \frac{1}{N} \sum_{i=1}^N x_i$$

$$= a + b m(X)$$

6.2  $\text{cov}(X, X) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(x_i - m(x))$

$$= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2$$

6.3  $\text{cov}(X, a+bY) = \frac{1}{N} \sum_{i=1}^N (x_i - m(x))((a + by_i) - m(a + bY))$

from 6.1  $\rightarrow m(a+bY) = a + b m(Y)$

$$= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(b(y_i - m(Y))) \quad * \text{ substitute}$$

$$= b \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(Y)) \quad * \text{ factor out } b$$

$$= b \text{cov}(X, Y)$$

6.4  $\text{cov}(a+bX, a+bY) = b^2 \text{cov}(X, Y)$

$$\text{cov}(a+bX, a+bY) = b \text{cov}(X, a+bY) \quad * \text{ part 6.3 twice}$$

$$= b(b \text{cov}(X, Y)) = b^2 \text{cov}(X, Y) \quad * \text{ part 6.3 again}$$

$$= \text{cov}(bX, bY) = b^2 \text{cov}(X, X) = b^2 s^2$$

6.5 assume  $b > 0$

MEDIAN:

a positive linear transformation preserves order

$$\text{med}(a+bX) = a + b \text{med}(X)$$

IQR:

$$\text{IQR}(X) = Q_3 - Q_1 \quad * \text{ apply transformation}$$

$$\text{IQR}(a+bX) = (a + bQ_3) - (a + bQ_1) = b(Q_3 - Q_1) = b \text{IQR}(X)$$

$$6.6 \quad X = \{1, 4\} \quad X^2 = \{1, 16\} \quad \sqrt{X} = \{1, 2\}$$

mean of  $X$

$$m(X) = \frac{1+4}{2} = 2.5$$

mean of  $X^2$

$$m(X^2) = \frac{1+16}{2} = 8.5$$

but:

$$(m(X))^2 = 2.5^2 = 6.25$$

mean of  $\sqrt{X}$

$$m(\sqrt{X}) = \frac{1+2}{2} = 1.5$$

but:

$$\sqrt{m(X)} = \sqrt{2.5} \approx 1.58$$

not equal

not equal