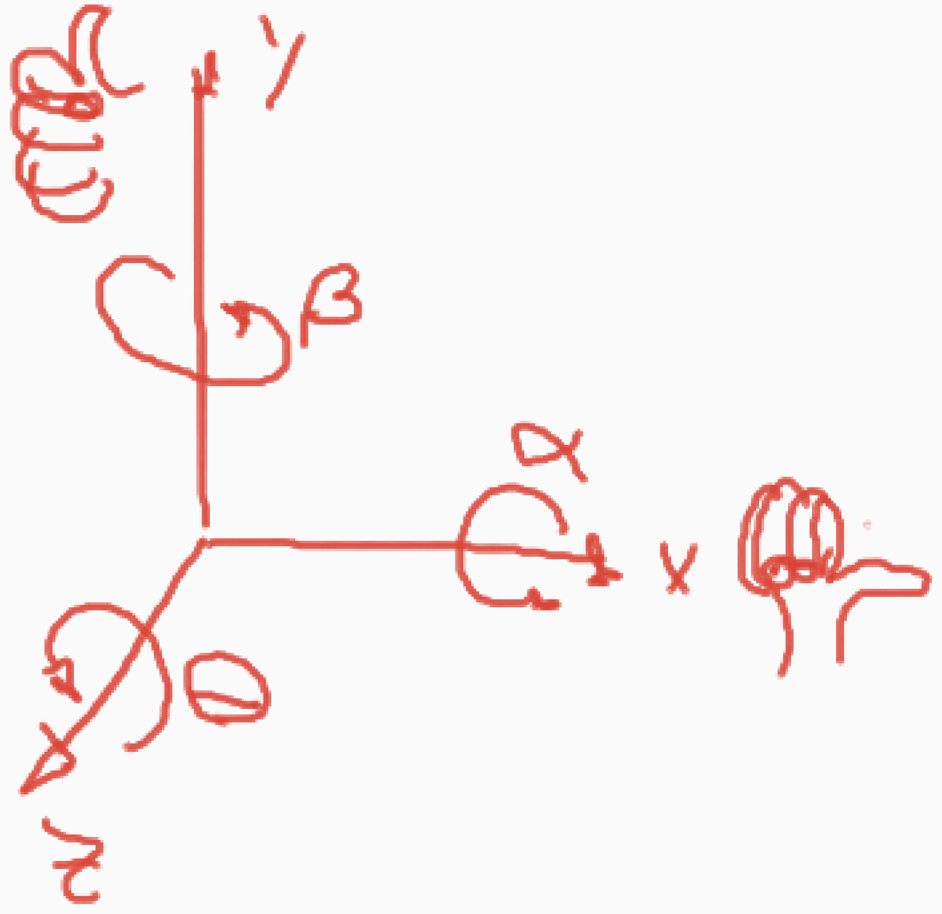
Matrices de transformación



Matrices de rotación

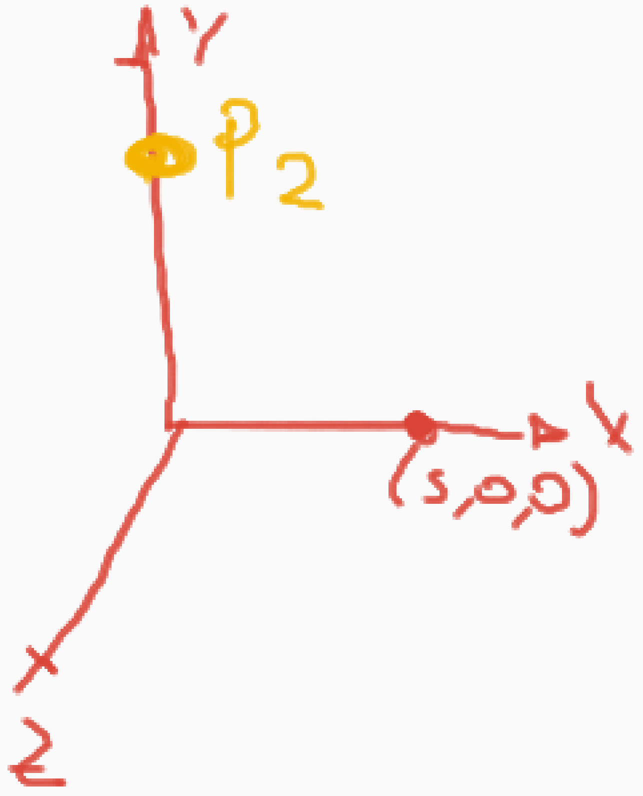
$$R(x,\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R(y,\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin & 0 & \cos\beta \end{bmatrix}$$

