GALACTIC ROTATION AND DARK MATTER

Carrera Ortiz, Georgia Institute of Technology, School of Physics

ABSTRACT

Cosmologists, in their endeavor to answer philosophical questions in physical terms, such as "where do we come from?", and "what governs our universe?", have noticed unexplained phenomena regarding the acceleration of our universe. By studying rotation curves, a possible answer to what's causing this acceleration is a hidden mass - unobservable with our eyes or current equipment, but able to make a difference on astronomical and cosmological scales. In this report, we discuss the possibility of 'Dark Matter'.

1. INTRODUCTION AND PURPOSE

Many years later, and astrophysicists are still uncovering more about the very essence of our universe each day that passes by. With technological advances such as the James Webb Space Telescope, GAIA, and more, we hope to gather more data to reveal secrets hidden in the physics of the universe. From our current information, the large bound of matter in our Milky Way Galaxy seemingly does not hold enough mass to match the strength of its gravitational field. If there is more mass, it is not luminous or interactive electromagnetically - only gravitationally, as far as we know. Here, we aim to use GAIA data for 15,000 stars within our galaxy to geometrically and mathematically analyze how it orbits around the galactic center. By clarifying rotation curves for a significant amount of data, as well as defining mass within certain rotation curves, we can compare this to the expected acceleration values with or without dark matter to determine how likely the dark matter theory is realistically and physically.

This work has made use of data from the European Space Agency (ESA) mission *Gaia* (https://www.cosmos.esa.int/gaia), processed by the *Gaia* Data Processing and Analysis Consortium (DPAC, https://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the *Gaia* Multilateral Agreement.

2. PROCEDURE

Thanks to the GAIA Archive, astronomers and astrophysicists have access to finely accurate data on a vast array of stars. The European Space Agency (ESA) in charge of the Gaia Mission provides data for millions of stars, including parameters such as "deduced positions, parallaxes, proper motions, radial velocities, and brightness measurements." For some sources, there is additional information regarding other astrophysical facets. For the purpose of this report, being able to access the galactic longitude, parallax, and the radial velocity for each star allows for an in-depth geometrical analysis to further understand each star's movement in our galaxy.

Parallax, measures in arcseconds (arcs), is a trigonometric measurement that helps to understand the position of star's as they seem to shift against background stars due to the relative motion of the Earth around the Sun. As technology becomes more advanced, instruments like Gaia can accurately measure the parallax of stars to milli-arcseconds (mas). With this information, we can use Equation 1 to find the distance of the star to the Gaia spacecraft in parsecs (pc), which can easily be converted to kiloparsecs (kpc).

$$d = \frac{1}{p"(\text{ arcs })}(\text{ pc }) \tag{1}$$

In order to determine the galactic distance (distance of each star to the center of the galaxy), a geometrical analysis of the situation is necessary. Dr. Sowell has created a simplified version to envision the geometry of the situation, shown in Figure 1 regarding distances and relative velocities. This graph uses dark black lines to represent distances and colorful lines to represent different velocities (RV: Radial Velocity, TV: Tangential Velocity, SV: Space Velocity, and Vorb: Orbital Velocity), and angles represented in green letters. From Gaia, we are given the longitude angle and are able to calculate R_g - the distance from the Sun to the star - as shown in Equation 1. We additionally know that the Sun is approximately 8.2 kpc away from the Galactic Center.

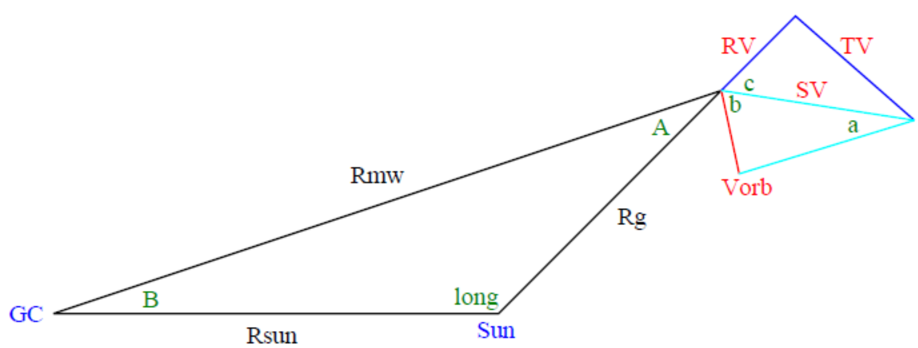

Figure 1. Velocities and Distances as Seen from the Galactic Center

As helpful as this is, it is important to note that this represents only 1 of 8 possible geometries that can occur, as the range of the longitude and direction of the radial velocity, and its magnitude in compared to the tangential velocity. Thereby, the calculations for some angles are not as simple as indicated by Figure 1.

