Energy Densities of Photon Fields Subject to Inverse Compton Scattering in Jetted AGN

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May 25, 2020

1 Introduction

In this document I compute the integrated energy densities $u\,[\mathrm{erg\,cm^{-3}}]$ for different photon fields available for Inverse Compton scattering in a jetted AGN. The energy densities are computed both in a stationary reference frame whose origin coincides with the galaxy Black Hole and in a reference frame comoving with the Blob, which is streaming along the jet with velocity $\mathcal{B}$ and Lorentz factor Γ.

2 Notation

Differential quantities are implicit, i.e. $X(x_1, x_2; y) = \frac{\partial X}{\partial x_1\, \partial x_2}(y)$, where after the ; we specify parameters. We aim to calculate the integral energy density, of a given photon field, i.e.

$$u(r) = \int \mathrm{d}\epsilon \int \mathrm{d}\Omega \; u(\epsilon, \Omega; r), \tag{1}$$

where $u(\epsilon, \Omega; r) = \frac{\partial u}{\partial \epsilon\, \partial \Omega}(r)$ is the differential energy density, $\epsilon = E/m_e c^2$ is the dimensionless energy of the photon (in units of the electron rest mass), $\Omega = (\mu, \phi)$ is the solid angle and r specifies the distance of the blob along the jet axis.

2.1 Transformations

Quantities specified in the Blob comoving frame are prime. We recall the following energy and cosine transformations, from the stationary to the comoving frame:

$$\begin{aligned} \epsilon' &= \Gamma \epsilon (1 - \mathcal{B}\mu), \\ \mu' &= \frac{\mu - \mathcal{B}}{1 - \mathcal{B}\mu}; \end{aligned} \tag{2}$$

and viceversa from the comoving to the stationary

$$
\begin{aligned}
\epsilon &= \Gamma \epsilon' (1 + \mathcal{B} \mu'), \\
\mu &= \frac{\mu' + \mathcal{B}}{1 + \mathcal{B} \mu'}.
\end{aligned}
\tag{3}
$$

It can be proved [DS02] that the quantity $u(\epsilon, \Omega)/\epsilon^3$ is a relativistic invariant, from which it follows

$$
u'(\epsilon', \Omega') = \frac{u(\epsilon, \Omega)}{\Gamma^3 (1 + \mathcal{B} \mu')^3}. \tag{4}
$$

3 Energy Densities for Different Photon Fields

3.1 Isotropic Monochromatic

Let us consider an isotropic monochromatic (ϵ_0) radiation field with energy density $u_0 \,/\, \mathrm{erg\,cm^{-3}}$,

$$
u(\epsilon, \Omega) = \frac{u_0 \, \delta(\epsilon - \epsilon_0)}{4\pi} \tag{5}
$$

3.1.1 Galaxy Frame

$$
u = \int_0^{\infty} \mathrm{d}\epsilon \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \mathrm{d}\mu \; u(\epsilon, \Omega) = u_0. \tag{6}
$$

3.1.2 Comoving Frame

$$
\begin{aligned}
u' &= \int_0^{\infty} \mathrm{d}\epsilon' \int_0^{2\pi} \mathrm{d}\phi' \int_{-1}^{1} \mathrm{d}\mu' \; \frac{u_0 \, \delta(\epsilon - \epsilon_0)}{4\pi} \, \frac{1}{\Gamma^3 (1 + \mathcal{B} \mu')^3} \\
&= 2\pi \int_0^{\infty} \frac{\mathrm{d}\epsilon}{\Gamma (1 + \mathcal{B} \mu')} \int_{-1}^{1} \mathrm{d}\mu' \; \frac{u_0 \, \delta(\epsilon - \epsilon_0)}{4\pi} \, \frac{1}{\Gamma^3 (1 + \mathcal{B} \mu')^3} \\
&= \frac{u_0}{2\Gamma^4} \int_{-1}^{1} \mathrm{d}\mu' \; \frac{1}{(1 + \mathcal{B} \mu')^4} = \frac{u_0}{2\Gamma^4} \left[-\frac{1}{3\mathcal{B} (1 + \mathcal{B} \mu')^3} \right]_{-1}^{1} \\
&= \frac{u_0}{2\Gamma^4} \left[\frac{(1 + \mathcal{B})^3 - (1 - \mathcal{B})^3}{3\mathcal{B} \Gamma^{-6}} \right] = \boxed{u_0 \, \Gamma^2 \left(1 + \frac{\mathcal{B}^3}{3} \right).}
\end{aligned}
\tag{7}
$$

And we have reobtained the result in Eq. 5 of [DS94] and Eq. 10 of [DS02].