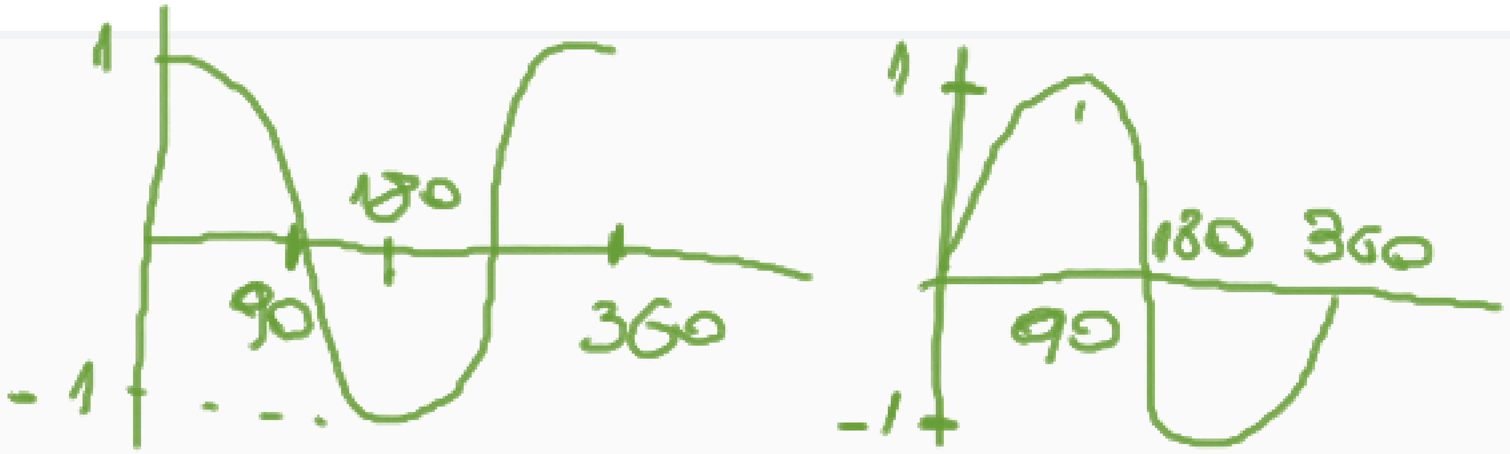
$$R(z,\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ejemplo

$$P_2 = R(z, 90^\circ)\, P_1 \qquad P_1 = (5,0,0) = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$



$$R(z,\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$P_2 = \begin{bmatrix} \overbrace{\cos 90}^{=0} & \underbrace{-\sin 90}_{=-1} & 0 \\ \underbrace{\sin 90}_{=1} & \underbrace{\cos 90}_{=0} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

$$P_2 = (0,5,0)$$

Rotar en los 3 ejes

$$P_2 = R(z,90)\, R(x,45)\, R(y,10)\, P_1$$

$$T = \left[\begin{array}{ccc|c} \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Matriz de Transformación

Rotación

Traslación

Escalado

Perspectiva

si no rota

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

desplaza 2 en X

Ejemplo: $P_1 = (1, 2, 1)$ desplazar en $X = 2$

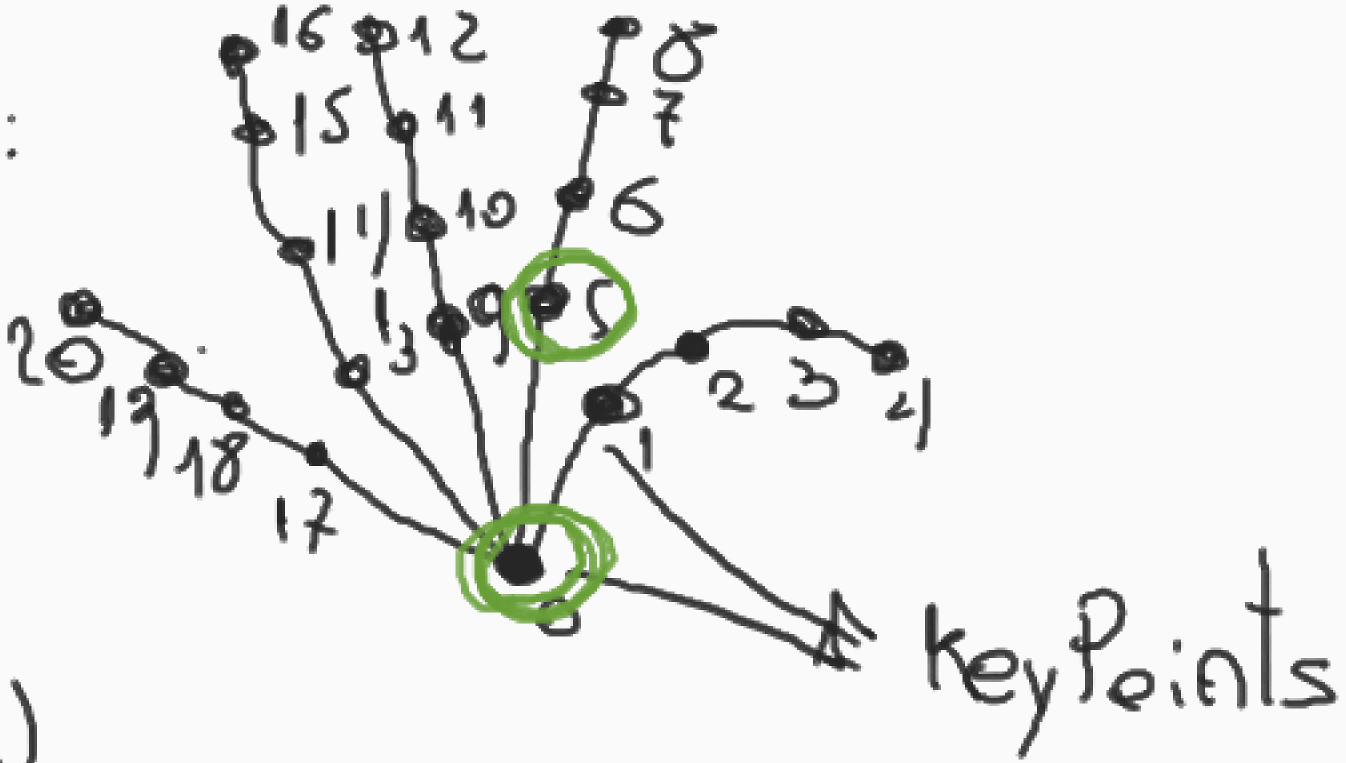
$$T = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_2 = T \cdot P_1$$

