

Question 6

$$m(X) = \frac{1}{N} \sum_{i=1}^N x_i \quad \text{cov}(X, Y) = \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(y_i - m(Y))$$

$$s^2 = \frac{1}{N} \sum_{i=1}^N (x_i - m(X))^2 \quad Y = a + bX$$

6.1

$$\begin{aligned} m(a + bX) &= \frac{1}{N} \sum_{i=1}^N (a + bx_i) \\ &= \frac{1}{N} \left(\sum_{i=1}^N a + \sum_{i=1}^N bx_i \right) \\ &= \frac{1}{N} (Na) + b \frac{1}{N} \sum_{i=1}^N x_i \\ &= a + bm(X) \end{aligned}$$

6.2

$$\begin{aligned} \text{cov}(X, X) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(x_i - m(X)) \\ &= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))^2 \end{aligned}$$

6.3

$$\text{cov}(X, a + bY) = \frac{1}{N} \sum_{i=1}^N (x_i - m(X))((a + by_i) - m(a + bY))$$

from 6.1 $\Rightarrow m(a + bY) = a + bm(Y)$

$$= \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(b(y_i - m(Y))) \quad * \text{substitute}$$

$$= b \frac{1}{N} \sum_{i=1}^N (x_i - m(X))(y_i - m(Y)) \quad * \text{factor out } b$$

$$= b \text{cov}(X, Y)$$

6.4

$$\text{cov}(a + bX, a + bY) = b^2 \text{cov}(X, Y)$$

$$\text{cov}(a + bX, a + bY) = b \text{cov}(X, a + bY) \quad * \text{Part 6.3 twice}$$

$$= b(b \text{cov}(X, Y)) = b^2 \text{cov}(X, Y) \quad * \text{Part 6.3 again}$$

$$= \text{cov}(bX, bX) = b^2 \text{cov}(X, X) = b^2 s^2$$

6.5

assume $b > 0$

MEDIAN:

a positive linear transformation preserves order

$$\text{med}(a + bX) = a + b \text{med}(X)$$

IQR:

$$\text{IQR}(X) = Q_3 - Q_1 \quad * \text{apply transformation}$$

$$\text{IQR}(a + bX) = (a + bQ_3) - (a + bQ_1) = b(Q_3 - Q_1) = b \text{IQR}(X)$$

6.6 $x = \{1, 4\}$ $x^2 = \{1, 16\}$ $\sqrt{x} = \{1, 2\}$

mean of x

$$m(x) = \frac{1+4}{2} = 2.5$$

mean of x^2

$$m(x^2) = \frac{1+16}{2} = 8.5$$

but:

$$(m(x))^2 = 2.5^2 = 6.25$$

not equal

mean of \sqrt{x}

$$m(\sqrt{x}) = \frac{1+2}{2} = 1.5$$

but:

$$\sqrt{m(x)} = \sqrt{2.5} \approx 1.58$$

not equal