Uncapacitated Fixed Charge Facility Location Problem

Lagrangian Relaxation

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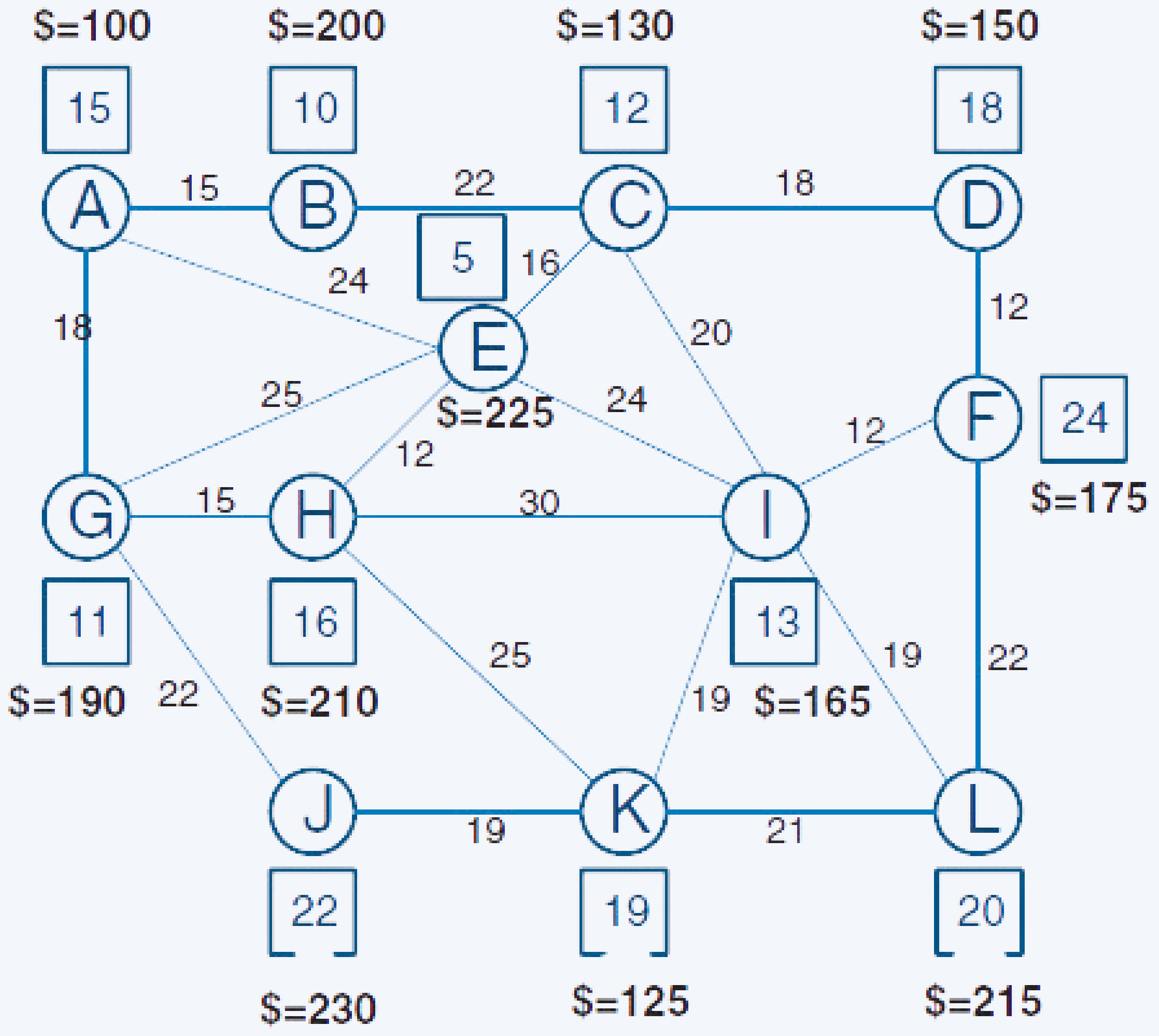
Problem Description

Given a set of nodes, which represent the demand among certain locations. We try locate the needed amount of facilities by minimizing the fixed and transportation costs. We take into consideration the demand per each node and the distance between each node.