3.2 Monochromatic Point Source Behind the Jet

Let us consider a source of luminosity L_0 at a distance r from the jet,

$$
u(\epsilon, \Omega; r) = \frac{L_0}{4\pi r^2 c} \frac{\delta(\mu - 1)}{2\pi} \delta(\epsilon - \epsilon_0) \tag{8}
$$

where we label $u_0 = \frac{L_0}{4\pi r^2 c}$ for convenience.

3.2.1 Galaxy Frame

$$u = \int_0^\infty \mathrm{d}\epsilon \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \mathrm{d}\mu\, u_0 \frac{\delta(\mu-1)}{2\pi} \delta(\epsilon-\epsilon_0) = u_0 \left(= \frac{L_0}{4\pi r^2 c}\right), \tag{9}$$

where the normalisation 2π cancels out the result of the integration in $\mathrm{d}\phi$ and the delta in μ removes the integration in μ.

3.2.2 Comoving Frame

$$u' = \int_0^\infty \mathrm{d}\epsilon' \int_0^{2\pi} \mathrm{d}\phi' \int_{-1}^{1} \mathrm{d}\mu'\, u_0 \frac{\delta(\mu-1)}{2\pi} \delta(\epsilon-\epsilon_0) \frac{1}{\Gamma^3(1+\mathcal{B}\mu')^3}, \tag{10}$$

Now we convert the differentials in ϵ' and μ' in ϵ and, in order to simplify them with the deltas, we note that from Eq. 2

$$\frac{\mathrm{d}\mu'}{\mathrm{d}\mu} = \frac{(1-\mathcal{B}\mu)+(\mu-\mathcal{B})\mathcal{B}}{(1-\mathcal{B}\mu)^2} \Rightarrow \mathrm{d}\mu' = \frac{1}{\Gamma^2(1-\mathcal{B}\mu)^2}\mathrm{d}\mu, \tag{11}$$

therefore

$$\begin{aligned}
u' &= 2\pi \int_0^\infty \frac{\mathrm{d}\epsilon}{\Gamma(1+\mathcal{B}\mu')} \int_{-1}^{1} \frac{\mathrm{d}\mu}{\Gamma^2(1-\mathcal{B}\mu)^2} \frac{u_0}{2\pi} \delta(\epsilon-\epsilon_0)\delta(\mu-1) \frac{1}{\Gamma^3(1+\mathcal{B}\mu')^3} \\
&= \frac{u_0}{\Gamma^6} \int_{-1}^{1} \frac{\mathrm{d}\mu}{(1-\mathcal{B}\mu)^2(1+\mathcal{B}\mu')^4}\, \delta(\mu-1) \\
&= \frac{u_0}{\Gamma^6} \frac{1}{(1-\mathcal{B})^2(1+\mathcal{B})^4} = \boxed{\frac{u_0}{\Gamma^2(1+\mathcal{B})^2},}
\end{aligned} \tag{12}$$

where in the penultimate equality we have used $\mu = 1 \Rightarrow \mu' = 1$ from Eq. 2 and the condition imposed by the dirac delta. We have reobtained Eq. 6 of [DS94]. **Note** we will use Eq. 9 and Eq. 12 as a crosscheck for the radiation fields of more complicate objects (for distances much larger than their dimensions they should appear as a point source behind the jet).

3.3 Spherical Shell Broad Line Region

Let us consider the BLR as a monochromatic (ϵ_{li}) infinitesimally thin (R_{li}) shell, as in [Fin16]

$$u(\epsilon, \Omega; r) = \frac{\xi_{\mathrm{li}} L_{\mathrm{disk}}}{(4\pi)^2 c} \delta(\epsilon - \epsilon_{\mathrm{li}}) \int_{-1}^{1} \frac{\mathrm{d}\mu_{\mathrm{re}}}{x^2} \delta(\mu - \mu_*), \tag{13}$$

where

$$\begin{aligned}
\mu_*^2 &= 1 - \left(\frac{R_{\mathrm{li}}}{x}\right)^2 (1-\mu_{\mathrm{re}}^2), \\
x^2 &= R_{\mathrm{li}}^2 + r^2 - 2rR_{\mathrm{li}}\mu_{\mathrm{re}},
\end{aligned} \tag{14}$$

and the geometry of the reprocessing material is illustrated in Fig. 1.

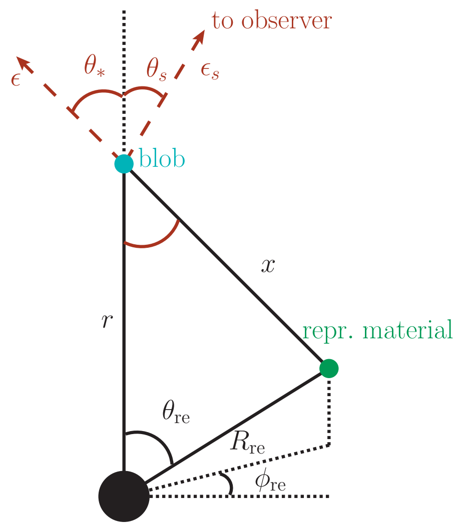

Figure 1: Geometry for inverse Compton scattering on reprocessing radiation field.