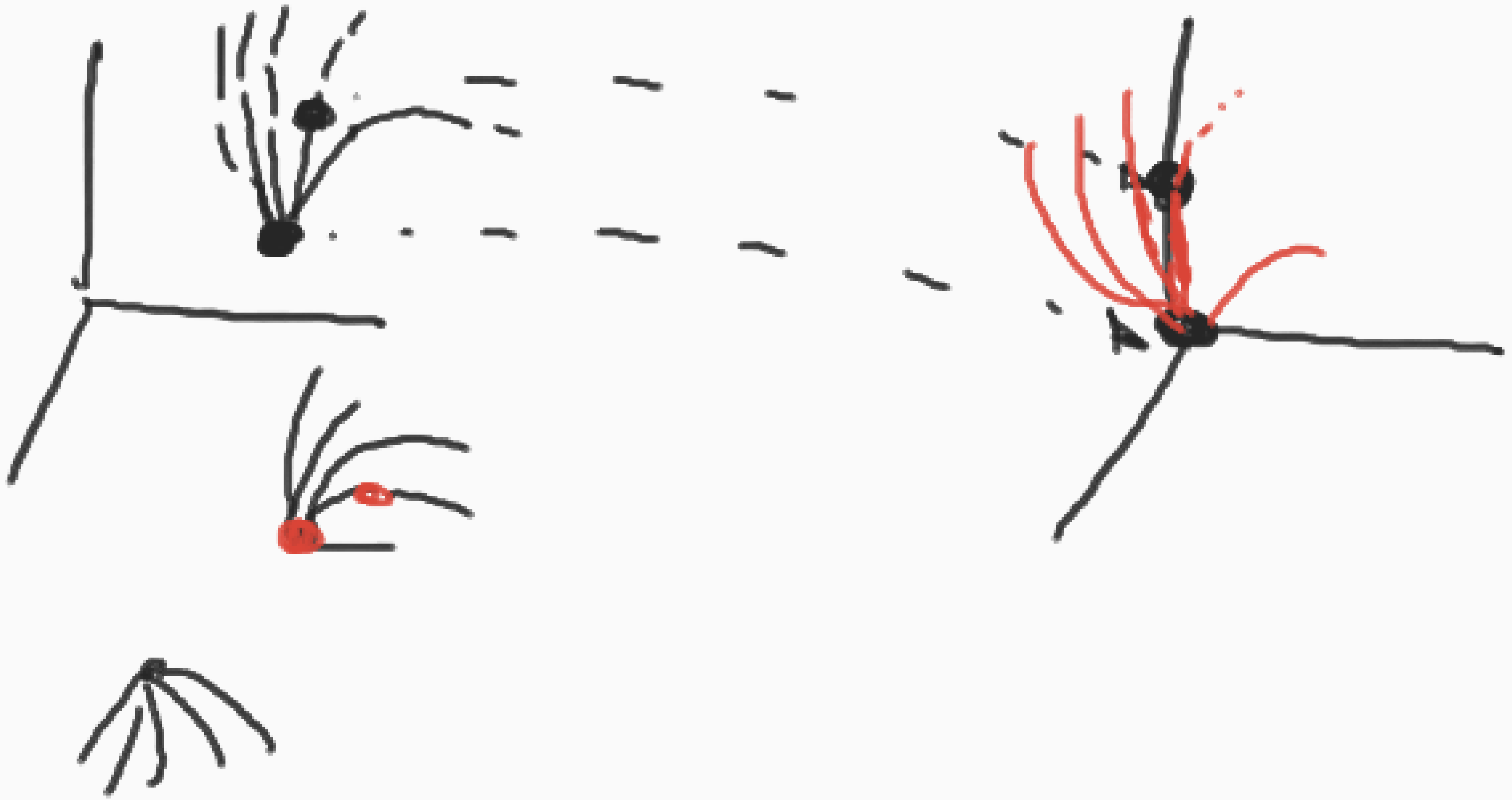
$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} \Rightarrow P_2 = (3, 2, 1)$$