In order to solve these geometries, we need the Law of Sines (Equation 2) and the Pythagorean Theorem.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}, \text{where a, b, c are distances, and A, B, C represent angles} \tag{2}$$

To find R_{mw}, the distance of the star to the galactic center, we use Equation 3.

$$R_{mw} = \sqrt{R_g^2 + R_{sun}^2 - (2 * R_g * R_{sun} * \cos(long))} \tag{3}$$

With all distances known in the first triangle as well as the provided longitude from Gaia, the Law of Sines leads to calculations for angles A and B in Figure 1. Onto the first 'velocity triangle', the Gaia data provides values for the radial velocity and proper motion of each star. This information allows for calculations for each velocity term by the relations of radial velocity, tangential velocity, and space velocity.

The tangential velocity is dependent on the proper motion (μ), here in units of mas/yr, and distance - represented in kpc in relation to Gaia.

$$TV(\frac{km}{s}) = 4.74\mu d \tag{4}$$

The space velocity is directly related to the radial velocity and tangential velocity, as shown in Equation 5.

$$SV(\frac{km}{s}) = \sqrt{RV^2 + TV^2} \tag{5}$$

The difference in geometries largely comes in play when finding angle a, as mentioned previously. This angle is necessary for its role in calculating the orbital velocity of a star, shown in Equation 6.

$$V_{orb} = SV * \frac{\sin a}{\sin 90} = SV * \sin a \tag{6}$$

Thus, with the orbital velocities calculated, the details of rotation curves can be explored, plotted, and analyzed.

3. STELLAR VALUES

We use a polar plot to picture the distribution of stars in the universe in terms of its distance and longitudinal angle to the galactic center (see Figure 2).

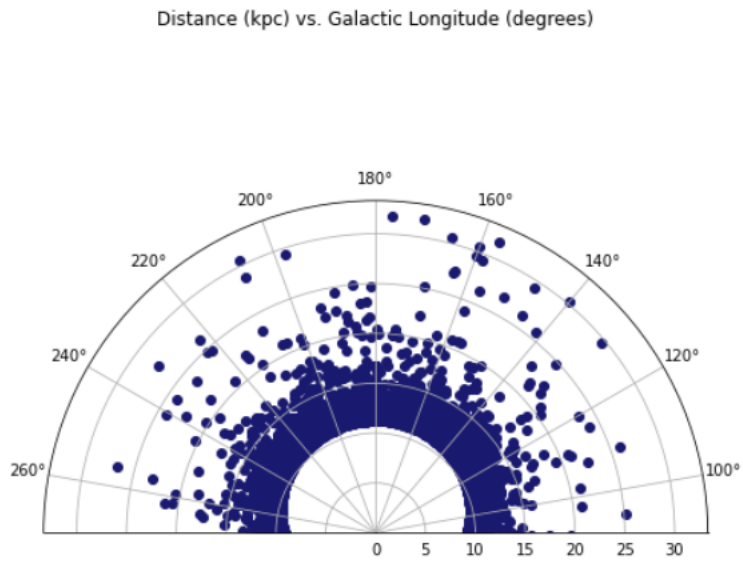

Figure 2. Polar Plot of Gaia Data

4. ANALYSIS RESULTS

Table 1. Average Velocities & Interior Masses

Ring (kpc)	Number	Avg Velocity (km/s)	Avg. Velocity (abs. val) (km/s)	Solar Mass (millions)
10	7788	1.298	68.74	82.66
12	3081	3.201	67.30	56.06
14	3315	1.408	68.75	73.36
16	527	7.974	91.94	26.68
18	140	-10.01	121.8	15.28
20	44	13.80	147.9	6.42
22	46	-108.11	309.5	241.1
24	19	56.37	230.4	8.02
26	16	-2.021	179.4	4.55
28	8	84.26	238.3	4.15
30	4	-266.49	321.1	4.21