3.3.1 Galaxy Frame

$$\begin{aligned} u &= \int_0^\infty \mathrm{d}\epsilon \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \mathrm{d}\mu \, \frac{\xi_{\rm li} L_{\rm disk}}{(4\pi)^2 c} \delta(\epsilon - \epsilon_{\rm li}) \int_{-1}^{1} \frac{\mathrm{d}\mu_{\rm re}}{x^2} \delta(\mu - \mu_*) \\ &= \frac{\xi_{\rm li} L_{\rm disk}}{8\pi c} \int_{-1}^{1} \frac{\mathrm{d}\mu_{\rm re}}{x^2}. \end{aligned} \tag{15}$$

Let's examine if for large distances ($r \gg R_{\rm li}$) Eq. 15 $\rightarrow$ Eq. 9, i.e. if the BLR appears as a point source behind the jet. Since $x \xrightarrow[r \gg R_{\rm li}]{} r$, we have

$$u = \frac{\xi_{\rm li} L_{\rm disk}}{8\pi c} \int_{-1}^{1} \frac{\mathrm{d}\mu_{\rm re}}{r^2} = \frac{\xi_{\rm li} L_{\rm disk}}{4\pi c r^2}, \tag{16}$$

which is Eq. 9, i.e. the energy density of a monochromatic point source behind the jet, with $u_0 = \frac{\xi_{\rm li} L_{\rm disk}}{4\pi c r^2}$ ($L_0 = \xi_{\rm li} L_{\rm disk}$).

3.3.2 Comoving Frame

$$\begin{aligned} u' &= \int_0^\infty \mathrm{d}\epsilon' \int_0^{2\pi} \mathrm{d}\phi' \int_{-1}^{1} \mathrm{d}\mu' \, \frac{\xi_{\rm li} L_{\rm disk}}{(4\pi)^2 c} \delta(\epsilon - \epsilon_{\rm li}) \int_{-1}^{1} \frac{\mathrm{d}\mu_{\rm re}}{x^2} \delta(\mu - \mu_*) \frac{1}{\Gamma^3 (1 + \mathcal{B}\mu')^3} \\ &= 2\pi \int_0^\infty \frac{\mathrm{d}\epsilon}{\Gamma(1 + \mathcal{B}\mu')} \int_{-1}^{1} \frac{\mathrm{d}\mu}{\Gamma^2 (1 - \mathcal{B}\mu)^2} \frac{\xi_{\rm li} L_{\rm disk}}{(4\pi)^2 c} \delta(\epsilon - \epsilon_{\rm li}) \int_{-1}^{1} \frac{\mathrm{d}\mu_{\rm re}}{x^2} \delta(\mu - \mu_*) \frac{1}{\Gamma^3 (1 + \mathcal{B}\mu')^3} \\ &= \frac{\xi_{\rm li} L_{\rm disk}}{8\pi c} \int_{-1}^{1} \frac{\mathrm{d}\mu}{\Gamma^2 (1 - \mathcal{B}\mu)^2 \Gamma^4 (1 + \mathcal{B}\mu')^4} \int_{-1}^{1} \frac{\mathrm{d}\mu_{\rm re}}{x^2} \delta(\mu - \mu_*), \end{aligned} \tag{17}$$

using the delta condition $\mu = \mu_* \Rightarrow \mu' = \frac{\mu_* - \mathcal{B}}{1 - \mathcal{B}\,\mu_*}$. The latter in turns imply $1 + \mathcal{B}\mu' = \frac{1}{\Gamma^2(1 - \mathcal{B}\mu_*)}$, therefore

$$\begin{aligned} u' &= \frac{\xi_{\rm li} L_{\rm disk}}{8\pi c} \frac{\Gamma^8 (1 - \mathcal{B}\mu_*)^4}{\Gamma^6 (1 - \mathcal{B}\mu_*)^2} \int_{-1}^{1} \frac{\mathrm{d}\mu_{\rm re}}{x^2} \\ &= \Gamma^2 (1 - \mathcal{B}\mu_*)^2 \frac{\xi_{\rm li} L_{\rm disk}}{8\pi c} \int_{-1}^{1} \frac{\mathrm{d}\mu_{\rm re}}{x^2}. \end{aligned} \tag{18}$$