Parameters



- I, J = sets of nodes: {A,B,C,...L}
- h: demand in each node
- d: distance between each node
- f: fixed cost in each node
- α: cost per unit distance per unit demand
- λ: lagrange multiplier

$$\lambda = 10 \times \frac{\sum_{i \in I} d_i}{|I|} + 10 \times \frac{\sum_{j \in J} f_j}{|J|}$$

Problem Development

Minimizing cost of supply location and demand transportation costs

$$\sum_{j \in J} f_j X_j + \alpha \sum_{i \in I} \sum_{j \in J} h_i d_{ij} Y_{ij}$$

Constraints

All facilities' demand must be satisfied	$\sum_{j \in J} Y_{ij} = 1$	$\forall i \in I$
The demand in each facility must be	$Y_{ij} \leqslant X_j$	$\forall i \in I; j \in J$
The demand in each facility must be	$X_j \in \{0, 1\}$	$\forall j \in J$
The demand in each facility must be	$Y_{ij} \geqslant 0$	$\forall i \in I; j \in J$

Relaxed Problem

Minimizing cost of supply location and demand transportation costs

$$\sum_{j\in J} f_j X_j + \alpha \sum_{i\in I}\sum_{j\in J} h_i d_{ij} Y_{ij} + \sum_{i\in I} \lambda_i \left[1 - \sum_{j\in J} \quad Y_{ij}\right]$$

$$= \sum_{j\in J} f_j X_j + \sum_{i\in I}\sum_{j\in J} (\alpha h_i d_{ij} - \lambda_i) Y_{ij} + \sum_{i\in I} \lambda_i$$

Constraints

The demand in each facility must be	$Y_{ij} \leqslant X_j$	$\forall i \in I; j \in J$
The demand in each facility must be	$X_j \in \{0,1\}$	$\forall j \in J$
The demand in each facility must be	$Y_{ij} \geqslant 0$	$\forall i \in I; j \in J$

Procedure

1. Calculate the lower bound using the relaxed function
2. Identify and update X and Y values that violate the constraints
3. Calculate the upper bound
4. Update lambda values

Calculate Lower Bound

Phase 1

Calculate V values to minimize the objective function of locating a facility in node j

$$V_j = f_j + \Sigma_{i \in I} min\left(0, ah_i d_{ij} - \lambda_i\right)$$

Phase 2

Place as supplier Xj that minimize the cost based on the Vj values

$$X_j = \left\{ \begin{matrix} 1 \text{ if } V_j < 0 \\ 0 \text{ if not} \end{matrix} \right\}$$

Phase 3

Place demand in Yij if there is a supplier Xj and its transportation cost minimizes the total price

$$Y_j = \left\{ \begin{matrix} 1 \text{ if } X_j = 1 \text{ and } ah_i d_{ij} - \lambda_i < 0 \\ 0 \text{ if not} \end{matrix} \right\}$$

Modify Xj and Yij for feasable solution

No suppliers Xj located

$$\sum_{j \in J} X_j = 0$$

$$X_j = \begin{cases} 1 & \text{if } V_j = min\,(V) \\ 0 & \text{if not} \end{cases}$$

Demand Yi covered by more than 1 supplier Xj

$$\sum_{j \in J} Y_{ij} >= 2$$

No demand Yi covered

$$\sum_{j \in J} Y_{ij} = 0$$

$$i \in I \qquad Y_{ij} = \begin{cases} 1 & \text{if } d_{ij} \text{ is the minimum of all opened } X_j \\ 0 & \text{if not} \end{cases}$$

Calculate the Upper Bound

Phase 1 Use the solution value of Xj and Yij to fix the value on the original problem

Phase 2 Calculate the solutions for each decision variable and objective value solving the original function with the original constraints

$$\sum_{j \in J} f_j X_j + \alpha \sum_{i \in I} \sum_{j \in J} h_i d_{ij} Y_{ij}$$

Update lambda values

1. Calculate the stepsize

$$t^n = \frac{a^n \left(UB - L^n\right)}{\sum_{i \in I} \left\{ \sum_{j \in J} Y_{ij}^n - 1 \right\}^2}$$

2. Update lambda depending to constraints violated

$$\begin{cases} \lambda_j = \lambda_j - t^n & \text{if} \quad \sum_{j \in J} Y_{ij} >= 2 \\ \lambda_j = \lambda_j + t^n & \text{if} \quad \sum_{j \in J} Y_{ij} = 0 \end{cases}$$

Results

Book’s problem

CPLEX solver

Time	0.04656 seconds
Objective Function Value	$1234.95
Facilities are located in	['A', 'D', 'K']

