Mock AMC 10

jj_ca888 and cosmicgenius

1 Rules

1. **DO NOT READ THE PROBLEMS UNTIL YOUR TIMER HAS BEGUN**
2. This is an ***individual contest*** only.
3. This is a 25 - question multiple choice test. The answer to each question will be one of the following: A, B, C, D or E. The problems are generally in increasing order of difficulty, although this is not guaranteed.
4. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smart-watches, or computing devices are allowed. No problems on the test will require the use of a calculator.
5. This test is 75 minutes long.
6. SCORING: You will receive 6 points for a correct answer, 1.5 points for no answer, and 0 points for an incorrect answer.
7. Have fun!

1 (jj_ca888). A zeek is equal to 10 zaaks. 5 zaaks are equal to 7 zuuks. If a zuuk is equal to 3 euros, then how many euros are equal to a zeek?

(A) 21 **(B)** 42 **(C)** 84 **(D)** 105 **(E)** 210

2 (jj_ca888). Abinav took a quiz with 50 questions. Each correct answer is worth 2 points and each wrong answer is worth 1 point. Assuming Abinav did all of the questions and got a score of 79, how many questions did he get wrong?

(A) 7 **(B)** 20 **(C)** 21 **(D)** 29 **(E)** 30

3 (jj_ca888). What is the sum of the prime factors of 9951?

(A) 23 **(B)** 77 **(C)** 141 **(D)** 200 **(E)** 225

4 (jj_ca888). Let the set S be the set of the first 10 positive integers. How many subsets of S have at least one prime number?

(A) 64 **(B)** 896 **(C)** 960 **(D)** 992 **(E)** 1008

5 (cosmicgenius). How many permutations of the string "BANANAS" are there such that there is exactly one character between the two characters 'B' and 'S'?

(A) 60 **(B)** 100 **(C)** 144 **(D)** 240 **(E)** 288

6 (jj_ca888). An ordered triple of three nonzero integers (a, b, c) is called an interesting triple if the following is true: $a^2 + b^2 = c^2$. How many interesting triples are there such that $|c| < 14$?

(A) 3 **(B)** 6 **(C)** 12 **(D)** 24 **(E)** 48

7 (jj_ca888). Mr.Hoang's math class has between 50 and 150 students. If he puts his students in groups of 3, there is 1 left over. If he puts his students in groups of 4, there are 2 left over. If he puts his students in groups of 5, there are 3 left over. What is the sum of the possible number of students Mr. Hoang has in his class?

(A) 58 **(B)** 96 **(C)** 124 **(D)** 154 **(E)** 176

8 (jj_ca888). Chris and Leo both took the 2005 AMC 10C. The scoring system is as follows: A correct answer is worth 6 points, and unanswered question is worth 1.5 points, and an incorrect answer is worth 0 points. There are 25 problems on the test. Due to Chris's high IQ, he scored 60 points above Leo. It is also known that Leo's score had a ones digit of 4. Find the sum of all the possible scores Chris could have received on that test.

(A) 342 **(B)** 531 **(C)** 561 **(D)** 625.5 **(E)** 649.5

9 (jj_ca888). The 4 boys, 2 girls, and the teacher of the math club at Redwood Middle School are lining up for a photo. How many ways are there to line them up if the teacher must stay in the middle and the two girls cannot be adjacent?

(A) 426 **(B)** 480 **(C)** 504 **(D)** 516 **(E)** 528

10 (jj_ca888). The quadratic equation $x^2 - (2a+5)x + 2a^2 - 9a$ has two real roots that differ by one for some real number a. What is the sum of all possible values of a?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

11 (cosmicgenius). Lucas, Tyler, and Simon are happily distributing 49 pieces of indistinguishable jelly beans among themselves. Lucas insists in receiving at least 4 jelly beans, Tyler only wants to receive an odd number of jelly beans, and Simon insists on receiving an even number of jelly beans. How many ways are there to distribute the 49 jelly beans?

(A) 276 **(B)** 300 **(C)** 325 **(D)** 351 **(E)** 378

12 (jj_ca888). Find the sum of all real values of k such that the equation

$$|x^2 - 4| = 2x + k$$

has three real solutions in x.

(A) 5 **(B)** 9 **(C)** 14 **(D)** 23 **(E)** 37

13 (jj_ca888). Regular hexagon $ABCDEF$ has side length 4. Point P is chosen at random inside the hexagon. We connect point P to each of the points A, B, C, D, E, and F to form six triangles. What is the probability that none of the six triangles have areas that exceed $6\sqrt{3}$?

(A) $\frac{2}{27}$ **(B)** $\frac{4}{27}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{3}$

14 (jj_ca888). Gary the Gorilla is traveling from point $(-3, -3)$ to $(3, 3)$ on the coordinate plane. Due to danger, Gary cannot go to points $(-2, -1)$ and $(1, 2)$. Given that Gary can only go up or right one unit at a time, how many paths can he take that will guarantee a safe trip?

(A) 84 **(B)** 348 **(C)** 464 **(D)** 576 **(E)** 924

15 (jj_ca888). Equilateral triangle $\triangle ABC$ has area 576. Let points M and N be the midpoints of line segments AB and AC respectively, and let point P be the midpoint of line segment AM. Line CP intersects line MN at Q. What is the area of $\triangle MPQ$?