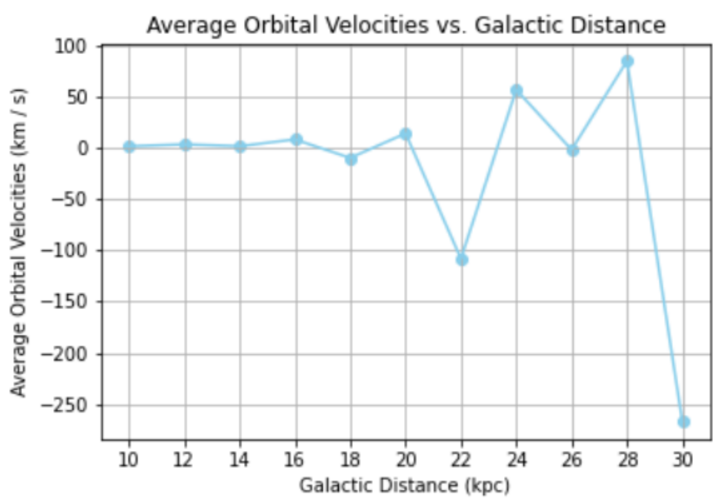

Figure 3. Rotation Curve - Galactic Distance vs. Average Orbital Velocities

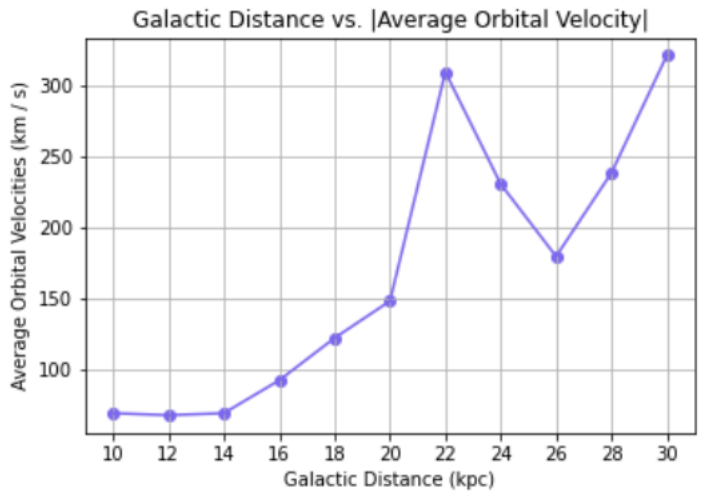

Figure 4. Rotation Curve - Galactic Distance vs. Average Orbital Velocities (abs. val)