Problema:

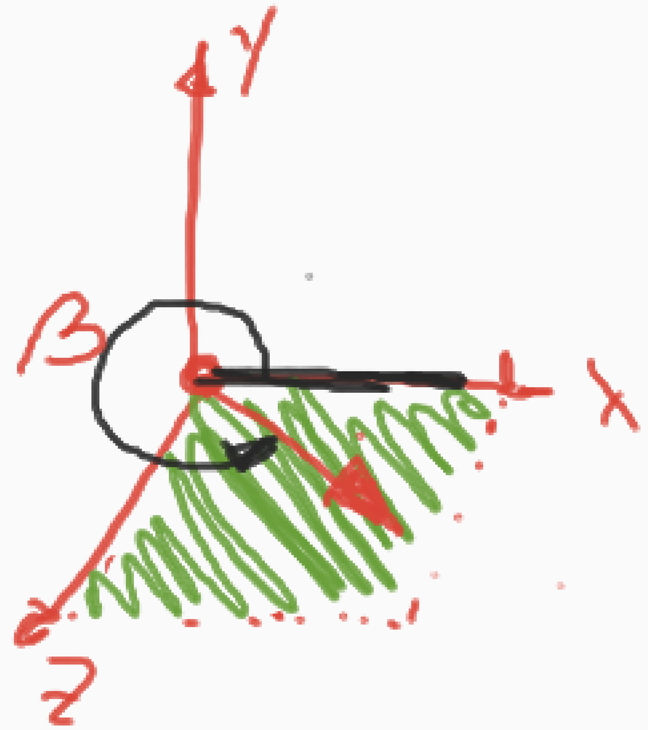
$kp0 = (x_0, y_0, z_0)$
$kp1 = (x_1, y_1, z_1)$
$kp2 = (x_2, y_2, z_1)$
$\vdots$