(A) 36 **(B)** 48 **(C)** 54 **(D)** 56 **(E)** 64

16 (jj_ca888). Johnny and David plan to meet at a restaurant for dinner. Each will arrive at a time between 6:00 P.M and 9:00 P.M. Once Johnny arrives, he will stay at the restaurant for 45 minutes. Once David arrives, he will stay at the restaurant for 60 minutes. They will have dinner together if they are both at the restaurant at some moment in time. Given that they have dinner together, the probability that Johnny arrives before David can be expressed as a common fraction in the form $\frac{a}{b}$ where a and b are relatively prime. What is $a + b$?

(A) 23 **(B)** 39 **(C)** 206 **(D)** 223 **(E)** 489

17 (Missionmath). There is an unique real number x that satisfies the following equation:

$$\frac{\sqrt{16 + \sqrt{x}}}{16} + \frac{\sqrt{16 + \sqrt{x}}}{\sqrt{x}} = \frac{\sqrt[4]{x}}{2}$$

Then, x can be written in the form $\frac{a}{b}$ where a and b are relatively prime. What is $a + b$?

(A) 19 **(B)** 41 **(C)** 67 **(D)** 137 **(E)** 265

18 (jj_ca888). Let p be a prime number such that there exists a triplet (a, b, c) of three distinct primes a, b, and c where

$$p = a^2 + b^2 + c^2$$

Find the sum of the two smallest possible values of p.

(A) 150 **(B)** 196 **(C)** 238 **(D)** 262 **(E)** 324

19 (jj_ca888). Square $ABCD$ has side length 40. Point O is drawn on line segment AB such that $AO = BO$. Points P and Q are drawn on line segment CD such that $CP = DQ = 10$. Let S be the locus of all points inside square $ABCD$ that are closer to point O than to point P and point Q. What is the area of the locus S?

(A) 600 **(B)** 640 **(C)** 700 **(D)** 750 **(E)** 850

20 (jj_ca888). Let r_1, r_2, and r_3 be roots of the cubic polynomial $P(x) = x^3 - 6x^2 + b$ where b is real. It is also known that $r_1^3 + r_2^3 + r_3^3 = 72$. What is the value of $P(10)$?

(A) 352 **(B)** 448 **(C)** 654 **(D)** 896 **(E)** 1024

21 (jj_ca888). Scalene triangle $\triangle ABC$ has area 45. Points P_1 and P_2 are located on side AB such that $AP_1 = P_1P_2 = BP_2$. Additionally, the points Q_1 and Q_2 are located on side AC such that $AQ_1 = Q_1Q_2 = CQ_2$. The area of the intersection of triangles BQ_1Q_2 and CP_1P_2 can be expressed as a common fraction $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 15 **(B)** 47 **(C)** 79 **(D)** 95 **(E)** 257

22 (cosmicgenius). The set of 100 integers $\{1, 2, 3, \ldots 100\}$ are written on a whiteboard. Every minute, a student comes up and erases two distinct numbers a and b from the board, then proceeds to write the quantity $\sqrt{\frac{1}{2}(a^2+b^2)}$. After 99 minutes, there is only one number left on the board, N. The expected value of N^2 can be expressed as a common fraction $\frac{m}{n}$ where m and n are relatively prime integers. What are the last two digits of $m-n$?

(A) 03 **(B)** 65 **(C)** 69 **(D)** 81 **(E)** 99

23 (jj_ca888). Triangle $\triangle ABC$ has side lengths $AB = 13$, $BC = 14$, and $AC = 15$. Let P be a point on line segment BC, and H_1 and H_2 be the orthocenters of triangles $\triangle ABP$ and $\triangle ACP$ respectively. The are exactly two possible points P such that $H_1H_2 = 1$. The distance between these two points can be expressed as $\frac{m}{n}$ for relatively prime positive integers m and n. What is $m+n$?

(A) 5 **(B)** 11 **(C)** 19 **(D)** 37 **(E)** 63

24 (jj_ca888 and cosmicgenius). Denote $|S|$ as the number of elements in set S. Let K be a set of elements such that $|K| = 1458$. Then, three not necessarily distinct subsets S_1, S_2, and S_3 of K are chosen uniformly and randomly under the condition

$$|S_1| = |S_2| = |S_3| = 567$$

What is the expected value of $|(S_1 \cap S_2) \cup (S_1 \cap S_3) \cup (S_2 \cap S_3)|$?

(A) 486 **(B)** 490 **(C)** 525 **(D)** 567 **(E)** 588

25 (jj_ca888). Isosceles triangle $\triangle ABC$ has side lengths $BC = 8$ and $AB = AC$. Let the angle bisector of $\angle ABC$ meet line AC at point D, and let the angle bisector of $\angle ACB$ meet line AB at point E. Let the two lines BD and CE intersect at point P. It is given that the area of quadrilateral $ADPE$ is exactly 45% of the area of triangle $\triangle ABC$. The area of $\triangle ABC$ can be expressed as $m\sqrt{n}$ where n is not divisible by the square of any prime. What is $m+n$?

(A) 34 **(B)** 42 **(C)** 50 **(D)** 56 **(E)** 66