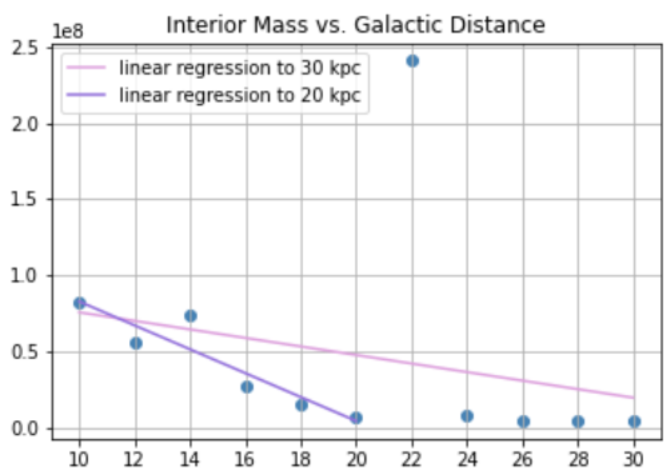

Figure 5. Mass as a Function of Distance Rings

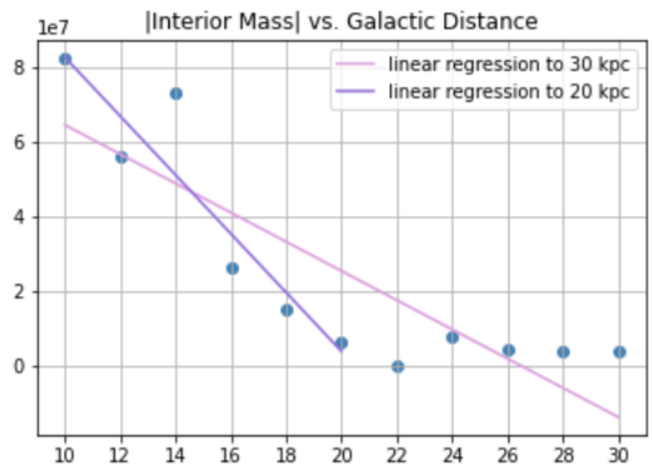

Figure 6. Mass as a Function of Distance Rings w/ $|v_{orb}|$

Table 2. Angular Sectors and Avg. Orbital Velocities for Stars Within the 14-16 kpc Ring

Sector (deg)	Number	Avg. Orbital Velocity (km/s)	Avg. Orbital Velocity (abs. val) (km/s)
$90° - 110°$	193	26.749	122.825
$110° - 130°$	131	- 7.904	92.451
$130° - 150°$	74	0.164	70.284
$150° - 170°$	78	- 1.136	62.827
$170° - 190°$	51	2.977	49.737

5. DISCUSSION

When analyzing the data, I consistently made 2 versions of answers: with and without taking the absolute value of the orbital velocities of each star. Though it may be accurate to the stars' movement with directions in mind, by looking at the speed different patterns may emerge that more clearly explain the stars' behaviors.

In both Figures 3 and 4, the velocity is relatively similar until either around the 16 or 18 kpc ring, and then large changes in the average orbital velocities occur. When focusing only on the magnitude rather than direction, such as in Figure 4, an approximately exponentially increasing pattern emerges, with an odd outlier in the 22 kpc ring. The differential motions of the stars seem to align more with random motion rather than mean motions, as discussed in Dr. Sowell's exercise on 'GALACTIC ROTATION AND DARK MATTER'.

We analyze the data further by looking at the results in Table 1. With each increasing ring, we see a decreasing amount of stars and thus decreasing amount of mass (represented in millions of solar masses). Though this is expected, it's incredible to note the likelihood that stars further away from the Galactic Center have much more mass on average compared to those closer. For example, not only do we see the last 3 rings consistently halve the number of stars (16 to 8 to 4), but the amount of mass contained within the ring is relatively close. Furthermore, each ring represents about a quarter of the mass held in the 18 kpc ring, but the closer rings up to 35 times more stars than the farther rings. Looking closer at the average orbital velocities in each ring, we notice the generally increasing nature, physically represented in Figures 3 and 4. Ultimately, it is quite notable for the rings to generally increase their velocity and momentum with each increasing ring despite the decreasing number of stars within each sector. From my understanding, this seems to match largely with our understanding of Keplerian orbits and help to explain how the arms of the Milky Way remain the same as they rotate around the Galactic Center.

From Table 1, the average velocity found in the 16 kpc ring is 7.974 km/s, and with taking the absolute value, it is 91.94 km/s. In an effort to verify these results with a different method focusing on the angular sectors within the 16 kpc ring. When taking the average of the velocities listed in Table 1, the average orbital velocity is 4.6244 km/s, and the (abs. val) average orbital velocity is 79.63 km/s. Respectively, this represents an approximate 42% error and 14% error. Though the first percent error seems large, at this magnitude, a difference of about 3 km/s is not exactly significant to discount the results found in this report, nor the means by which they were achieved.

A similar report using Gaia data, from first author Hai-Feng Wang, reports a table of average circular velocities from radial distances of almost 10 kpc to 28 kpc being consistently around 200 ± 25 km/s. This is a stark difference from my calculations, where the average velocities did not reach near 200 km/s until around the 20 kpc ring, and then varied greatly after this point with a max of around 309 km/s. There was no tight-knit mean motion from my analysis as there seems to be found from Wang's data and analysis.

In another report utilizing data from Gaia, first author Crosta found, by analyzing and applying an intricate coordinate system, the result of a lower and upper bound for the radial limit, as well as normalized orbital velocity within the region. The radial limit from Crosta is defined as $0.9 < r < 35.8$ kpc, a slightly more generous range than studied in this report. The normalized velocity found by Crosta is 296 km/s. When averaging each average orbital velocity (abs. val) found in my report, Table 1, the normalized velocity within the entire radial range is approximately 165 km/s. The difference is notable, with about a 44% 'error' in our normalized orbital velocity values.