Lagrangian Relaxation

Time	1.7069 seconds
Objective Value	$1234.95
Facilities Locations	['A', 'D', 'K']
Best Lower Bound	$1234.95
Best Upper Bound	$1234.95
No. Iterations	4034

Large problem (30 nodes)

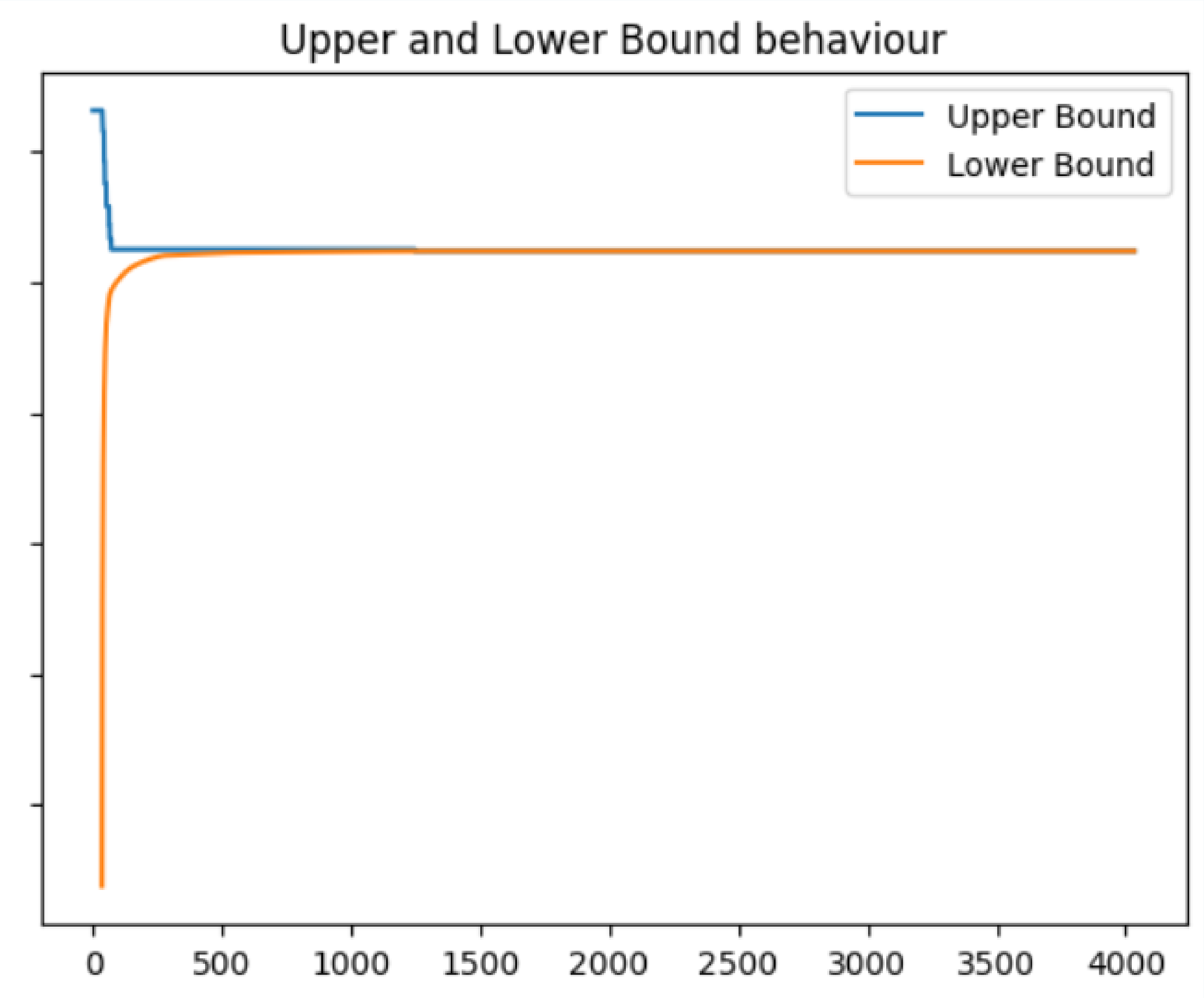
CPLEX solver

Time	0.1088 seconds
Objective Value	$2,328.9
Facilities Locations	[0, 1, 5, 6, 9, 12, 13, 27, 28]