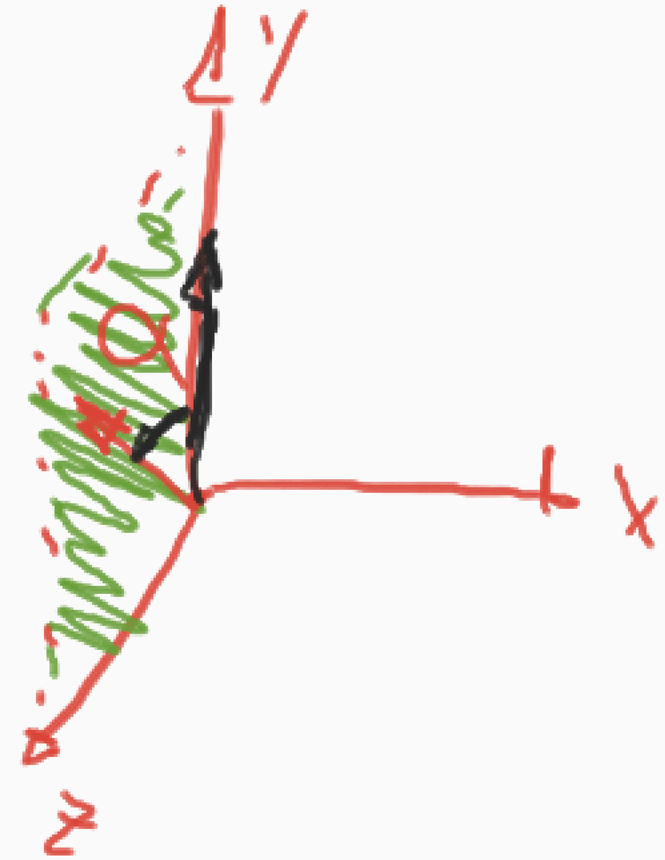


Normalización



Ángulos de rotación α, β, θ



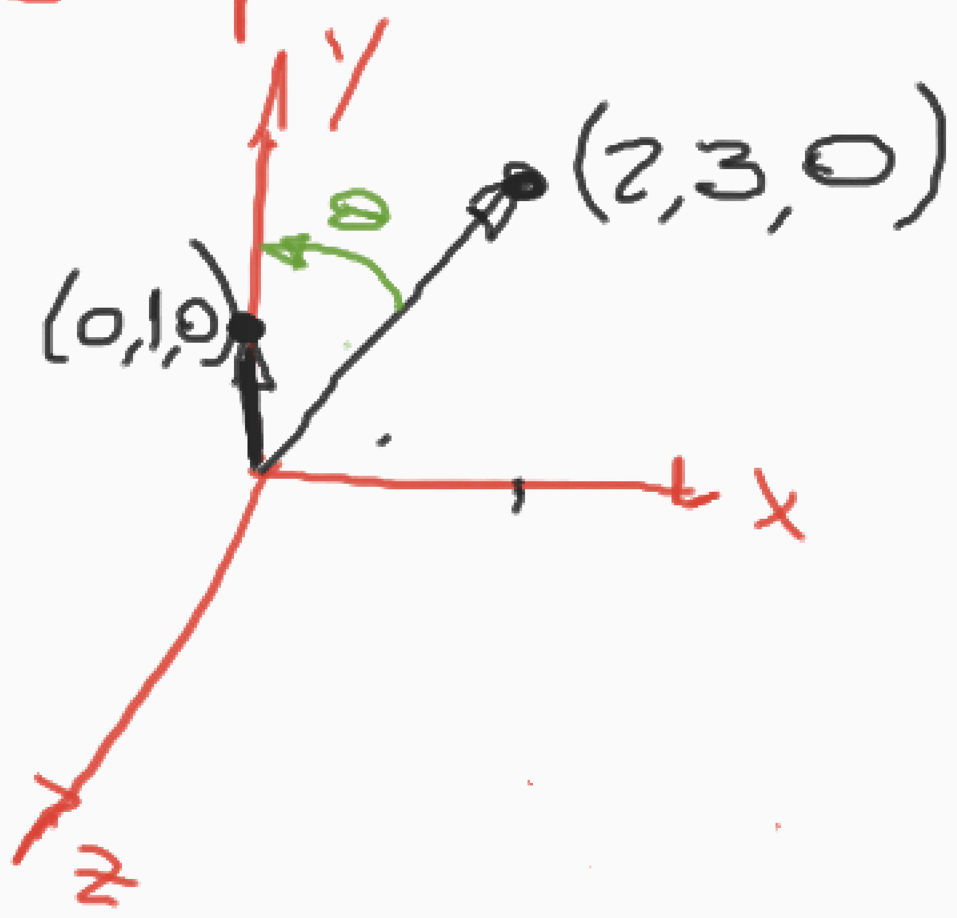


$$U = (U_x, U_y, U_z)$$
$$V = (V_x, V_y, V_z)$$

$$\cos \gamma = \frac{U \cdot V}{|U|\,|V|} = \frac{U_x V_x + U_y V_y + U_z V_z}{\sqrt{U_x^2 + U_y^2 + U_z^2}\,\sqrt{V_x^2 + V_y^2 + V_z^2}}$$