As mentioned previously, the stars motion seem to behaving in a mostly random motion that adheres to Kepler's laws. However, analyzing comparisons with other reports shows the orbital velocities found from my analysis seem to be much lower, indicating either an error in my calculations or a suggestion of a possible 'missing' mass that would increase the acceleration and result in a smoother and more accurate rotation curve.

Yet, if the problem is not my analysis or any 'missing mass', there are still other likely sources of error. One example would be that Gaia collects data on radial velocities only for stars with surface temperatures in the range of 3550 to 6900 K. Using a quick reference to an HR Diagram, we can note that this is largely going to include only relatively low-mass main sequence stars and a few supergiants. There is little variation and does not entirely represent all the stars that we could expect to find within these rings, which could likely contribute much more mass to the equations.

Additionally, the proper motions from the Gaia data were assumed to be in the plane and in the direction of galactic rotation. With this assumption, the perceived velocity can be significantly affected. Because proper motion is an observation of the apparent rate of position change across the sky, a change in this direction can affect the geometry of the situation which significantly affects the value of the orbital velocity, as discussed in Section II. The geometry of the astronomical system is a crucial piece of this analysis, making details such as relative angles a defining piece of each data point. The calculations based on the data could entirely change by taking into account direction of the proper motion.

Similarly, idealized assumptions such as perfectly circular orbits neglects key astronomical behaviors and true motions of stars as they revolve around the galactic center. Though an in-depth analysis was undertaken for the general geometry, the rotational geometry realistically includes a major and minor axis where the velocity at different points on these ellipsoids vary in tune with Kepler's laws. By assuming a perfectly circular orbit, we are allowed the assumption of a uniform circular velocity - which is likely a very special case and not the norm for most stars in our galaxy.

From a data perspective, the distribution of stars was not equal. As seen in Table 1, the number of stars in each ring decreased from almost 8,000 stars to 4 stars. The sample data in the further rings is nearing, if not already, statistically insignificant due to being so small. In a collection of 15,000 stars, an even distribution makes a huge difference. By the law of large numbers, the standard deviation between the orbital velocity for stars in the 12 kpc ring is much lower due to comparing over 3,000 stars than it is for the 28 kpc ring where 8 stars can have wildly different orbital velocities and the average velocity is a hardly relevant stat. This begs the question - are these stars in the further rings actually more massive on average and travelling much faster on average, or are there a similar amount of these high-mass, high-velocity stars in the closer rings that just have other supporting data points to paint an accurate picture?

In order to fix some of the statistical errors possibly introduced from the previously acknowledge source of error, a weighting of rings by number of stars could be considered. Without this, the range that the uneven distribution of stars in the data could extend to revoke a large part of this analysis.

Gaia spacecraft is located within our solar system, which on the galactic scale is about 8 kpc away from the center near the edge of a spiraling arm. Though the distances of these stars to the galactic center is calculated, the variation in stars is very different than if we were able to look directly from the center of the galaxy to evenly and isotropically measure each star around us. There are millions of stars and a large galactic bulge preventing our view of stars on the other side of the galaxy, introducing another assumption on the uniformity of stars across the galaxy, such that the distribution of number of stars, mass, and velocities are the same at each part of the Milky Way.

Similar to how weighting the rings would help mitigate the statistical errors, without performing any error propagation, such as noting the effects of the outlier found in the 22 kpc ring, the errors in the report are allowed to compound infinitely without mitigation.

6. CONCLUSIONS

Ultimately, the question of dark matter cannot be answered in this single report. A more diverse set of data to analyze should be kept in mind for future reference. Additionally, as we've seen from other papers such as Wang's and Crosta's, there are multiple ways to view the problem of rotation curves and each analysis can provide a differing result. Hopefully, with each budding scientist, we innovate new ways to apply physics to the problem and either understand how to find mass we cannot see or reveal more secrets of the universe we have yet to uncover. Additionally, assumptions around the data and issues with the distribution of data should be addressed and remediated to paint an accurate picture on the issue, and answer the question on dark matter once and for all.

7. CITATIONS

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