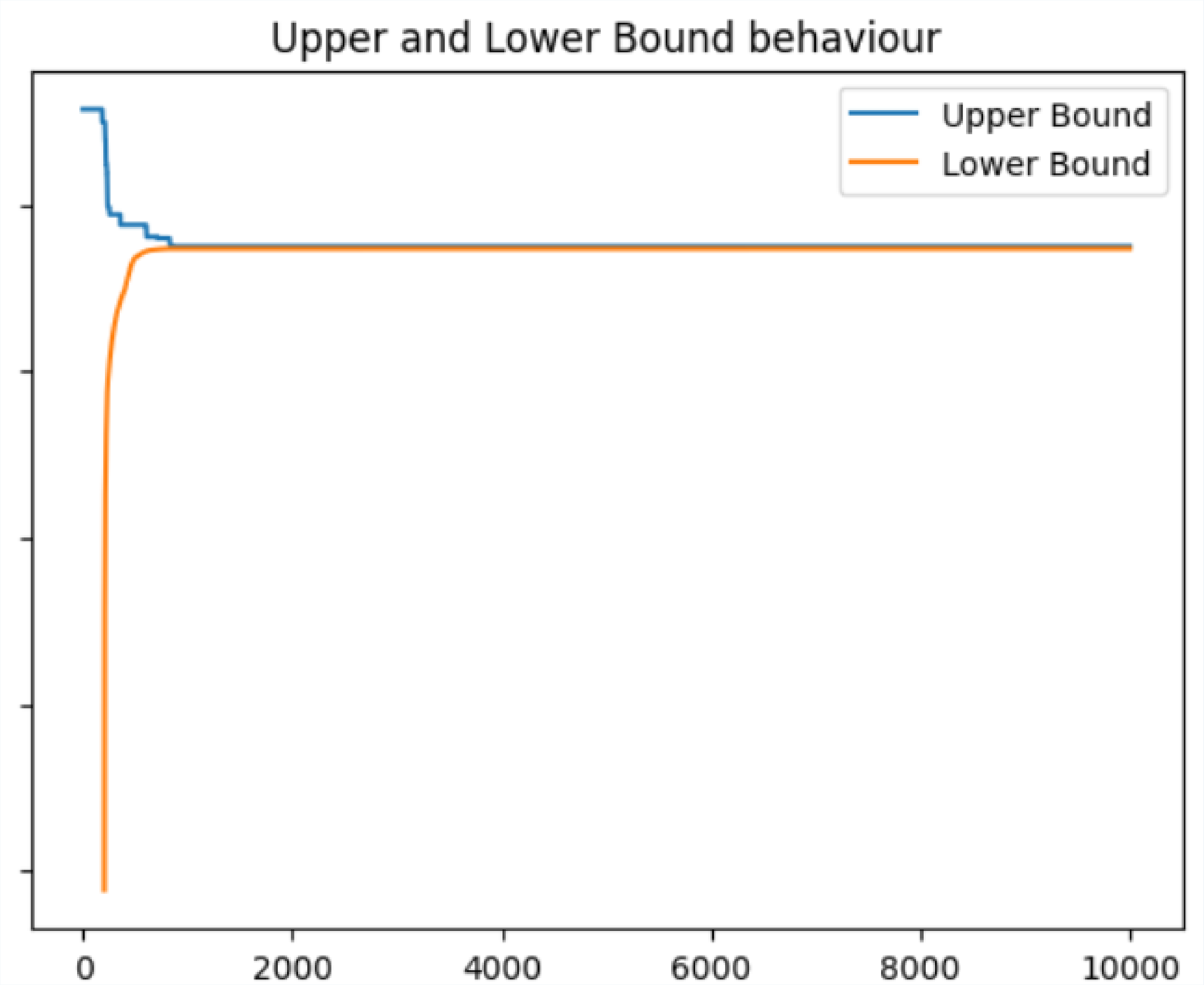
Lagrangian Relaxation

Time	8.7035 seconds
Objective Value	$2,296.73
Facilities Locations	[0, 1, 5, 6, 9, 12, 13, 27, 28]
Best Lower Bound	$2,296.73
Best Upper Bound	$2,328.9
No. Iterations	10,000

Book’s problem



Large problem



CODE

THANK YOU!

References

Daskin, M. (2013) Network and Discrete Location: Models, Algorithms and Applications. (2nd ed.) John Wiley & Sons (297-314)