If the calculation was done correctly, in the limit of large distances ($r \gg R_{\rm li}$) Eq. 18 $\rightarrow$ Eq. 12, i.e. the BLR should appears as a point source behind the jet (also in the comoving frame). For $r \gg R_{\rm li}$, $x^2 \rightarrow r^2$ and $\mu_* \rightarrow 1$, so

$$u' = \Gamma^2 (1 - \mathcal{B})^2 \frac{\xi_{\rm li} L_{\rm disk}}{8\pi c} \frac{2}{r^2} = \frac{\Gamma^2 (1 - \mathcal{B}^2)^2}{(1 + \mathcal{B})^2} \frac{\xi_{\rm li} L_{\rm disk}}{4\pi c r^2} = \frac{1}{\Gamma^2 (1 + \mathcal{B})^2} \frac{\xi_{\rm li} L_{\rm disk}}{4\pi c r^2}. \tag{19}$$

where in the penultimate equality we have multiplied and divided by $(1 + \mathcal{B})^2$. We have reobtained Eq. 12 with $u_0 = \frac{\xi_{\rm li} L_{\rm disk}}{4\pi c r^2}$ ($L_0 = \xi_{\rm li} L_{\rm disk}$).

3.4 Ring Dust Torus

Following [Fin16],

$$u(\epsilon, \Omega; r) = \frac{\xi_{\rm dt} L_{\rm disk}}{(4\pi)^2 c x^2} \delta(\mu - r/x) \delta(\epsilon - 2.7\Theta), \tag{20}$$

where now

$$x^2 = R_{\rm dt}^2 + r^2 \tag{21}$$

3.4.1 Galaxy Frame

$$\begin{aligned} u &= \int_0^\infty \mathrm{d}\epsilon \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \mathrm{d}\mu \, \frac{\xi_{\rm dt} L_{\rm disk}}{(4\pi)^2 c} \delta(\mu - r/x) \delta(\epsilon - 2.7\Theta) \\ &= \frac{\xi_{\rm dt} L_{\rm disk}}{8\pi c x^2} \end{aligned} \tag{22}$$

There must be a problem with the expression Eq. 20 as it is clear that Eq. 22 misses a factor 2 to reduce to Eq. 9 in the limit of large distances ($r \gg R_{\rm dt}$).

3.4.2 Comoving Frame

$$\begin{aligned} u' &= \int_0^\infty \mathrm{d}\epsilon' \int_0^{2\pi} \mathrm{d}\phi' \int_{-1}^{1} \mathrm{d}\mu' \, \frac{\xi_{\rm dt} L_{\rm disk}}{(4\pi)^2 c x^2} \delta(\mu - r/x) \delta(\epsilon - 2.7\Theta) \frac{1}{\Gamma^3 (1 + \mathcal{B}\mu')^3} \\ &= 2\pi \int_0^\infty \frac{\mathrm{d}\epsilon}{\Gamma(1 + \mathcal{B}\mu')} \int_{-1}^{1} \frac{\mathrm{d}\mu}{\Gamma^2 (1 - \mathcal{B}\mu)^2} \frac{\xi_{\rm dt} L_{\rm disk}}{(4\pi)^2 c x^2} \delta(\mu - r/x) \delta(\epsilon - 2.7\Theta) \frac{1}{\Gamma^3 (1 + \mathcal{B}\mu')^3} \\ &= \frac{\xi_{\rm dt} L_{\rm disk}}{8\pi c} \int_{-1}^{1} \frac{\mathrm{d}\mu}{\Gamma^2 (1 - \mathcal{B}\mu)^2 \Gamma^4 (1 + \mathcal{B}\mu')^4} \delta(\mu - r/x), \end{aligned} \tag{23}$$

as we have seen for the BLR case, using the delta condition $\mu = r/x \Rightarrow \mu' = \frac{r/x-\mathcal{B}}{1-\mathcal{B}\ r/x}$ and it follows that $1+\mathcal{B}\mu' = \frac{1}{\Gamma^2(1-\mathcal{B}r/x)}$. Follows that

$$u' = \frac{\xi_{\rm dt} L_{\rm disk}}{8\pi c} \frac{\Gamma^8(1-\mathcal{B}r/x)^4}{\Gamma^6(1-\mathcal{B}r/x)^2} = \Gamma^2(1-\mathcal{B}r/x)^2 \frac{\xi_{\rm dt} L_{\rm disk}}{8\pi c}. \tag{24}$$

We notice that as for the Galaxy frame, this expression misses a factor 2 to reduce to Eq. 12, the case of point source behind the jet, in the limit of large distances. $x \to r$ for $r \gg R_{\rm dt}$, so

$$u' = \Gamma^2(1-\mathcal{B})^2 \frac{\xi_{\rm dt} L_{\rm disk}}{8\pi c} = \frac{1}{\Gamma^2(1+\mathcal{B})^2} \frac{\xi_{\rm dt} L_{\rm disk}}{8\pi c r^2}. \tag{25}$$

References

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