Question 6

$$m(X) = \frac{1}{N} \sum_{i=1}^{N} X_i \qquad cov(X,Y) = \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))(Y_i - m(Y))$$

$$s^2 = \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))^2 \qquad Y = a + bX$$

6.1

$$m(a+bX) = \frac{1}{N} \sum_{i=1}^{N} (a + bx_i)$$

$$= \frac{1}{N} \left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i \right)$$

$$= \frac{1}{N} (Na) + b \frac{1}{N} \sum_{i=1}^{N} X_i$$

$$= a + bm(X)$$

6.2

$$cov(X,X) = \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))(X_i - m(X))$$

$$= \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))^2$$

6.3

$$cov(X, a+bY) = \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))((a + by_i) - m(a+bY))$$

from 6.1 → $m(a+bY) = a + bm(Y)$

$$= \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))(b(y_i - m(Y))) \qquad *\text{substitute}$$

$$= b \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))(y_i - m(Y)) \qquad *\text{factor out } b$$

$$= b \, cov(X,Y)$$

6.4

$$cov(a+bX, a+bY) = b^2 cov(X,Y)$$
$$cov(a+bX, a+bY) = b \, cov(X, a+bY) \qquad *\text{part 6.3 twice}$$
$$= b(b \, cov(X,Y)) = b^2 cov(X,Y) \qquad *\text{Part 6.3 again}$$
$$= cov(bX, bX) = b^2 cov(X,X) = b^2 s^2$$

6.5

assume $b > 0$

MEDIAN:

a positive linear transformation preserves order

$$med(a+bX) = a + b \, med(X)$$

IQR:

$$IQR(X) = Q_3 - Q_1 \qquad *\text{apply transformation}$$
$$IQR(a+bX) = (a+bQ_3) - (a+bQ_1) = b(Q_3 - Q_1) = b \, IQR(X)$$

6.6 $X = \{1, 4\}$ $X^2 = \{1, 16\}$ $\sqrt{X} = \{1, 2\}$

mean of X

$$m(X) = \frac{1+4}{2} = 2.5$$

mean of X^2

$$m(X^2) = \frac{1+16}{2} = 8.5$$

but:

$$(m(X))^2 = 2.5^2 = 6.25$$

not equal

mean of $\sqrt{X}$

$$m(\sqrt{X}) = \frac{1+2}{2} = 1.5$$

but:

$$\sqrt{m(X)} = \sqrt{2.5} \approx 1.58$$

not equal