$$\gamma = \cos^{-1}\left(\frac{U \cdot V}{|U|\,|V|}\right)$$

Ejemplo: Calcula θ



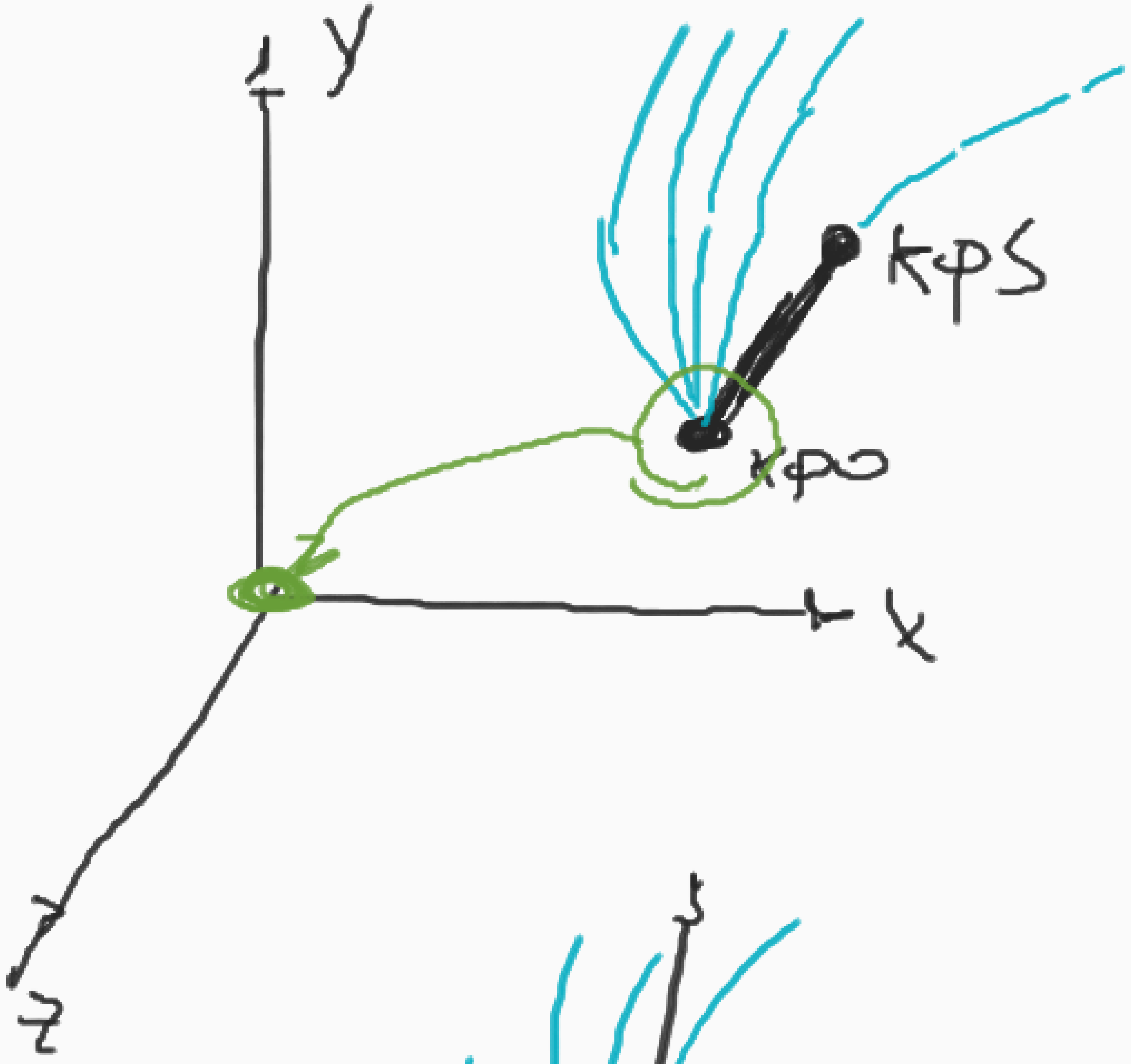
$$\cos\theta = \frac{2\times 0 + 3\times 1 + 0\times 0}{\sqrt{2^2+3^2+0}\,\sqrt{0+1^2+0}} = \frac{3}{\sqrt{13}}$$

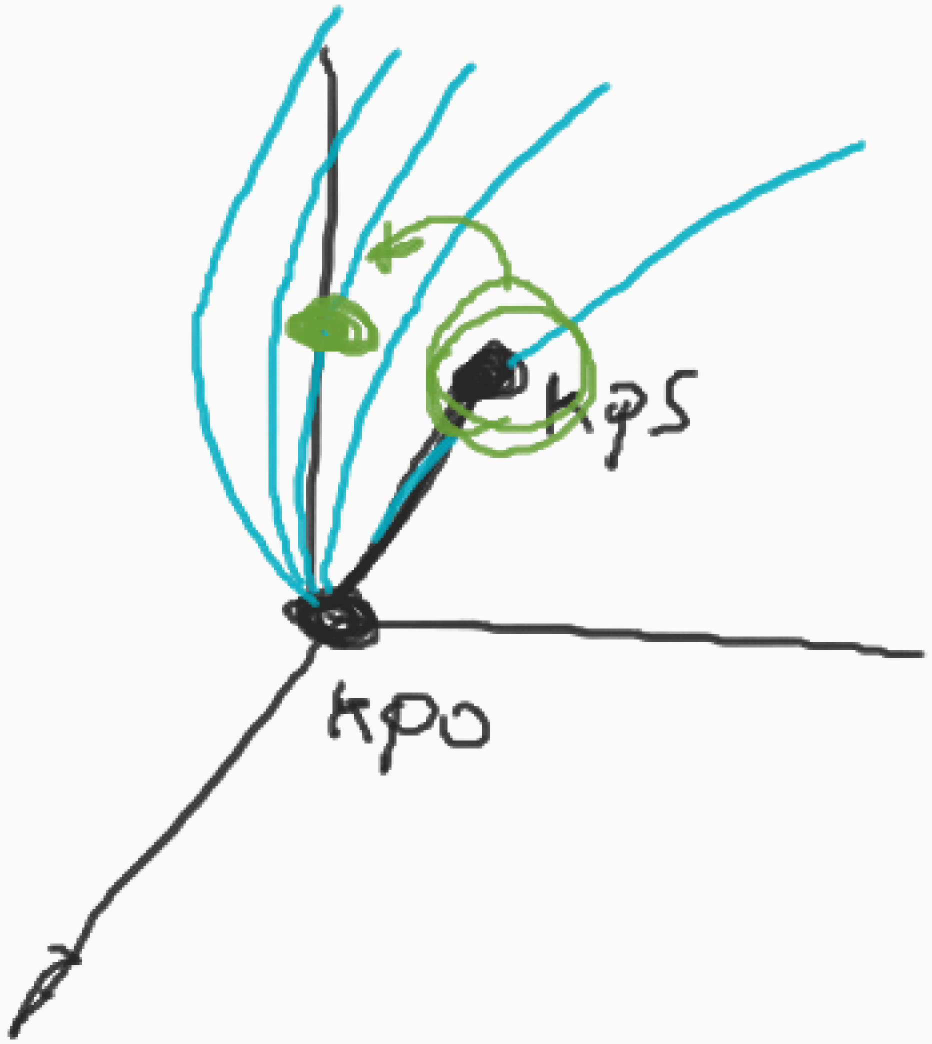
$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right) = 33{,}69^\circ$$

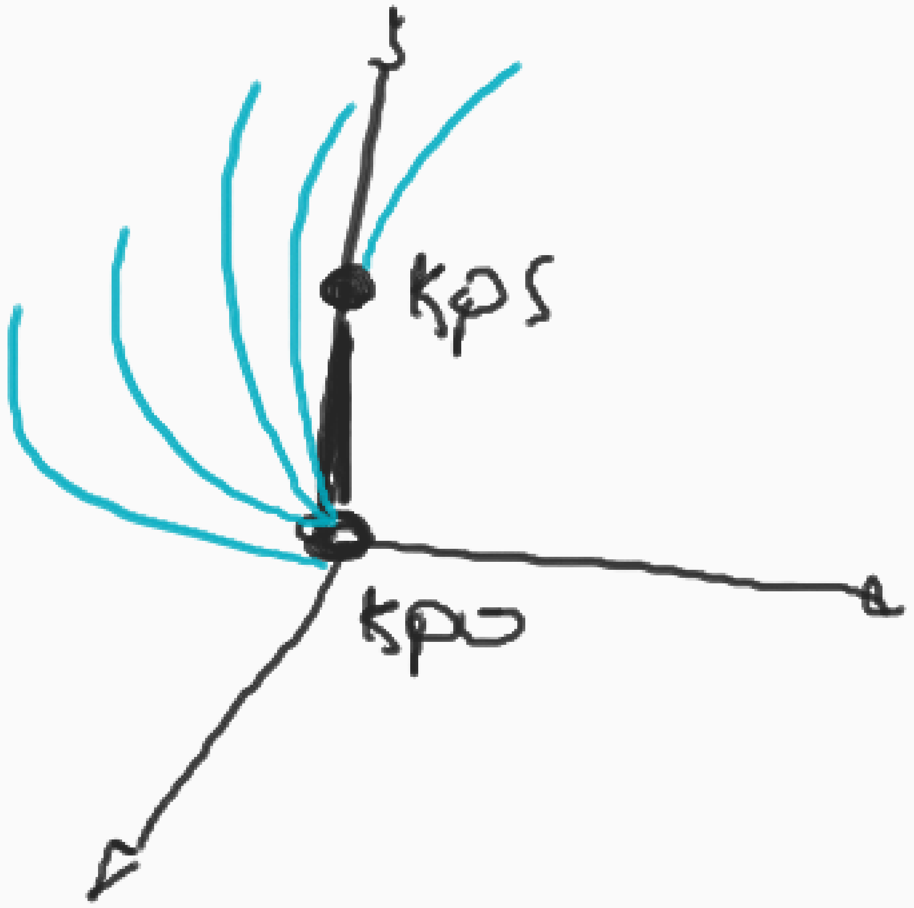
Propuesta para normalización

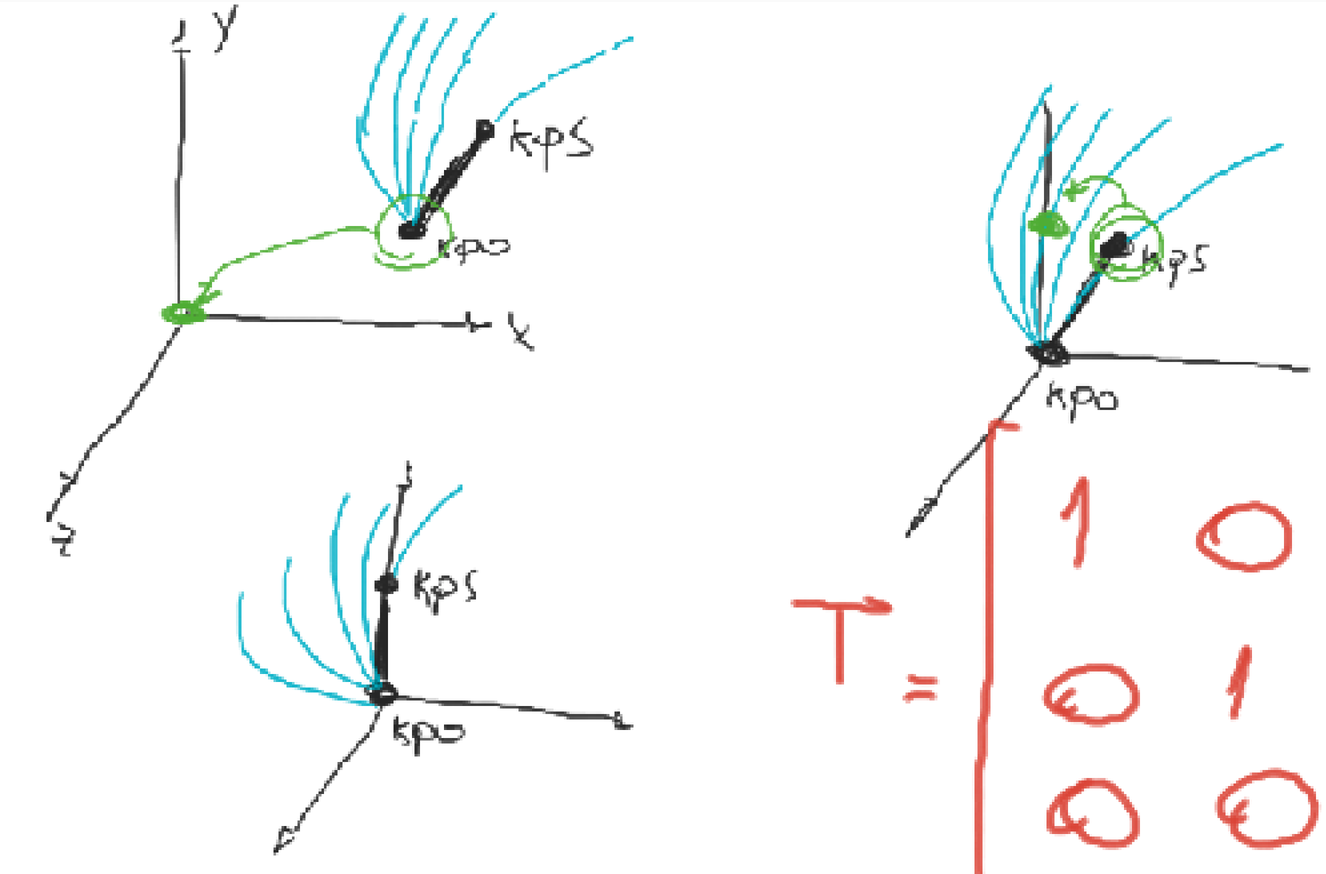
Encontrar la matriz de traslación y las $R(x,\alpha)$, $R(y,\beta)$ y $R(z,\theta)$

$$P_{nuevo} = \underset{4\times 4}{\overset{R(x,\alpha)}{\begin{bmatrix} & & & 0\\ & & & 0\\ & & & 0\\ 0 & 0 & 0 & 1\end{bmatrix}}} \overset{R(y,\beta)}{\begin{bmatrix} & & & 0\\ & & & 0\\ & & & 0\\ 0 & 0 & 0 & 1\end{bmatrix}} \overset{R(z,\theta)}{\begin{bmatrix} & & & 0\\ & & & 0\\ & & & 0\\ 0 & 0 & 0 & 1\end{bmatrix}} \begin{bmatrix} 1 & 0 & 0 & T_x\\ 0 & 1 & 0 & T_y\\ 0 & 0 & 1 & T_z\\ 0 & 0 & 0 & 1\end{bmatrix} \overset{P}{\begin{bmatrix} x\\ y\\ z\\ 1\end{bmatrix}}$$


y
kφS
kφo
x
z


kφS
kφo


kφS
kφo



$$kpO = (3,1,0) \qquad kpS = (4,2,0)$$

$$T = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}}_{kpO} = \begin{bmatrix} 3-3 \\ 1-1 \\ 0 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{kpO'}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 4 \\ 2 \\ 0 \\ 1 \end{bmatrix}}_{kpS} = \begin{bmatrix} 4-3 \\ 2-1 \\ 0 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}}_{kpS'}$$


x
y
z
Kpo
1
5
-4
Kps
10
7
3
R(x, α)
0 0 0 1
R(y, β)
0 0 0 1
R(z, θ)
0 0 0 1
T
1 0 0
0 1 0
0 0 1
0 0 0 1